Modeling of Electric Power Supply Chain Networks with Fuel Suppliers via Variational Inequalities

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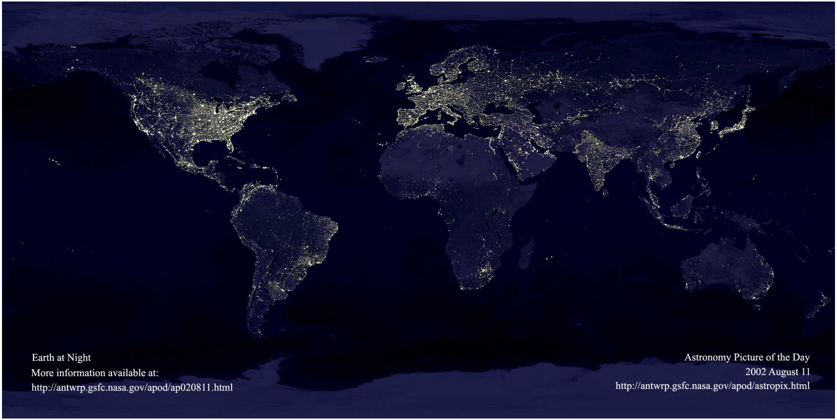
Motivation

- In US: half a trillion dollars worth of net assets
- Electric power supply chains provide the foundations of the functioning of our modern economies and societies
  - Communication, transportation, heating, lighting, cooling, computing, entertainment, etc.
- Deregulation: from vertically integrated to competitive market
  - In US, EU and many other countries
- Inelastic, seasonal demand

Motivation

- “[In recent years] the adequacy of the bulk power transmission system has been challenged to support the movement of power in unprecedented amounts and in unexpected directions” (NERC (1998))
- “Electricity transformed the past century, and it will be even more crucial in the years to come” (EPRI (2003))

Earth at Night



Supply Chain Perspective

- Network equilibrium
- Several classes of decision-makers
- Optimization problem for every decision-maker is unique

Related Literature

- Beckmann, M. J., McGuire, C. B., and Winsten, C. B. (1956), **Studies in the Economics of Transportation**. Yale University Press, New Haven, Connecticut
- Nagurney, A (1999), **Network Economics: A Variational Inequality Approach**, Second and Revised Edition, Kluwer Academic Publishers, Dordrecht, The Netherlands
- Nagurney, A. and Matsypura, D. (2004), A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption, *Proceedings of the International Conference in Computing, Communications and Control Technologies*, Austin, Texas, Volume VI, 127-134.

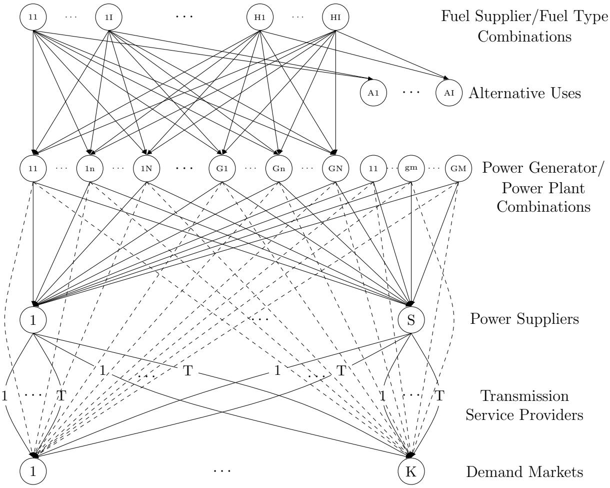
Related Literature cont'd

- Nagurney, A., Dong, J., and Zhang, D. (2002), A supply chain network equilibrium model, *Transportation Research E* 38, 281-303.
- Nagurney, A (2006), On the Relationship Between Supply Chain and Transportation Network Equilibria: A Supernetwork Equivalence with Computations, *Transportation Research E* 42, 293-316.
- Wu, K., Nagurney, A., Liu, Z., and Stranlund, J. (2006), Modeling Generator Power Plant Portfolios and Pollution Taxes in Electric Power Supply Chain Networks: A Transportation Network Equilibrium Transformation, *Appears in Transportation Research D* 11, 171-190.

Characteristics of the Model

- Explicit modeling of fuel suppliers
- Spatially distributed generation plants owned by one company
- ‘Self-supply’ generation
- Inelastic demand

Electric Power Supply Chain Network



Decision-Making Behavior of Fuel Supplier h

Maximize Profit

Maximize

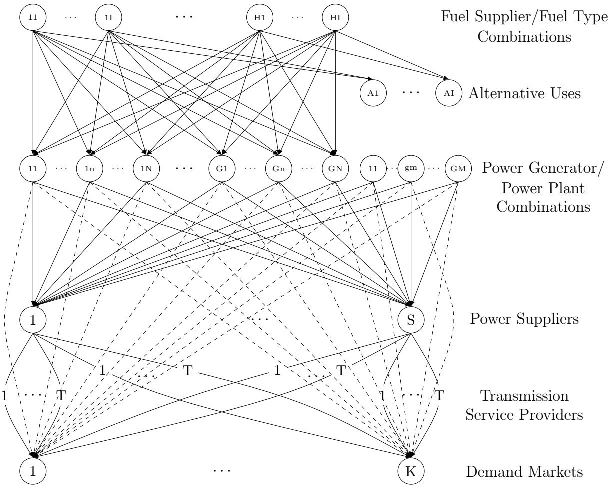
$$\sum_{i=1}^{I}\left[\sum_{g=1}^{G}\sum_{n=1}^{N}\rho_{0hi}^{gn*}q_{hi}^{gn}-c_{hi}(q_{hi})-\sum_{g=1}^{G}\sum_{n=1}^{N}c_{hi}^{gn}(q_{hi}^{gn})\right]$$

subject to:

$$\sum_{g=1}^{G}\sum_{n=1}^{N}q_{hi}^{gn}\leq U_{hi},\quad \forall i,$$

$$q_{hi}^{gn}\geq 0,\quad \forall i,g,n.$$

Electric Power Supply Chain Network



Decision-Making Behavior of Power Generator g

Maximize Profit

Maximize

$$\sum_{n=1}^{N}\left[\sum_{s=1}^{S}\rho_{1gns}^{*}q_{gns}+\sum_{k=1}^{K}\rho_{1gnk}^{*}q_{gnk}-\sum_{h=1}^{H}\sum_{i=1}^{I}\rho_{0hi}^{gn*}q_{hi}^{gn}-f_{gn}(q_{gn})-\sum_{s=1}^{S}c_{gns}(q_{gns})-\sum_{k=1}^{K}c_{gnk}(q_{gnk})\right]$$
$$+\sum_{m=1}^{M}\left[\sum_{s=1}^{S}\rho_{1gms}^{*}q_{gms}+\sum_{k=1}^{K}\rho_{1gmk}^{*}q_{gmk}-f_{gm}(q_{gm})-\sum_{s=1}^{S}c_{gms}(q_{gms})-\sum_{k=1}^{K}c_{gmk}(q_{gmk})\right]$$

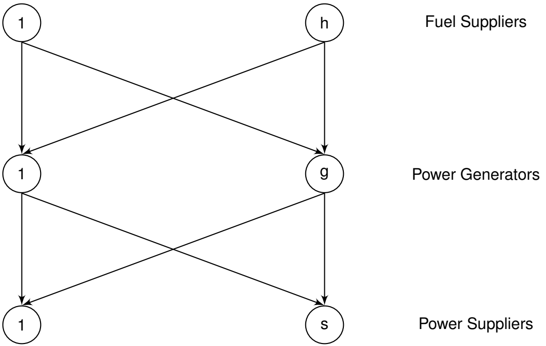
subject to:

$$\sum_{s=1}^{S}q_{gns}+\sum_{k=1}^{K}q_{gnk}=\sum_{h=1}^{H}\sum_{i=1}^{I}\alpha_{hi}^{gn}q_{hi}^{gn},\quad \forall n,$$
$$q_{gns}\geq 0,\quad q_{gnk}\geq 0,\quad q_{gms}\geq 0,\quad q_{gmk}\geq 0,\quad \forall n,m,s,k.$$

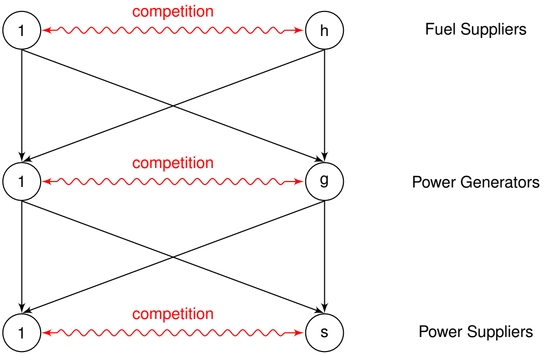
Game Theory

- Competition game between decision-makers of the same class
- Coordination game between decision-makers of different classes

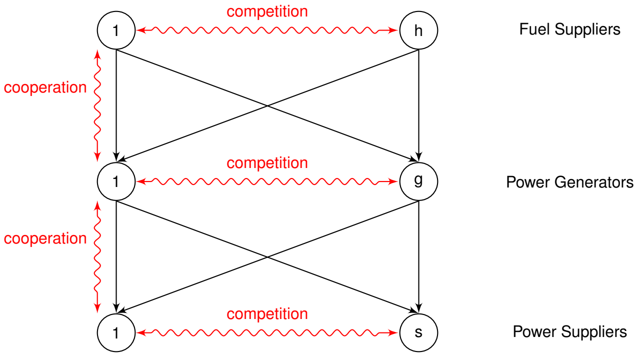
Example: Electric Power Supply Chain Network



Example: Electric Power Supply Chain Network



Example: Electric Power Supply Chain Network



Decision-Making Behavior of Power Supplier s

Maximize Profit

Maximize

$$\sum_{k=1}^{K}\sum_{t=1}^{T}\rho_{2sk}^{t*}q_{sk}^{t} - c_s(Q^1, Q^2, Q^4) - \sum_{g=1}^{G}\sum_{m=1}^{M}\rho_{1gms}^{*}q_{gms} - \sum_{g=1}^{G}\sum_{n=1}^{N}\rho_{1gns}^{*}q_{gns}$$
$$- \sum_{g=1}^{G}\sum_{m=1}^{M}\hat{c}_{gms}(q_{gms}) - \sum_{g=1}^{G}\sum_{n=1}^{N}\hat{c}_{gns}(q_{gns}) - \sum_{k=1}^{K}\sum_{t=1}^{T}c_{sk}^{t}(q_{sk}^{t})$$

subject to:

$$\sum_{k=1}^{K}\sum_{t=1}^{T}q_{sk}^{t} = \sum_{g=1}^{G}\sum_{m=1}^{M}q_{gms} + \sum_{g=1}^{G}\sum_{n=1}^{N}q_{gns},$$

$$q_{sk}^{t} \geq 0, \quad \forall k, t, \quad q_{gms} \geq 0, \quad \forall g, m, \quad q_{gns} \geq 0, \quad \forall g, n.$$

Equilibrium Conditions for Demand Market k

Prices

$$\rho_{2sk}^{t*} + \hat{c}_{sk}^{t}(Q^{2*}) \begin{cases} = \rho_{3k}^{*}, & \text{if} \quad q_{sk}^{t*} > 0 \\ \geq \rho_{3k}^{*}, & \text{if} \quad q_{sk}^{t*} = 0 \end{cases} \qquad \forall s, t$$

$$\rho_{1gnk}^{*} + \hat{c}_{gnk}(Q^{3*}) \begin{cases} = \rho_{3k}^{*}, & \text{if} \quad q_{gnk}^{*} > 0 \\ \geq \rho_{3k}^{*}, & \text{if} \quad q_{gnk}^{*} = 0 \end{cases} \qquad \forall g, n$$

$$\rho_{1gmk}^{*} + \hat{c}_{gmk}(Q^{5*}) \begin{cases} = \rho_{3k}^{*}, & \text{if} \quad q_{gmk}^{*} > 0 \\ \geq \rho_{3k}^{*}, & \text{if} \quad q_{gmk}^{*} = 0 \end{cases} \qquad \forall g, m$$

Flows

$$d_k = \sum_{s=1}^{S} \sum_{t=1}^{T} q_{sk}^{t} + \sum_{g=1}^{G} \sum_{m=1}^{M} q_{gmk} + \sum_{g=1}^{G} \sum_{n=1}^{N} q_{gnk}$$

Variational Inequality Formulation

Determine $(Q^{0*}, Q^{1*}, Q^{2*}, Q^{3*}, Q^{4*}, Q^{5*}) \in \mathcal{K}$ satisfying:

$$\sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{g=1}^{G}\sum_{n=1}^{N}\left[\frac{\partial c_{hi}(q_{hi}^{*})}{\partial q_{hi}^{gn}}+\frac{\partial c_{hi}^{gn}(q_{hi}^{gn*})}{\partial q_{hi}^{gn}}+\frac{\partial f_{gn}(q_{gn}^{*})}{\partial q_{hi}^{gn}}\right]\times\left[q_{hi}^{gn}-q_{hi}^{gn*}\right]$$

$$+\sum_{g=1}^{G}\sum_{n=1}^{N}\sum_{s=1}^{S}\left[\frac{\partial c_{gns}(q_{gns}^{*})}{\partial q_{gns}}+\frac{\partial c_{s}(Q^{1*},Q^{2*},Q^{4*})}{\partial q_{gns}}+\frac{\partial \hat{c}_{gns}(q_{gns}^{*})}{\partial q_{gns}}\right]\times\left[q_{gns}-q_{gns}^{*}\right]$$

$$+\sum_{g=1}^{G}\sum_{n=1}^{N}\sum_{k=1}^{K}\left[\frac{\partial c_{gnk}(q_{gnk}^{*})}{\partial q_{gnk}}+\hat{c}_{gnk}(Q^{3*})\right]\times\left[q_{gnk}-q_{gnk}^{*}\right]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{s=1}^{S}\left[\frac{\partial f_{gm}(q_{gm}^{*})}{\partial q_{gms}}+\frac{\partial c_{gms}(q_{gms}^{*})}{\partial q_{gms}}+\frac{\partial c_{s}(Q^{1*},Q^{2*},Q^{4*})}{\partial q_{gms}}+\frac{\partial \hat{c}_{gms}(q_{gms}^{*})}{\partial q_{gms}}\right]\times\left[q_{gms}-q_{gms}^{*}\right]$$

$$+\sum_{g=1}^{G}\sum_{m=1}^{M}\sum_{k=1}^{K}\left[\frac{\partial f_{gm}(q_{gm}^{*})}{\partial q_{gmk}}+\frac{\partial c_{gmk}(q_{gmk}^{*})}{\partial q_{gmk}}+\hat{c}_{gmk}(Q^{5*})\right]\times\left[q_{gmk}-q_{gmk}^{*}\right]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{s}(Q^{1*},Q^{2*},Q^{4*})}{\partial q_{sk}^{t}}+\frac{\partial c_{sk}^{t}(q_{sk}^{t*})}{\partial q_{sk}^{t}}+\hat{c}_{sk}^{t}(Q^{2*})\right]\times\left[q_{sk}^{t}-q_{sk}^{t*}\right]\geq 0,$$

$$\forall (Q^{0},Q^{1},Q^{2},Q^{3},Q^{4},Q^{5})\in\mathcal{K}$$

The Euler Method (Dupuis and Nagurney (1993))

Step 0: Initialization Set $X^0 \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter. Let $\mathcal{T} = 1$ and set the sequence $\{\alpha_\mathcal{T}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_\mathcal{T} = \infty$, $\alpha_\mathcal{T} > 0$, $\alpha_\mathcal{T} \to 0$, as $\mathcal{T} \to \infty$.

Step 1: Computation Compute $X^\mathcal{T} \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^\mathcal{T} + \alpha_\mathcal{T} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^\mathcal{T} \rangle \geq 0, \qquad \forall X \in \mathcal{K}.$$

Step 2: Convergence Verification If $|X^\mathcal{T} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Questions? Comments?

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http://www.bepress.com/ijeeps/vol8/iss1/art5

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