Sustainable Supply Chain and Transportation Networks

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The Virtual Center for Supernetworks
Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

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Topics

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- Transportation Network Equilibrium Model
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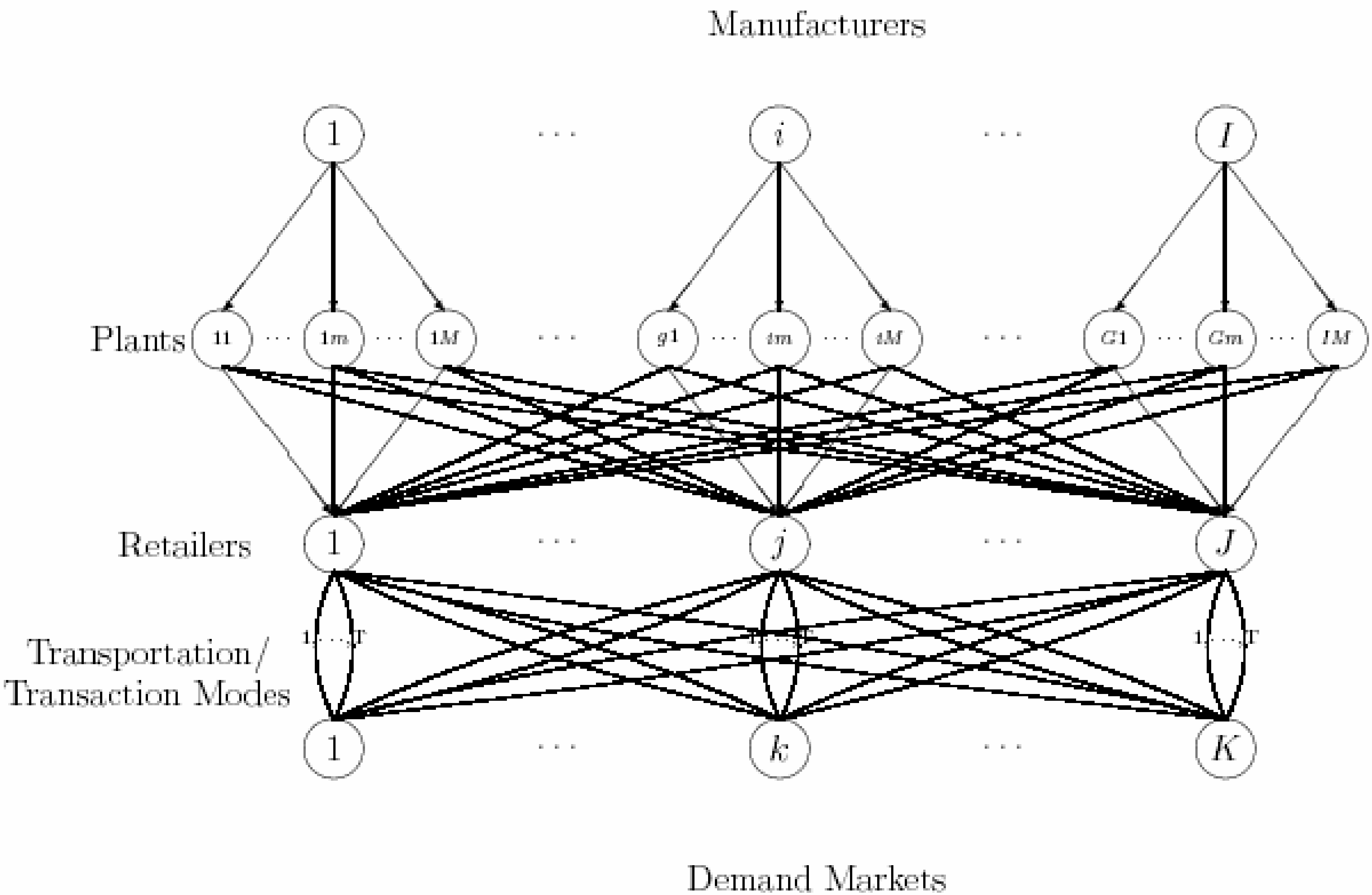
Motivation for Research

- Green Supply Chains
- Environmental impacts
- Regulatory Implications
- Multicriteria Decision Makers
- Why transform to a Transportation Network?
  - Theoretical insights
  - Computational Efficiency

Literature

- Cline (1992), Poterba (1993), and Painuly (2001)
- Beckman, McGuire, and Winsten (1956)
- Dafermos (1982)
- Nagurney (2000, 2006a, 2006b)
- Nagurney, Dong, and Zhang (2002)
- Nagurney and Toyasaki (2003)
- Nagurney and Liu (2005)
- Wu et al (2006)


Manufacturers
1
i
I
Plants
11
1m
1M
g1
im
iM
G1
Gm
IM
Retailers
1
j
J
Transportation/
Transaction Modes
1, ..., T
1
k
K
Demand Markets

Multicriteria Decision-Making Behavior of the Manufacturers and Their Optimality Conditions

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{j=1}^{J}\rho_{1imj}^{*}q_{imj}-\sum_{m=1}^{M}f_{im}(q_m)-\sum_{m=1}^{M}\sum_{j=1}^{J}c_{imj}(q_{imj}).$$

$$\text{Minimize} \sum_{m=1}^{M}e_{im}q_{im}+\sum_{m=1}^{M}\sum_{j=1}^{J}e_{imj}q_{imj}.$$

Multicriteria Decision-Making Behavior of the Manufacturers and Their Optimality Conditions

$$\text{Maximize} \quad \sum_{m=1}^{M}\sum_{j=1}^{J}\rho^*_{1imj}q_{imj}-\sum_{m=1}^{M} f_{im}(q_m)-\sum_{m=1}^{M}\sum_{j=1}^{J} c_{imj}(q_{imj})-\alpha_i\Big(\sum_{m=1}^{M} e_{im}q_{im}+\sum_{m=1}^{M}\sum_{j=1}^{J} e_{imj}q_{imj}\Big)$$

$$\sum_{j=1}^{J} q_{imj} = q_{im}, \quad m=1,\ldots,M, \qquad (2)$$

$$q_{imj} \geq 0, \quad m=1,\ldots,M; j=1,\ldots,J.$$

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Multicriteria Decision-Making Behavior of the Manufacturers and Their Optimality Conditions

The optimality conditions for all manufacturers simultaneously, under the above assumptions coincide with the solution of the following variational inequality: determine $(q^*, Q^{1*}) \in K^1$ satisfying

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q_m^*)}{\partial q_{im}} + \alpha_i e_{im}\right] \times [q_{im} - q_{im}^*]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\frac{\partial c_{imj}(q_{imj}^*)}{\partial q_{imj}} + \alpha_i e_{imj} - \rho_{1imj}^*\right] \times [q_{imj} - q_{imj}^*] \geq 0, \quad \forall (q, Q^1) \in K^1, \qquad (4)$$

where $K^1 \equiv \{(q, Q^1) | (q, Q^1) \in R_+^{IM+IMJ} \text{ and (2) holds}\}$.

Multicriteria Decision-Making Behavior of the Retailers and Their Optimality Conditions

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{t=1}^{T} \rho_{2jk}^{t*} q_{jk}^{t} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{m=1}^{M} \rho_{1imj}^{*} q_{imj} - \sum_{k=1}^{K}\sum_{t=1}^{T} c_{jk}^{t}(q_{jk}^{t}).$$

$$\text{Minimize} \quad e_j h_j + \sum_{k=1}^{K}\sum_{t=1}^{T} e_{jk}^{t} q_{jk}^{t}.$$

for notational convenience, we let

$$h_j \equiv \sum_{i=1}^{I}\sum_{m=1}^{M} q_{imj}, \quad j = 1, \ldots, J. \tag{7}$$

Multicriteria Decision-Making Behavior of the Retailers and Their Optimality Conditions

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{t=1}^{T} \rho_{2jk}^{t*} q_{jk}^{t} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{m=1}^{M} \rho_{1imj}^{*} q_{imj} - \sum_{k=1}^{K}\sum_{t=1}^{T} c_{jk}^{t}(q_{jk}^{t})$$

$$-\beta_j(e_j h_j + \sum_{k=1}^{K}\sum_{t=1}^{T} e_{jk}^{t} q_{jk}^{t})$$

$$\sum_{k=1}^{K}\sum_{t=1}^{T} q_{jk}^{t} = \sum_{i=1}^{I}\sum_{m=1}^{M} q_{imj} \qquad (10)$$

$$q_{imj} \geq 0, \quad i = 1, \ldots, I, \quad m = 1, \ldots, M,$$

$$q_{jk}^{t} \geq 0, \quad k = 1, \ldots, K; t = 1, \ldots, T.$$

Multicriteria Decision-Making Behavior of the Retailers and Their Optimality Conditions

The optimality conditions for all retailers simultaneously, under the above assumptions coincide with the solution of the following variational inequality: determine $(h^*, Q^{1*}, Q^{2*}) \in K^3$ satisfying

$$\sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j}+\beta_j e_j\right]\times[h_j-h_j^*]+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{jk}^t(q_{jk}^{t*})}{\partial q_{jk}^t}+\beta_j e_{jk}^t-\rho_{2jk}^{t*}\right]\times[q_{jk}^t-q_{jk}^{t*}]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\rho_{1imj}^*\right]\times\left[q_{imj}-q_{imj}^*\right]\geq 0, \qquad \forall (h, Q^1, Q^2,) \in K^3, \qquad (13)$$

where $K^3 \equiv \{(h, Q^2, Q^1) | (h, Q^2, Q^1) \in R_+^{J(1+TK+IM)}$ and (7) and (10) hold$\}$.

Equilibrium Conditions for the Demand Markets

At each demand market k; $k = 1, \ldots, K$, the following conservation of flow equation must be satisfied:

$$d_k = \sum_{j=1}^{J} \sum_{t=1}^{T} q^t_{jk}. \tag{14}$$

$$\rho^{t*}_{2jk} + \hat{c}^t_{jk}(Q^{2*}) + \eta_k e^t_{jk} \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q^{t*}_{jk} > 0, \\ \geq \rho_{3k}(d^*), & \text{if} \quad q^{t*}_{jk} = 0. \end{cases}$$

Equilibrium Conditions for the Demand Markets

The equivalent variational inequality governing all the demand markets takes the form: determine $(Q^{2*}, d^*) \in K^4$ satisfying

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\rho_{2jk}^{t*}+\hat{c}_{jk}^{t}(Q^{2*})+\eta_k e_{jk}^{t}\right]\times\left[q_{jk}^{t}-q_{jk}^{t*}\right]-\sum_{k=1}^{K}\rho_{3k}(d^*)\times\left[d_k-d_k^*\right]\geq 0, \quad \forall (Q^2,d)\in \mathcal{K}^4, \tag{16}$$

where $\mathcal{K}^4 \equiv \{(Q^2,d)|(Q^2,d)\in R_+^{K(JT+1)}$ and (14) holds$\}$.

The Equilibrium Conditions for the Supply Chain Network with Manufacturing Plants and Environmental Concerns

In equilibrium, the optimality conditions for all the manufacturers, the optimality conditions for all the retailers, and the equilibrium conditions for all the demand markets must be simultaneously satisfied so that no decision-maker has any incentive to alter his transactions.

Definition 1: Supply Chain Network Equilibrium with Manufacturing Plants and Environmental Concerns

The equilibrium state of the supply chain network with manufacturing plants and environmental concerns is one where the product flows between the tiers of the network coincide and the product flows and prices satisfy the sum of conditions (4), (13), and (16).

Theorem 1: Variational Inequality Formulation of the Supply Chain Network Equilibrium with Manufacturing Plants and Environmental Concerns

The equilibrium conditions governing the supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in \mathcal{K}^5$ *satisfying:*

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q_m^*)}{\partial q_{im}} + \alpha_i e_{im}\right] \times [q_{im} - q_{im}^*] + \sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j} + \beta_j e_j\right] \times [h_j - h_j^*]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\frac{\partial c_{imj}(q_{imj}^*)}{\partial q_{imj}} + \alpha_i e_{imj}\right] \times [q_{imj} - q_{imj}^*]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{jk}^t(q_{jk}^{t*})}{\partial q_{jk}^t} + \hat{c}_{jk}^t(Q^{2*}) + (\beta_j + \eta_k)e_{jk}^t\right] \times [q_{jk}^t - q_{jk}^{t*}] - \sum_{k=1}^{K}\rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d) \in \mathcal{K}^5,$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d) | (q, h, Q^1, Q^2, d) \in R_+^{IM+J+IMJ+TJK+K} \text{ and } (2), \textbf{ and } (10) \text{ hold}\}.$$

The Transportation Network Equilibrium Model with Elastic Demands

The link flows are related to the path flows through the following conservation of flow equations:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{20}$$

where $\delta_{ap} = 1$ if link a is contained in path p, and $\delta_{ap} = 0$, otherwise. Hence, the flow on a link is equal to the sum of the flows on paths that contain that link.

The user costs on paths are related to user costs on links through the following equations:

$$C_p = \sum_{a \in L} c_a \delta_{ap}, \quad \forall p \in P,$$

that is, the user cost on a path is equal to the sum of user costs on links that make up the path.

The Transportation Network Equilibrium Model with Elastic Demands

We have the following conservation of flow equations:

$$\sum_{p \in P_w} x_p = d_w, \quad \forall w.$$

In equilibrium, the following conditions must hold for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) - \lambda_w(d^*) \begin{cases} = 0, & \text{if} \quad x_p^* > 0, \\ \geq 0, & \text{if} \quad x_p^* = 0. \end{cases} \tag{25}$$

The Transportation Network Equilibrium Model with Elastic Demands

As proved in Dafermos (1982), the transportation network equilibrium conditions (25) are equivalent to the following variational inequality in path flows: determine $(x^*, d^*) \in K^6$ satisfying:

$$\sum_{w \in W} \sum_{p \in P_w} C_p(x^*) \times \left[x_p - x_p^*\right] - \sum_{w \in W} \lambda_w(d^*) \times [d_w - d_w^*] \geq 0, \quad \forall (x, d) \in K^6,$$

where $K^6 \equiv \{(x, d) | (x, d) \in R_+^{n_P + n_W}$ and $d_w = \sum_{p \in P_w} x_p, \forall w\}$.

The Transportation Network Equilibrium Model with Elastic Demands

A link flow pattern and associated travel demand pattern is a transportation network equilibrium if and only if it satisfies the variational inequality problem: determine (f^*, d^*) є K^7 satisfying:

$$\sum_{a\in L} c_a(f^*)\times(f_a - f_a^*) - \sum_{w\in W}\lambda_w(d^*)\times(d_w - d_w^*) \geq 0, \quad \forall (f,d)\in K^7,$$

where $K^7 \equiv \{(f,d)\in R_+^{n_L+n_W} \,|\, \textit{there exists an } x \textit{ satisfying} (20) \textit{ and } d_w = \sum_{p\in P_w} x_p, \forall w\}$.

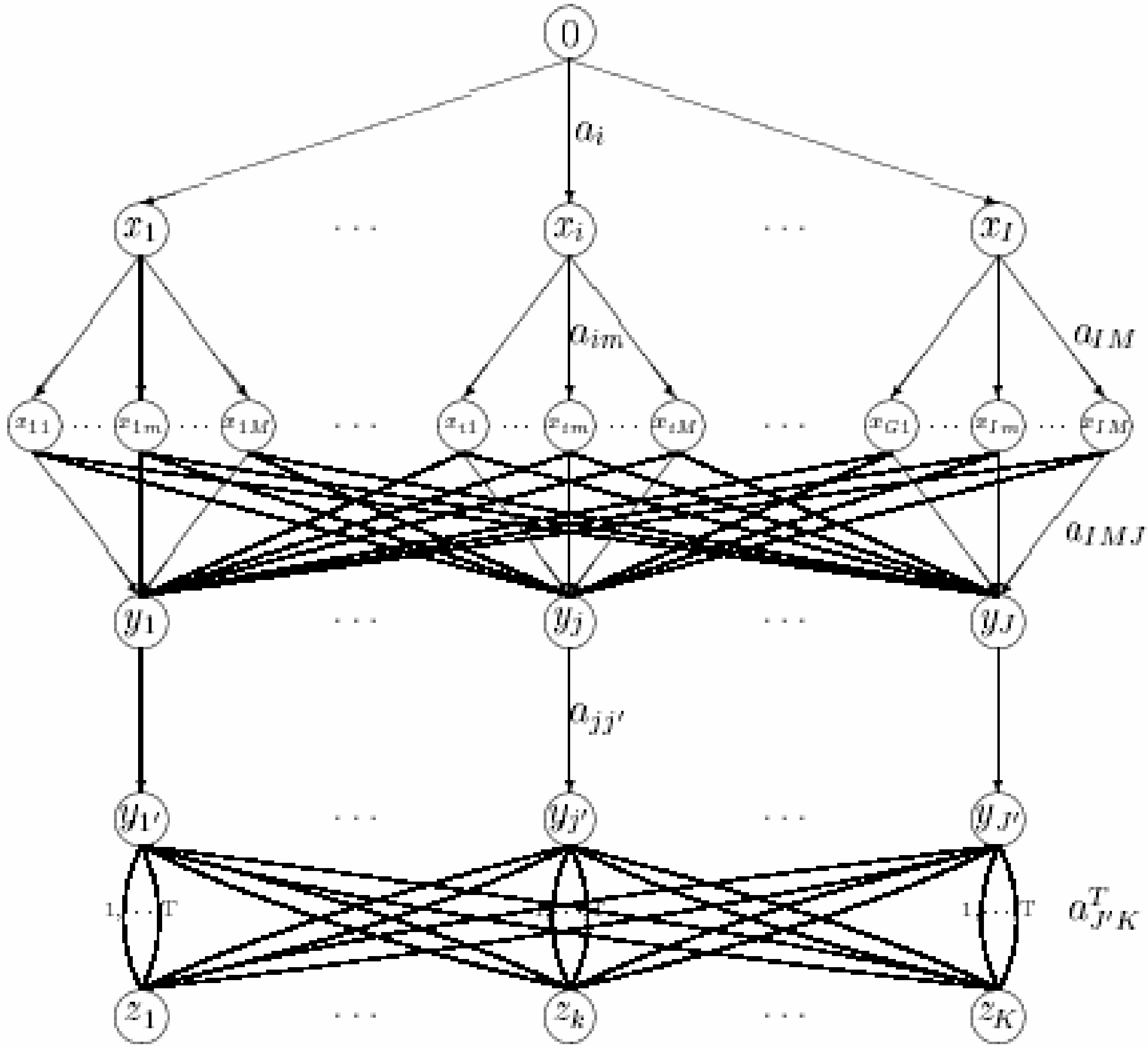

Figure 2: The $\mathcal{G}_S$ Supernetwork Representation of Supply Chain Network Equilibrium with Manufacturing Plants

Note that the following conservation of flow equations must hold on the network $\mathcal{G}_S$:

$$f_{a_i} = \sum_{m=1}^{M} \sum_{j=1}^{J} \sum_{j'=1}^{J'} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad i = 1, \ldots, I,$$

$$f_{a_{im}} = \sum_{j=1}^{J} \sum_{j'=1'}^{J'} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad i = 1, \ldots, I; m = 1, \ldots, M,$$

$$f_{a_{imj}} = \sum_{j'=1'}^{J'} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad i = 1, \ldots, I; m = 1, \ldots, M; j = 1, \ldots, J,$$

$$f_{a_{jj'}} = \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad jj' = 11', \ldots, JJ',$$

$$f_{a^t_{j'k}} = \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{j=1}^{J} x_{p^t_{imjj'k}}, \quad j' = 1', \ldots, J'; t = 1, \ldots, T; k = 1, \ldots, K.$$

Also, we have that

$$d_{w_k} = \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{jj'=11'}^{JJ'} \sum_{t=1}^{T} x_{p^t_{imjj'k}}, \quad k = 1, \ldots, K.$$

We can construct a feasible link flow pattern for $\mathcal{G}_S$ based on the corresponding feasible supply chain flow pattern in the supply chain network model, $(q, h, Q^1, Q^2, d) \in \mathcal{K}^5$, in the following way:

$$q_i \equiv f_{a_i}, \quad i = 1, \ldots, I,$$

$$q_{im} \equiv f_{a_{im}}, \quad i = 1, \ldots, I; m = 1, \ldots, M,$$

$$q_{imj} \equiv f_{a_{imj}}, \quad i = 1, \ldots, I; m = 1, \ldots, M; j = 1, \ldots, J,$$

$$h_j \equiv f_{a_{jj'}}, \quad jj' = 11', \ldots, JJ',$$

$$q^t_{jk} = f_{a^t_{j'k}}, \quad j = 1, \ldots, J; j' = 1', \ldots, J'; t = 1, \ldots, T; k = 1, \ldots, K,$$

$$d_k = \sum_{j=1}^{J} \sum_{t=1}^{T} q^t_{jk}, \quad k = 1, \ldots, K.$$

Observe that although q_i is not explicitly stated in the model in Section 2, it is inferred in that

$$q_i = \sum_{m=1}^{M} q_{im}, \quad i = 1, \ldots, I,$$

and simply represents the total amount of product produced by manufacturer i.

The Transportation Network Equilibrium Model with Elastic Demands

In equilibrium, the following conditions must hold for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) - \lambda_w(d^*) \begin{cases} = 0, & \textit{if} \quad x_p^* > 0, \\ \geq 0, & \textit{if} \quad x_p^* = 0. \end{cases} \tag{25}$$

Consequently, the equilibrium conditions (25) for the transportation network equilibrium model on the network $\mathcal{G}_S$ state that for every O/D pair w_k and every path connecting the O/D pair w_k:

$$C_{p^t_{imjj'k}} - \lambda_{w_k}$$

$$= \frac{\partial f_{im}}{\partial q_{im}} + \alpha_i e_{im} + \frac{\partial c_{imj}}{\partial q_{imj}} + \alpha_i e_{imj} + \frac{\partial c_j}{\partial h_j} + \beta_j e_j + \frac{\partial c^t_{jk}}{\partial q^t_{jk}} + \hat{c}^t_{jk} + (\beta_j + \eta_k) e^t_{jk} - \lambda_{w_k} \begin{cases} = 0, & \text{if} \quad x^*_{p^t_{imjj'k}} > 0, \\ \geq 0, & \text{if} \quad x^*_{p^t_{imjj'k}} = 0. \end{cases} \tag{49}$$

The Transportation Network Equilibrium Model with Elastic Demands

For the transportation network equilibrium problem on GS, according to Theorem 2, we have that a link flow and travel disutility pattern (f*, d*) є K^7 is an equilibrium (according to (49)), if and only if it satisfies the variational inequality:

$$\sum_{i=1}^{I} c_{a_i}(f^{1*})\times(f_{a_i}-f^*_{a_i}) + \sum_{i=1}^{I}\sum_{m=1}^{M} c_{a_{im}}(f^{2*})\times(f_{a_{im}}-f^*_{a_{im}}) + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J} c_{a_{imj}}(f^{3*})\times(f_{a_{imj}}-f^*_{a_{imj}})$$

$$+\sum_{jj'=11'}^{JJ'} c_{a_{jj'}}(f^{4*})\times(f_{a_{jj'}}-f^*_{a_{jj'}}) + \sum_{j'=1'}^{J'}\sum_{k=1}^{K}\sum_{t=1}^{T} c_{a^t_{j'k}}(f^{5*})\times(f_{a^t_{j'k}}-f^*_{a^t_{j'k}})$$

$$-\sum_{k=1}^{K} \lambda_{w_k}(d^*)\times(d_{w_k}-d^*_{w_k}) \geq 0, \quad \forall (f,d)\in K^7. \qquad (50)$$

The Transportation Network Equilibrium Model with Elastic Demands

After the substitution of constraints into (50), we have the following variational inequality: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in K^5$ satisfying:

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\frac{\partial f_{im}(q_i^*)}{\partial q_{im}} + \alpha_i e_{im}\right] \times [q_{im} - q_{im}^*] + \sum_{j=1}^{J}\left[\frac{\partial c_j(h^*)}{\partial h_j} + \beta_j e_j\right] \times [h_j - h_j^*]$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[\frac{\partial c_{imj}(q_{imj}^*)}{\partial q_{imj}} + \alpha_i e_{imj}\right] \times [q_{imj} - q_{imj}^*]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{jk}^t(q_{jk}^{t*})}{\partial q_{jk}^t} + \hat{c}_{jk}^t(Q^{2*}) + (\beta_j + \eta_k) e_{jk}^t\right] \times [q_{jk}^t - q_{jk}^{t*}] - \sum_{k=1}^{K} \rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0,$$

$$\forall (q, h, Q^1, Q^2, d) \in K^5.$$

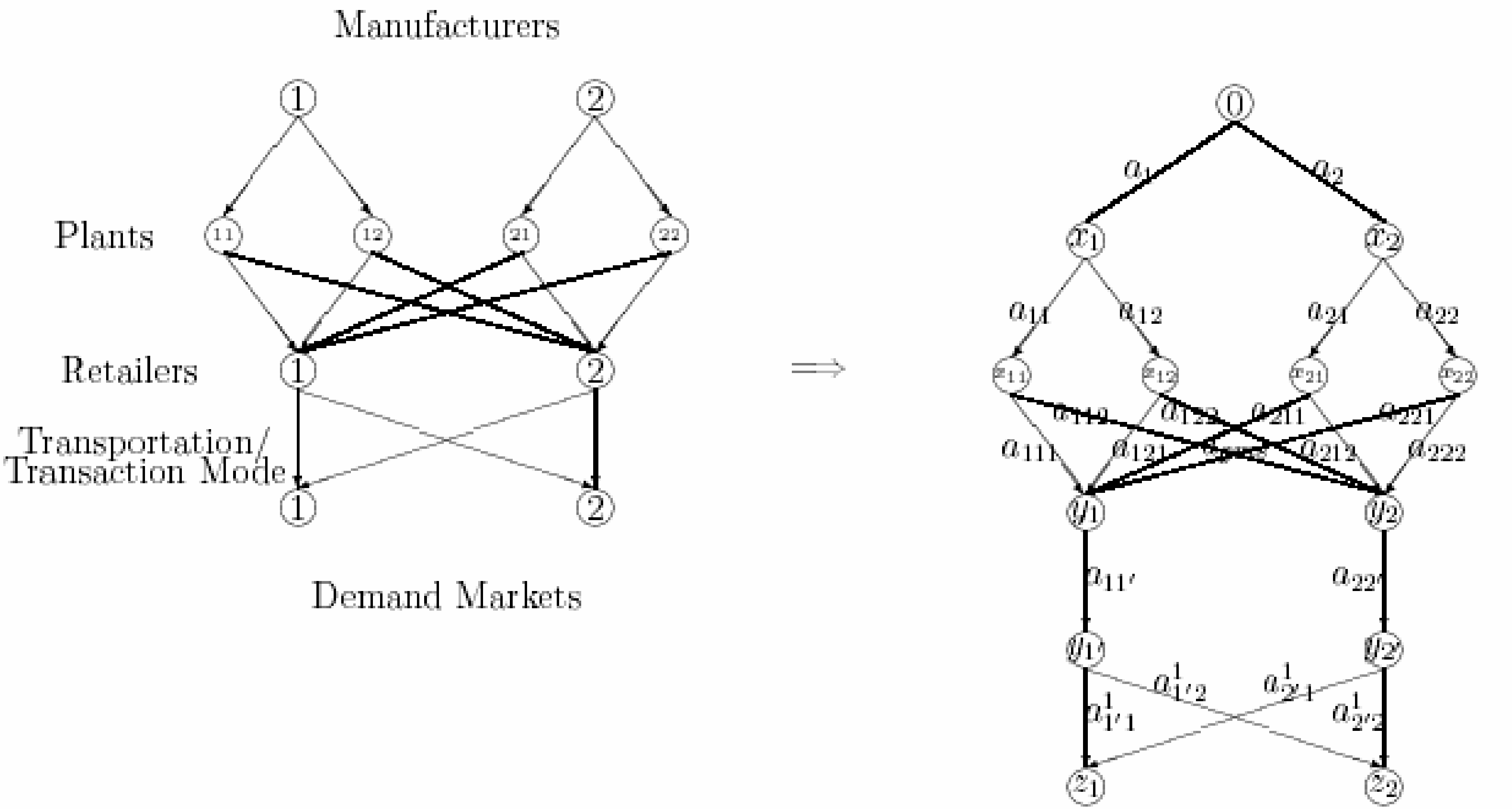

Figure 3: Supply Chain Network and Corresponding Supernetwork $\mathcal{G}_S$ for the Numerical Examples

Numerical Examples

Example 1

$$\alpha_1 = \alpha_2 = 0,\ \beta_1 = \beta_2 = 0,\ \text{and}\ \eta_1 = \eta_2 = 0.$$

The production cost functions for the manufacturers were given by:

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21},$$

$$f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

The transportation/transaction cost functions faced by the manufacturers and associated with transacting with the retailers were given by:

$$c_{imj}(q_{imj}) = .5q_{imj}^2 + 3.5q_{imj}, \quad i = 1; m = 1,2; j = 1,2;$$

$$c_{imj}(q_{imj}) = .5q_{imj}^2 + 2q_{imj}, \quad i = 2; m = 1,2; j = 1,2.$$

The operating costs of the retailers, in turn, were given by:

$$c_1(Q^1) = .5(\sum_{i=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2} q_{i2})^2.$$

The demand market price functions at the demand markets were:

$$\rho_{31}(d) = -d_1 + 500, \quad \rho_{32} = -d_2 + 500,$$

and the unit transportation/transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}_{jk}^1(q_{jk}^1) = q_{jk}^1 + 5, \quad j = 1,2; k = 1,2.$$

All other transportation/transaction costs were assumed to be equal to zero. We assumed that the manufacturing plants emitted pollutants where $e_{11} = e_{12} = e_{21} = e_{22} = 5$.

Numerical Examples

Example 2

$\alpha_1 = \alpha_2 = 1$, with all other weights equal to zero.

Example 3

$e^1_{11} = e^1_{12} = 10.$ $\eta_1 = \eta_2 = 1.$

Example 4

$\eta_1 = \eta_2 = 32$

(with $\eta_1 = \eta_2 = 30$ there were still positive flows on those polluting links)

Table 2: Equilibrium Solutions of Examples 1, 2, 3, and 4

Equilibrium Values	Example 1	Example 2	Example 3	Example 4
$f^*_{a_1} = q^*_1$	48.17	47.68	47.17	42.04
$f^*_{a_2} = q^*_2$	169.62	167.89	166.20	109.34
$f^*_{a_{11}} = q^*_{11}$	33.37	33.03	32.69	25.87
$f^*_{a_{12}} = q^*_{12}$	14.80	14.65	14.48	16.37
$f^*_{a_{21}} = q^*_{21}$	33.71	33.37	33.02	26.17
$f^*_{a_{22}} = q^*_{22}$	135.91	134.53	133.17	83.17
$f^*_{a_{11'}} = h^*_1$	108.90	107.79	103.82	0.00
$f^*_{a_{22'}} = h^*_2$	108.90	107.79	109.54	151.58
$f^*_{a_{111}} = q^*_{111}$	16.69	16.52	15.60	0.00
$f^*_{a_{112}} = q^*_{112}$	16.69	16.52	17.09	25.87
$f^*_{a_{121}} = q^*_{121}$	7.40	7.32	6.61	0.00
$f^*_{a_{122}} = q^*_{122}$	7.40	7.32	7.87	16.37
$f^*_{a_{211}} = q^*_{211}$	16.85	16.68	15.77	0.00
$f^*_{a_{212}} = q^*_{212}$	16.85	16.68	17.25	26.17
$f^*_{a_{221}} = q^*_{221}$	67.96	67.26	65.84	0.00
$f^*_{a_{222}} = q^*_{222}$	67.96	67.26	67.33	83.17
$f^*_{a^1_{1'1}} = q^{1*}_{11}$	54.45	53.89	51.91	0.00
$f^*_{a^1_{1'2}} = q^{1*}_{12}$	54.45	53.89	51.91	0.00
$f^*_{a^1_{2'1}} = q^{1*}_{21}$	54.45	53.89	54.77	75.79
$f^*_{a^1_{2'2}} = q^{1*}_{22}$	54.45	53.89	54.77	75.79
$d^*_{w_1} = d^*_1$	108.90	107.79	106.68	75.79
$d^*_{w_2} = d^*_2$	108.90	107.79	106.68	75.79
$\lambda_{w_1} = \rho_{31}$	391.11	392.23	393.30	424.21
$\lambda_{w_2} = \rho_{32}$	391.11	392.23	393.30	424.21

Numerical Examples

Example 1

The total amount of emissions in this example was: $e_{11}q_{11}^* + e_{12}q_{12}^* + e_{21}q_{21}^* + e_{22}q_{22}^* = 1,089.$

Example 2

The total emissions generated were equal to: 1,077.85

Example 3

$e_{11}q_{11}^* + e_{12}q_{12}^* + e_{21}q_{21}^* + e_{22}q_{22}^* + e_{11}^{1*}q_{11}^1 + e_{12}^{1*}q_{12}^{1*} = 1,585.95.$

Example 4

The total amount of emissions were now: 756.70.

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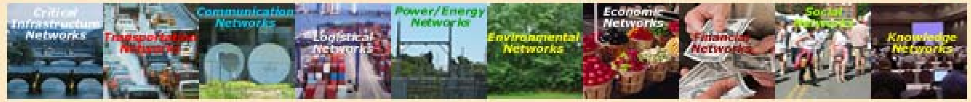
The Virtual Center for Supernetworks



Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

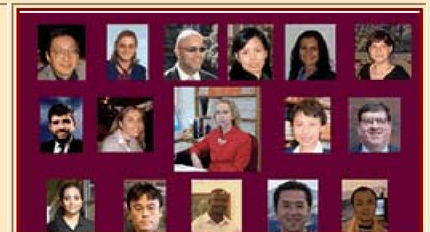
Mission: The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, supply chains, telecommunication, and electric power networks to economic, environmental, financial, knowledge and social networks.

The Applications of Supernetworks Include: multimodal transportation networks, critical infrastructure, energy and the environment, the Internet and electronic commerce, global supply chain management, international financial networks, web-based advertising, complex networks and decision-making, integrated social and economic networks, network games, and network metrics.

Announcements and Notes from the Center Director Professor Anna Nagurney

Updated: April 12, 2007

BusinessWeek online
Imperial College London
DUSP MIT
AWIS Association for Women in Science
In the Media



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Sustainable Supply Chain and Transportation Networks

Appears in International Journal of Sustainable Transportation (2007), vol. 1, pp29-51.

Anna Nagurney, Zugang Liu, and Trisha Woolley