The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions:

A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels

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Outline of Presentation

- Research motivation
- Literature overview
- Description of a supernetwork consisting of a financial and a social network
- Numerical examples
- Summary and conclusions

This Presentation is Based on the Paper:

"The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels"

A. Nagurney, T. Wakolbinger, and L. Zhao, to appear in *Computational Economics* (2006)

Research Motivation

- Increasing importance of electronic financial transactions:
  - In 2001 15 million Americans paid their bills online with up to 46 million expected by 2005.
  - $160 billion in mortgages were taken out online in the US (cf. Mullaney and Little (2002)).
- Strong importance of personal relationships in financial transactions

Importance of Relationships in Financial Transactions

- Examples in the context of micro-financing
  - Ghatak (2002), Anthony (1997)

- Examples in the context of lending
  - Sharpe (1990), Petersen and Rajan (1994, 1995), Berger and Udell (1995), Uzzi (1997, 1999), DiMaggio and Louch (1998), Arrow (1998), Wilner (2000), Burt (2000), Boot and Thakor (2000)

Some of the Related Financial Network Literature

– Nagurney, A. and Siokos S. (1997), **Financial Networks: Statics and Dynamics**, Springer-Verlag, Heidelberg, Germany.

– Nagurney, A. and Ke, K. (2001), “Financial Networks with Intermediation,” *Quantitative Finance* **1**, 441-451.

– Nagurney, A. and Ke, K. (2003), “Financial Networks with Electronic Transactions: Modeling, Analysis, and Computations,” *Quantitative Finance* **3**, 71-87.

Some of the Related Financial Network Literature

- Nagurney, A. and Cruz, J. (2003), "International Financial Networks with Electronic Transactions," in **Innovations in Financial and Economic Networks**, A. Nagurney, Editor, Edward Elgar Publishers, Cheltenham, England, pp. 136-168.

- Nagurney, A. and Cruz, J. (2004), "Dynamics of International Financial Networks with Risk Management," *Quantitative Finance* **4**, 276-291.

Some of the Related Supernetwork Literature

- Wakolbinger, T., and Nagurney, A. (2004), "Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations," *Netnomics* **6**, 153-185.
- Cruz, J. M., Nagurney, A., and Wakolbinger. T. (2004), "Financial Engineering of the Integration of Global Supply Chain Networks and Social Networks with Risk Management," see: http://supernet.som.umass.edu.

Some of the Related Supernetwork Literature

- Nagurney, A., Cruz, J., and Wakolbinger, T. (2004), "The Co-Evolution and Emergence of Integrated International Financial Networks and Social Networks: Theory, Analysis, and Computations," invited chapter for **Globalization and Regional Economic Modeling**, R. Cooper, K. P. Donaghy, G. J. D. Hewings, Springer, Berlin, Germany.

Supernetwork Integrating Social Networks with Financial Networks

- Models the interaction of financial and social networks
- Captures interactions among individual sectors
- Includes electronic transactions
- Allows for non-investment
- Incorporates transaction costs and risk
- Shows the dynamic evolution of
  - Financial flows and associated prices on the financial network with intermediation
  - Relationship levels on the social network

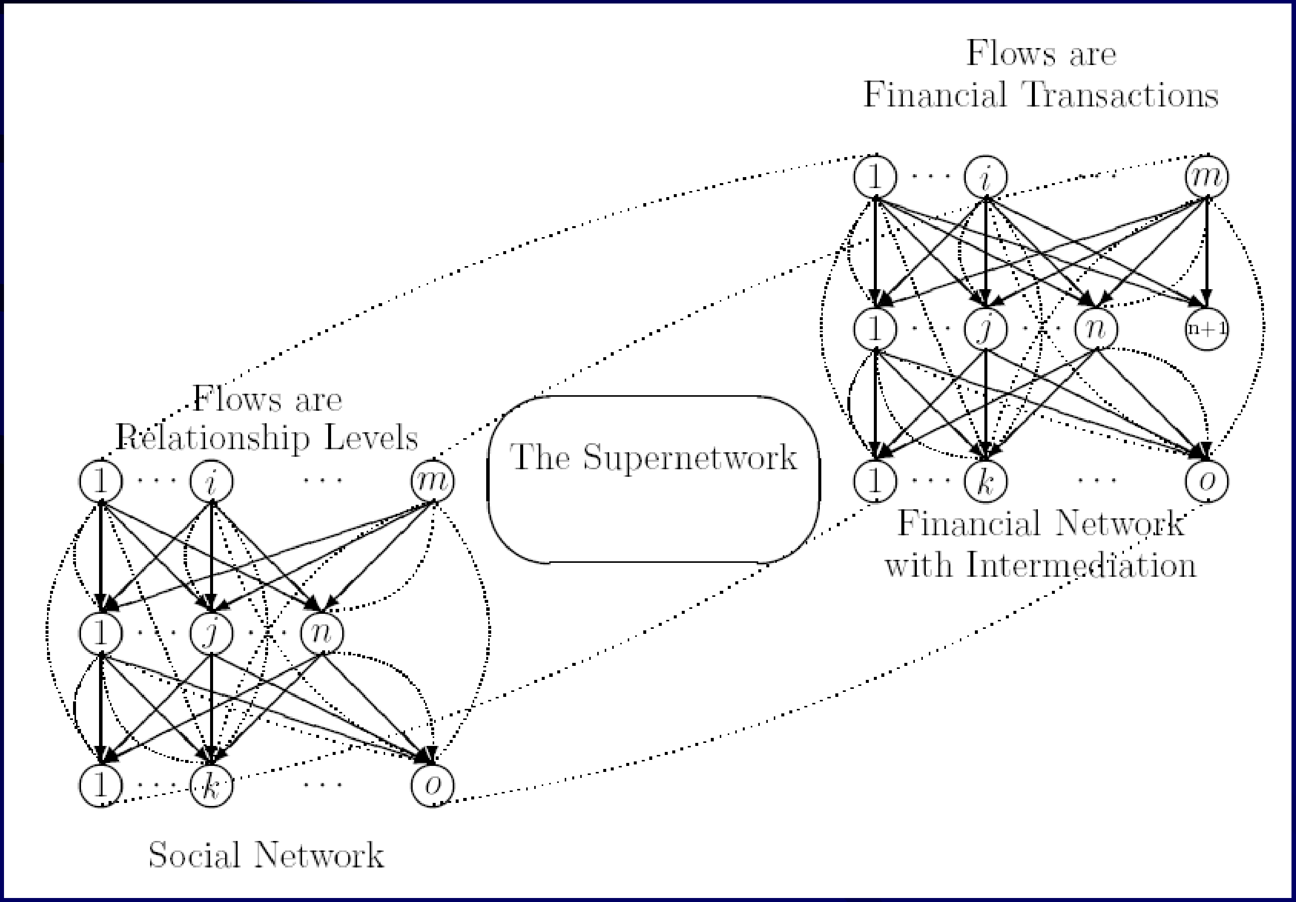
Model Assumptions

- 3 tiers of decision-makers: source agents, intermediaries and demand markets
- Source agents can transact either physically or electronically with the intermediaries
- Source agents can transact directly with the demand markets via internet links
- Intermediaries can transact either physically or electronically with the demand markets

Multicriteria Decision-Makers

- Source agents and intermediaries
  - Maximize net revenue
  - Minimize risk
  - Maximize relationship value
  - Individual weights assigned to the different criteria

Supernetwork Structure: Integrated Financial/ Social Network System



Role of Relationships

- Decision-makers in the network can decide about the relationship levels [0,1] that they want to establish.
- Establishing relationship levels incurs some costs.
- Higher relationship levels
  - Reduce transaction costs
  - Reduce risk
  - Have some additional value ("relationship value")

Novelty of Our Research

- Supernetworks show the dynamic co-evolution of financial (financial product, price and even informational) flows and the social network structure
- Financial flows and social network structure are interrelated
- Network of relationships with a measurable economic value

Variable Notation

q_{ijl}	flow on link joining source agent *i* with intermediary *j* through mode *l*
q_{ik}	flow on internet link joining source agent *i* with demand market *k*
q_{jkl}	flow on link joining intermediary *j* with node *k* through mode *l*
ρ_{1ijl}	price associated with the financial product transacted between source agent *i* and intermediary *j* via mode *l*
ρ_{1ik}	price associated with the financial product transacted between source agent *i* and demand market *k*
ρ_{2jkl}	price associated with the financial product transacted between intermediary *j* and demand market *k* in mode *l*
ρ_{3k}	price of the financial product at demand market *k*
$\square_j$	shadow price at intermediary *j*

Variable and Weight Notation

h_{ijl}	relationship level between source agent *i* and intermediary *j* in transaction mode *l*
h_{ik}	relationship level associated with the virtual mode of transaction between source agent *i* and demand market *k*
h_{jkl}	relationship level between intermediary *j* and demand market *k* transacting through mode *l*
α_i	nonnegative weight that source agent *i* assigns to risk
β_i	nonnegative weight that source agent *i* assigns to relationship value
δ_j	nonnegative weight that intermediary *j* assigns to risk
γ_j	nonnegative weight that intermediary *j* assigns to relationship value

Function Notation for Source Agent *i*

$b_{ijl}(h_{ijl})$	relationship production cost function with intermediary *j* in mode *l*
$b_{ik}(h_{ik})$	relationship production cost function with demand market *k*
$c_{ijl}(q_{ijl}, h_{ijl})$	transaction cost function with intermediary *j* in mode *l*
$c_{ik}(q_{ik}, h_{ik})$	transaction cost function with demand market *k*
$r_{ijl}(q_{ijl}, h_{ijl})$	risk function with intermediary *j* in mode *l*
$r_{ik}(q_{ik}, h_{ik})$	risk function with demand market *k*
$v_{ijl}(h_{ijl})$	relationship value function with intermediary *j* in mode *l*
$v_{ik}(h_{ik})$	relationship value function with demand market *k*

Function Notation for Intermediary *j*

$\hat{b}_{ijl}(h_{ijl})$	relationship production cost function with source agent *i* in mode *l*
$b_{jkl}(h_{jkl})$	relationship production cost function with node *k* in mode *l*
$\hat{c}_{ijl}(q_{ijl}, h_{ijl})$	transaction cost function with source agent *i* in mode *l*
$c_{jkl}(q_{jkl}, h_{jkl})$	transaction cost function with demand market *k* in mode *l*
$\hat{r}_{ijl}(q_{ijl}, h_{ijl})$	risk function with source agent *i* in mode *l*
$r_{jkl}(q_{jkl}, h_{jkl})$	risk function with demand market *k* in mode *l*
$\hat{v}_{ijl}(h_{ijl})$	relationship value function with source agent *i* in mode *l*
$v_{jkl}(h_{jkl})$	relationship value function with demand market *k* in mode *l*
$c_j(Q^1)$	handling cost function of intermediary *j*

Function Notation for Demand Market *k*

$d_k(\rho_3)$	demand function
$\hat{c}_{jkl}(Q^2, Q^3, h^2, h^3)$	transaction cost function with intermediary *j* in mode *l*
$\hat{c}_{ik}(Q^2, Q^3, h^2, h^3)$	transaction cost function with source agent *i*

A Source Agent's Multicriteria Decision-Making Problem

$$\text{Maximize } U^i = \sum_{j=1}^{n}\sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} + \sum_{k=1}^{o} \rho^*_{1ik} q_{ik} - \sum_{j=1}^{n}\sum_{l=1}^{2} c_{ijl}(q_{ijl}, h_{ijl}) - \sum_{k=1}^{o} c_{ik}(q_{ik}, h_{ik})$$

$$-\sum_{j=1}^{n}\sum_{l=1}^{2} b_{ijl}(h_{ijl}) - \sum_{k=1}^{o} b_{ik}(h_{ik}) - \alpha_i\left(\sum_{j=1}^{n}\sum_{l=1}^{2} r_{ijl}(q_{ijl}, h_{ijl}) + \sum_{k=1}^{o} r_{ik}(q_{ik}, h_{ik})\right)$$

$$+\beta_i\left(\sum_{j=1}^{n}\sum_{l=1}^{2} v_{ijl}(h_{ijl}) + \sum_{k=1}^{o} v_{ik}(h_{ik})\right)$$

subject to:

$$q_{ijl} \geq 0, \quad \forall j, l, \quad q_{ik} \geq 0, \quad \forall k,$$

$$0 \leq h_{ijl} \leq 1, \quad \forall j, l, \quad 0 \leq h_{ik} \leq 1, \quad \forall k,$$

Optimality Condition of Source Agents

determine $(Q^{1*}, Q^{2*}, h^{1*}, h^{2*}) \in \mathcal{K}_1$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} - \rho^*_{1ijl}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik}, h^*_{ik})}{\partial q_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik}, h^*_{ik})}{\partial q_{ik}} - \rho^*_{1ik}\right] \times [q_{ik} - q^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial b_{ijl}(h^*_{ijl})}{\partial h_{ijl}} - \beta_i \frac{\partial v_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}}\right] \times \left[h_{ijl} - h^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik}, h^*_{ik})}{\partial h_{ik}} + \frac{\partial b_{ik}(h^*_{ik})}{\partial h_{ik}} - \beta_i \frac{\partial v_{ik}(h^*_{ik})}{\partial h_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik}, h^*_{ik})}{\partial h_{ik}}\right] \times [h_{ik} - h^*_{ik}] \geq 0,$$

$$\forall (Q^1, Q^2, h^1, h^2) \in \mathcal{K}_1, \qquad (21)$$

where

$$\mathcal{K}_1 \equiv \left[(Q^1, Q^2, h^1, h^2) \mid q_{ijl} \geq 0,\ q_{ik} \geq 0,\ 0 \leq h_{ijl} \leq 1,\ 0 \leq h_{ik} \leq 1,\ \forall i, j, l, k, \text{and (1) holds}\right].$$

A Financial Intermediary's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad U^j = \sum_{k=1}^{o}\sum_{l=1}^{2} \rho^*_{2jkl} q_{jkl} - c_j(Q^1) - \sum_{i=1}^{m}\sum_{l=1}^{2} \hat{c}_{ijl}(q_{ijl}, h_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} c_{jkl}(q_{jkl}, h_{jkl})$$

$$-\sum_{i=1}^{m}\sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} - \sum_{i=1}^{m}\sum_{l=1}^{2} \hat{b}_{ijl}(h_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} b_{jkl}(h_{jkl}) - \delta_j\Big(\sum_{i=1}^{m}\sum_{l=1}^{2} \hat{r}_{ijl}(q_{ijl}, h_{ijl}) + \sum_{k=1}^{o}\sum_{l=1}^{2} r_{jkl}(q_{jkl}, h_{jkl})\Big)$$

$$+\gamma_j\Big(\sum_{i=1}^{m}\sum_{l=1}^{2} \hat{v}_{ijl}(h_{ijl}) + \sum_{k=1}^{o}\sum_{l=1}^{2} v_{jkl}(h_{jkl})\Big) \tag{41}$$

subject to:

$$\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} \le \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl} \tag{42}$$

$$q_{ijl} \ge 0, \quad \forall i, l, \quad q_{jkl} \ge 0, \quad \forall k, l, \tag{43}$$

$$0 \le h_{ijl} \le 1, \quad \forall i, l, \quad 0 \le h_{jkl} \le 1, \quad \forall k, l. \tag{44}$$

Optimality Conditions of Intermediaries

determine $(Q^{1*}, Q^{3*}, h^{1*}, h^{3*}, \epsilon^*) \in \mathcal{K}_2$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} + \rho^*_{1ijl} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} - \epsilon^*_j\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial q_{jkl}} - \rho^*_{2jkl} + \epsilon^*_j + \delta_j \frac{\partial r_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial q_{jkl}}\right] \times \left[q_{jkl} - q^*_{jkl}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial \hat{c}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} - \gamma_j \frac{\partial \hat{v}_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial \hat{b}_{ijl}(h^*_{ijl})}{\partial h_{ijl}}\right] \times \left[h_{ijl} - h^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial h_{jkl}} - \gamma_j \frac{\partial v_{jkl}(h^*_{jkl})}{\partial h_{jkl}} + \delta_j \frac{\partial r_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial h_{jkl}} + \frac{\partial b_{jkl}(h^*_{jkl})}{\partial h_{jkl}}\right] \times \left[h_{jkl} - h^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{o}\sum_{l=1}^{2} q^*_{jkl}\right] \times \left[\epsilon_j - \epsilon^*_j\right] \geq 0, \quad \forall (Q^1, Q^3, h^1, h^3, \epsilon) \in \mathcal{K}_2, \qquad (45)$$

where

$$\mathcal{K}_2 \equiv \left[(Q^1, Q^3, h^1, h^3, \epsilon) \mid q_{ijl} \geq 0, q_{jkl} \geq 0, 0 \leq h_{ijl} \leq 1, 0 \leq h_{jkl} \leq 1, \epsilon_j \geq 0, \forall i, j, l, k\right],$$

Equilibrium Conditions for the Demand Markets

for all intermediaries: j; $j = 1, ..., n$ and all modes l; $l = 1, 2$:

$$\rho^*_{2jkl} + \hat{c}_{jkl}(Q^{2*}, Q^{3*}, h^{2*}, h^{3*}) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^*_{jkl} > 0 \\ \geq \rho^*_{3k}, & \text{if} \quad q^*_{jkl} = 0, \end{cases}$$

and for all source agents i; $i = 1, ..., m$:

$$\rho^*_{1ik} + \hat{c}_{ik}(Q^{2*}, Q^{3*}, h^{2*}, h^{3*}) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^*_{ik} > 0 \\ \geq \rho^*_{3k}, & \text{if} \quad q^*_{ik} = 0. \end{cases}$$

In addition, we must have that

$$d_k(\rho^*_3) \begin{cases} = \sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik}, & \text{if} \quad \rho^*_{3k} > 0 \\ \leq \sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik}, & \text{if} \quad \rho^*_{3k} = 0. \end{cases}$$

VI Formulation of the Equilibrium Conditions for the Demand Markets

determine $(Q^{2^*}, Q^{3^*}, \rho_3^*) \in R^{2no+mo+n}$, such that

$$\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\rho_{2jkl}^* + \hat{c}_{jkl}(Q^{2^*}, Q^{3^*}, h^{2*}, h^{3*}) - \rho_{3k}^*\right] \times \left[q_{jkl} - q_{jkl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\rho_{1ik}^* + \hat{c}_{ik}(Q^{2^*}, Q^{3^*}, h^{2*}, h^{3*}) - \rho_{3k}^*\right] \times [q_{ik} - q_{ik}^*]$$

$$+\sum_{k=1}^{o}\left[\sum_{l=1}^{2}\sum_{j=1}^{n} q_{jkl}^* + \sum_{i=1}^{m} q_{ik}^* - d_k(\rho_3^*)\right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \quad \forall (Q^2, Q^3, \rho_3) \in R_+^{mo+2no+n}.$$

The Equilibrium State

Definition 1: The equilibrium state of the supernetwork integrating the financial network with the social network is one where the financial flows and relationship levels between the tiers of the network coincide and the financial flows, relationship levels, and prices satisfy the sum of conditions (21), (45), and (53).

The equilibrium state is equivalent to a VI of the form:

determine $X^* \in \mathcal{K}$ satisfying

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

Projected Dynamical System

- The dynamic models can be rewritten as a projected dynamical system (Nagurney and Zhang (1996a)) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{80}$$

where $\Pi_{\mathcal{K}}$ denotes the projection of $-F(X)$ onto $\mathcal{K}$ at X and X_0 is equal to the point corresponding to the initial financial product transactions, relationship levels, shadow prices, and the demand market prices.

- The set of stationary points of the projected dynamical system (80) coincides with the set of solutions of the variational inequality problem (54) and, thus, with the set of equilibrium points as defined in Definition 1.

The Disequilibrium Dynamics

- The trajectory of the PDS describes the dynamic evolution of:
  - The financial product transactions on the financial network
  - The relationship levels on the social network
  - The demand market prices
  - The Lagrange multipliers or shadow prices associated with the intermediaries
- The projection operation guarantees that the constraints underlying the supernetwork system are not violated.

Dynamics of Demand Market Prices

- The demand market prices evolve according to the difference between the demand at the market (as a function of the prices at the demand markets at that time) and the amount of the financial product transactions.

- The projection operator guarantees that the prices do not take on negative values.

Dynamics of Shadow Prices

- The Lagrange multipliers/shadow prices associated with the intermediaries evolve according to the difference between the sum of the financial product transacted with the demand markets and that obtained from the source agents.

- The projection operator guarantees that these prices do not become negative.

Dynamics of Relationship Levels

- The relationship levels evolve on the social network links of the supernetwork according to the difference between the corresponding weighted relationship value, the sum of the various marginal costs and weighted marginal risks.

- The relationship levels are guaranteed to remain within the range zero to one.

Dynamics of Financial Product Transactions

- The financial product transactions evolve on the financial network links according to the difference between the characteristic price and various marginal and unit costs plus the weighted marginal risks.

- These flows are guaranteed to not assume negative values due to the projection operation.

Computational Procedure

We use the Euler Method to solve the Variational Inequality (VI) problem and to track the dynamic trajectories associated with the projected dynamical systems. The VI is in standard form:

determine $X^* \in \mathcal{K}$ satisfying

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \to 0$, as $T \to \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

$$y = P_{\mathcal{K}} X = \arg\min_{z \in \mathcal{K}} \|X - z\|.$$

Step 2: Convergence Verification

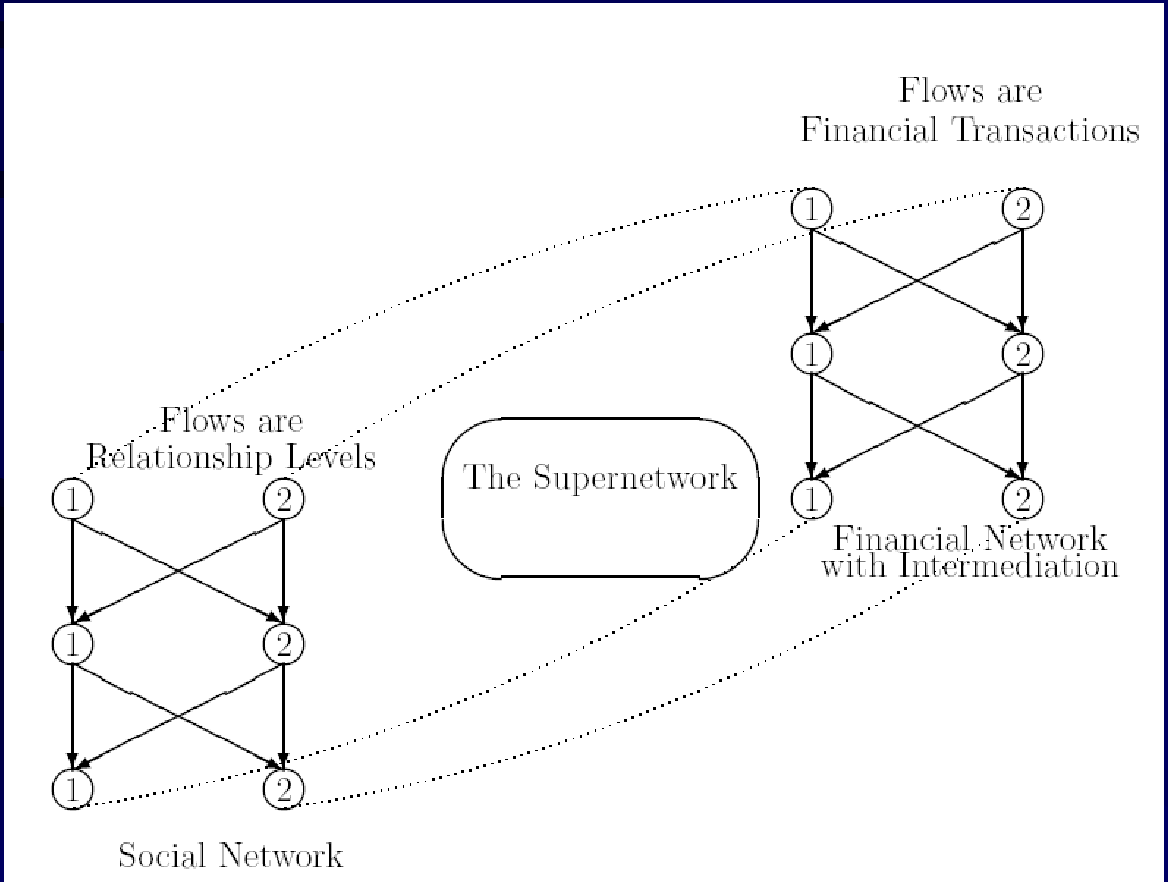
If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1.

Qualitative Properties

We have established

- Existence of a solution to the VI
- Uniqueness of a solution to the VI
- Conditions for the existence of a unique trajectory to the projected dynamical system
- Convergence of the Euler method

Supernetwork Structure of the Numerical Examples



Numerical Examples

- 2 source agents, 2 intermediaries, 2 demand markets
- No electronic transactions
- Transactions only between source agents and intermediaries and between intermediaries and demand markets
- Financial holdings of each source agent are 20
- Variance-covariance matrices are equal to identity matrices

Numerical Examples

- Transaction cost functions of source agents

$$c_{ij1}(q_{ij1}, h_{ij1}) = .5q_{ijl}^2 + 3.5q_{ijl} - h_{ij1}, \quad \text{for } i = 1, 2; j = 1, 2.$$

- Handling cost functions of intermediaries

$$c_j(Q^1) = .5(\sum_{i=1}^{2} q_{ij1})^2, \quad \text{for } j = 1, 2.$$

- Transaction cost functions of intermediaries

$$\hat{c}_{ij1}(q_{ij1}, h_{ijl}) = 1.5q_{ijl}^2 + 3q_{ijl}, \quad \text{for } i = 1, 2; j = 1, 2.$$

Numerical Examples

- Demand functions

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

- Transaction cost functions for demand markets

$$\hat{c}_{jk1}(Q^2, Q^2, h^2, h^3) = q_{jk1} - h_{jk1} + 5, \quad \text{for } j = 1, 2; k = 1, 2.$$

- Relationship value functions

$$v_{ij1}(h_{ij1}) = h_{ij1}, \quad \forall i, j; \quad v_{jk1}(h_{jk1}) = h_{jk1}, \quad \forall j, k,$$

- Relationship cost functions

$$b_{ijl}(h_{ijl}) = 2h_{ijl} + 1, \quad \forall i, j, l = 1; \quad b_{jkl}(h_{jk1}) = h_{jk1} + 1, \quad \forall j, k.$$

Differences between Examples

- Example 1
  - The weight for risk and relationship value is equal to 1.
- Example 2
  - The weight for relationship value for the two source agents increased from 1 to 10.
- Example 3
  - Like Example 2 but demand function changed to

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1100.$$

Numerical Examples: Equilibrium Financial Product Transactions

- Examples 1 and 2

$$Q^{1*} := q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 1.00,$$
$$Q^{3*} := q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 1.00.$$

- Example 3

$$Q^{1*} := q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 1.0449,$$
$$Q^{3*} := q^*_{111} = 1.3952, \quad q^*_{121} = 0.6680, \quad q^*_{211} = 1.3952, \quad q^*_{221} = 0.6680.$$

Numerical Examples: Equilibrium Prices

- Examples 1 and 2
  - At intermediaries

    $$\epsilon_1^* = \epsilon_2^* = 14.5808,$$

  - At demand markets

    $$\rho_{31}^* = \rho_{32}^* = 285.1427.$$

- Example 3
  - At intermediaries

    $$\epsilon_1^* = \epsilon_2^* = 14.8486,$$

  - At demand markets

    $$\rho_{31}^* = 397.9481, \quad \rho_{32}^* = 200.8729.$$

Numerical Examples: Equilibrium Relationship Levels

- Example 1
  - All relationship levels are equal to 0.
- Example 2 and 3
  - Relationship levels of source agents are equal to 1.

Types of Simulations

- We can simulate
  - Changes in transaction, handling, and relationship production cost functions
  - Changes in demand and risk functions
  - Changes in weights for relationship value and risk
  - Addition and removal of actors
  - Addition and removal of multiple transaction modes

Summary

- We model the behavior of the decision-makers, their interactions, and the dynamic evolution of the associated variables.
- We study the problems qualitatively as well as computationally.
- We develop algorithms, implement them, and establish conditions for convergence.
- We have studied to-date "good behavior“. Fascinating questions arise when there may be situations of instability, multiple equilibria, chaos, cycles, etc.

The full text of the related papers can be found under Downloadable Articles at:

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Thank you!

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