Statics and Dynamics of Global Supply Chain Networks with Environmental Decision-Making

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Motivation for Globalization

- Growing competition
- Emphasis on efficiency
- Cost reduction
- Satisfaction of consumer demands
- Advances in technology (E-commerce)

New Challenges for Global Supply Chains

Uncertainty (disruption of supply chains)

- Threat of illness: SARS (cf. Engardio et al. (2003))
- Terrorist threats (cf. Sheffi (2001))
- War

Background

Criticism of globalization has increased from the environmentalists.

The production of pollution-intensive goods shifts from countries with strict environmental regulations towards those with lax regulations.

Environmental regulations are a form of protectionism.

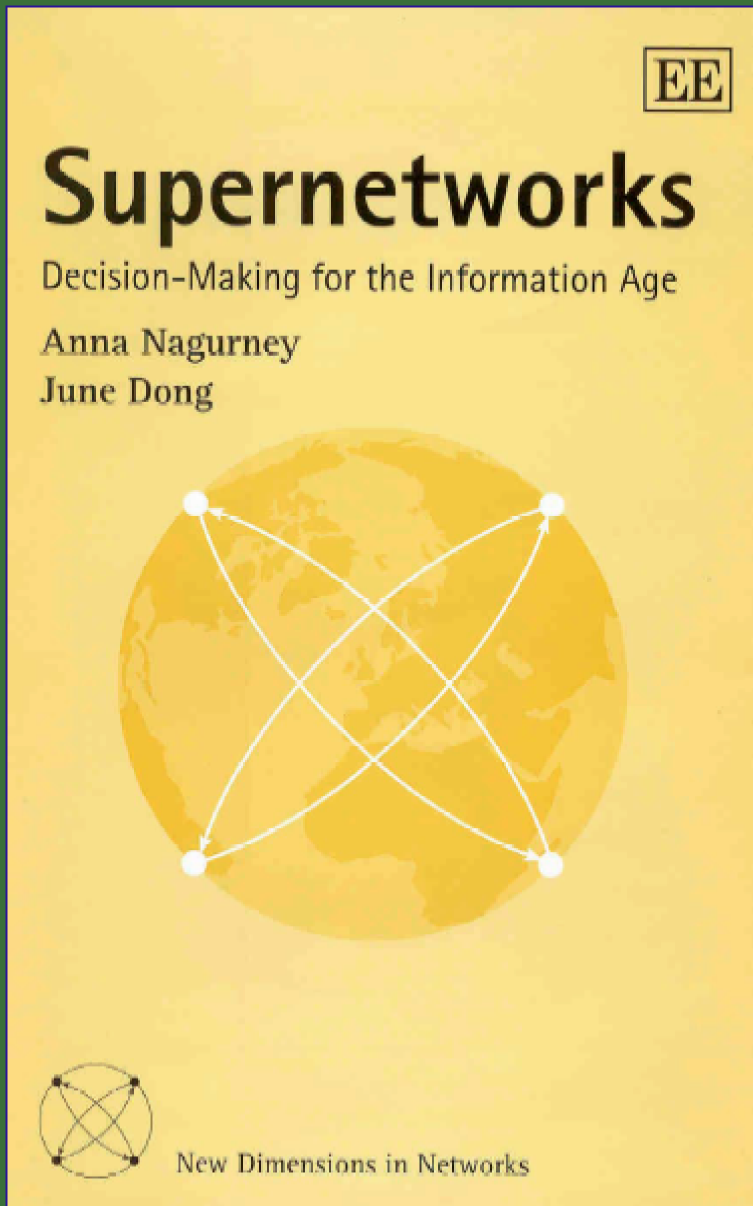
Background

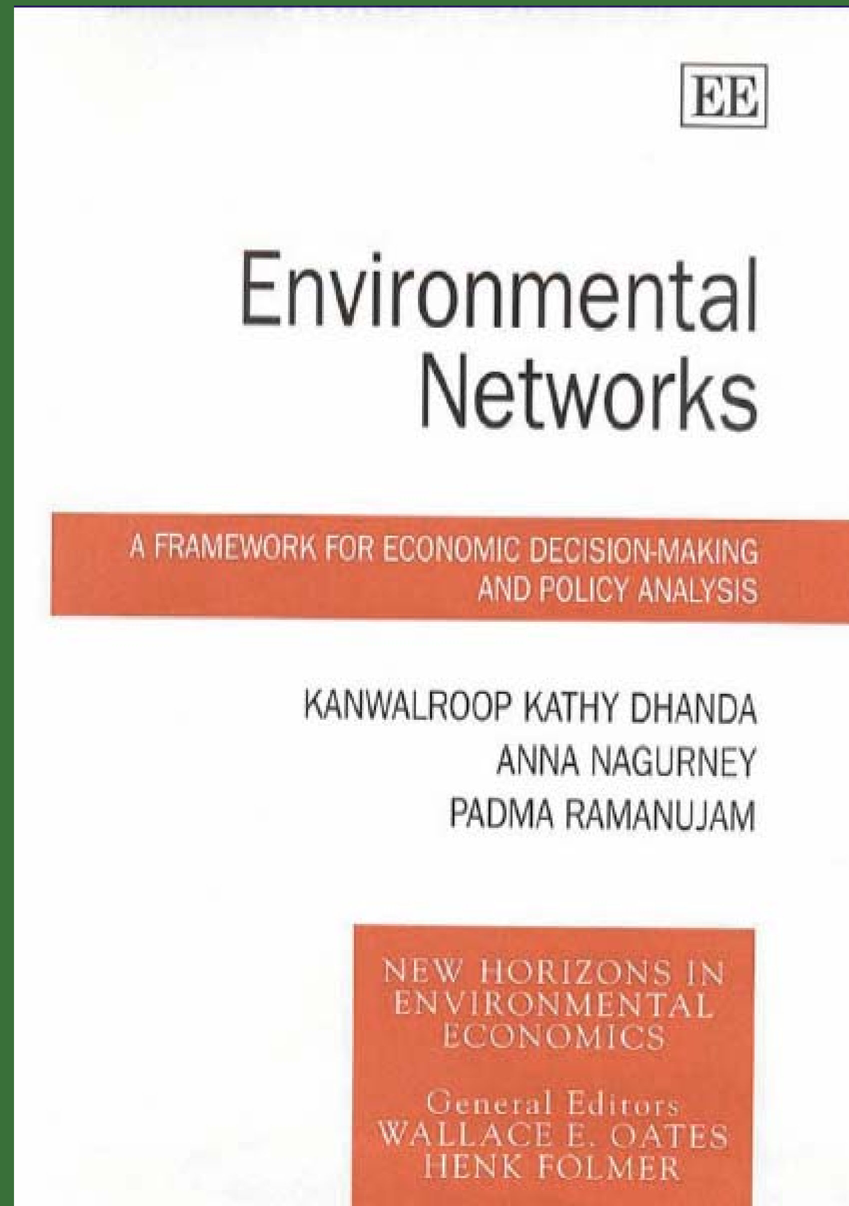
The introduction of electronic commerce (e-commerce) has had an immense effect on the manner of ordering and transporting goods.

Gains from e-commerce could reach $450 billion a year by 2005.

E-commerce offers potential for reducing risks associated with physical transportation.

Relevant Books





Mathematical models for supply chain networks with environmental concerns:

Nagurney and Toyasaki (2003) "Supply Chain Supernetworks and Environmental Criteria," *Transportation Research D* **8,** 185-213.

Nagurney and Toyasaki (2003) "Electronic Waste Management and Recycling: A Multitiered Network Equilibrium Framework for E-cycling," to appear in *Transportation Research E*.

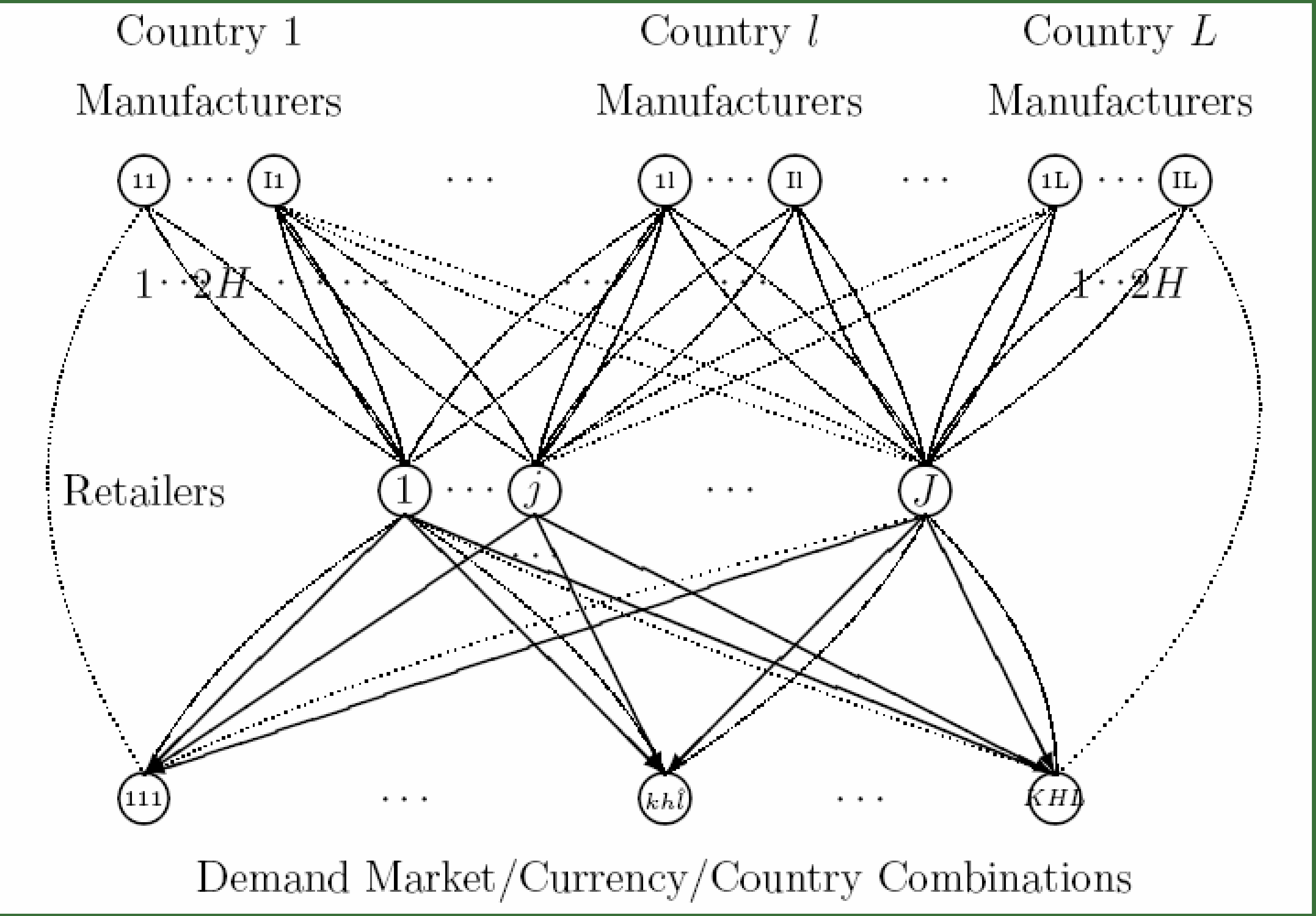
Risk management in a global supply chain context with a centralized decision-making

Cohen, M.A (1996), Cohen, M.A, and Mallick (1997), and Fabian (2000)

Risk management in a global supply chain context with a decentralized decision-making

Nagurney, Cruz, and Matsypura (2003)

The Structure of the Global Supply Chain Supernetwork



Characteristics of the Global Supply Chain Network Model

Global supply chain network model with decentralized decision-makers

Presence of e-commerce

Multicriteria decision-making

* Net revenue maximization
* Emission minimization
* Risk minimization

A Manufacturer's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho_{1jhm}^{il*}+e_h^*)q_{jhm}^{il}+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho_{1kh\hat{l}}^{il*}+e_h^*)q_{kh\hat{l}}^{il}-\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}c_{jhm}^{il}(q_{jhm}^{il})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il})-f^{il}(Q^1,Q^3)$$

$$-\alpha^{il}(\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(h^{il}+h_{jm}^{il})q_{jhm}^{il}+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(h^{il}+h_{k\hat{l}}^{il})q_{kh\hat{l}}^{il})-\omega^{il}r^{il}(Q^1,Q^3)$$

The Optimality Conditions of the Manufacturers

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}}+\frac{\partial c^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}}+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}}+\alpha^{il}(\eta^{il}+\eta^{il}_{jm})\right.$$

$$\left.-\rho^{il*}_{1jhm}\;-e^{*}_{h}\right]\times\left[q^{il}_{jhm}-q^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}}+\frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}}+\omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}}+\alpha^{il}(\eta^{il}+\eta^{il}_{k\hat{l}})\right.$$

$$\left.-\rho^{il*}_{1kh\hat{l}}\;-e^{*}_{h}\right]\times\left[q^{il}_{kh\hat{l}}-q^{il*}_{kh\hat{l}}\right]\geq 0,\quad\forall(Q^{1},Q^{3})\in\mathcal{K}^{1},$$

where the feasible set $\mathcal{K}^1\equiv\{(Q^1,Q^3)|(Q^1,Q^3)\in R_+^{IL(2JH+KHL)}\}$.

A Retailer's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad \sum_{k=1}^{k}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}(\rho^{j*}_{2kh\hat{l}m}+e^*_h)q^j_{kh\hat{l}m} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{c}^{il}_{jhm}(q^{il}_{jhm})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{m=1}^{2}\sum_{\hat{l}=1}^{L}c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho^{il*}_{1jhm}+e^*_h)q^{il}_{jhm}$$

$$-\vartheta_j r^j(Q^1,Q^2) - \beta_j(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(h^{il}+h^{il}_{jm})q^{il}_{jhm})$$

subject to : nonnegative constraints and

$$\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}q^j_{kh\hat{l}m} \le \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}q^{il}_{jhm}$$

Optimality Conditions of the Retailers

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} + \vartheta_j \frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}} + \beta_j(h^{il}+h^{il}_{jm}) + \rho^{il*}_{1jhm} + e^*_h \right.$$

$$\left. + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}} - \gamma^*_j\right]\times\left[q^{il}_{jhm}-q^{il*}_{jhm}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} + \vartheta_j \frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q^j_{kh\hat{l}m}} - \rho^{j*}_{2kh\hat{l}m} - e^*_h + \gamma^*_j\right]\times\left[q^j_{kh\hat{l}m}-q^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il*}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^{j*}_{kh\hat{l}m}\right]\times\left[\gamma_j-\gamma^*_j\right]\geq 0,$$

$$\forall (Q^1, Q^2, \gamma) \in R_+^{2ILJH+2JKHL+J}$$

The Equilibrium Conditions for the Demand Markets

for all intermediaries: $j = 1, \ldots, J$ and all mode m; $m = 1, 2$:

$$\rho^{j*}_{2kh\hat{l}m} + e^*_h + \hat{c}^j_{kh\hat{l}m}(Q^{2*}) + \eta_{kh\hat{l}} h^j_{k\hat{l}m} \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{j*}_{kh\hat{l}m} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{j*}_{kh\hat{l}m} = 0, \end{cases}$$

for all source agents il;$i = 1, \ldots, I$ and $l = 1, \ldots, L$:

$$\rho^{il*}_{1kh\hat{l}} + e^*_h + \hat{c}^{il}_{kh\hat{l}}(Q^{3*}) + \eta_{kh\hat{l}}(h^{il} + h^{il}_{k\hat{l}}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{il*}_{kh\hat{l}} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{il*}_{kh\hat{l}} = 0, \end{cases}$$

$$d_{kh\hat{l}}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} \sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} > 0 \\ \leq \sum_{j=1}^{J} \sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} = 0. \end{cases}$$

Global Supply Chain Network Equilibrium

Definition:

The equilibrium state of the supply chain supernetwork is one where the product transactions between the tiers of the network coincide and the product transactions and prices satisfy the sum of conditions presented above.

Variational Inequality Formulation

determine $(Q^{1*}, Q^{2*}, Q^{3*}, \gamma^*, \rho_3^*) \in \mathcal{K}$, *satisfying:*

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}} + \frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm})}{\partial q^{il}_{jhm}}\right.$$

$$\left. + \omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{jhm}} + \vartheta_j \frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}} + (\alpha^{il}+\beta_j)(\eta^{il}+\eta^{il}_{jm}) - \gamma_j^*\right] \times \left[q^{il}_{jhm} - q^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} + \hat{c}^{il}_{kh\hat{l}}(Q^{3*})\right.$$

$$\left. + \omega^{il}\frac{\partial r^{il}(Q^{1*},Q^{3*})}{\partial q^{il}_{kh\hat{l}}} + (\alpha^{il}+\delta_{kh\hat{l}})(\eta^{il}+\eta^{il}_{k\hat{l}}) - \rho^*_{3kh\hat{l}}\right] \times \left[q^{il}_{kh\hat{l}} - q^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\frac{\partial c^{j}_{khl}(q^{j*}_{kh\hat{l}m})}{\partial {q_{kh\hat{l}m}}^{j}} + \hat{c}^{j}_{kh\hat{l}m}(Q^{2*}) + \vartheta_j\frac{\partial r^j(Q^{1*},Q^{2*})}{\partial q^{j}_{kh\hat{l}m}}\right.$$

$$\left. + \delta_{kh\hat{l}}\eta^{j}_{k\hat{l}m} + \gamma_j^* - \rho^*_{3kh\hat{l}}\right] \times \left[q^{j}_{kh\hat{l}m} - q^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il*}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^{j*}_{kh\hat{l}m}\right] \times \left[\gamma_j - \gamma_j^*\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il*}_{kh\hat{l}} - d_{kh\hat{l}}(\rho_3^*)\right] \times \left[\rho_{3kh\hat{l}} - \rho^*_{3kh\hat{l}}\right] \geq 0,$$

Variational Inequality Formulation

The variational inequality can be expressed in standard form as:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}$$

where $X \equiv (Q^1, Q^2, Q^3, \gamma, \rho_3)$ and $F(X) \equiv (F_{iljhm}, F_{ilkh\hat{l}}, F_{jkh\hat{l}m}, F_j, F_{kh\hat{l}})_{i=1,\ldots,I;\hat{l}=l=1,\ldots,L;j=1,\ldots,J;}$
$_{h=1,\ldots,H;m=1,2}$

The Dynamic Global Supply Chain Model

Demand market price dynamics:

$$\dot{\rho}_{3kh\hat{l}} = \begin{cases} d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J}\sum_{m=1}^{2} q^{j}_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}}, & \text{if} \quad \rho_{3kh\hat{l}} > 0 \\ \max\{0, d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J}\sum_{m=1}^{2} q^{j}_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}}\} & \text{if} \quad \rho_{3kh\hat{l}} = 0. \end{cases}$$

The Dynamic Global Supply Chain Model

The dynamics of the product transactions between the retailers and the demand markets:

$$\dot{q}^j_{kh\hat{l}m} = \begin{cases} \rho_{3kh\hat{l}} - \frac{\partial c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^j_{kh\hat{l}m}} - \hat{c}^j_{kh\hat{l}m}(Q^2) - \delta_{kh\hat{l}} \eta^j_{k\hat{l}m} - \gamma_j, & \text{if } \quad q^j_{kh\hat{l}m} > 0 \\ \max\{0, \rho_{3kh\hat{l}} - \frac{\partial c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^j_{kh\hat{l}m}} - \hat{c}^j_{kh\hat{l}m}(Q^2) - \delta_{kh\hat{l}} \eta^j_{k\hat{l}m} - \gamma_j\}, & \text{if } \quad q^j_{kh\hat{l}m} = 0. \end{cases}$$

The Dynamic Global Supply Chain Model

The dynamics of the prices at the retailers:

$$\dot{\gamma}_j = \begin{cases} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} q^{j}_{kh\hat{l}m} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} q^{il}_{jhm}, & \text{if} \quad \gamma_j > 0 \\ \max\{0, \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} q^{j}_{kh\hat{l}m} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} q^{il}_{jhm}\}, & \text{if} \quad \gamma_j = 0 \end{cases}$$

The Dynamic Global Supply Chain Model

The dynamics of the product transactions between the manufacturers and retailers :

$$\dot{q}^{il}_{jhm} = \begin{cases} \gamma_j - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} & \\ -\omega^{il} \frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^{il}_{jhm}} - (\alpha^{il} + \beta_j)(\eta^{il} + \eta^{il}_{jm}), & \text{if} \quad q^{il}_{jhm} > 0 \\ \max\{0, \gamma_j - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm})}{\partial q^{il}_{jhm}} & \\ -\omega^{il} \frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{jhm}} - \vartheta_j \frac{\partial r^j(Q^1,Q^2)}{\partial q^{il}_{jhm}} - (\alpha^{il} + \beta_j)(\eta^{il} + \eta^{il}_{jm})\}, & \text{if} \quad q^{il}_{jhm} = 0. \end{cases}$$

The Dynamic Global Supply Chain Model

The dynamics of the product transactions between the manufacturers and the demand markets :

$$\dot{q}^{il}_{kh\hat{l}} = \begin{cases} \rho_{3kh\hat{l}} - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} - \hat{c}^{il}_{kh\hat{l}}(Q^3) & \\ -\omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - (\alpha^{il} + \delta_{kh\hat{l}})(\eta^{il} + \eta^{il}_{k\hat{l}}), & \text{if} \quad q^{il}_{kh\hat{l}} > 0 \\ \max\{0, \rho_{3kh\hat{l}} - \frac{\partial f^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} - \hat{c}^{il}_{kh\hat{l}}(Q^3) & \\ -\omega^{il}\frac{\partial r^{il}(Q^1,Q^3)}{\partial q^{il}_{kh\hat{l}}} - (\alpha^{il} + \delta_{kh\hat{l}})(\eta^{il} + \eta^{il}_{k\hat{l}})\}, & \text{if} \quad q^{il}_{kh\hat{l}} = 0 \end{cases}$$

Projected Dynamical System

The dynamic model can be rewritten as a projected dynamical system (Dupuis and Nagurney (1993)).

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = (Q^{10}, Q^{20}, Q^{30}, \gamma^0, \rho_3^0)$

The feasible set $\mathcal{K}$ is simply the nonnegative orthant.

The set of stationary points of the projected dynamical system coincides with the set of solutions of the variational inequality (Dupuis and Nagurney (1993)).

Additional Theoretical Results

We have established:

- The ***Existence*** of the solution of the VI
- The ***Uniqueness*** of the solution of the VI
- The ***Convergence*** of the algorithm.

The Algorithm

The algorithm that we propose is an Euler-type method, which is induced by the general iterative scheme of Dupuis and Nagurney [1993].

This is a discrete-time algorithm and serves as an approximation to the continuous time trajectories generated by the dynamic model.

The notable feature of this type of algorithm is that the VI subproblems in this model are resolved into network optimization problems with a special structure that can be solved exactly in closed form.

The Euler Method

Step 0: Initialization

Set $X^0 = (Q^{10}, Q^{20}, Q^{30}, \gamma^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter and set $\mathcal{T} = 1$. Set the sequence $\{\alpha_\mathcal{T}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_\mathcal{T}$, $\alpha_\mathcal{T} > 0$, $\alpha_\mathcal{T} \to 0$, as $\mathcal{T} \to \infty$ (which is a requirement for convergence).

Step 1: Computation

Compute $X^\mathcal{T} = (Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, Q^{3\mathcal{T}}, \gamma^\mathcal{T}, \rho_3^\mathcal{T}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^\mathcal{T} + \alpha_\mathcal{T} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^\mathcal{T} \rangle \geq 0, \qquad \forall X \in \mathcal{K}.$$

Step 2: Convergence Verification

If $|X^\mathcal{T} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Computation of the Product Transactions

$$q_{jhm}^{il\mathcal{T}} = \max\{0, q_{jhm}^{il\mathcal{T}-1} + \alpha_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \frac{\partial c_{jhm}^{il}(q_{jhm}^{il\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \frac{\partial c_j(Q^{1\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \frac{\partial \hat{c}_{jhm}^{il}(q_{jhm}^{il\mathcal{T}-1})}{\partial q_{jhm}^{il}}$$
$$+\omega^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \vartheta_j \frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q_{jhm}^{il}} + (\alpha^{il} + \beta_j)(h^{il} + h_{jm}^{il}) - \gamma_j^{\mathcal{T}-1})\}$$
$$\forall i, l, j, h, m$$

$$q_{kh\hat{l}}^{il\mathcal{T}} = \max\{0, q_{kh\hat{l}}^{il\mathcal{T}-1} + \alpha_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}} + \frac{\partial c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}} + \hat{c}_{kh\hat{l}}^{il}(Q^{3\mathcal{T}-1})$$
$$+\omega^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}} + (\alpha^{il} + \eta_{kh\hat{l}})(h^{il} + h_{k\hat{l}}^{il}) - \rho_{3kh\hat{l}}^{\mathcal{T}-1})\} \qquad \forall i, l, k, h, \hat{l}$$

$$q_{kh\hat{l}m}^{j\mathcal{T}} = \max\{0, q_{kh\hat{l}m}^{j\mathcal{T}-1} + \alpha_{\mathcal{T}}(\frac{\partial c_{kh\hat{l}m}^{j}(q_{kh\hat{l}m}^{j\mathcal{T}-1})}{\partial q_{kh\hat{l}m}^{j}} + \vartheta_j \frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q_{kh\hat{l}m}^{j}} + \hat{c}_{kh\hat{l}m}^{j}(Q^{2\mathcal{T}-1})$$
$$+\eta_{kh\hat{l}}h_{k\hat{l}m}^{j} + \gamma_j^{\mathcal{T}-1} - \rho_{3kh\hat{l}}^{\mathcal{T}-1})\}, \forall j, k, h, \hat{l}, m.$$

Computation of the Product Transactions

$$\gamma_j^{\mathcal{T}} = \max\{0, \gamma_j^{\mathcal{T}-1} - \alpha_{\mathcal{T}}(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q_{jhm}^{il\mathcal{T}-1} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q_{kh\hat{l}m}^{j\mathcal{T}-1})\}, \quad \forall j$$

$$\rho_{3kh\hat{l}}^{\mathcal{T}} = \max\{0, \rho_{3kh\hat{l}}^{\mathcal{T}-1} - \alpha_{\mathcal{T}}(\sum_{j=1}^{J}\sum_{m=1}^{2} q_{kh\hat{l}m}^{j\mathcal{T}-1} + \sum_{i=1}^{I}\sum_{l=1}^{L} q_{kh\hat{l}}^{il} - d_{kh\hat{l}}(\rho_3^{\mathcal{T}-1}))\}, \quad \forall k, h, \hat{l}.$$

Numerical Examples

We present single-country examples, for simplicity and illustration purposes

- The production cost functions

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2$$

- The transaction cost functions of the manufacturers

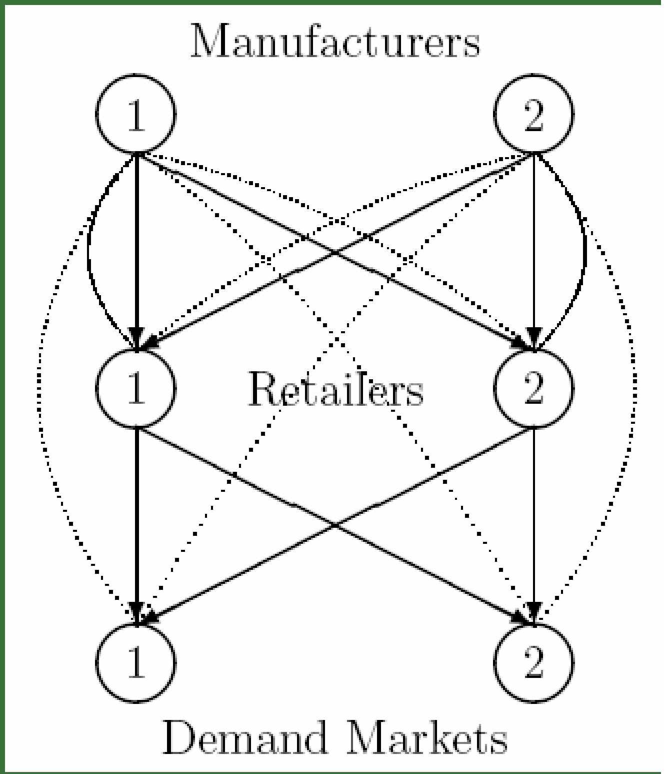
$$c_{ij1}(q_{ij1}) = .5q_{ij1}^2 + 3.5q_{ij1}, \quad \text{for } i = 1,2; j = 1,2,$$
$$c_{ij2}(q_{ij2}) = 1.5q_{ij2}^2 + 3q_{ij2}, \quad \text{for } i = 1,22; j = 1,2.$$
$$c_{ik}(q_{ik}) = q_{ik}^2 + 2q_{ik}, \quad \text{for } i = 1,2; k = 1,2$$

- The handling costs of the retailers

$$c_1(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i1l})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i2l})^2$$

- The transaction costs of the retailers

$$\hat{c}_{ijl}(q_{ijl}) = 1.5q_{ijl}^2 + 3q_{ijl}, \quad \text{for } i = 1,2; j = 1,2; l = 1,2.$$
$$\hat{c}_{jk1}(Q^2, Q^3) = q_{jk1} + 5, \quad \text{for } j = 1,2; k = 1,2$$



$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000,$$
$$d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000$$

When the Weights Associated with Environmental Criteria Vary

	Example 1	Example 2	Example 3	Example 4
	$\alpha^{11}=\alpha^{21}=0$ $\beta_1=\beta_2=0$ $\delta_{111}=\delta_{211}=0$	$\alpha^{11}=\alpha^{21}=1$ $\beta_1=\beta_2=0$ $\delta_{111}=\delta_{211}=0$	$\alpha^{11}=\alpha^{21}=1$ $\beta_1=\beta_2=1$ $\delta_{111}=\delta_{211}=0$	$\alpha^{11}=\alpha^{21}=1$ $\beta_1=\beta_2=1$ $\delta_{111}=\delta_{211}=1$
$Q_1^*: q_{111}^*=q_{121}^*=q_{211}^*=q_{221}^*$, $q_{112}^*=q_{122}^*=q_{212}^*=q_{222}^*$	3.4611 2.3907	3.3214 2.4309	3.1136 2.4250	3.1270 2.4347
Q_2^*	13.033	13.3127	13.4861	13.396
Q_3^*	5.8513	5.7509	5.5362	5.5603
γ^*	263.908	264.047	264.309	263.623
ρ^*	274.701	274.820	274.843	274.881
Total Emissions	114.089	112.918	111.381	111.213

When the Weights are Further Modified

	Example 4	Example 5	Example 6	Example 7
	$\alpha^{11} = \alpha^{21} = 1$ $\beta_1 = \beta_2 = 1$ $\delta_{111} = \delta_{211} = 1$	$\alpha^{11} = \alpha^{21} = 1$ $\beta_1 = \beta_2 = 1$ $\delta_{111} = \delta_{211} = 5$	$\alpha^{11} = \alpha^{21} = 5$ $\beta_1 = \beta_2 = 1$ $\delta_{111} = \delta_{211} = 5$	$\alpha^{11} = \alpha^{21} = 5$ $\beta_1 = \beta_2 = 5$ $\delta_{111} = \delta_{211} = 5$
Total Emissions	111.2138	110.5442	105.8604	99.7104

Conclusions

We modeled:

- The global supply chain network with a focus on decentralized decision-making
- Environmental concerns
- Multicriteria decision-makers (revenue, emission, and risk)
- The presence of e-commerce
- The linking the equilibrium points of the static version and the stationary points of the dynamic version
- The explicit formula for the product transactions and the prices.

Thank You!

For more information on this paper and the

Virtual Center for Supernetworks see:

http://supernet.som.umass.edu

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