

**Presentations Prepared for the Visit of Professor  
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Tina Wakolbinger, Zugang (Leo) Liu, Trisha Woolley, Patrick Qiang

September 27, 2006

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# A Dynamic Theory for the Integration of Social and Economic Networks with Applications to Supply Chain and Financial Networks

Tina Wakolbinger  
September 27, 2006

“Economic action is embedded in social relations that sometimes facilitate and at other times derail exchange.” (Uzzi, 1996, p. 674)

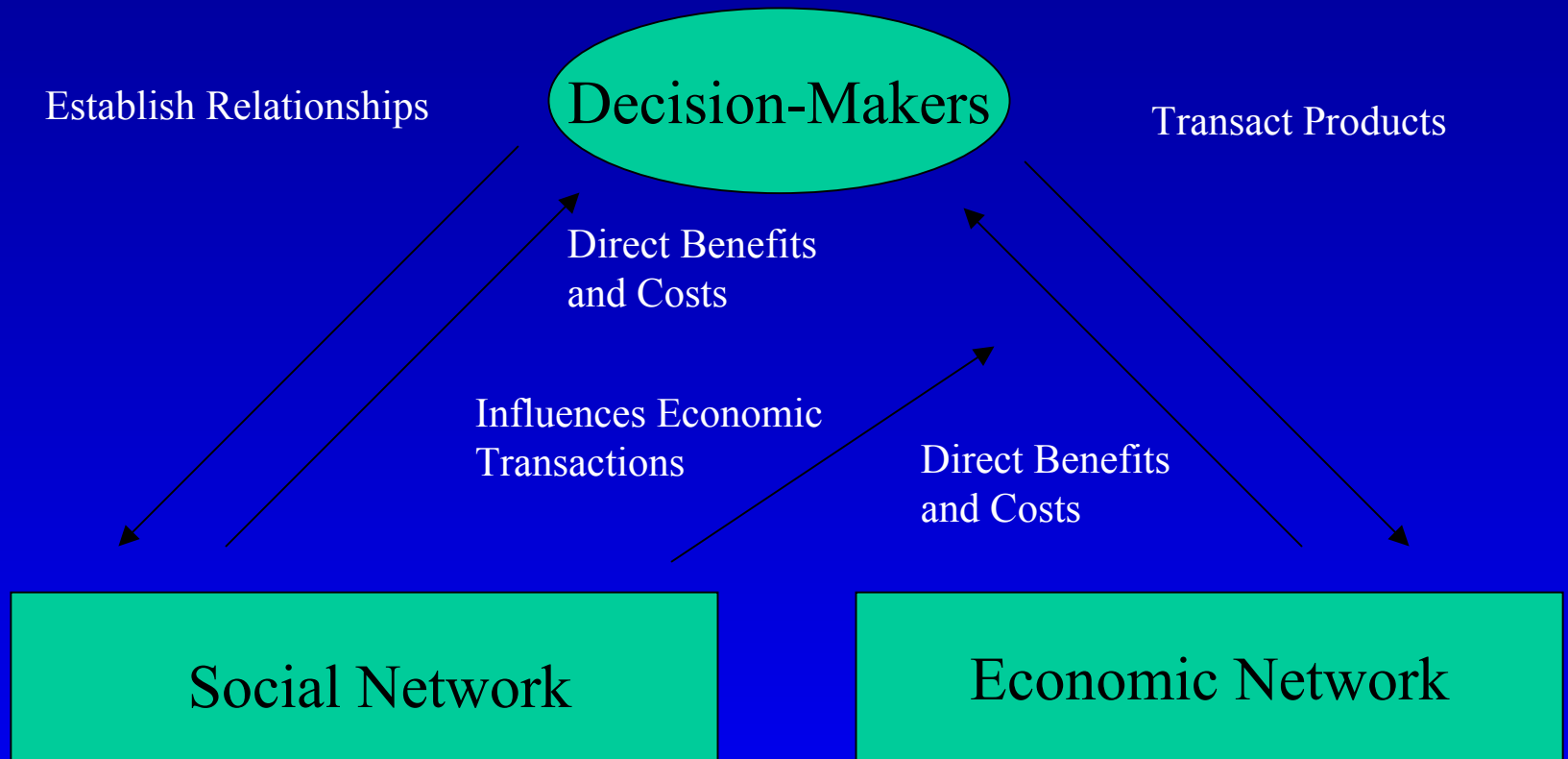
# Relationships in Supply Chains

- Can reduce risk (cf. Baker and Faulkner (2004, p. 92))
  - By reducing “information asymmetry between buyer and seller”
  - By reducing “opportunism due to imposed social obligations and effective sanctions on the seller”
- Can reduce transaction costs
  - By increasing levels of trust (cf. Dyer (2000))

# Relationships in Financial Transactions

- Make firms more likely to get loans and to receive lower interest rates on loans (cf. Uzzi (1999))
- Play an important role in micro-financing (cf. Ghatak (2002) and Anthony (1997))
- Can protect investors in partly fraudulent businesses (cf. Baker and Faulkner (2004))

# Research Framework

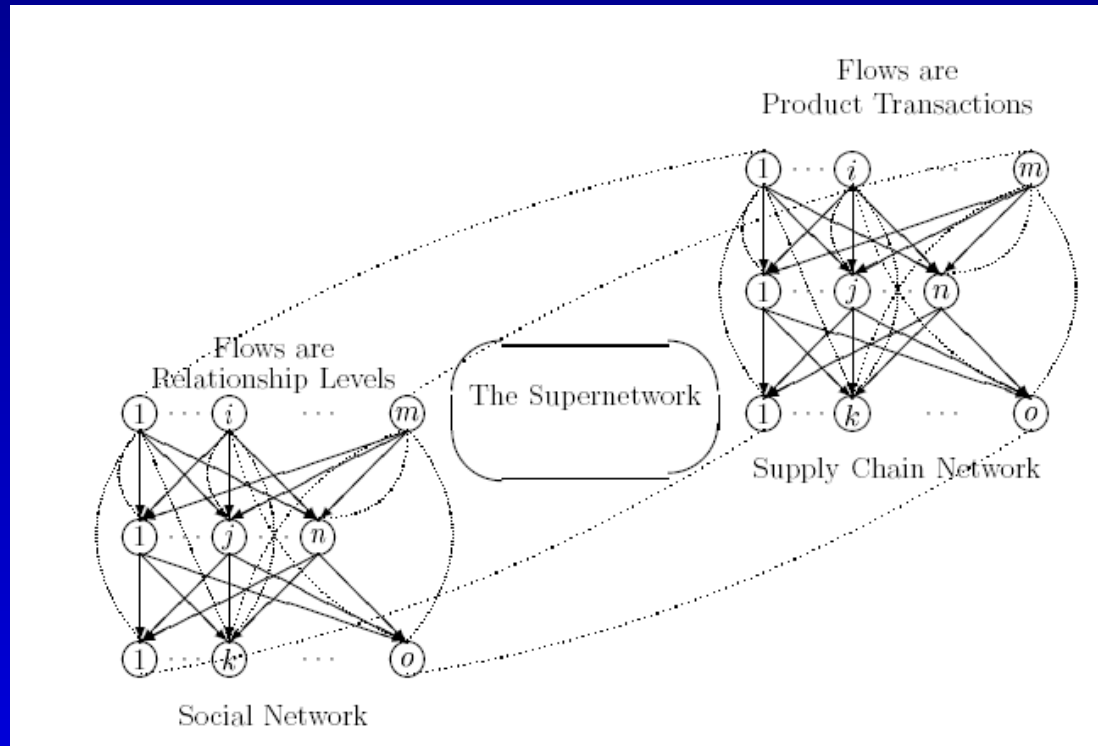


# Methodologies

- Variational inequality theory
  - To analyze network equilibria
  - Nagurney (1999)
- Projected dynamical systems
  - To analyze the dynamics
  - Dupuis and Nagurney (1993)
  - Nagurney and Zhang (1996a)



# Supernetwork Integrating a Social with a Supply Chain Network



“Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations,” Tina Wakolbinger and Anna Nagurney, *Netnomics* 6: (2004), 153-185.

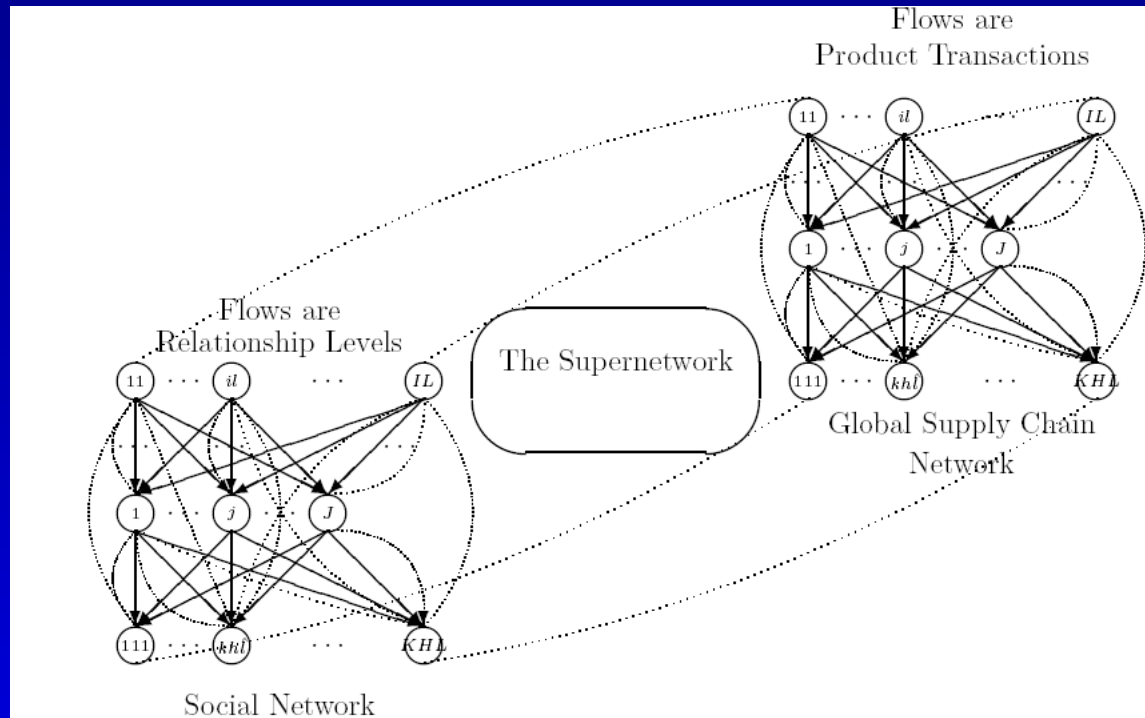
# Supernetwork Integrating a Social with a Supply Chain Network

- Models the interaction between a supply chain and a social network
- Captures interactions among individual sectors
- Includes electronic transactions, transaction costs, and risk
- Shows the dynamic evolution of:
  - Product flows and associated prices on the supply chain network
  - Relationship levels on the social network

# Supernetwork Integrating a Social with a Supply Chain Network

- Decision-makers in the network can decide about the amount of product they wish to transact and the relationship levels  $[0,1]$  they wish to establish
- Establishing relationship levels incurs some costs
- Relationship levels
  - Influence transaction costs
  - Influence risk
  - Have some additional value (“relationship value”)

# Supernetwork Integrating a Global Supply Chain with a Social Network



“Financial Engineering of the Integration of Global Supply Chain Networks and Social Networks with Risk Management,” Jose M. Cruz, Anna Nagurney, and Tina Wakolbinger *Naval Research Logistics* **53**: (2006), 674-696.

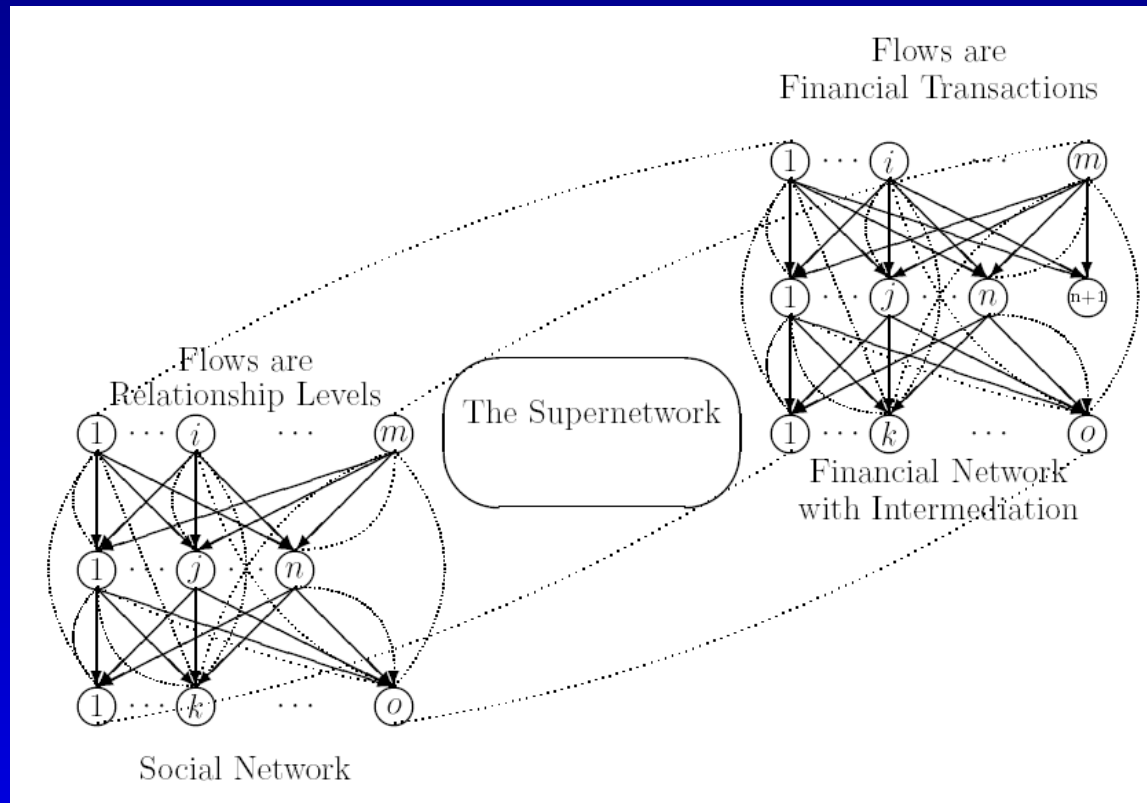
# Supernetwork Integrating a Global Supply Chain with a Social Network

- Extends the previous model to the international domain
  - Introduction of multiple countries and currencies
- Includes more general risk and relationship value functions

|                                    |  |
|------------------------------------|--|
| $r^{il}(Q^1, Q^2, \eta^1, \eta^2)$ | the risk incurred by manufacturer $il$ in his transactions |
| $r^j(Q^1, Q^3, \eta^1, \eta^3)$    | the risk incurred by retailer $j$ in his transactions      |

|                                    |   |
|------------------------------------|---|
| $v^{il}(\eta^1, \eta^2, Q^1, Q^2)$ | the relationship value function associated with manufacturer $il$ |
| $v^j(\eta^1, \eta^3, Q^1, Q^3)$    | the relationship value function associated with retailer $j$      |

# Supernetwork Integrating a Social with a Financial Network

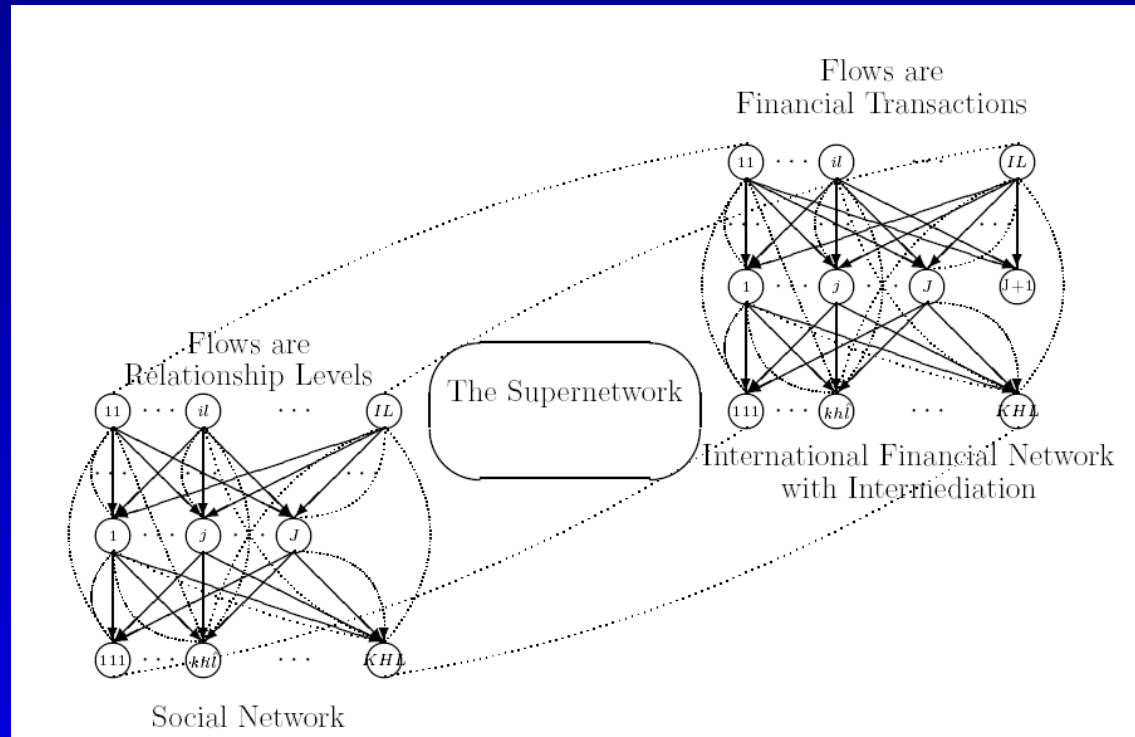


“The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels,” A. Nagurney, T. Wakolbinger, and L. Zhao, *Computational Economics* 27: (2006), 353-393.

# Supernetwork Integrating a Social Network with a Financial Network

- Models the interaction between financial and social networks
- Captures interactions among individual sectors
- Includes electronic transactions
- Allows for non-investment
- Incorporates transaction costs and risk
- Shows the dynamic evolution of:
  - Financial flows and associated prices on the financial network with intermediation
  - Relationship levels on the social network

# Supernetwork Integrating a Global Financial with a Social Network



“The Co-Evolution and Emergence of Integrated International Financial Networks and Social Networks: Theory, Analysis, and Computations,” Anna Nagurney, Jose M. Cruz, and Tina Wakolbinger (2004); invited chapter for Globalization and Regional Economic Modeling, edited by R. Cooper, K. P. Donaghy, G. J. D. Hewings, Springer.



# Supernetwork Integrating a Global Financial with a Social Network

- Extends the previous model to the international domain
  - Introduction of multiple countries and currencies
  - Highlights the importance of relationships in global financial transactions

# Novelty of the Research

- Economic flows and the social network structure are interrelated
- Supernetworks show the dynamic co-evolution of economic flows and the social network structure
- Relationships have different strength
- Networks of relationships have a measurable economic value

*Thank you!*



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# Dynamic Supply Chains, Transportation Network Equilibria, and Evolutionary Variational Inequalities

Zugang Liu

September 27, 2006

# Motivation

- Many researchers have described the various networks that underlie supply chain analysis and management with the goal being primarily that of optimization.
- In 2002, Nagurney, Dong and Zhang in Transportation Research E presented apparently the first supply chain network equilibrium model.
- The objective of this research was to develop a general dynamic supply chain network equilibrium model with exogenous time-varying demand.

# Motivation

- The theory that has originated from the study of transportation networks was utilized to construct this time-dependent equilibrium modeling framework for supply chain networks.
- The new dynamic supply chain network model that we developed in this research is also motivated by the unification of projected dynamical systems theory and evolutionary (infinite-dimensional) variational inequalities.

# Outline

- The supply chain network model with fixed demands
  - Overview of the supply chain network equilibrium models
  - The development of the supply chain network equilibrium model with fixed demands
- The supernetwork equivalence of the supply chain networks and the transportation networks
  - Overview of the transportation network equilibrium models
  - The supernetwork equivalence of the transportation networks and the supply chain networks with fixed demand
- The supply chain network model with time-varying demands
  - Evolutionary variational inequalities and projected dynamical systems; Applications to transportation network equilibrium
  - The computation of the supply chain network equilibrium model with time-varying demands.

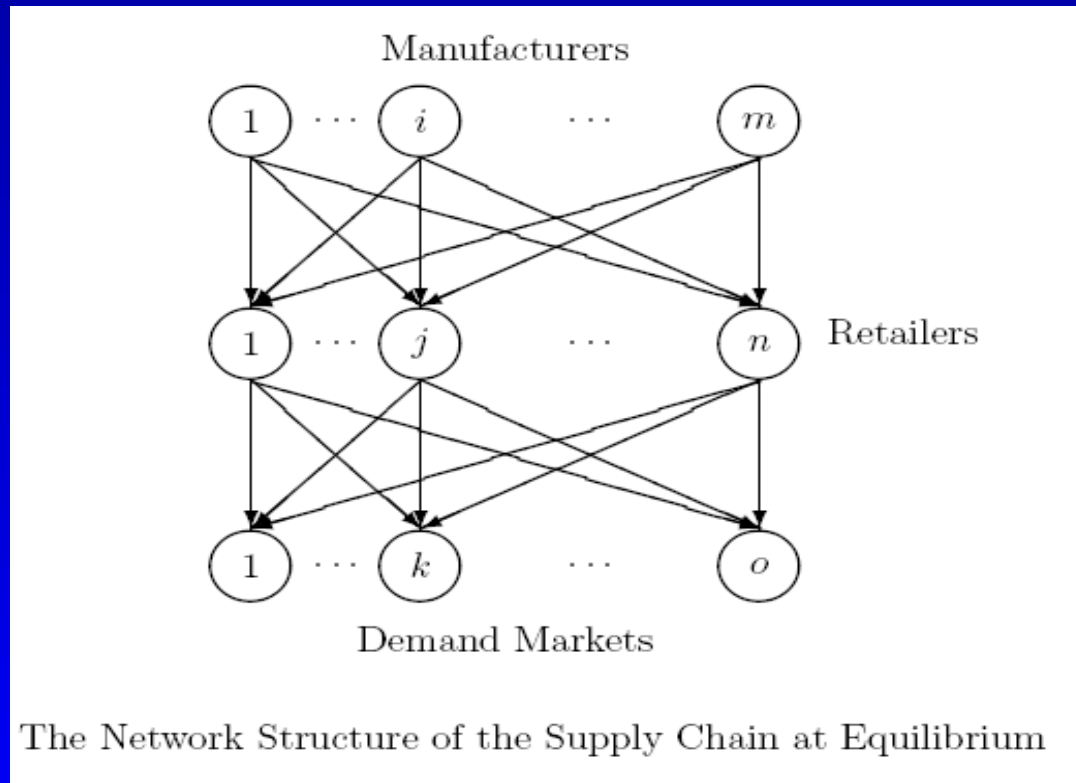
# Some of the Related Network Economics Literature

- Beckmann, M. J., McGuire, C. B., and Winsten, C. B. (1956), *Studies in the Economics of Transportation*. Yale University Press, New Haven, Connecticut.
- Nagurney, A (1999), *Network Economics: A Variational Inequality Approach*, Second and Revised Edition, Kluwer Academic Publishers, Dordrecht, The Netherlands.
- Nagurney, A., Dong, J., and Zhang, D. (2002), A Supply Chain Network Equilibrium Model, *Transportation Research E* 38, 281-303.



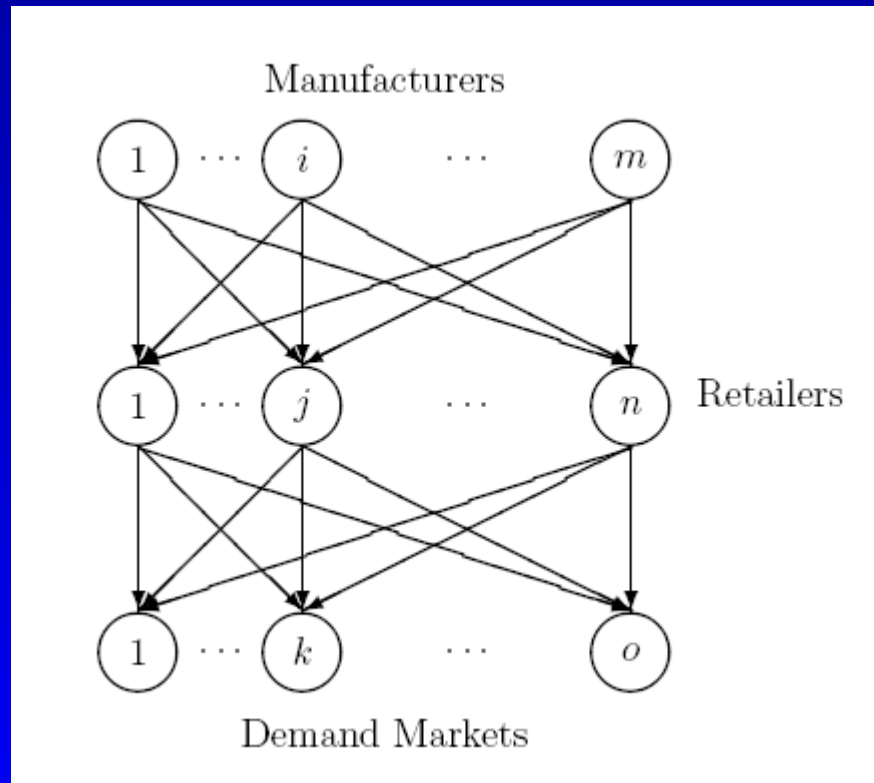
# Overview of the Supply Chain Network Equilibrium Model

- Nagurney, A., Dong, J. and Zhang, D. (2002), A Supply Chain Network Equilibrium Model, *Transportation Research E* 38, 281-303.



# The Development of the Supply Chain Network Equilibrium Model with Fixed Demands

- Commodities with price-insensitive demand
  - Electricity, gasoline, milk, etc.



# The Behavior of Manufacturers and their Optimality Conditions

- Manufacturer's optimization problem

$$\text{Maximize} \quad \sum_{j=1}^n \rho_{1ij}^* q_{ij} - f_i(Q^1) - \sum_{j=1}^n c_{ij}(q_{ij}),$$

- The Optimality conditions of the manufacturers

$$\sum_{i=1}^m \sum_{j=1}^n \left[ \frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} - \rho_{1ij}^* \right] \times [q_{ij} - q_{ij}^*] \geq 0, \quad \forall Q^1 \in R_+^{mn}.$$

# The Behavior of Retailers and their Optimality Conditions

- Retailer's optimization problem

$$\begin{aligned} & \text{Maximize} \quad \sum_{k=1}^o \rho_{2j}^* q_{jk} - c_j(Q^1) - \sum_{i=1}^m \rho_{1ij}^* q_{ij} \\ & \text{subject to:} \quad \sum_{k=1}^o q_{jk} \leq \sum_{i=1}^m q_{ij}, \end{aligned}$$

- The Optimality conditions of the retailers

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{\partial c_j(Q^{1*})}{\partial q_{ij}} + \rho_{1ij}^* - \gamma_j^* \right] \times [q_{ij} - q_{ij}^*] + \sum_{j=1}^n \sum_{k=1}^o \left[ -\rho_{2j}^* + \gamma_j^* \right] \times [q_{jk} - q_{jk}^*] \\ & + \sum_{j=1}^n \left[ \sum_{i=1}^m q_{ij}^* - \sum_{k=1}^o q_{jk}^* \right] \times [\gamma_j - \gamma_j^*] \geq 0, \quad \forall (Q^1, Q^2, \gamma) \in R_+^{mn+no+n}. \end{aligned}$$

# The Equilibrium Conditions at the Demand Markets

- Conservation of flow equations must hold

$$d_k = \sum_{j=1}^n q_{jk}, \quad k = 1, \dots, o,$$

- We say that vector  $(Q^{2*}, \rho_3^*)$  is an equilibrium vector if for each  $s, k$  pair:

$$\rho_{2j}^* + c_{jk}(Q^{2*}) \begin{cases} = \rho_{3k}^*, & \text{if } q_{jk}^* > 0, \\ \geq \rho_{3k}^*, & \text{if } q_{jk}^* = 0. \end{cases}$$

# Supply Chain Network Equilibrium (For Fixed Demands at the Markets)

**Definition:** The equilibrium state of the supply chain network is one where the product flows between the tiers of the network coincide and the product flows satisfy the conservation of flow equations, the sum of optimality conditions of the manufacturers and the retailers, and the equilibrium conditions at the demand markets.

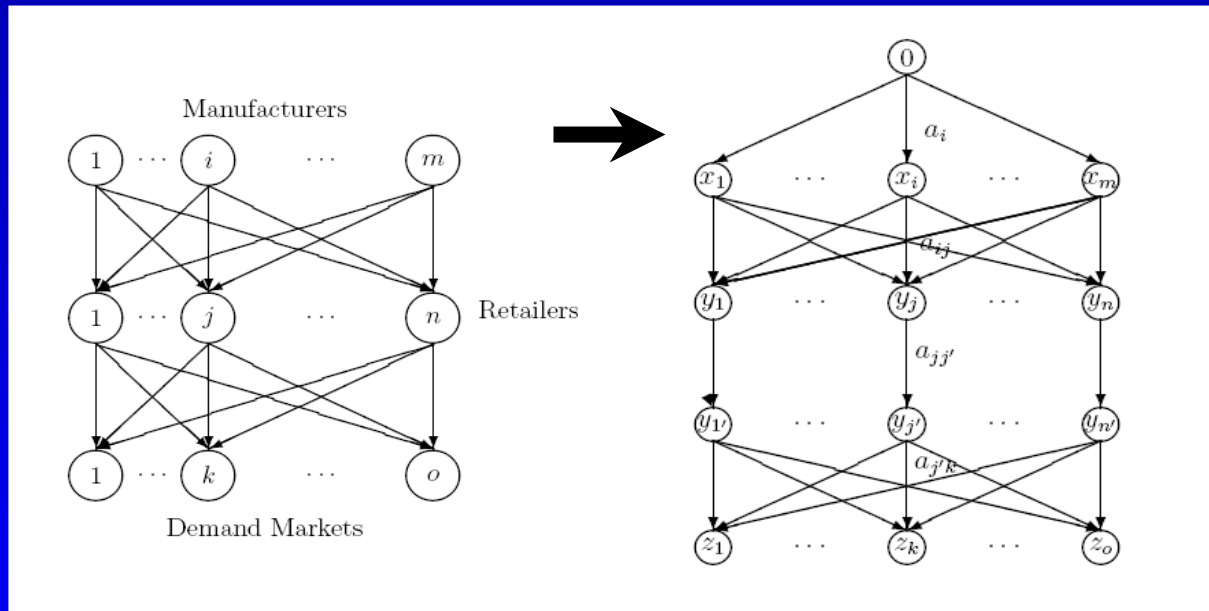
# Variational Inequality Formulation

- Determine  $(Q^{1*}, Q^{2*}, \gamma^*) \in \mathcal{K}^2$  satisfying

$$\begin{aligned} & \sum_{i=1}^m \sum_{j=1}^n \left[ \frac{\partial f_i(Q^{1*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ij}} - \gamma_j^* \right] \times [q_{ij} - q_{ij}^*] \\ & + \sum_{j=1}^n \sum_{k=1}^o [c_{jk}(Q^{2*}) + \gamma_j^*] \times [q_{jk} - q_{jk}^*] + \sum_{j=1}^n \left[ \sum_{i=1}^m q_{ij}^* - \sum_{k=1}^o q_{jk}^* \right] \times [\gamma_j - \gamma_j^*] \geq 0, \\ & \forall (Q^1, Q^2, \gamma) \in \mathcal{K}^2. \end{aligned}$$

# The Supernetwork Equivalence of Supply Chain Network Equilibrium and Transportation Network Equilibrium

- Nagurney, A. (2005) “On the Relationship Between Supply Chain and Transportation Network Equilibria: A Supernetwork Equivalence with Computations” (*To appear in Transportation Research E* (2006); published online by Elsevier May 3 (2005).)





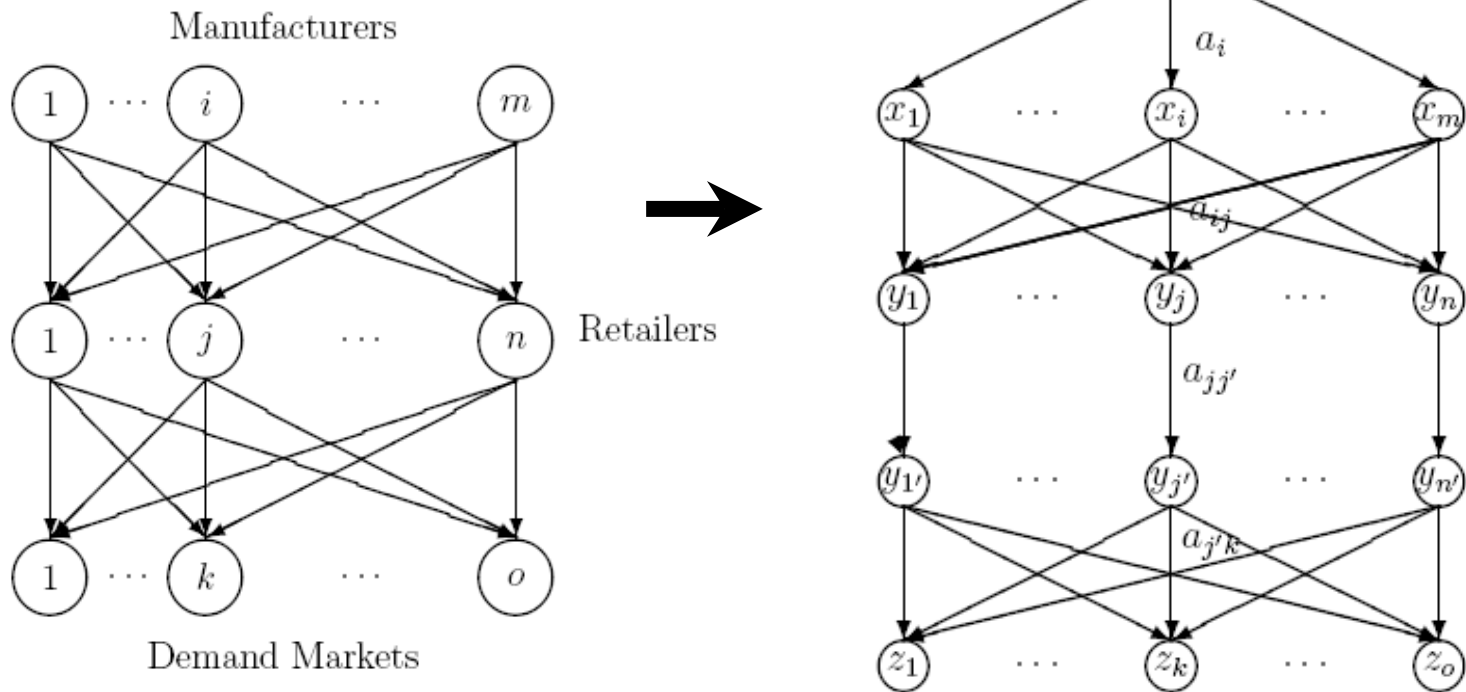
# Overview of the Transportation Network Equilibrium Model with Fixed Demand

- Smith, M. J., 1979. Existence, uniqueness, and stability of traffic equilibria. *Transportation Research* 13B, 259-304.
- Dafermos, S., 1980. Traffic equilibrium and variational inequalities. *Transportation Science* 14, 42-54.
- In equilibrium, the following conditions must hold for each O/D pair and each path.

$$C_p(x^*) - \lambda_w^* \begin{cases} = 0, & \text{if } x_p^* > 0, \\ \geq 0, & \text{if } x_p^* = 0. \end{cases}$$
- A path flow pattern is a transportation network equilibrium if and only if it satisfies the variational inequality:

$$\sum_{w \in W} \sum_{p \in P_w} C_p(x^*) \times [x_p - x_p^*] \geq 0, \quad \forall x \in \mathcal{K}^5.$$

# Transportation Network Equilibrium Reformulation of the Supply Chain Network Model with Fixed Demands



# The Evolutionary Variational Inequalities and Projected Dynamical Systems Literature

- Cojocaru, M.-G., Daniele, P., Nagurney, A., 2005a.  
Projected dynamical systems and evolutionary variational inequalities via Hilbert spaces with applications. *Journal of Optimization Theory and Applications* 27, no. 3, 1-15.
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- Barbagallo, A., 2005. Regularity results for timedependent variational and quasivariational inequalities and computational procedures. *To appear in Mathematical Models and Methods in Applied Sciences*.

# The Evolutionary Variational Inequalities and Projected Dynamical Systems Literature

- Daniele, P., Maugeri, A., Oettli, W., 1998. Variational inequalities and time-dependent traffic equilibria. *Comptes Rendue Academie des Science*, Paris 326, serie I, 10591062.
- Daniele, P., Maugeri, A., Oettli, W., 1999. Time-dependent traffic equilibria. *Journal of Optimization Theory and its Applications* 103, 543-555.

# Finite-Dimensional Variational Inequalities and Projected Dynamical Systems Literature

- Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. *Annals of Operations Research* 44, 9-42.
- Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Boston, Massachusetts.
- Nagurney, A., Zhang, D., 1997. Projected dynamical systems in the formulation, stability analysis, and computation of fixed demand traffic network equilibria. *Transportation Science* 31, 147-158.

# Projected Dynamical Systems and Evolutionary Variational Inequalities

- Projected Dynamical Systems (PDSs) (Dupuis and Nagurney (1993))
  - PDS describes how the state of the network system approaches an equilibrium point on the curve of equilibria.
  - For almost every moment ‘t’ on the equilibria curve, there is a  $PDS_t$  associated with it.
  - A  $PDS_t$  is usually applied to study small scale time dynamics, i.e  $[t, t+\tau]$

# Projected Dynamical Systems

PDSs: 
$$\frac{dx(t)}{dt} = \Pi_{\mathcal{K}}(x(t), -F(x(t))).$$

In this formulation,  $\mathcal{K}$  is a convex polyhedral set in  $R^n$ ,  $F : \mathcal{K} \rightarrow R^n$  is a Lipschitz continuous function with linear growth and  $\Pi_{\mathcal{K}} : R \times \mathcal{K} \rightarrow R^n$  is the Gateaux directional derivative

$$\Pi_{\mathcal{K}}(x, -F(x)) = \lim_{\delta \rightarrow 0^+} \frac{P_{\mathcal{K}}(x - \delta F(x)) - x}{\delta}$$

of the projection operator  $P_{\mathcal{K}} : R^n \rightarrow \mathcal{K}$ , given by

$$\|P_{\mathcal{K}}(z) - z\| = \inf_{y \in \mathcal{K}} \|y - z\|$$



# Evolutionary Variational Inequalities

- Evolutionary Variational Inequalities (EVIs)
  - EVI provides a curve of equilibria of the network system over a finite time interval  $[0, T]$
  - An EVI is usually used to model large scale time, i.e,  $[0, T]$
  - EVIs have been applied to time-dependent equilibrium problems in transportation, and in economics and finance.

# Evolutionary Variational Inequalities

Define  $\ll \phi, u \gg := \int_0^T \langle \phi(t), u(t) \rangle dt,$

find  $v \in \mathcal{K}$  such that  $\ll F(v), u - v \gg \geq 0, \forall u \in \mathcal{K}.$

where

$$\mathcal{K} = \bigcup_{t \in [0, T]} \left\{ u \in L^p([0, T], \mathbb{R}^q) \mid \lambda(t) \leq u(t) \leq \mu(t) \text{ a.e. in } [0, T]; \right.$$

$$\left. \sum_{i=1}^q \xi_{ji} u_i(t) = \rho_j(t) \text{ a.e. in } [0, T], \xi_{ji} \in \{0, 1\}, i \in \{1, \dots, q\}, j \in \{1, \dots, l\} \right\}.$$

# Projected Dynamical Systems and Evolutionary Variational Inequalities

- Cojocaru, Daniele, and Nagurney (2005b) showed the following:

## Theorem

*Assume that  $\hat{\mathcal{K}} \subseteq H$  is non-empty, closed, and convex. Assume also that  $F : \hat{\mathcal{K}} \rightarrow H$  is a pseudo-monotone vector field, that is, for every pair of points  $x, y \in \hat{\mathcal{K}}$ , we have that*

$$\langle F(x), y - x \rangle \geq 0 \implies \langle F(y), y - x \rangle \geq 0,$$

*and that  $F$  is Lipschitz continuous, where  $H$  is a Hilbert space. Then the solutions of EVI (47) are the same as the critical points of the projected differential equation (48), that is, they are the functions  $x^* \in \hat{\mathcal{K}}$  such that*

$$\Pi_{\hat{\mathcal{K}}}(x^*(t), -F(x^*(t))) = 0,$$

*and vice-versa.*



# Numerical Solution of Evolutionary Variational Inequalities

- The vector field  $F$  satisfies the requirement in Theorem.
- We first discretize time horizon  $T$ .
- At each fixed time point, we solve the associated projected dynamical system  $PDS_t$
- We use the Euler method to solve the projected dynamical system  $PDS_t$ .

# The Euler Method

## Step 0: Initialization

Set  $X^0 \in \mathcal{K}$  and set  $T = 0$ .  $T$  is an iteration counter which may also be interpreted as a time period.

## Step 1: Computation

Compute  $X^{T+1}$  by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where  $\{a_T\}$  is a sequence of positive scalars satisfying:  $\sum_{T=0}^{\infty} a_T = \infty$ ,  $a_T \rightarrow 0$ , as  $T \rightarrow \infty$

and  $P_{\mathcal{K}}$  is the projection of  $X$  on the set  $\mathcal{K}$  defined as:

$$y = P_{\mathcal{K}}X = \arg \min_{z \in \mathcal{K}} \|X - z\|.$$

## Step 2: Convergence Verification

If  $\|X^{T+1} - X^T\| \leq \epsilon$ , for some  $\epsilon > 0$ , a prespecified tolerance, then stop; else, set  $T = T + 1$ , and go to Step 1,

# The Solution to the Transportation Network Model with Time-Varying Demands

## Feasible set

$$\hat{\mathcal{K}} = \left\{ x \in L^2([0, T], R^Q) : 0 \leq x(t) \leq \mu \text{ a.e. in } [0, T]; \sum_{p \in P_w} x_p(t) = d_w(t), \forall w, \text{ a.e. in } [0, T] \right\}.$$

## Define

EVI:

$$\langle \langle \Phi, x \rangle \rangle = \int_0^T \langle \Phi(t), x(t) \rangle dt$$

determine  $x^* \in \hat{\mathcal{K}}$  such that:

$$\langle \langle F(x^*), x - x^* \rangle \rangle \geq 0, \quad \forall x \in \hat{\mathcal{K}}.$$

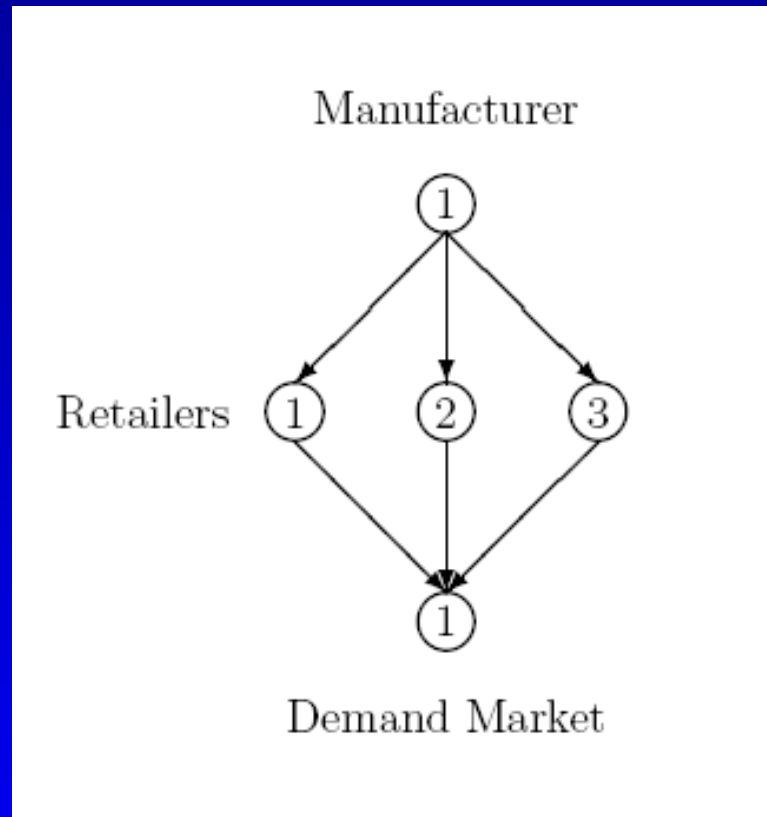
where  $F(x)$  denotes the vector of **path costs** as a function of path flows.

# Solving Supply Chain Network Model with Time-Varying Demands

- First, construct the equivalent transportation network equilibrium model
- Solve the transportation network equilibrium model with time varying demands
- Convert the solution of the transportation network into the time-dependent supply chain network equilibrium model

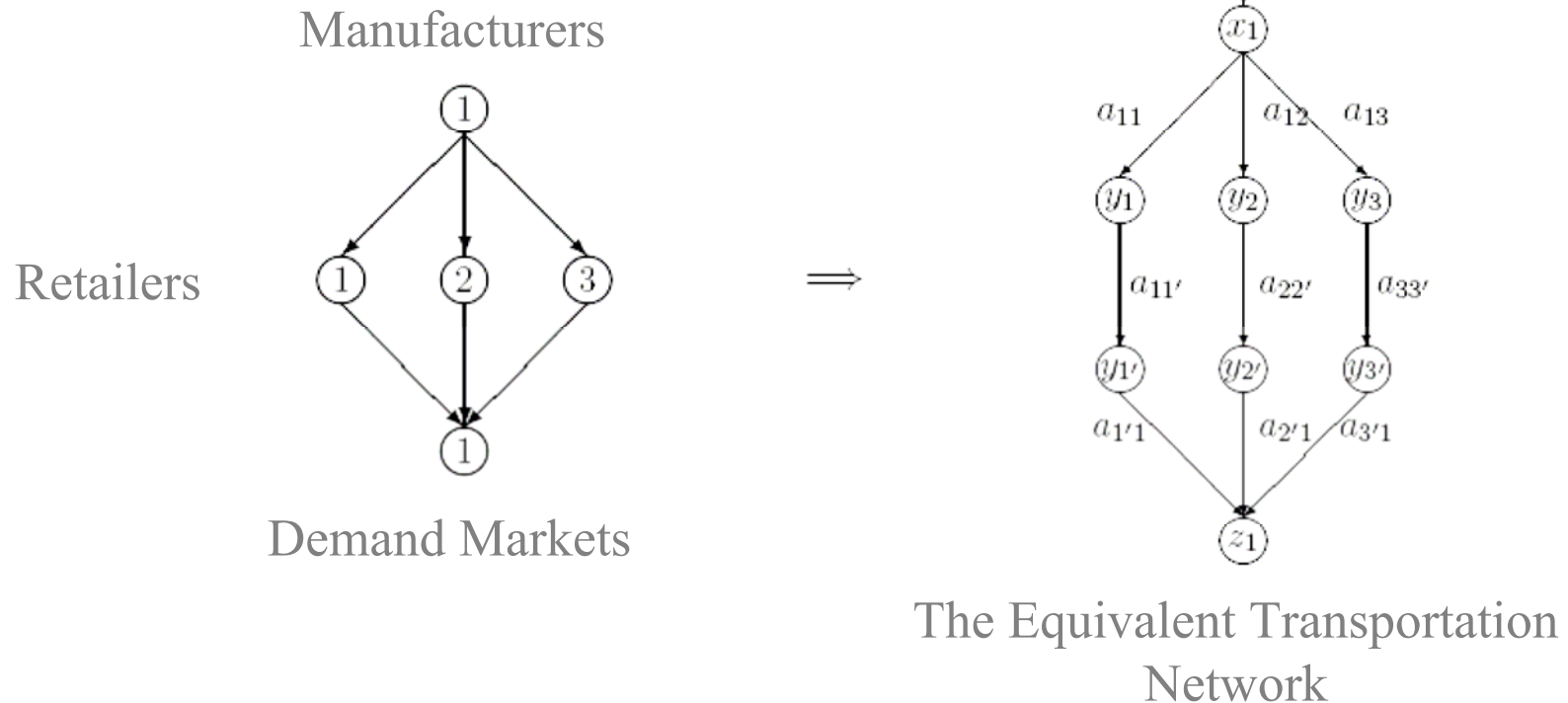
# Dynamic Supply Chain Network Examples with Computations

- Example 1





# Numerical Example 1



# Numerical Example 1

- Production cost functions

$$f_1(q_1(t)) = 2.5(q_1(t))^2 + 2q_1(t).$$

- Transaction cost functions of the products

$$c_{11}(q_{11}(t)) = .5(q_{11}(t))^2 + 3.5q_{11}(t), \quad c_{12}(q_{12}(t)) = .5(q_{12}(t))^2 + 2.5q_{12}(t),$$

$$c_{13}(q_{13}(t)) = .5(q_{13}(t))^2 + 1.5q_{13}(t).$$

# Numerical Example 1

- Handling cost functions of the retailers

$$c_1(Q^1(t)) = .5(q_{11}(t))^2, \quad c_2(Q^1(t)) = .5(q_{12}(t))^2, \quad c_3(Q^1(t)) = .5(q_{13}(t))^2.$$

- Unit transaction cost between the retailers and the demand markets

$$\hat{c}_{11}^1(Q^2(t)) = q_{11}^1(t) + 1, \quad \hat{c}_{21}^1(Q^2(t)) = q_{21}^1(t) + 5, \quad \hat{c}_{31}^1(Q^2(t)) = q_{31}^1(t) + 10.$$

# Numerical Example 1

- Three paths

$$p_1 = (a_1, a_{11}, a_{11'}, a_{1'1}), \quad p_2 = (a_1, a_{12}, a_{22'}, a_{2'1}), \quad p_3 = (a_1, a_{13}, a_{33'}, a_{3'1}).$$

- The time-varying demand function

$$d_{w_1}(t) = d_1(t) = 41 + 10t.$$

# The Solution of Numerical Example 1

- Explicit Solution
  - Path flows

$$x_{p1}^*(t) = 3.33t + 14.78,$$

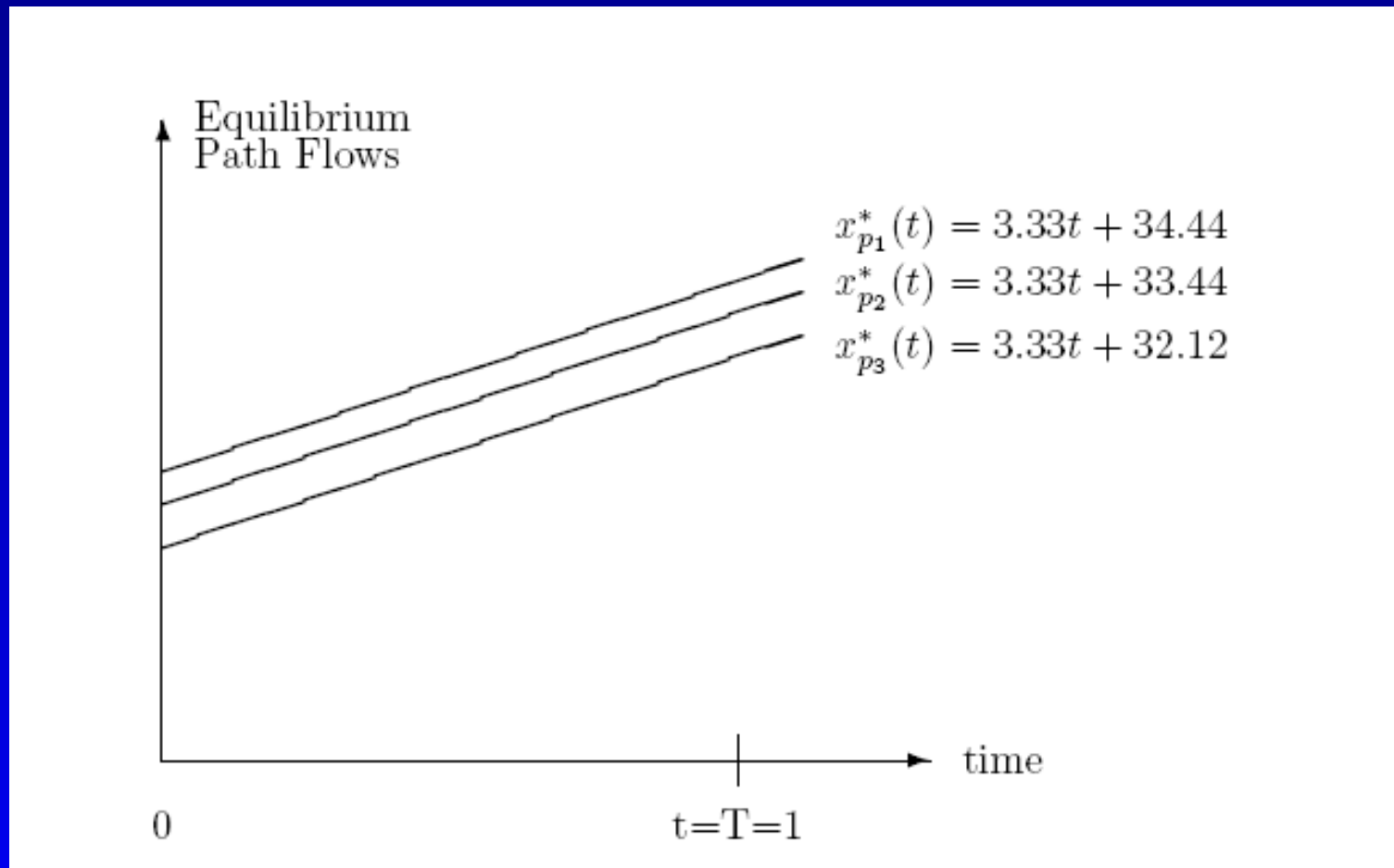
$$x_{p2}^*(t) = 3.33t + 13.78,$$

$$x_{p3}^*(t) = 3.33t + 12.45,$$

- Travel disutility

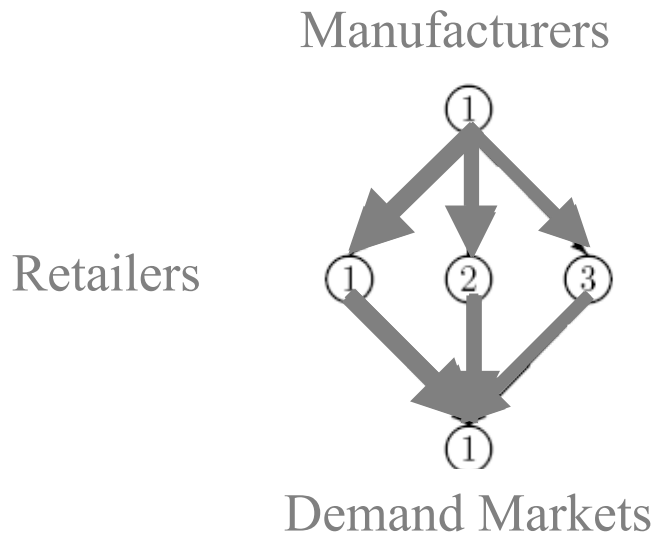
$$\lambda_{w_1}^*(t) = 60t + 255.83, \quad \text{for } t \in [0, T].$$

# Time-Dependent Equilibrium Path Flows for Numerical Example 1

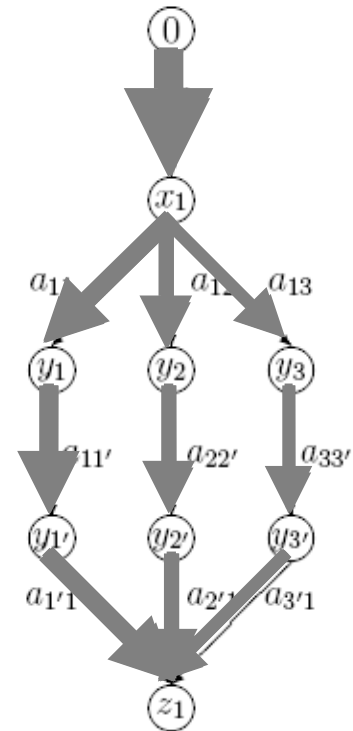


# The Solution of Numerical Example 1

$t=1$



$\Rightarrow$



The Equivalent Transportation  
Network



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# Conclusions

- We established the supernetwork equivalence of the supply chain networks with transportation networks.
- We utilized this isomorphism in the computation of the supply chain network equilibrium with time-varying demands.



# Conclusions

- We are also investigating the applications to electric power networks.
  - Nagurney, A. and Matsypura, D. A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption, Proceedings of the International Conference on Computing, Communications and Control Technologies, Austin, Texas, Volume VI: (2004) pp 127-134.)
  - Nagurney, A., Liu, Z., Cojocaru, M-G., and Daniele, P., Dynamic Electric Power Supply Chains and Transportation Networks: An Evolutionary Variational Inequality Formulation (To appear in *Transportation Research E*.)

*Thank You!*

# Optimal Endogenous Carbon Taxes for Electric Power Supply Chains with Power Plants

Trisha Woolley  
September 27, 2006

# Topics

- **Acknowledgements**
- **Paper Outline**
- **Motivation**
- **Literature**
- **Notation for Models**
- **Models**
- **Numerical Examples**
- **Conclusions**
- **References**

# Paper Outline

- Develop a modeling and computational framework that allows for the determination of optimal carbon taxes applied to electric power plants in the context of electric power supply chain (generation/distribution/consumption) networks.
- The general framework that we develop allows for three distinct types of carbon taxation environmental policies
  - A completely decentralized scheme in which taxes can be applied to each individual power generator/ power plant in order to guarantee that each assigned emission bound is not exceeded
  - A centralized scheme which assumes a fixed bound over the entire electric power supply chain in terms of total carbon emissions
  - A centralized scheme which assumes the bound to be a function of the tax.
- Twelve numerical examples are presented in which the optimal carbon taxes, as well as the equilibrium electric power flows and demands, are computed.

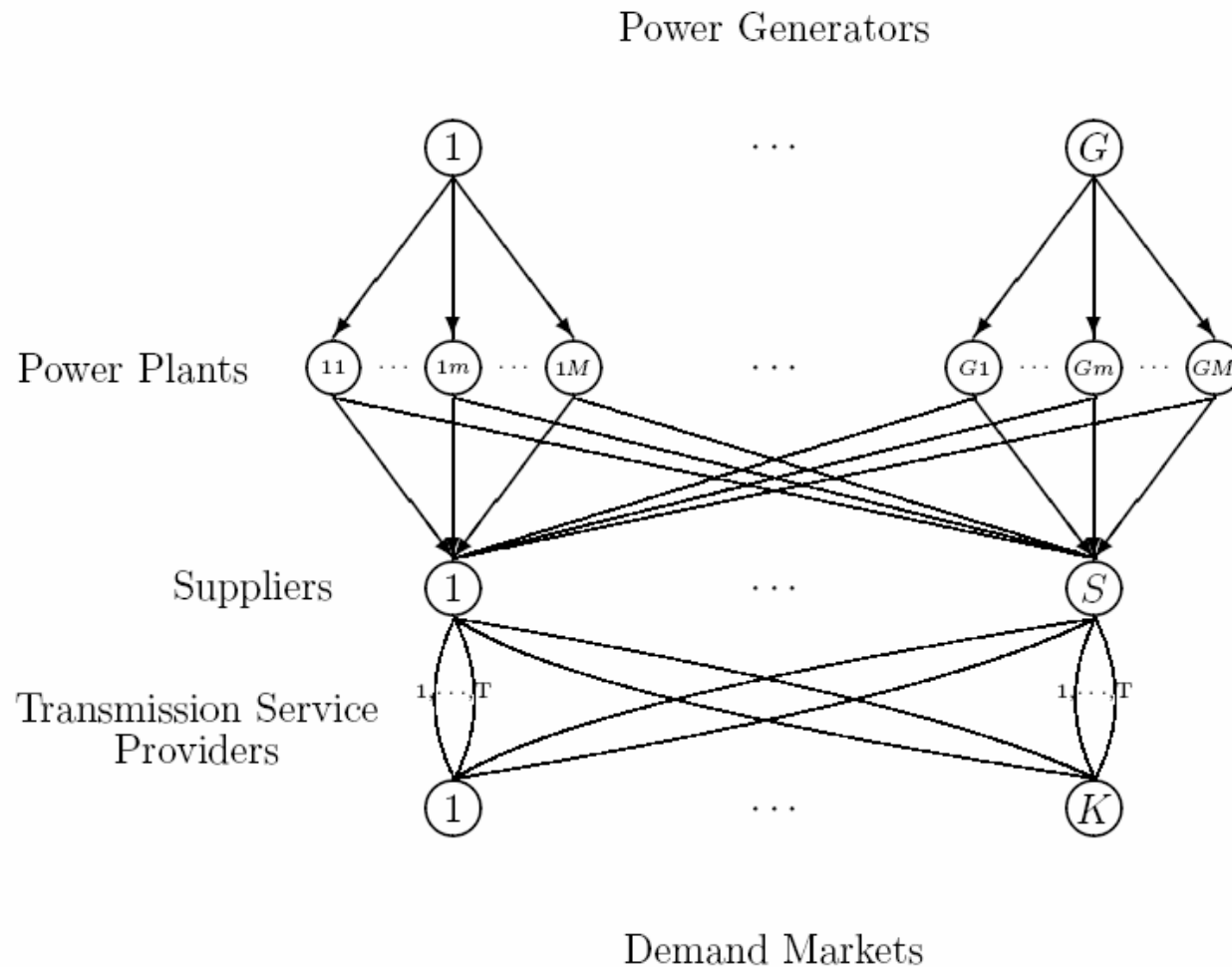
# Motivation

- The electrical industry is growing, with the total global consumption of electricity to reach 23.1 trillion kilowatt hours in 2025.
- Of the total U.S. emissions of carbon dioxide and nitrous oxide, more than a third arises from generating electricity.
- Accumulated evidence of global warming.
- Need for environmental-energy modeling which include carbon taxes to address market failures in energy.

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# The Electric Power Supply Chain Network with Power Plants





# Notation for the Electric Power Supply Chain Network Model

| Notation           | Definition  |
|--------------------|---|
| $q_{gm}$           | quantity of electricity produced by generator $g$ using power plant $m$ , where $g = 1, \dots, G; m = 1, \dots, M$  |
| $q_m$              | $G$ -dimensional vector of electric power generated by the gencos using power plant $m$ with components: $q_{1m}, \dots, q_{Gm}$  |
| $q$                | $GM$ -dimensional vector of all the electric power outputs generated by the gencos at the power plants  |
| $Q^1$              | $GMS$ -dimensional vector of electric power flows between the power plants of the power generators and the power suppliers with component $gms$ denoted by $q_{gms}$  |
| $Q^2$              | $STK$ -dimensional vector of power flows between suppliers and demand markets with component $stk$ denoted by $q_{sk}^t$ and denoting the flow between supplier $s$ and demand market $k$ via transmission provider $t$ |
| $d$                | $K$ -dimensional vector of market demands with component $k$ denoted by $d_k$   |
| $f_{gm}(q_m)$      | power generating cost function of power generator $g$ using power plant $m$ with marginal power generating cost with respect to $q_{gm}$ denoted by $\frac{\partial f_{gm}}{\partial q_{gm}}$                           |
| $c_{gms}(q_{gms})$ | transaction cost incurred by power generator $g$ using power plant $m$ in transacting with power supplier $s$ with marginal transaction cost denoted by $\frac{\partial c_{gms}(q_{gms})}{\partial q_{gms}}$            |



# Notation for the Electric Power Supply Chain Network Model

|                          |  |
|--------------------------|--|
| $e_{gm}$                 | amount of carbon emitted by genco $g$ using power plant $m$ per unit of electric power produced  |
| $h$                      | $S$ -dimensional vector of the power suppliers' supplies of the electric power with component $s$ denoted by $h_s$ , with $h_s \equiv \sum_{g=1}^G \sum_{m=1}^M q_{gms}$   |
| $c_s(h) \equiv c_s(Q^1)$ | operating cost of power supplier $s$ with marginal operating cost with respect to $h_s$ denoted by $\frac{\partial c_s}{\partial h_s}$ and the marginal operating cost with respect to $q_{gms}$ denoted by $\frac{\partial c_s(Q^1)}{\partial q_{gms}}$ |
| $c_{sk}^t(q_{sk}^t)$     | transaction cost incurred by power supplier $s$ in transacting with demand market $k$ via transmission provider $t$ with marginal transaction cost with respect to $q_{sk}^t$ denoted by $\frac{\partial c_{sk}^t(q_{sk}^t)}{\partial q_{sk}^t}$         |
| $\hat{c}_{gms}(q_{gms})$ | transaction cost incurred by power supplier $s$ in transacting with power generator $g$ for power generated by plant $m$ with marginal transaction cost denoted by $\frac{\partial \hat{c}_{gms}(q_{gms})}{\partial q_{gms}}$                            |
| $\hat{c}_{sk}^t(Q^2)$    | unit transaction cost incurred by consumers at demand market $k$ in transacting with power supplier $s$ via transmission provider $t$  |
| $\rho_{3k}(d)$           | demand market price function at demand market $k$  |

# A Decentralized Carbon Taxation Scheme

The Optimization problem of the power generator can be expressed as follows:

$$\text{Maximize} \quad \sum_{m=1}^M \sum_{s=1}^S \rho_{1gms}^* q_{gms} - \sum_{m=1}^M f_{gm}(q_m) - \sum_{m=1}^M \sum_{s=1}^S c_{gms}(q_{gms}) - \sum_{m=1}^M \tau_{gm}^* e_{gm} q_{gm}$$

subject to:

$$\begin{aligned} \sum_{s=1}^S q_{gms} &= q_{gm}, \quad m = 1, \dots, M, \\ q_{gms} &\geq 0, \quad m = 1, \dots, M; s = 1, \dots, S. \end{aligned}$$

The Optimization problem faced by the supplier can be expressed as follows:

$$\text{Maximize} \quad \sum_{k=1}^K \sum_{t=1}^T \rho_{2sk}^{t*} q_{sk}^t - c_s(Q^1) - \sum_{g=1}^G \sum_{m=1}^M \rho_{1gms}^* q_{gms} - \sum_{g=1}^G \sum_{m=1}^M \hat{c}_{gms}(q_{gms}) - \sum_{k=1}^K \sum_{t=1}^T c_{sk}^t(q_{sk}^t)$$

subject to:

$$\begin{aligned} \sum_{k=1}^K \sum_{t=1}^T q_{sk}^t &= \sum_{g=1}^G \sum_{m=1}^M q_{gms} \\ q_{gms} &\geq 0, \quad g = 1, \dots, G, \quad m = 1, \dots, M, \\ q_{sk}^t &\geq 0, \quad k = 1, \dots, K; t = 1, \dots, T. \end{aligned}$$



# A Decentralized Carbon Taxation Scheme

## Market Equilibrium Conditions at Demand Market k

$$\rho_{2sk}^{t*} + \hat{c}_{sk}^t(Q^{2*}) \begin{cases} = \rho_{3k}(d^*), & \text{if } q_{sk}^{t*} > 0, \\ \geq \rho_{3k}(d^*), & \text{if } q_{sk}^{t*} = 0. \end{cases}$$

## Decentralized Carbon Tax Equilibrium conditions

$$\bar{B}_{gm} - e_{gm}q_{gm}^* \begin{cases} = 0, & \text{if } \tau_{gm}^* > 0, \\ \geq 0, & \text{if } \tau_{gm}^* = 0. \end{cases}$$

# Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Decentralized Carbon Taxes

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine  $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, \tau^*) \in \mathcal{K}^5$  satisfying:

$$\begin{aligned} & \sum_{g=1}^G \sum_{m=1}^M \left[ \frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm}^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^S \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] \\ & + \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[ \frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*] \\ & + \sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[ \frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^K \rho_{3k}(d^*) \times [d_k - d_k^*] \\ & + \sum_{g=1}^G \sum_{m=1}^M [\bar{B}_{gm} - e_{gm} q_{gm}^*] \times [\tau_{gm} - \tau_{gm}^*] \geq 0, \end{aligned}$$

$$\forall (q, h, Q^1, Q^2, d, \tau) \in \mathcal{K}^5,$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d, \tau) | (q, h, Q^1, Q^2, d, \tau) \in R_+^{2GM+S+GMS+TSK+K} \text{ and the constraints hold.}\}$$



# A Centralized Carbon Taxation Scheme

## Centralized Carbon Tax Equilibrium conditions with a Fixed Bound

$$\bar{B} - \sum_{g=1}^G \sum_{m=1}^M e_{gm} q_{gm}^* \begin{cases} = 0, & \text{if } \mathcal{T}^* > 0, \\ \geq 0, & \text{if } \mathcal{T}^* = 0. \end{cases} \quad (23)$$

Clearly equilibrium conditions (23) can be formulated as the inequality:

$$\left[ \bar{B} - \sum_{g=1}^G \sum_{m=1}^M e_{gm} q_{gm}^* \right] \times [T - \mathcal{T}^*] \geq 0, \quad \forall T \geq 0.$$

# Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Centralized Carbon Taxes and a Fixed Upper Bound

The equilibrium conditions governing the electric power supply chain network according to Definition 2 coincide with the solution of the variational inequality given by: determine  $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, T^*) \in \mathcal{K}^6$  satisfying:

$$\begin{aligned} & \sum_{g=1}^G \sum_{m=1}^M \left[ \frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + T^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^S \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] \\ & + \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[ \frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*] \\ & + \sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[ \frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^K \rho_{3k}(d^*) \times [d_k - d_k^*] \\ & + \left[ \bar{B} - \sum_{g=1}^G \sum_{m=1}^M e_{gm} q_{gm}^* \right] \times [T - T^*] \geq 0, \quad \forall (q, h, Q^1, Q^2, d, T) \in \mathcal{K}^4, \end{aligned}$$

where

$$\mathcal{K}^5 \equiv \{(q, h, Q^1, Q^2, d, T) | (q, h, Q^1, Q^2, d, T) \in R_+^{GM+S+GMS+TSK+K+1} \text{ and the constraints hold.}\}$$

# A Centralized Carbon Taxation Scheme

## Centralized Carbon Tax Equilibrium conditions with an Elastic Bound

In this case, the carbon tax equilibrium conditions (cf. (23)) would be as follows.

$$B(T^*) - \sum_{g=1}^G \sum_{m=1}^M e_{gm} q_{gm}^* \begin{cases} = 0, & \text{if } T^* > 0, \\ \geq 0, & \text{if } T^* = 0. \end{cases}$$

Clearly equilibrium conditions (23) can be formulated as the inequality:

$$\left[ B(T^*) - \sum_{g=1}^G \sum_{m=1}^M e_{gm} q_{gm}^* \right] \times [T - T^*] \geq 0, \quad \forall T \geq 0.$$



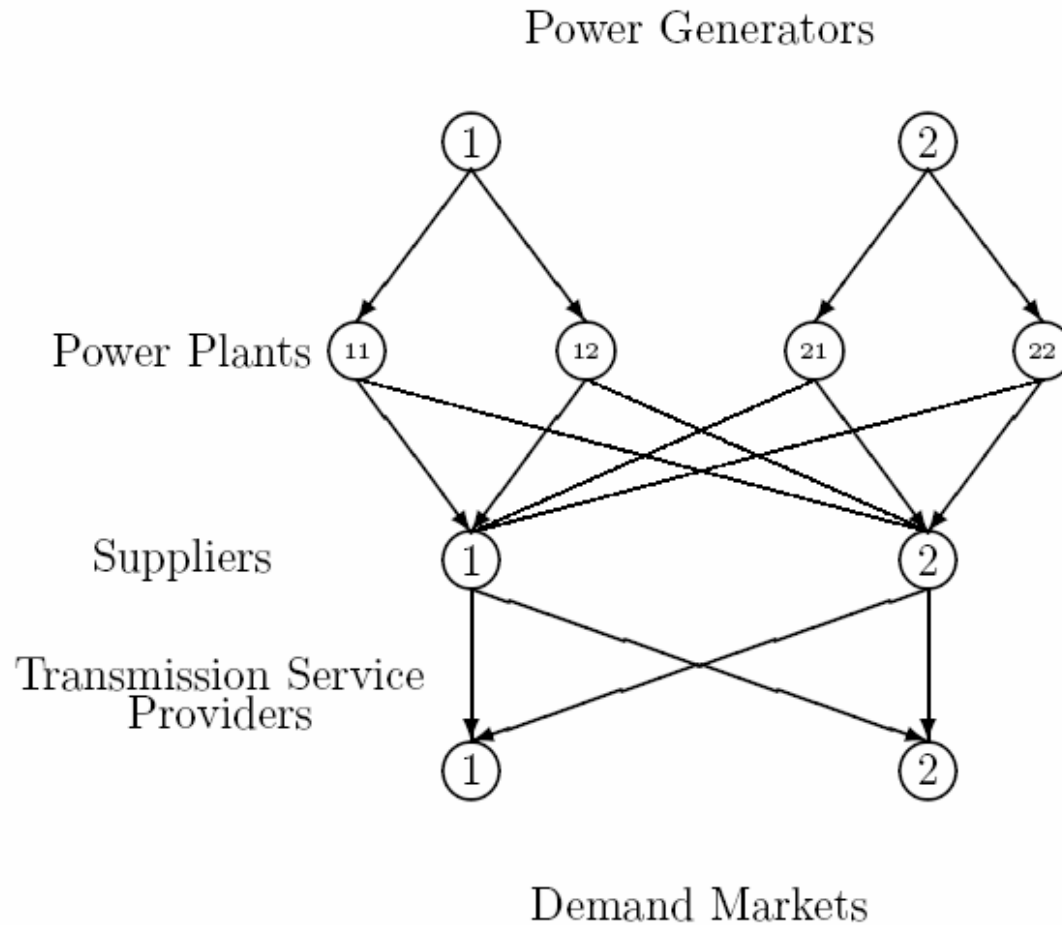
# Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium with Centralized Carbon Taxes and an Elastic Carbon Emission Bound

The equilibrium conditions governing the electric power supply chain network according to Definition 3 coincide with the solution of the variational inequality given by: determine  $(q^*, h^*, Q^{1*}, Q^{2*}, d^*, T^*) \in \mathcal{K}^6$  satisfying:

$$\begin{aligned}
 & \sum_{g=1}^G \sum_{m=1}^M \left[ \frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + T^* e_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^S \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] \\
 & + \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[ \frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*] \\
 & + \sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[ \frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^K \rho_{3k}(d^*) \times [d_k - d_k^*] \\
 & + \left[ B(T^*) - \sum_{g=1}^G \sum_{m=1}^M e_{gm} q_{gm}^* \right] \times [T - T^*] \geq 0, \\
 & \forall (q, h, Q^1, Q^2, d, T) \in \mathcal{K}^6.
 \end{aligned}$$



# 12 Numerical Examples



## Numerical Examples 1 ,2 ,3, and 4

Instances of an electric power supply chain network equilibrium model with a *Decentralized* Carbon taxation scheme.

## Numerical Examples 5, 6, 7, and 8

Instances of an electric power supply chain network equilibrium model with a *Centralized* Carbon taxation scheme with a *Fixed* bound on the carbon emissions.

## Numerical Examples 9, 10, 11, and 12

Instances of an electric power supply chain network equilibrium model with a *Centralized* Carbon taxation scheme with a *Variable* bound on the carbon emissions.

# Solutions to Numerical Examples 1 ,2 ,3, and 4

| Equilibrium Solution             | Example 1 | Example 2 | Example 3 | Example 4 |
|----------------------------------|-----------|-----------|-----------|-----------|
| Computed Equilibrium Power Flows |           |           |           |           |
| $q_{11}^*$                       | 22.56     | 29.86     | 23.00     | 11.51     |
| $q_{12}^*$                       | 9.93      | 31.17     | 23.00     | 23.02     |
| $q_{21}^*$                       | 22.90     | 30.20     | 23.00     | 23.02     |
| $q_{22}^*$                       | 92.38     | 23.00     | 23.00     | 23.05     |
| $q_{111}^*$                      | 11.28     | 14.93     | 11.50     | 5.76      |
| $q_{112}^*$                      | 11.28     | 14.93     | 11.50     | 5.76      |
| $q_{121}^*$                      | 4.97      | 15.59     | 11.50     | 11.51     |
| $q_{122}^*$                      | 4.97      | 15.59     | 11.50     | 11.51     |
| $q_{211}^*$                      | 11.45     | 15.10     | 11.50     | 11.51     |
| $q_{212}^*$                      | 11.45     | 15.10     | 11.50     | 11.51     |
| $q_{221}^*$                      | 46.19     | 11.50     | 11.50     | 11.52     |
| $q_{222}^*$                      | 46.19     | 11.50     | 11.50     | 11.52     |
| $h_1^*$                          | 73.89     | 57.12     | 46.00     | 40.30     |
| $h_2^*$                          | 73.89     | 57.12     | 46.00     | 40.30     |
| $q_{11}^{1*}$                    | 36.94     | 28.56     | 23.00     | 20.15     |
| $q_{12}^{1*}$                    | 36.94     | 28.56     | 23.00     | 20.15     |
| $q_{21}^{1*}$                    | 36.94     | 28.56     | 23.00     | 20.15     |
| $q_{22}^{1*}$                    | 36.94     | 28.56     | 23.00     | 20.15     |
| Computed Equilibrium Demands     |           |           |           |           |
| $d_1^*$                          | 73.89     | 57.12     | 46.00     | 40.30     |
| $d_2^*$                          | 73.89     | 57.12     | 46.00     | 40.30     |
| Computed Optimal Taxes           |           |           |           |           |
| $\tau_{11}^*$                    | 0.00      | 0.00      | 76.43     | 77.86     |
| $\tau_{12}^*$                    | 0.00      | 0.00      | 76.43     | 92.38     |
| $\tau_{21}^*$                    | 0.00      | 0.00      | 77.93     | 105.41    |
| $\tau_{22}^*$                    | 0.00      | 130.26    | 169.93    | 185.96    |

## Solutions to Numerical Examples 5, 6, 7, and 8

| Equilibrium Solution             | Example 5 | Example 6 | Example 7 | Example 8 |
|----------------------------------|-----------|-----------|-----------|-----------|
| Computed Equilibrium Power Flows |           |           |           |           |
| $q_{11}^*$                       | 15.20     | 7.48      | 2.85      | 2.87      |
| $q_{12}^*$                       | 6.63      | 3.17      | 1.10      | 1.10      |
| $q_{21}^*$                       | 15.53     | 7.82      | 3.19      | 3.20      |
| $q_{22}^*$                       | 62.65     | 31.53     | 12.86     | 12.91     |
| $q_{111}^*$                      | 7.60      | 3.74      | 1.43      | 1.43      |
| $q_{112}^*$                      | 7.60      | 3.74      | 1.43      | 1.43      |
| $q_{121}^*$                      | 3.31      | 1.59      | 0.55      | 0.55      |
| $q_{122}^*$                      | 3.31      | 1.59      | 0.55      | 0.55      |
| $q_{211}^*$                      | 7.76      | 3.91      | 1.59      | 1.60      |
| $q_{212}^*$                      | 7.76      | 3.91      | 1.59      | 1.60      |
| $q_{221}^*$                      | 31.32     | 15.77     | 6.43      | 6.46      |
| $q_{222}^*$                      | 31.32     | 15.77     | 6.43      | 6.46      |
| $h_1^*$                          | 50.00     | 25.00     | 10.00     | 10.00     |
| $h_2^*$                          | 50.00     | 25.00     | 10.00     | 10.00     |
| $q_{11}^{1*}$                    | 25.00     | 12.50     | 5.00      | 0.00      |
| $q_{12}^{1*}$                    | 25.00     | 12.50     | 5.00      | 10.00     |
| $q_{21}^{1*}$                    | 25.00     | 12.50     | 5.00      | 0.00      |
| $q_{22}^{1*}$                    | 25.00     | 12.50     | 5.00      | 10.00     |
| Computed Equilibrium Demands     |           |           |           |           |
| $d_1^*$                          | 50.00     | 25.00     | 10.00     | 0.00      |
| $d_2^*$                          | 50.00     | 25.00     | 10.00     | 20.00     |
| Computed Optimal Tax             |           |           |           |           |
| $\mathcal{T}^*$                  | 115.50    | 236.38    | 308.91    | 656.96    |

# Solutions to Numerical Examples 9, 10, 11, and 12

| Equilibrium Solution             | Example 9 | Example 10 | Example 11 | Example 12 |
|----------------------------------|-----------|------------|------------|------------|
| Computed Equilibrium Power Flows |           |            |            |            |
| $q_{11}^*$                       | 20.41     | 18.15      | 16.80      | 27.32      |
| $q_{12}^*$                       | 8.96      | 7.95       | 7.35       | 12.06      |
| $q_{21}^*$                       | 20.74     | 18.48      | 17.13      | 27.65      |
| $q_{22}^*$                       | 83.68     | 74.58      | 69.11      | 111.57     |
| $q_{111}^*$                      | 10.20     | 9.08       | 8.40       | 13.66      |
| $q_{112}^*$                      | 10.20     | 9.08       | 8.40       | 13.66      |
| $q_{121}^*$                      | 4.48      | 3.98       | 3.67       | 6.03       |
| $q_{122}^*$                      | 4.48      | 3.98       | 3.67       | 6.03       |
| $q_{211}^*$                      | 10.37     | 9.24       | 8.57       | 13.83      |
| $q_{212}^*$                      | 10.37     | 9.24       | 8.57       | 13.83      |
| $q_{221}^*$                      | 41.84     | 37.29      | 34.56      | 55.79      |
| $q_{222}^*$                      | 1.84      | 37.29      | 34.56      | 55.79      |
| $h_1^*$                          | 66.90     | 59.58      | 55.19      | 89.31      |
| $h_2^*$                          | 66.90     | 59.58      | 55.19      | 89.31      |
| $q_{11}^{1*}$                    | 33.45     | 29.79      | 27.60      | 0.00       |
| $q_{12}^{1*}$                    | 33.45     | 29.79      | 27.60      | 89.31      |
| $q_{21}^{1*}$                    | 33.45     | 29.79      | 27.60      | 0.00       |
| $q_{22}^{1*}$                    | 33.45     | 29.79      | 27.60      | 89.31      |
| Computed Equilibrium Demands     |           |            |            |            |
| $d_1^*$                          | 66.90     | 59.58      | 55.19      | 0.00       |
| $d_2^*$                          | 66.90     | 59.58      | 55.19      | 178.61     |
| Computed Optimal Tax             |           |            |            |            |
| $\mathcal{T}^*$                  | 33.79     | 69.16      | 90.38      | 128.61     |

# Conclusions

- The model presented in this paper may help policy-makers to determine the optimal carbon taxes on the power plants in the electric power generation industry.
- The first model, a completely decentralized scheme, allows the policy-makers to determine the optimal tax for each individual electric power plant which guarantees that the emission bound or quota of each plant is not exceeded.
- The second and third models, on the other hand, both enforce a “global” emission bound on the entire industry by imposing a uniform tax rate on the generating plants.

# Conclusions

- The numerical results demonstrate, as the theory predicts, that the carbon taxes achieve the desired goal, in that the imposed bounds on the carbon emissions are not exceeded.
- Moreover, they illustrate the spectrum of scenarios that can be explored in terms of changes in the bounds on the carbon emissions; changes in emission factors; changes in the demand functions, etc.



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*Thank you!*



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# Overview of Supply Chain Reliability and Robustness

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# Definition of Supply Chain (SC)

- “A supply chain is a coordinated system of organizations, people, activities, information and resources involved in moving a product or service in physical or virtual manner from supplier to customer.”(R.G. Poluha, 2006)

# Objective of SCs

- The primary objective of a supply chain is “...to maximize the overall value generated...” for the product/service providers in tiers of the SC. (Chopra, Sunil and Meindl 2004)

# Challenges of SC Design

- Decisions are costly and irreversible.
- Uncertainties and risk threat the operation efficiency of SCs.
- Supply side risk vs. demand side risk



# Relevance of SC Reliability and Robustness

- Several recent events have signified the importance of the research in SC reliability and robustness issue (Jüttner, Peck Christopher 2003).
  - Natural disaster (e.g. Hurricane Katrina)
  - Intentional disruptions (e.g. September 11)
  - Labor actions (e.g. West Coast Port Lockout)

# Definition of SC Reliability and Robustness

- No broad definition in the literature.
  - Existing definitions of SC reliability/robustness are presented from specific perspective (e.g. lead time, responsiveness etc.).
- Generally speaking, “a supply chain is *robust* if it performs well with respect to uncertain future conditions; a supply chain is *reliable* if it performs well when parts of the system fail”. (L.V. Snyder 2003)

# Other Definitions

- “Robustness is a design principle of natural, engineering, or social systems that have been designed or selected for stability.” (Santa Fe Institute)
- “A robust solution in an optimization problem is one that has the best performance under its worst case.” (Kouvelis and Yu 1997)
- “Supply chain reliability is defined as the probability of the chain meeting mission requirements to provide the required supplies to the critical transfer points within the system.” (Thomas 2002)

# Relevant Literature

- Case studies on resilient supply chain design (Y. Sheffi 2005).
- Facility location problem to minimize the total failure cost of a SC (Daskin, Snyder 2005).
- Contingency procumbent strategies under the uncertain supplies (Tomlin 2006).

# Strategies to Increase SC Reliability/Robustness

- Forming strong alliance with SC partners.
- Collaborating and sharing risk.
- Keeping “Strategic Emergency Stock”.
- Risk pooling.
- Increasing SC flexibility.

# Conflicted Goals

- The lowest-cost supplier vs. the known supplier
- Centralization vs. dispersion
- Redundancy vs. efficiency
- Information sharing vs. maintain competitiveness

# Transportation Network vs. SC

- Nagurney (2005) has shown that SC network can be reformulated as a transportation network.
- Reliability and robustness analysis can be applied to all relevant networks.

# Thank You

*For more information, please see the*



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