Dynamics of Global Supply Chain Supernetworks

Anna Nagurney, Jose Cruz, and Dmytro Matsypura

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The Virtual Center for Supernetworks
http://supernet.som.umass.edu

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THE ROCKEFELLER FOUNDATION

- We at the *Isenberg School of Management* have established the *Virtual Center for Supernetworks*, which along with our new Supernetworks Laboratory for Computation and Visualization, serves as a resource for researchers, educators, and practitioners.
- The center emphasizes the importance of critical infrastructure networks, their modeling, and analysis, and at the same time expands upon scientific network tools for decision-making.

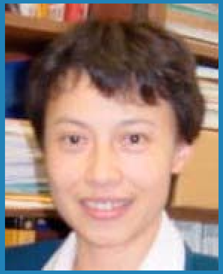
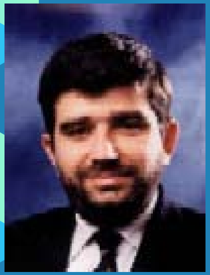
- The center team is multidisciplinary and multicultural and at present consists of doctoral students from three different countries.
- The center supports *undergraduates in research* since they are our future and provide new and fresh perspectives.
- Center associates from different academic institutions and industry work closely with the center director and student associates.

The Supernetwork Team









We are in a New Era of Decision-Making characterized by:

- increasing risk and uncertainty;
- importance of dynamics and realizing a fast and sound response to evolving events;
- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- global reach of many decisions, and
- high impact of many decisions.

The complexity of today's decision-making environments in organizations requires the development and harnessing of *appropriate and rigorous scientific tools* which must be based on *information technology* since only such technology provides one with the speed and accuracy needed to model complex interactions and to optimize accordingly.

The New Era is Network-Based with the Internet providing critical infrastructure along with transportation/logistical networks as well as other telecommunication networks and energy networks. *No longer are networks independent of one another but critically linked* with major questions arising regarding decision-making and appropriate management tools.

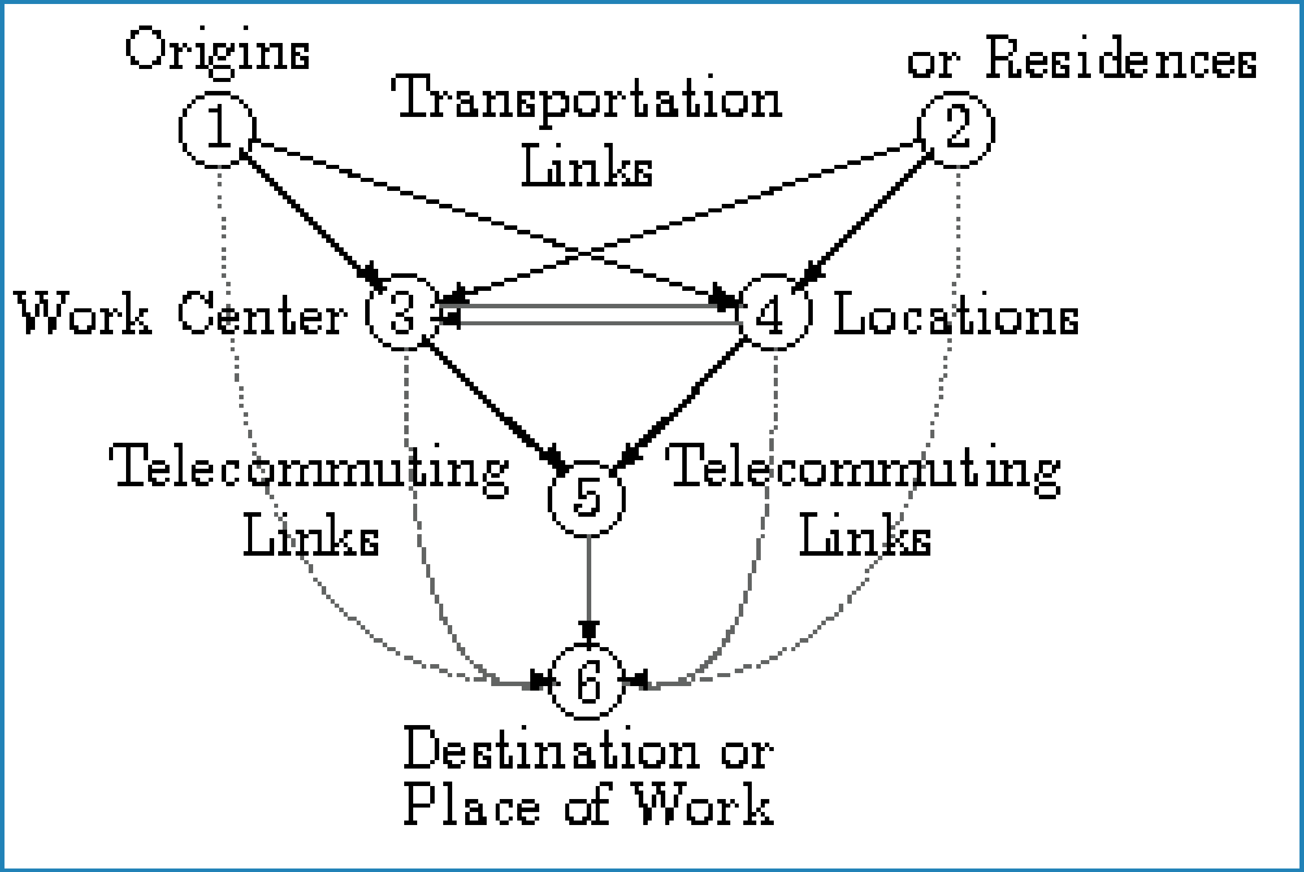
Indeed, the events of 9/11 coupled with the recent computer worm and viruses along with the biggest blackout in US history demonstrate irrevocably that we must as a nation harness the best and most powerful methodologies for the modeling, analysis, and solution of complex decision-making problems.

Novel Applications

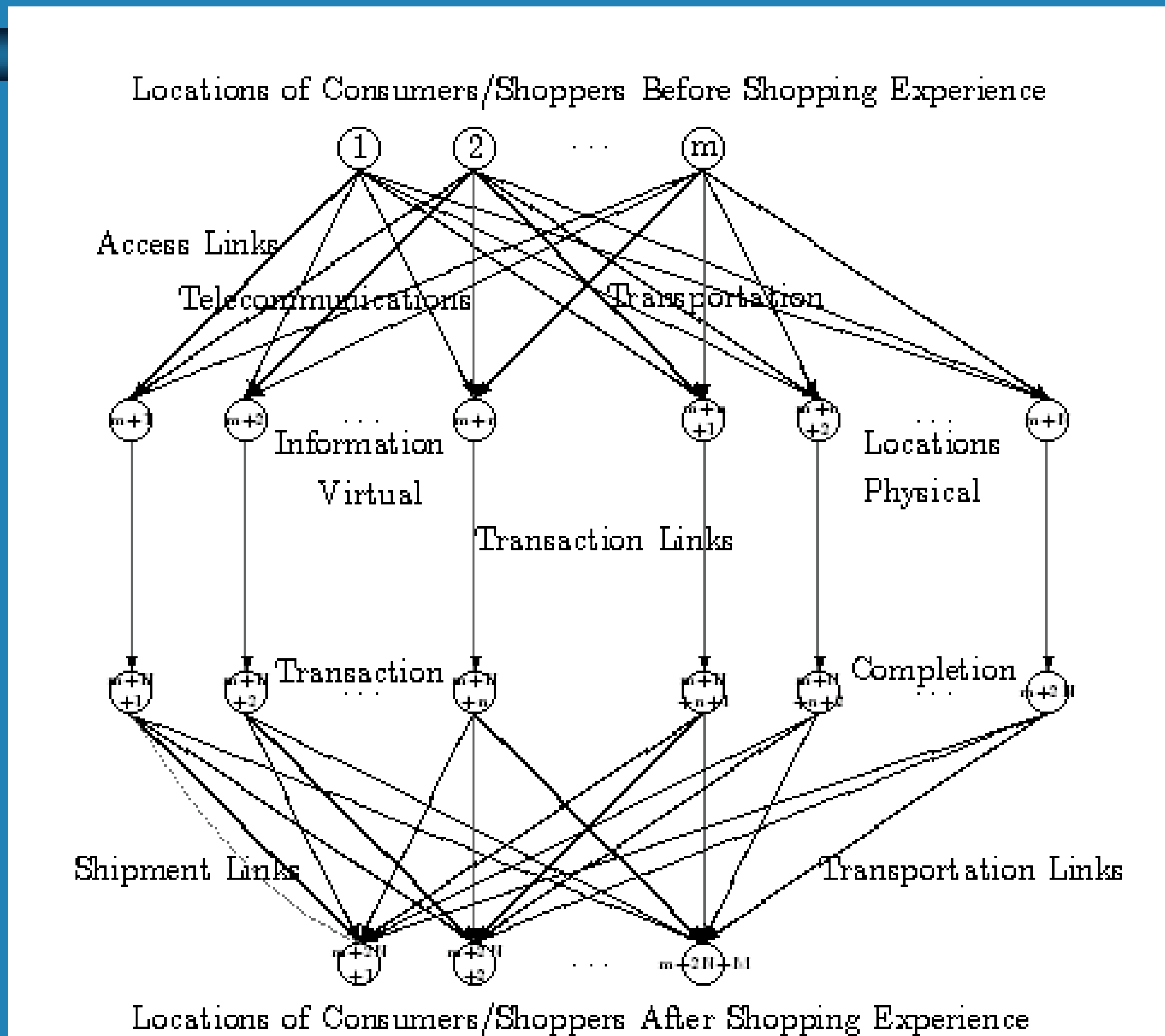
Applications of Supernetworks

- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Energy Networks/Power Grids

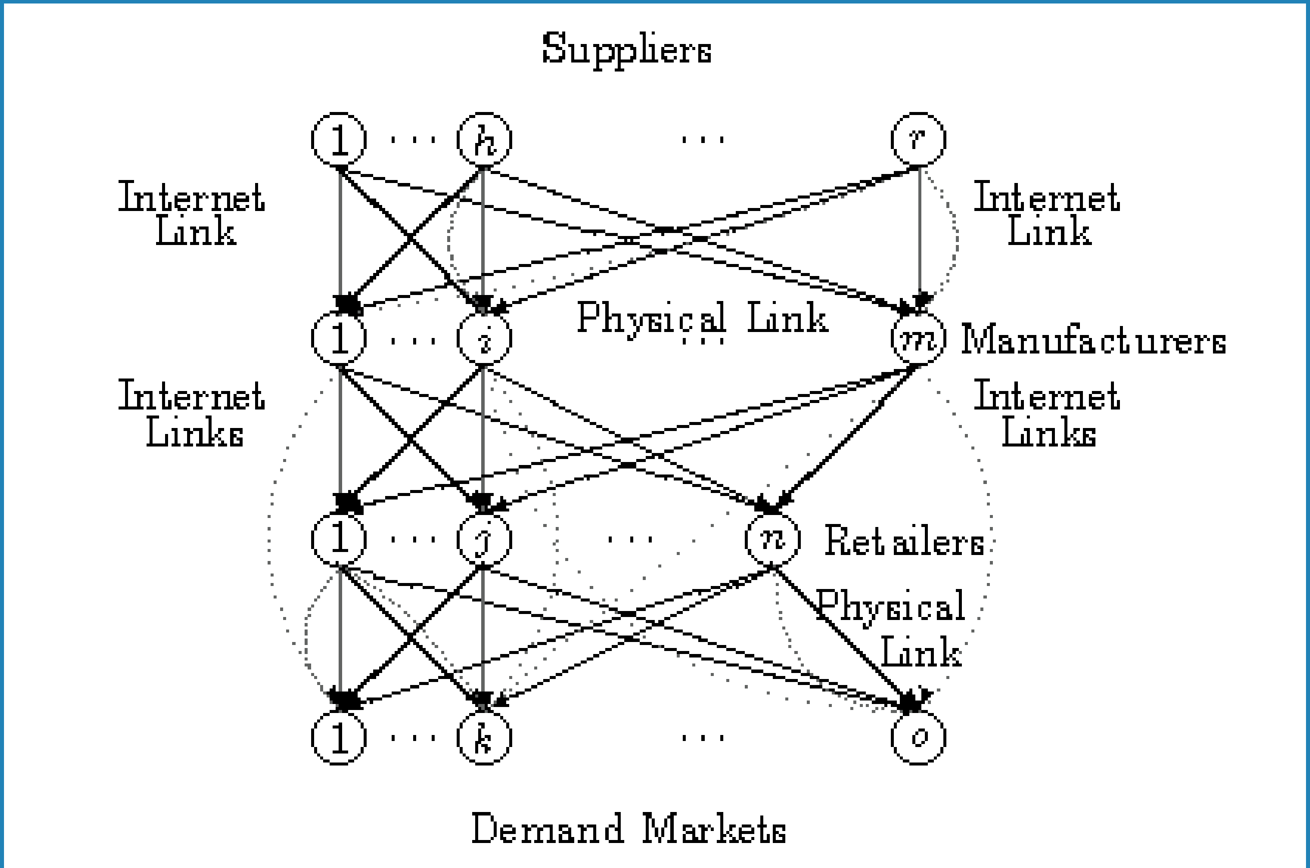
A Supernetwork Conceptualization of Commuting versus Telecommuting



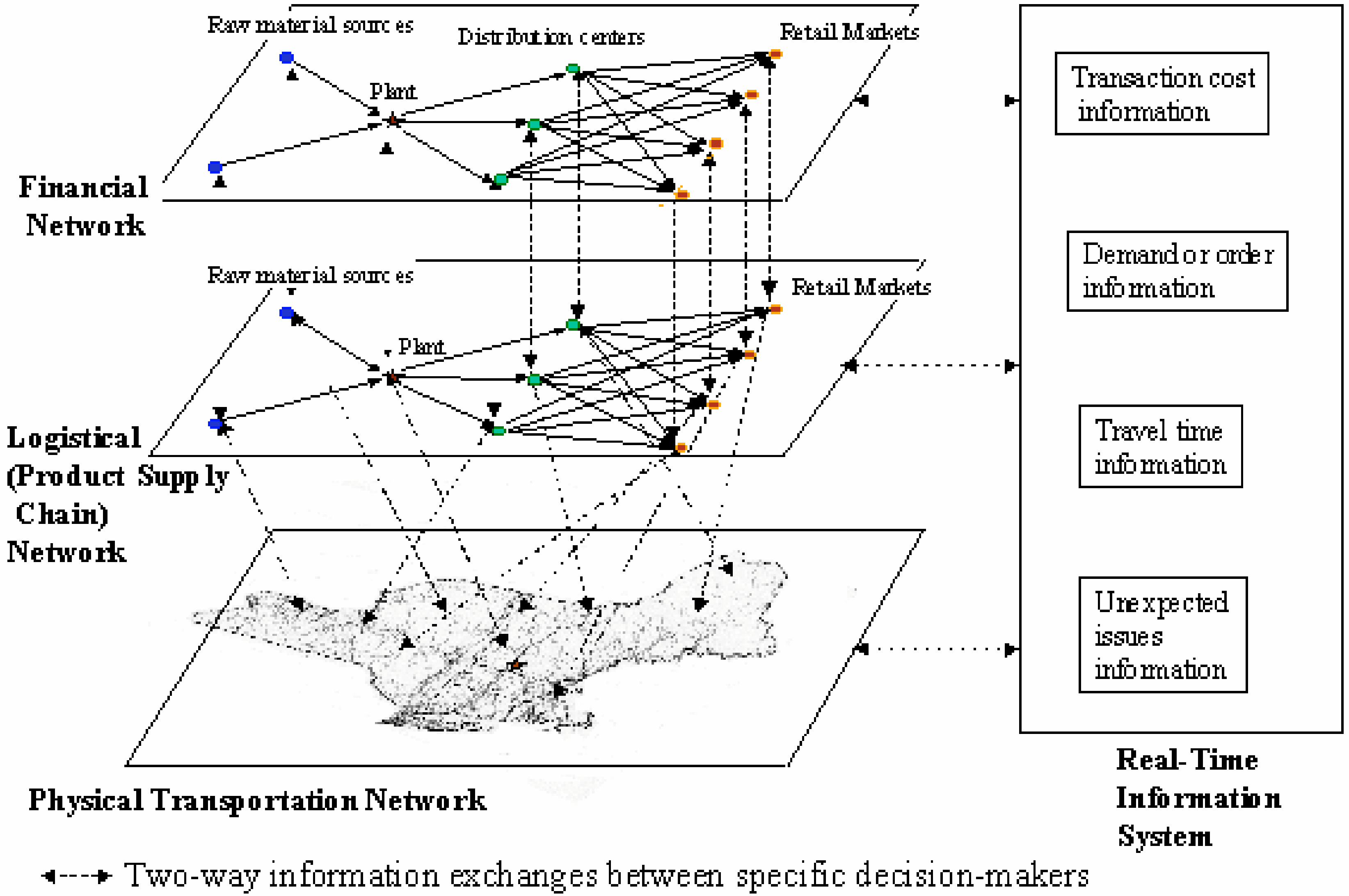
A Supernetwork Framework for Teleshopping versus Shopping



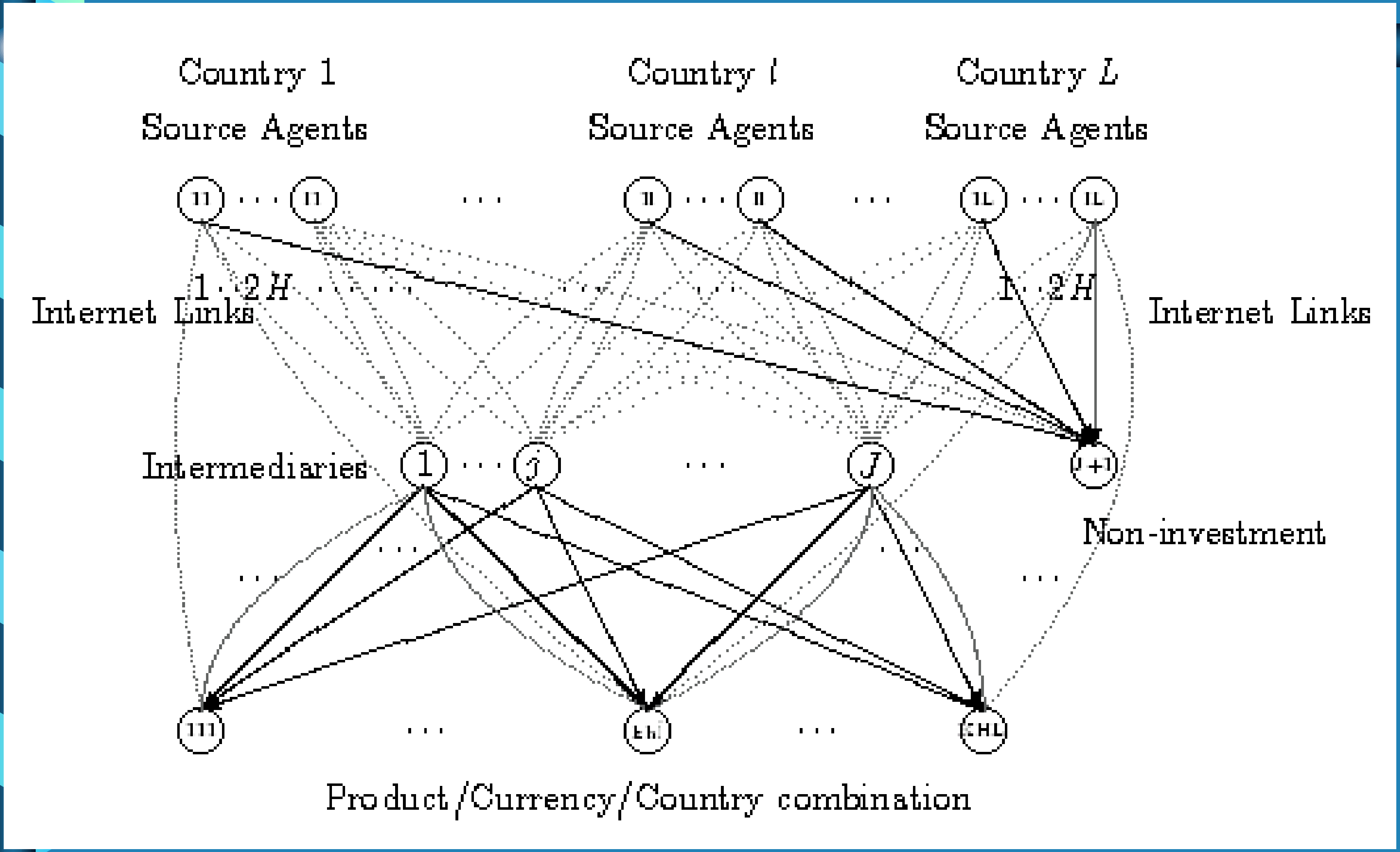
The Supernetwork Structure of a Supply Chain Network



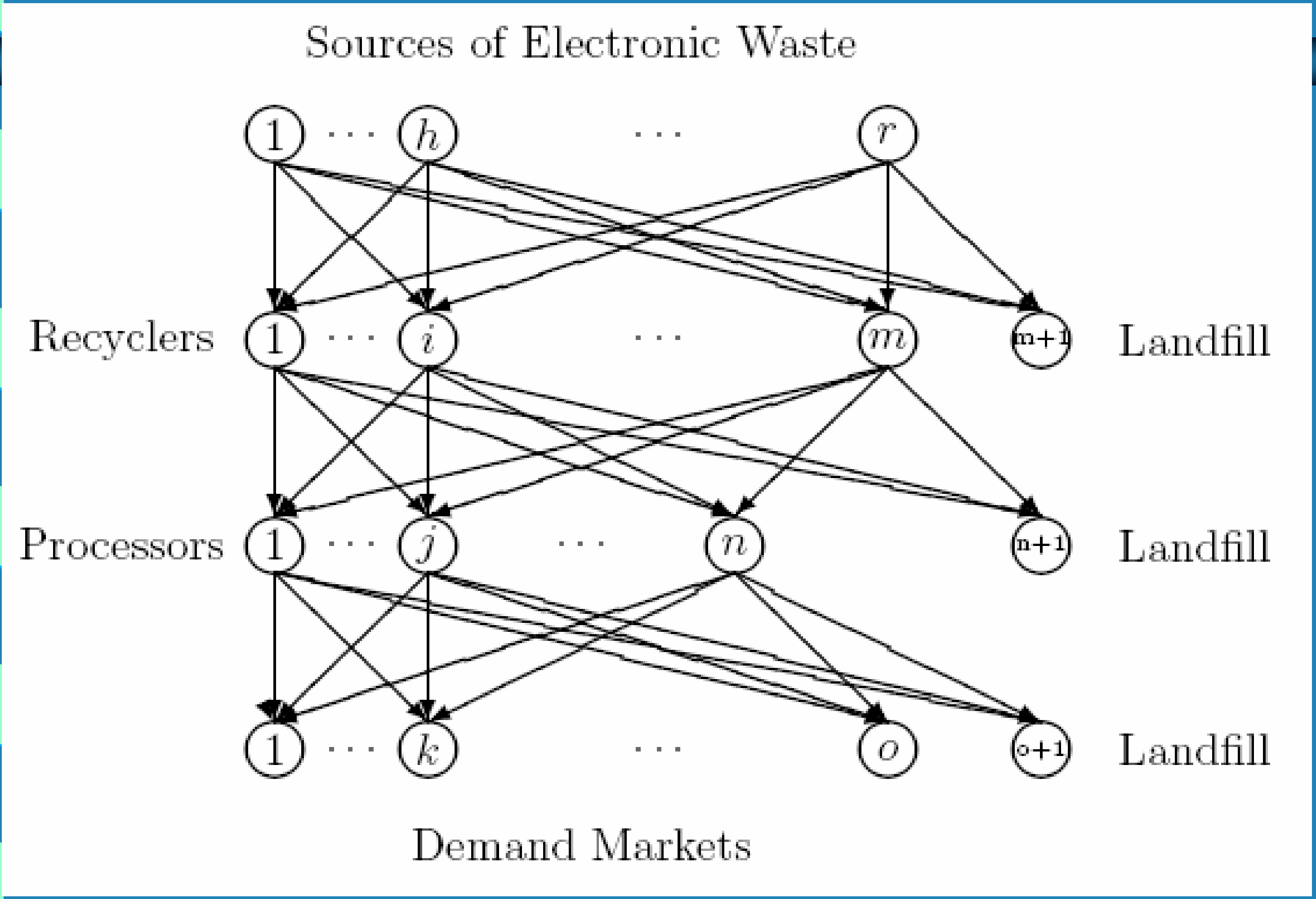
Supply Chain -Transportation Supernetwork Representation



International Financial Networks with Electronic Transactions



The 4-Tiered E-Cycling Network

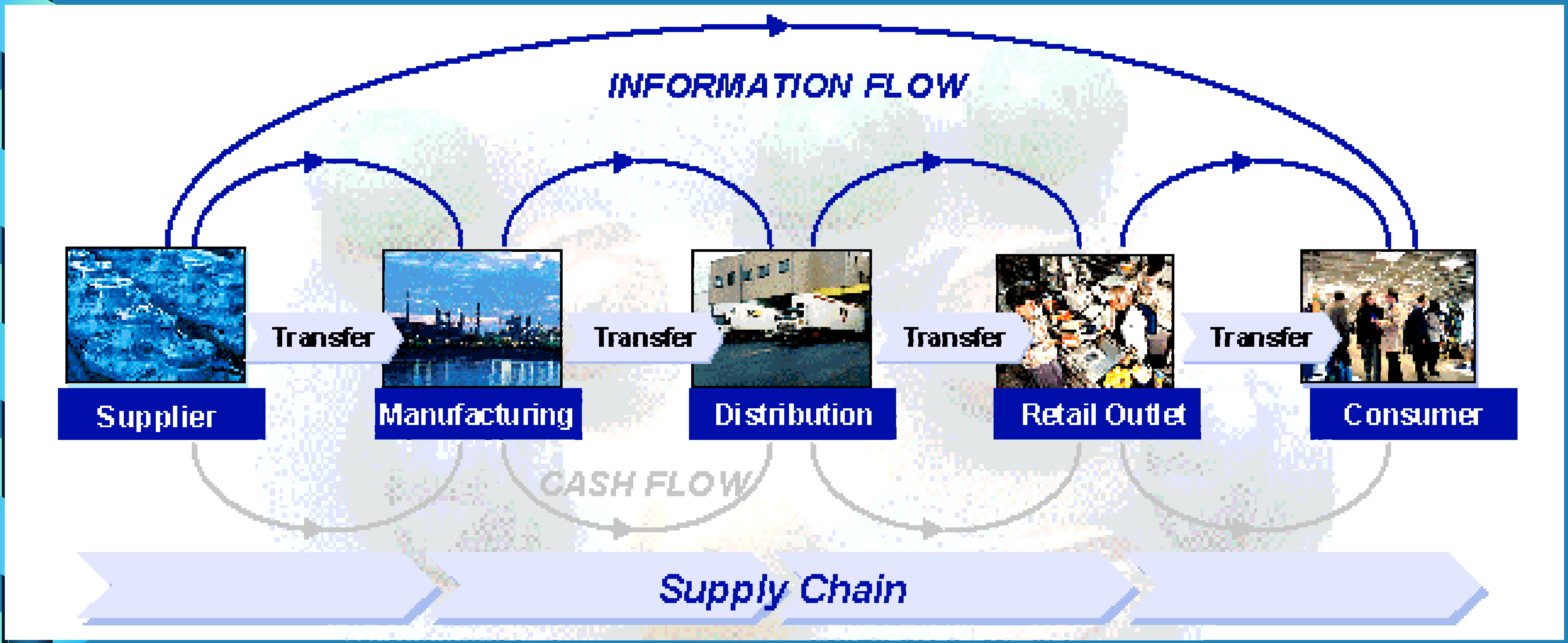


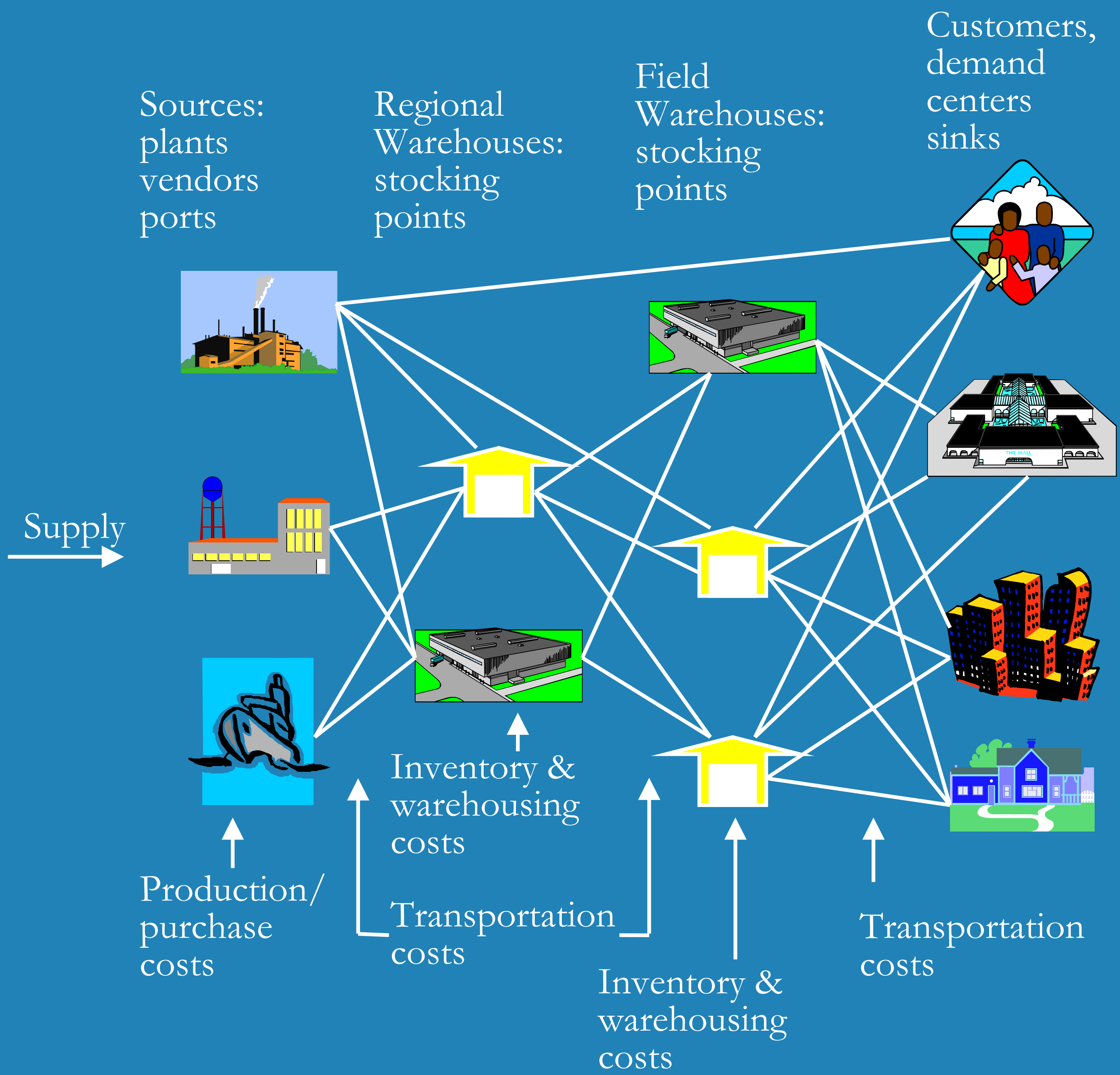
Some Center

Activities

- To further promote knowledge about networks we have established a new book series, ***New Dimensions in Networks***, with Edward Elgar Publishers.

- Professor Anna Nagurney has just been appointed co-editor of the journal ***Netnomics: Economic Research and Electronic Networking***, published by Kluwer Academic Publishers.


INFORMATION FLOW
Transfer
Transfer
Transfer
Transfer
Supplier
Manufacturing
Distribution
Retail Outlet
Consumer
CASH FLOW
Supply Chain



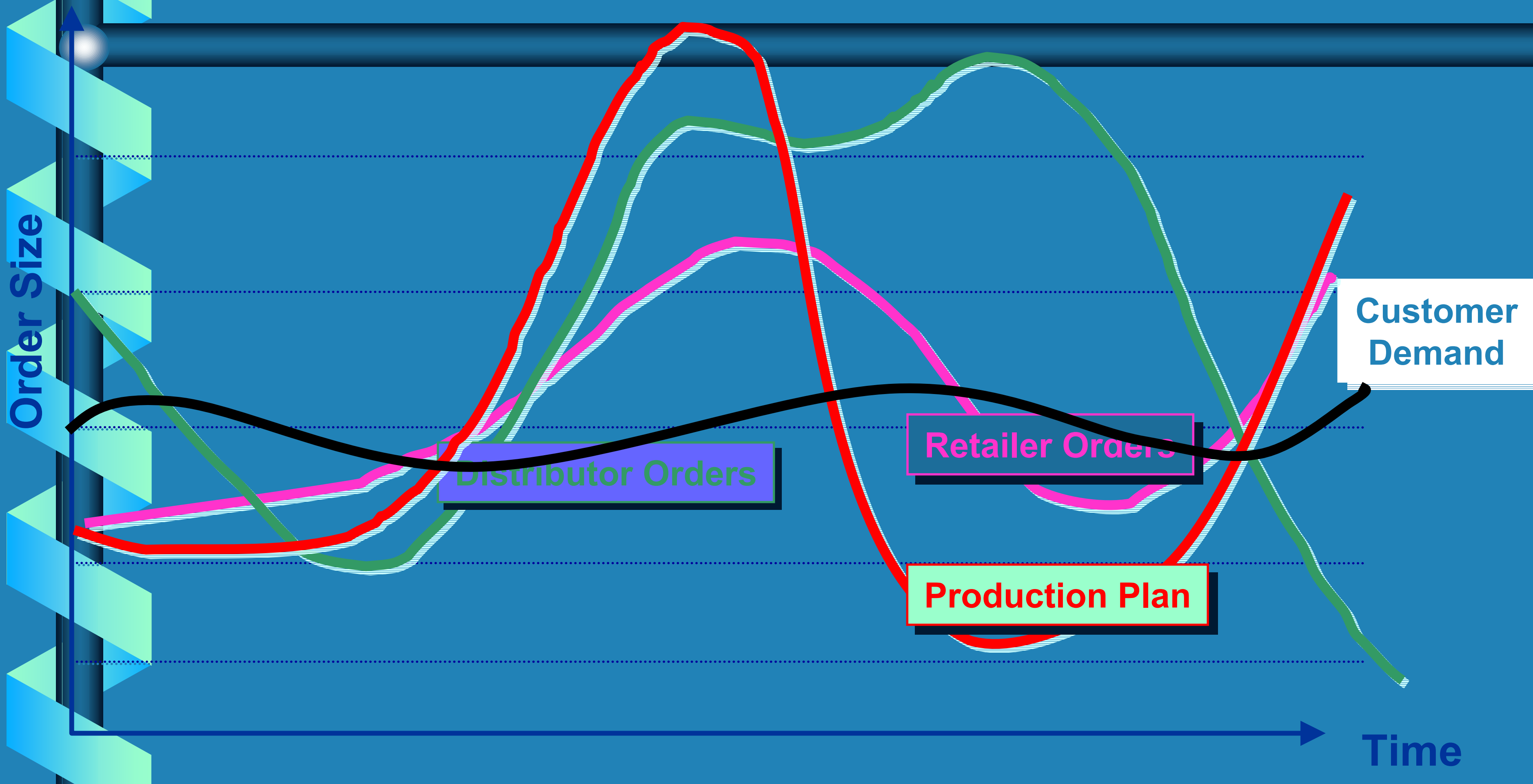
Sources:
plants
vendors
ports
Regional
Warehouses:
stocking
points
Field
Warehouses:
stocking
points
Customers,
demand
centers
sinks
Supply
Inventory &
warehousing
costs
Production/
purchase
costs
Transportation
costs
Inventory &
warehousing
costs
Transportation
costs

Supply Chain Management

- **Definition:**

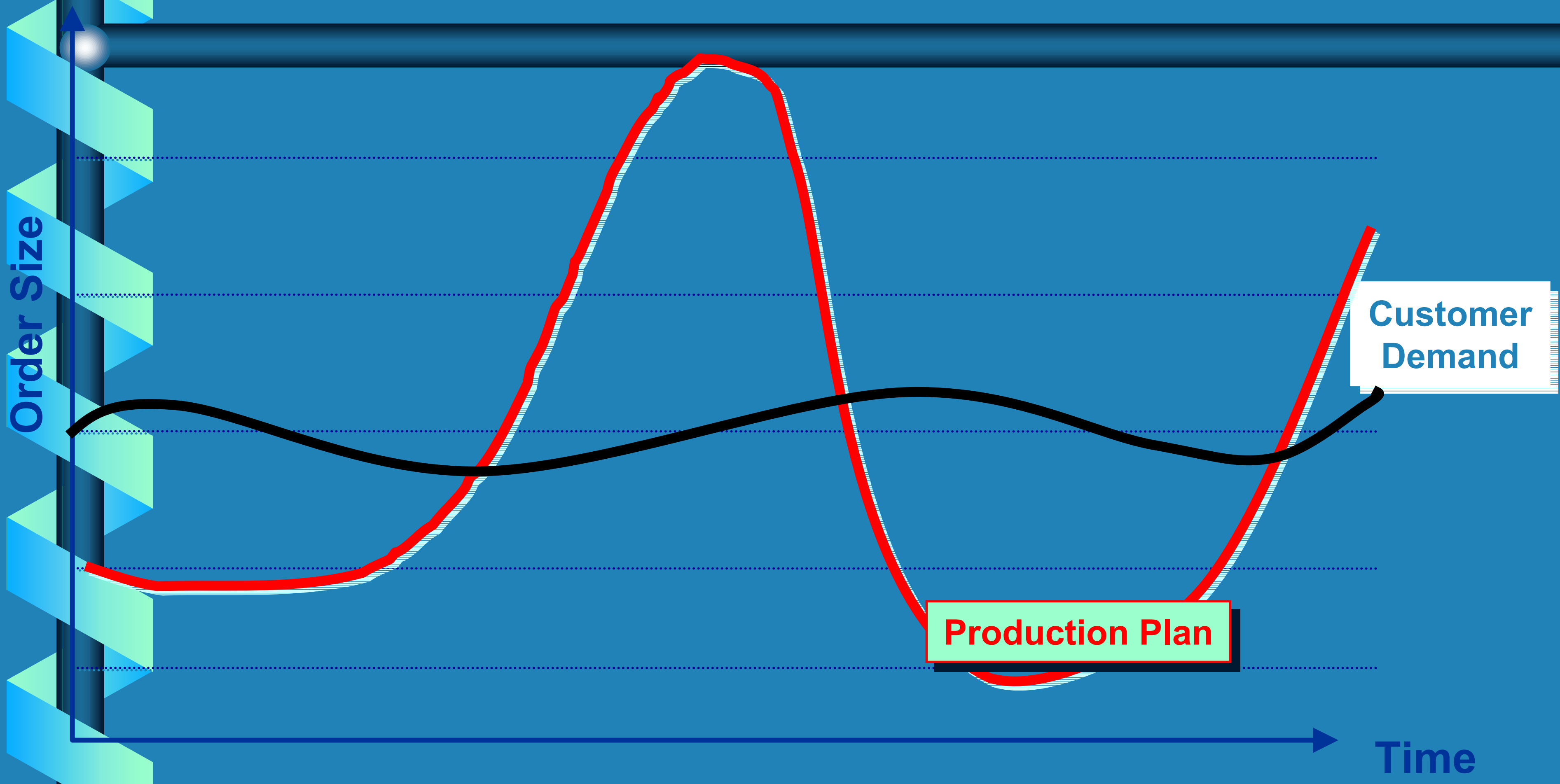
Supply Chain Management is primarily concerned with the efficient integration of *suppliers, factories, warehouses and stores* so that merchandise is produced and distributed in the right quantities, to the right locations and at the right time, and so as to minimize total system cost subject to satisfying service requirements.

The Dynamics of the Supply Chain



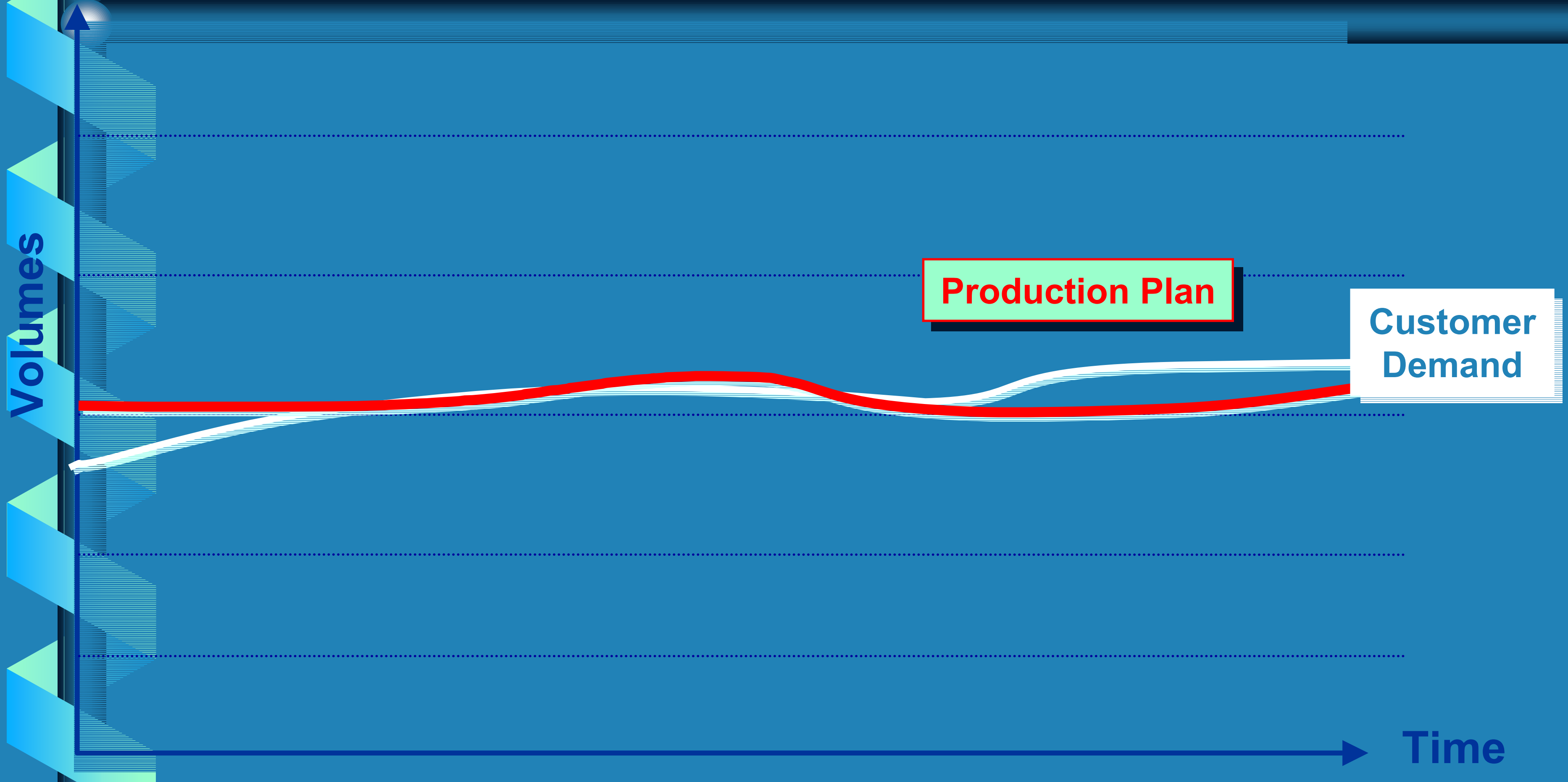
Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

What Management Gets...



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

What Management Wants…



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

Supply Chain: The Magnitude

- In 1998, American companies spent $898 billion in supply-related activities (or 10.6% of Gross Domestic Product).
  - Transportation 58%
  - Inventory 38%
  - Management 4%
- Third party logistics services grew in 1998 by 15% to nearly $40 billion

Supply Chain: The Complexity

National Semiconductors:

- **Production:**
  - Produces chips in six different locations: four in the US, one in Britain and one in Israel
  - Chips are shipped to seven assembly locations in Southeast Asia.
- **Distribution**
  - The final product is shipped to hundreds of facilities all over the world
  - 20,000 different routes
  - 12 different airlines are involved
  - 95% of the products are delivered within 45 days
  - 5% are delivered within 90 days.

What do we see?

Supernetworks!

Supply Chain Literature

- **Optimization framework of Centralized SCM**
  - Distribution network design
    - Geoffrion and Power 1995 "Twenty years of strategic distribution design…" *Interface* 25
    - Erenguc et al 1999 "Integrated production/distribution planning in supply chain: an invited review" *EJOR* 115
  - Inventory Management
    - Chen 1998 "Echelon reorder points, installation reorder points, and the value of centralized demand information" *Management Science* 44
    - Poirier 1996 **Supply Chain Optimization: Building a Total Business Network**

Supply Chain Literature

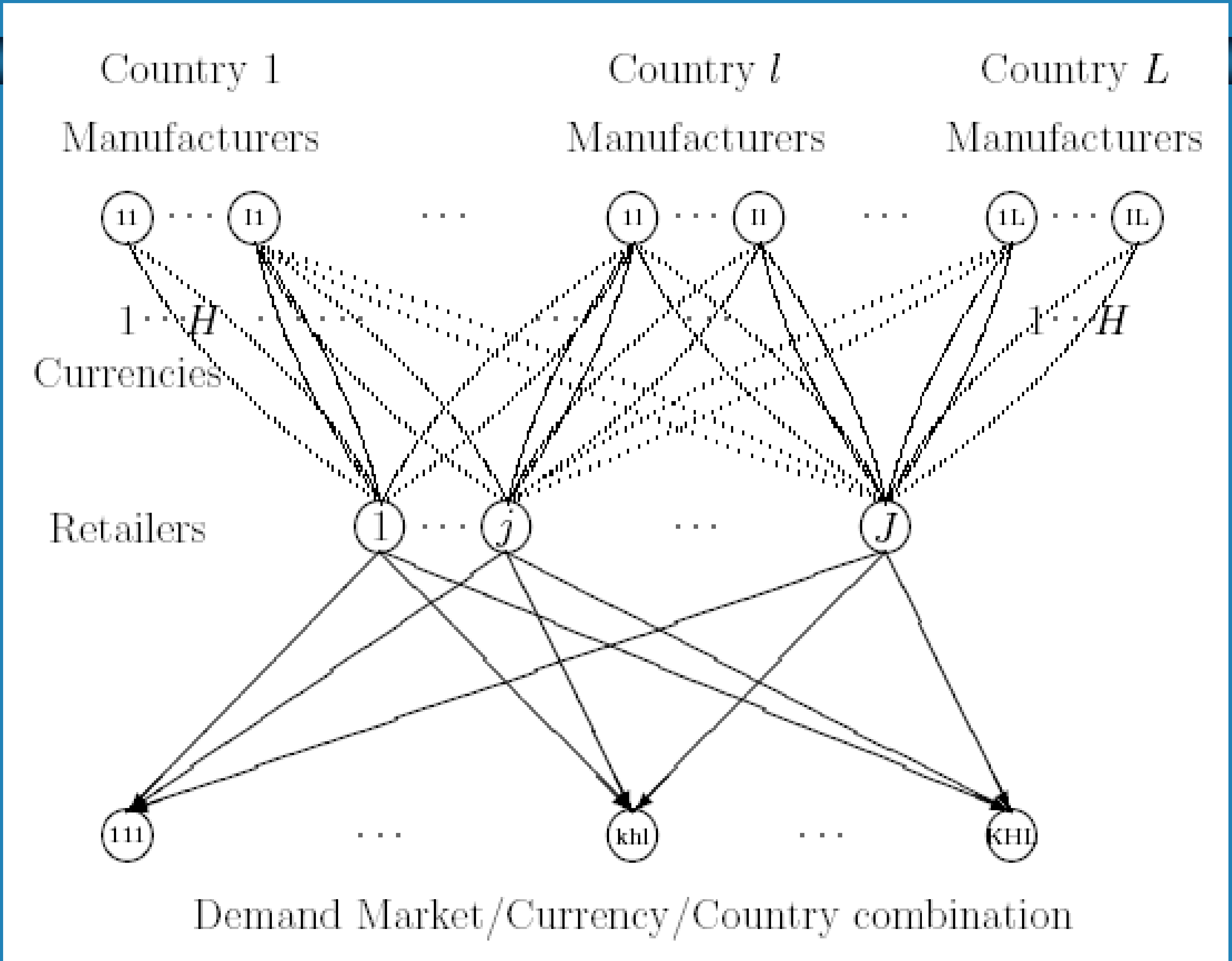
- **Decentralized Supply Chains with Noncooperative Entities**
  - Corbett and Karmarkar 2001 "Competition and structure in serial supply chains with deterministic demand" *Management Science* 47
  - Nagurney, Zhang and Dong 2002 "Supply chain networks and electronic commerce: A theoretical perspective" *Netnomics* 4
  - Nagurney, Zhang and Dong 2002 "A supply chain network equilibrium model" *Transportation Research E*
  - Nagurney, Ke, Cruz, Hancock, and Southworth, 2002 "Dynamics of Supply Chains: A Multilevel (Logistical/Informational/Financial) Network Perspective", *Environment and Planning B 29*
  - Nagurney, Cruz, and Matsypura, 2003 "Dynamics of global supply chain supernetworks," *Mathematical and Computer Modelling 37*
  - Nagurney, Zhang and Dong 2003 "A supply chain network equilibrium model with random demands" *European Journal of Operational Research*, to appear
  - Nagurney and Dong, 2002 **Supernetworks: Decision-making for the Information Age**, Edward Elgar Publishing

The full paper is available at:

Nagurney, Cruz, and Matsypura, 2003 "Dynamics of global supply chain supernetworks," *Mathematical and Computer Modelling* ***37****, 963-983.*

See also: http://supernet.som.umass.edu

Global Supply Chain Supernetwork



Model Notation

Consider:

- Homogeneous product
- *L* countries
- *I* manufacturers in each country
- *J* retailers, which can be either physical or virtual
- *K* demand markets in each country
- *H* currencies in the global economy

Assume:

- Manufacturer can transact with the retailers in different currencies
- Demand in a country can be associated with a particular currency

Denote:

- q^{il}_{jh} – flow on link *h* joining node *il* with node *j*
- q^{j}_{klh} – flow on link joining node *j* with node *klh*
- ρ^{il}_{1jh} – the price associated with the product in currency *h* transacted between manufacturer *il* and retailer *j*
- ρ^{j}_{2khl} – the price associated with retailer *j* and demand market *k* in currency *h* and country *l*
- ρ_{3khl} – the price of the product at demand market *k* in currency *h* and in country *l*
- e_h – the rate of appreciation of currency *h* against the basic currency

The Behavior of the Manufacturers and Their Optimality Conditions

transaction cost functions:

$$c^{il}_{jh} = c^{il}_{jh}(q^{il}_{jh}),\ \forall i, l, j, h$$

production cost functions:

$$f^{il} = f^{il}(Q^1),\ \forall i, l.$$

$$\text{Maximize} \quad U^{il} = \sum_{j=1}^{J}\sum_{h=1}^{H}(\rho^{il*}_{1jh} + e^*_h)q^{il}_{jh} - \sum_{j=1}^{J}\sum_{h=1}^{H} c^{il}_{jh}(q^{il}_{jh}) - f^{il}(Q^1)$$

The optimality conditions of all manufacturers *i* in all countries *l* simultaneously under the above assumptions can be compactly expressed as:

determine $Q^{1*} \in R^{IJHL}_+$ satisfying

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(Q^{1*})}{\partial q^{il}_{jh}} + \frac{\partial c^{il}_{jh}(q^{il*}_{jh})}{\partial q^{il}_{jh}} - \rho^{il*}_{1jh} - e^*_h\right] \times \left[q^{il}_{jh} - q^{il*}_{jh}\right] \geq 0, \quad \forall Q^1 \in R^{IJHL}_+$$

The Behavior of the Retailers

handling/conversion cost function: $c_j = c_j(Q^1), \forall j$

transaction cost function: $\underline{c}^{il}_{jh} = \underline{c}^{il}_{jh}(q^{il}_{jh}), \forall i, l, j, h$

$c^j_{khl} = c^j_{khl}(q^j_{khl}), \forall j, k, h, l$

$$\text{Maximize} \quad U^j = \sum_{k=1}^{k}\sum_{h=1}^{H}\sum_{l=1}^{L}(\rho^{j*}_{2khl} + e^*_h)q^{il}_{jh} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\hat{c}^{il}_{jh}(q^{il}_{jh})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}c^j_{khl}(q^j_{khl}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}(\rho^{il*}_{1jh} + e^*_h)q^{il}_{jh}$$

subject to $\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L} q^j_{khl} \le \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q^{il}_{jh}, q^j_{khl} \ge 0, q^{il}_{jh} \ge 0$

Retailers' Optimality Conditions

The variational inequality problem:
determine $(Q^{1*}; Q^{2*}; \gamma^*) \in R_+^{ILJH+JKHL+J}$, such that

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}} + \rho_{1jh}^{il*} + e_h^* + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*})}{\partial q_{jh}^{il}} - \gamma_j^*\right] \times \left[q_{jh}^{il} - q_{jh}^{il*}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\frac{\partial c_{khl}^{j}(q_{khl}^{j*})}{\partial q_{khl}^{j}} - \rho_{2khl}^{j*} - e_h^* + \gamma_j^*\right] \times \left[q_{khl}^{j} - q_{khl}^{j*}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q_{jh}^{il*} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{khl}^{j*}\right] \times \left[\gamma_j - \gamma_j^*\right] \geq 0, \quad \forall (Q^1, Q^2, \gamma) \in R_+^{ILJH+JKHL+J}$$

The Consumers at the Demand Markets and the Equilibrium Conditions

transaction cost functions: $\underline{c}^j_{khl} = \underline{c}^j_{khl}(Q^2), \forall j, k, h, l$

demand functions: $d_{khl} = d_{khl}(\rho_3), \forall k, h, l$

the equilibrium conditions for the consumers at demand market *khl*:

$$\rho^{j*}_{2khl} + e^*_h + \hat{c}^j_{khl}(Q^{2*}) \begin{cases} = \rho^*_{3khl}, & \text{if} \quad q^{j*}_{khl} > 0 \\ \geq \rho^*_{3khl}, & \text{if} \quad q^{j*}_{khl} = 0, \end{cases}$$

$$d_{khl}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} q^{j*}_{khl}, & \text{if} \quad \rho^*_{3khl} > 0 \\ \leq \sum_{j=1}^{J} q^{j*}_{khl}, & \text{if} \quad \rho^*_{3khl} = 0. \end{cases}$$

Variational Inequality Formulation

The variational inequality problem:
determine $(Q^{2*}; \rho^*_3) \in R_+^{(J+1)KHL}$, such that

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\rho_{2khl}^{j*} + e_h^* + \hat{c}_{khl}^j(Q^{2*}) - \rho_{3khl}^*\right] \times \left[q_{khl}^j - q_{khl}^{j*}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\sum_{j=1}^{J} q_{khl}^{j*} - d_{khl}(\rho_3^*)\right] \times \left[\rho_{3khl} - \rho_{3khl}^*\right] \geq 0, \quad \forall (Q^2, \rho_3) \in R_+^{(J+1)KHL}.$$

Variational Inequality Formulation of the Equilibrium Conditions for the Global Supply Chain Network Economy

The equilibrium conditions governing the global supply chain network are equivalent to the solution of the variational inequality given by:

determine $(Q^{1*}; Q^{2*}; \gamma^*; \rho^*_3) \in K$*, satisfying:*

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(Q^{1*})}{\partial q^{il}_{jh}}+\frac{\partial c^{il}_{jh}(q^{il*}_{jh})}{\partial q^{il}_{jh}}+\frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jh}}+\frac{\partial \hat{c}^{il}_{jh}(q^{il*}_{jh})}{\partial q^{il}_{jh}}-\gamma^*_j\right]\times\left[q^{il}_{jh}-q^{il*}_{jh}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\frac{\partial c^{j}_{khl}(q^{j*}_{khl})}{\partial q^{j}_{khl}}+\gamma^*_j+\hat{c}^{j}_{khl}(Q^{2*})-\rho^*_{3khl}\right]\times\left[q^{j}_{khl}-q^{j*}_{khl}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}q^{il*}_{jh}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}q^{j*}_{khl}\right]\times\left[\gamma_j-\gamma^*_j\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{l=1}^{L}\left[\sum_{j=1}^{J}q^{j*}_{khl}-d_{khl}(\rho^*_3)\right]\times\left[\rho_{3khl}-\rho^*_{3khl}\right]\geq 0,\quad \forall (Q^1,Q^2,\gamma,\rho_3)\in\mathcal{K},$$

$$\text{where } \mathcal{K}\equiv\{(Q^1,Q^2,\gamma,\rho_3)|(Q^2,\gamma,\rho_3)\in R_+^{ILJH+JKHL+J+KHL}\}.$$

The Dynamic Adjustment Process

- Demand Market Price Dynamics
  - we assume that the rate of change of the price ρ_{3khl} is equal to the difference between the demand for the product at the demand market and currency and country and the amount of the product actually available at that particular market:

$$\dot{\rho}_{3khl} = \begin{cases} d_{khl}(\rho_3) - \sum_{j=1}^{J} q^j_{khl}, & \text{if} \quad \rho_{3khl} > 0 \\ \max\{0, d_{khl}(\rho_3) - \sum_{j=1}^{J} q^j_{khl}\} & \text{if} \quad \rho_{3khl} = 0. \end{cases}$$

The Dynamic Adjustment Process

- The Dynamics of the Product Shipments between the Retailers and the Demand Markets
  - the rate of change of the product shipment q^j_{khl}, in turn, is assumed to equal to the difference between the price the consumers are willing to pay for the product at the demand market and currency and country minus the price charged and the various transaction costs:

$$\dot{q}^j_{khl} = \begin{cases} \rho_{3khl} - \frac{\partial c^j_{khl}(q^j_{khl})}{\partial q^j_{khl}} - \hat{c}^j_{khl}(Q^2) - \gamma_j, & \text{if} \quad q^j_{khl} > 0 \\ \max\{0, \rho_{3khl} - \frac{\partial c^j_{khl}(q^j_{khl})}{\partial q^j_{khl}} - \hat{c}^j_{khl}(Q^2) - \gamma_j\} & \text{if} \quad q^j_{khl} = 0. \end{cases}$$

The Dynamics of the Prices at the Retailers

- the prices at the retailers, whether they are physical or virtual, must reflect supply and demand conditions as well. we propose the following dynamic adjustment for every retailer *j*:

$$\dot{\gamma}_j = \begin{cases} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} q_{khl}^{j} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il}, & \text{if} \quad \gamma_j > 0 \\ \max\{0, \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} q_{khl}^{j} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il}\}, & \text{if} \quad \gamma_j = 0. \end{cases}$$

- The Dynamics of the Product Shipments between Manufacturers and Retailers
  - The dynamics of the product shipments transacted in described:

$$\dot{q}^{il}_{jh} = \begin{cases} \gamma_j - \frac{\partial f^{il}(Q^1)}{\partial q^{il}_{jh}} - \frac{\partial c^{il}_{jh}(q^{il}_{jh})}{\partial q^{il}_{jh}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jh}} - \frac{\partial \hat{c}^{il}_{jh}(q^{il}_{jh})}{\partial q^{il}_{jh}}, & \text{if} \quad q^{il}_{jh} > 0 \\ \max\{0, \gamma_j - \frac{\partial f^{il}(Q^1)}{\partial q^{il}_{jh}} - \frac{\partial c^{il}_{jh}(q^{il}_{jh})}{\partial q^{il}_{jh}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jh}} - \frac{\partial \hat{c}^{il}_{jh}(q^{il}_{jh})}{\partial q^{il}_{jh}}\}, & \text{if} \quad q^{il}_{jh} = 0. \end{cases}$$

The Projected Dynamical System

- The Projected Dynamical System
  - Consider now a dynamical system in which the demand market prices, the product shipments between retailers and demand markets, the prices at the retailers and the product shipments between manufacturers and retailers evolve according to the rules presented above
  - then the dynamic model described above can be rewritten as a projected dynamical system defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0,$$

Equivalence between Stationary Points and Solutions of the Variational Inequality

Result

- Theorem: Set of Stationary Points Coincides with Set of Equilibrium Points

The set of stationary points of the projected dynamical system coincides with the set of solutions of the variational inequality problem and, thus, with the set of equilibrium points as defined in Definition 1

Qualitative Properties

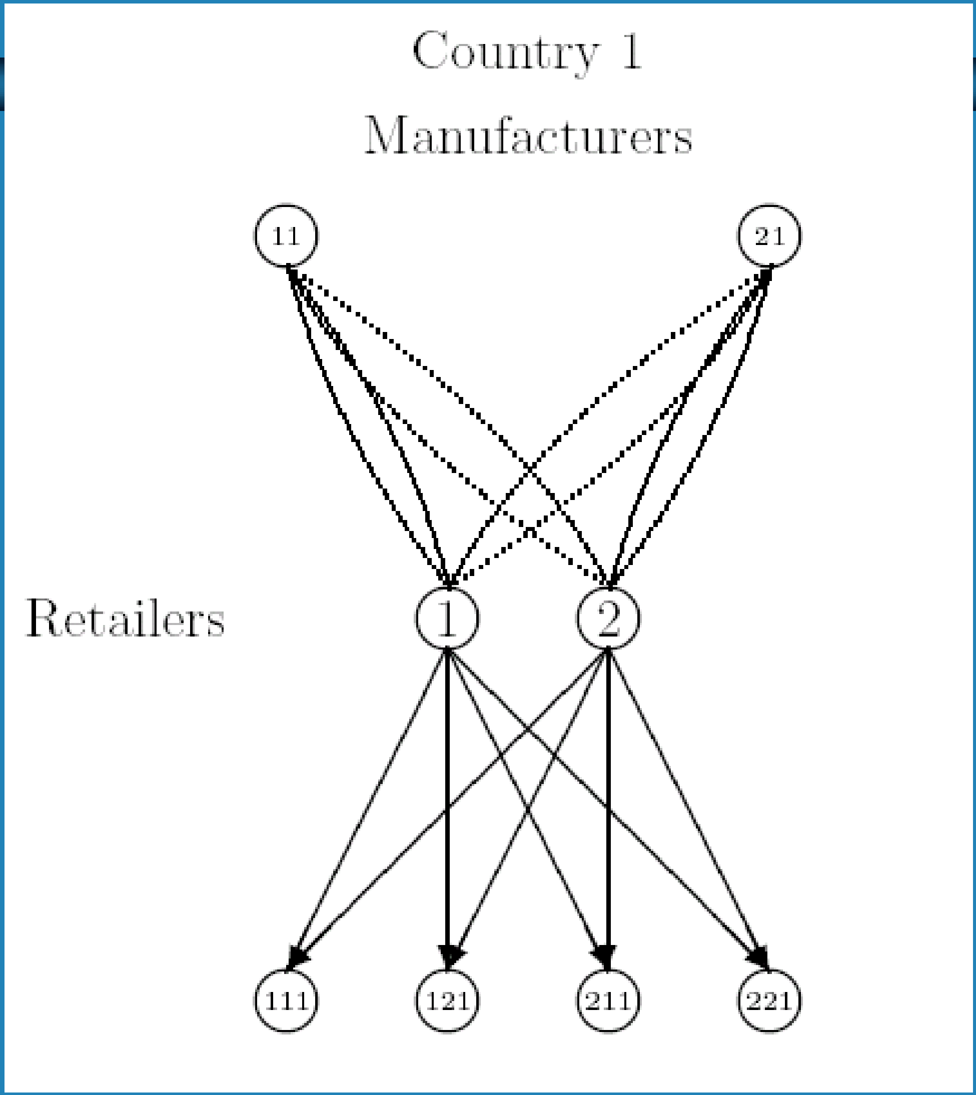
- **Theorem**: Existence (of the solution of the presented VI)
- **Theorem**: Uniqueness (of the solution of the presented VI)
- **Theorem**: Existence and Uniqueness (of the solution to the projected dynamical system)
- **Theorem**: Stability of the Global Supply Chain Network

The Algorithm

- The algorithm that we propose is the Euler-type method, which is induced by the general iterative scheme of Dupuis and Nagurney [1993]

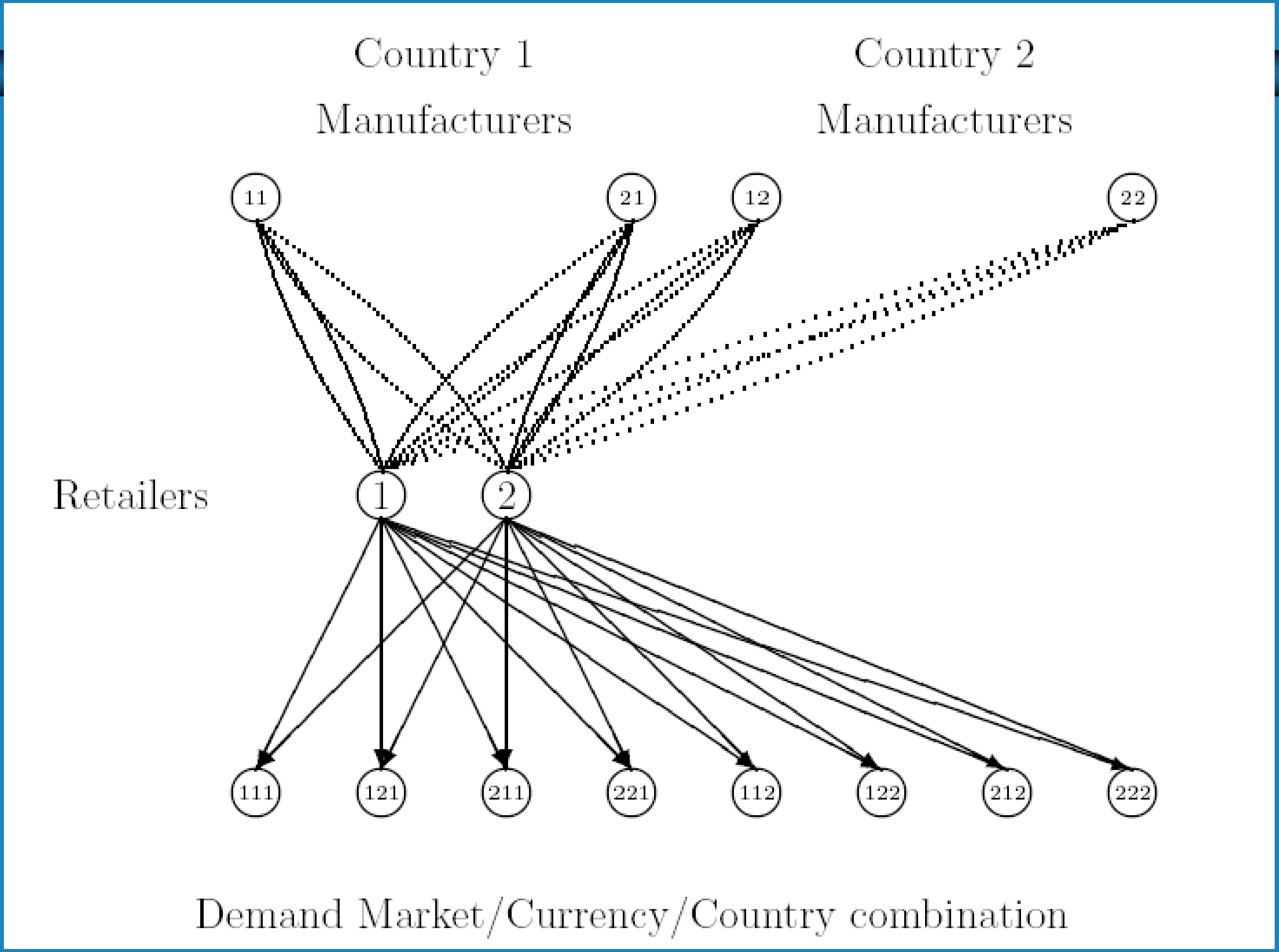
- See also Nagurney and Dong [2002]

Examples 1, 2, 3



	Example 1	Example 2	Example 3
$q^{il}_{jh}, \forall i, l, j, h$	15.605	15.643	10.751
q^{1*}_{111}	15.605	18.100	13.206
q^{1*}_{121}	15.605	14.100	9.206
$q^{1*}_{211}=q^{1*}_{221}$	15.605	15.191	10.297
q^{2*}_{111}	15.605	18.100	13.206
q^{2*}_{121}	15.605	14.100	9.206
$q^{2*}_{211}= q^{2*}_{221}$	15.605	15.191	10.297
$\gamma^*_1= \gamma^*_2$	256.190	256.840	264.534
ρ^*_{3111}	276.797	279.942	282.738
ρ^*_{3121}	276.797	275.943	278.739
$\rho^*_{3211}= \rho^*_{3221}$	276.797	277.033	279.830
$\rho^{il*}_{1jh}+e_h^*$	143.95	144.316	186.274

Examples 4, 5


Country 1
Manufacturers
Country 2
Manufacturers
11
21
12
22
Retailers
1
2
111
121
211
221
112
122
212
222
Demand Market/Currency/Country combination

	Example 4	Example 5
$q^{il}_{jh}, \forall i, l, j, h$	12.712	12.877
$q^{1*}_{111}=q^{1*}_{121}=q^{1*}_{211}=q^{1*}_{221}$	12.712	10.605
q^{1*}_{112}	12.712	11.332
q^{1*}_{122}	12.712	15.332
q^{1*}_{212}	12.712	14.968
q^{1*}_{222}	12.712	18.968
$q^{2*}_{111}=q^{2*}_{121}=q^{2*}_{211}=q^{2*}_{221}$	12.712	10.605
q^{2*}_{112}	12.712	11.332
q^{2*}_{122}	12.712	15.332
q^{2*}_{212}	12.712	14.968
q^{2*}_{222}	12.712	18.968

	Example 4	Example 5
$\gamma^*_1 = \gamma^*_2$	260.739	264.051
$\rho^*_{3111} = \rho^*_{3121} = \rho^*_{3211} = \rho^*_{3221}$	278.450	279.654
ρ^*_{3112}	278.450	280.381
ρ^*_{3122}	278.450	284.381
ρ^*_{3212}	278.450	284.017
ρ^*_{3222}	278.450	284.017
$\rho^{il*}_{1jh} + e_h^*$	117.908	119.198

Thank you!!!

For more information visit the Virtual Center for Supernetworks

http://supernet.som.umass.edu