Supply Chain Supernetworks With Random Demands

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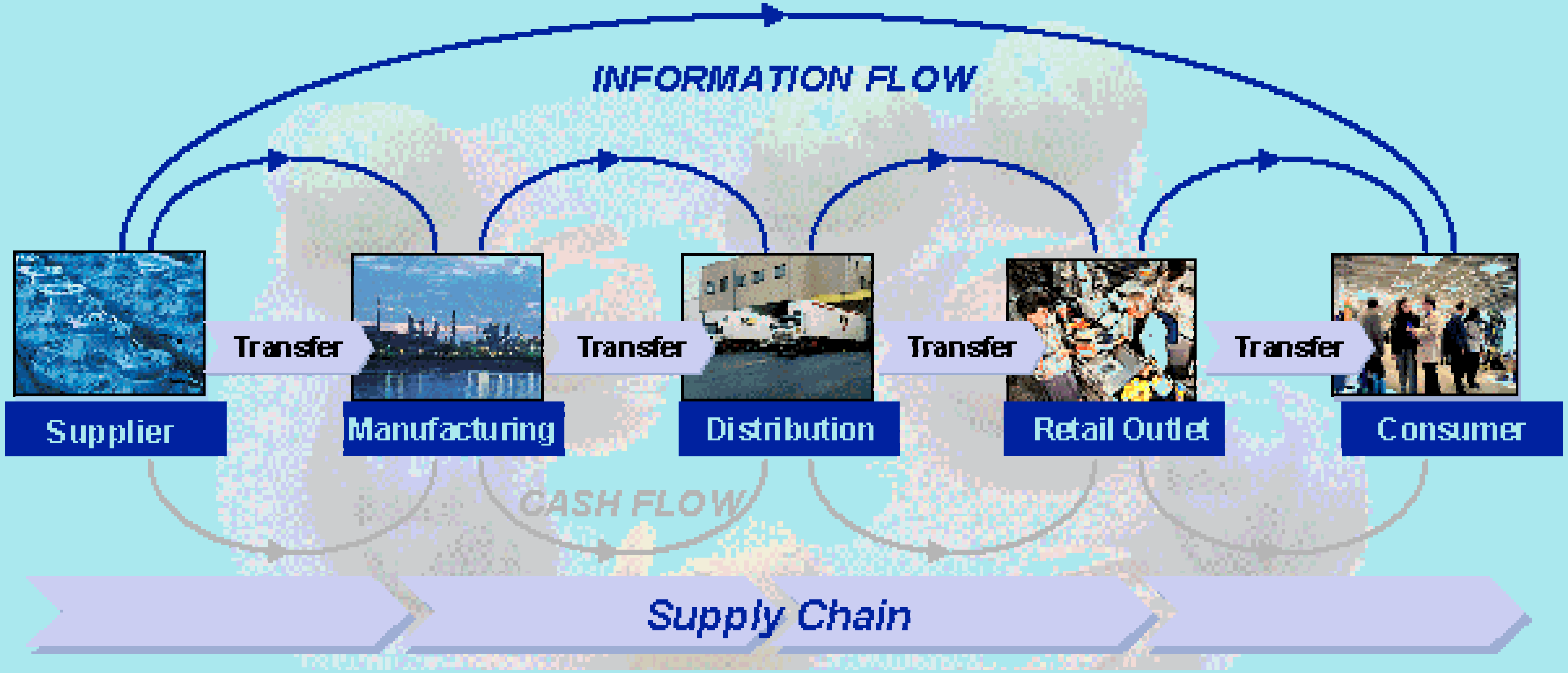
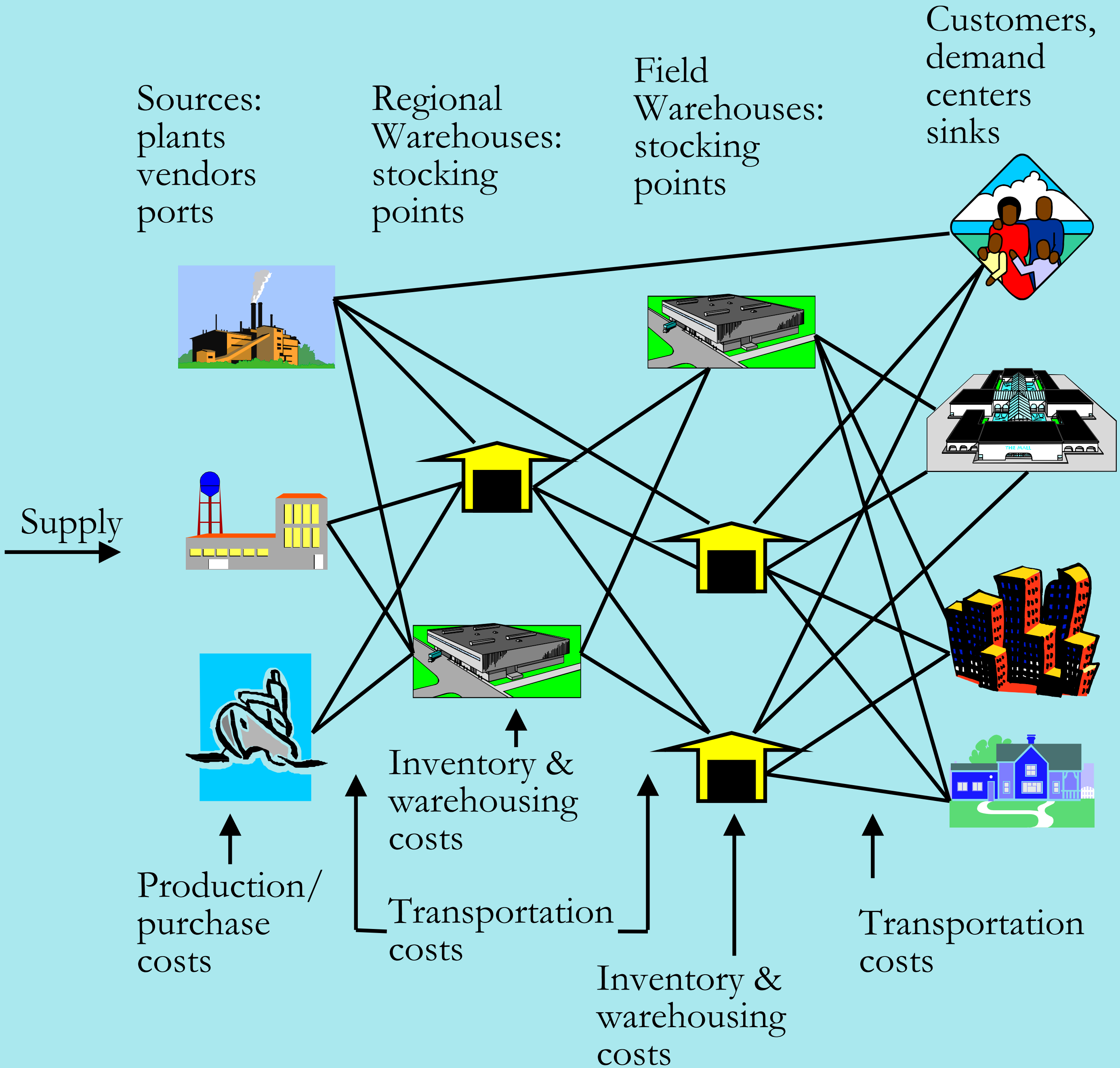
2005. 5. 28


INFORMATION FLOW
Transfer
Transfer
Transfer
Transfer
Supplier
Manufacturing
Distribution
Retail Outlet
Consumer
CASH FLOW
Supply Chain


Sources:
plants
vendors
ports
Regional
Warehouses:
stocking
points
Field
Warehouses:
stocking
points
Customers,
demand
centers
sinks
Supply
Production/
purchase
costs
Inventory &
warehousing
costs
Transportation
costs
Inventory &
warehousing
costs
Transportation
costs

Supply Chain Management

Definition:

Supply Chain Management is primarily concerned with the efficient integration of *suppliers, factories, warehouses and stores* so that merchandise is produced and distributed in the right quantities, to the right locations and at the right time, and so as to minimize total system cost subject to satisfying service requirements.

What do we see?

Coordination within a supply chain among manufacturer, distributor, and retailer

as well as

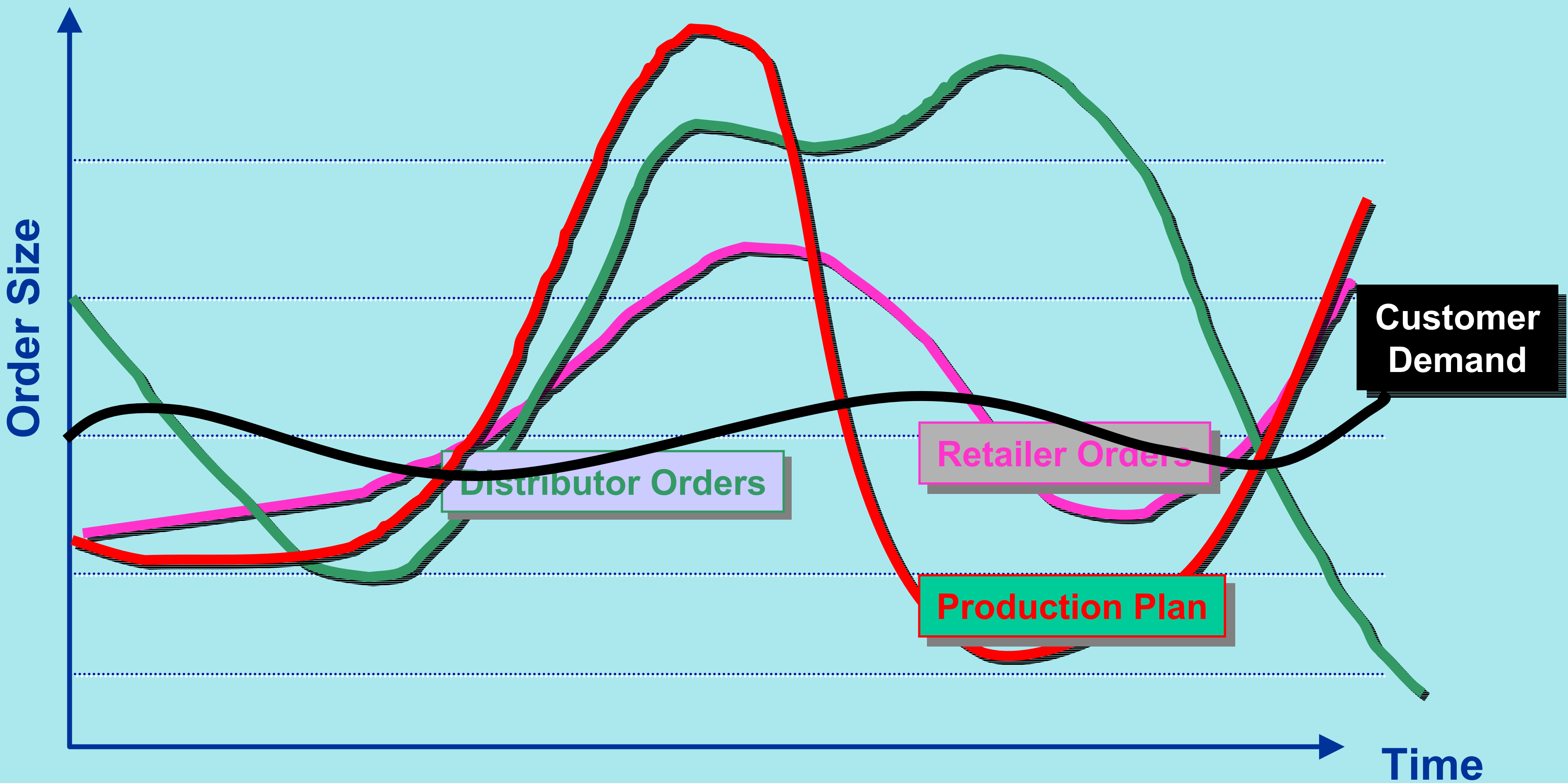
Competition among

manufacturers

distributors

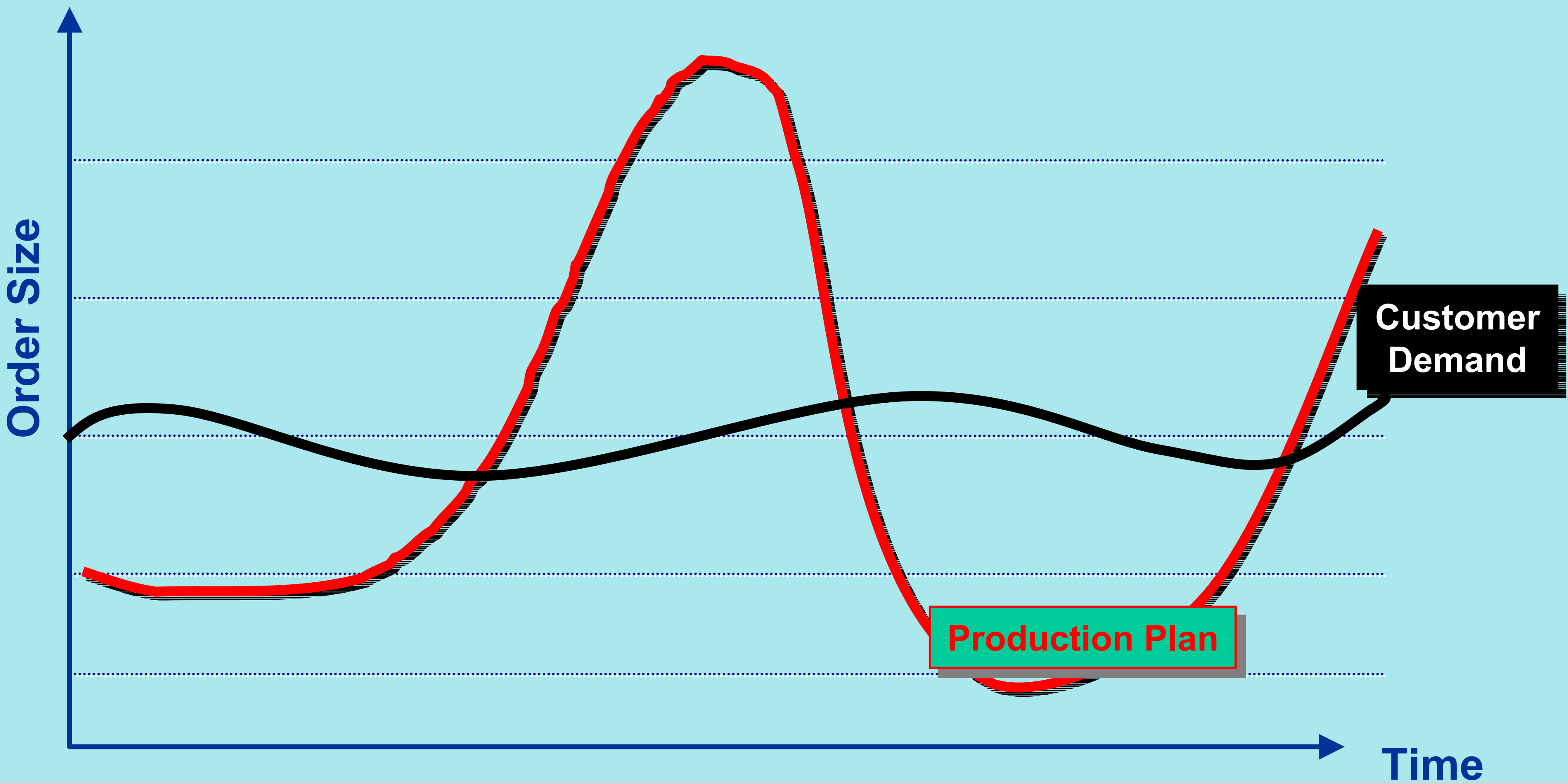
retailers

The Dynamics of the Supply Chain



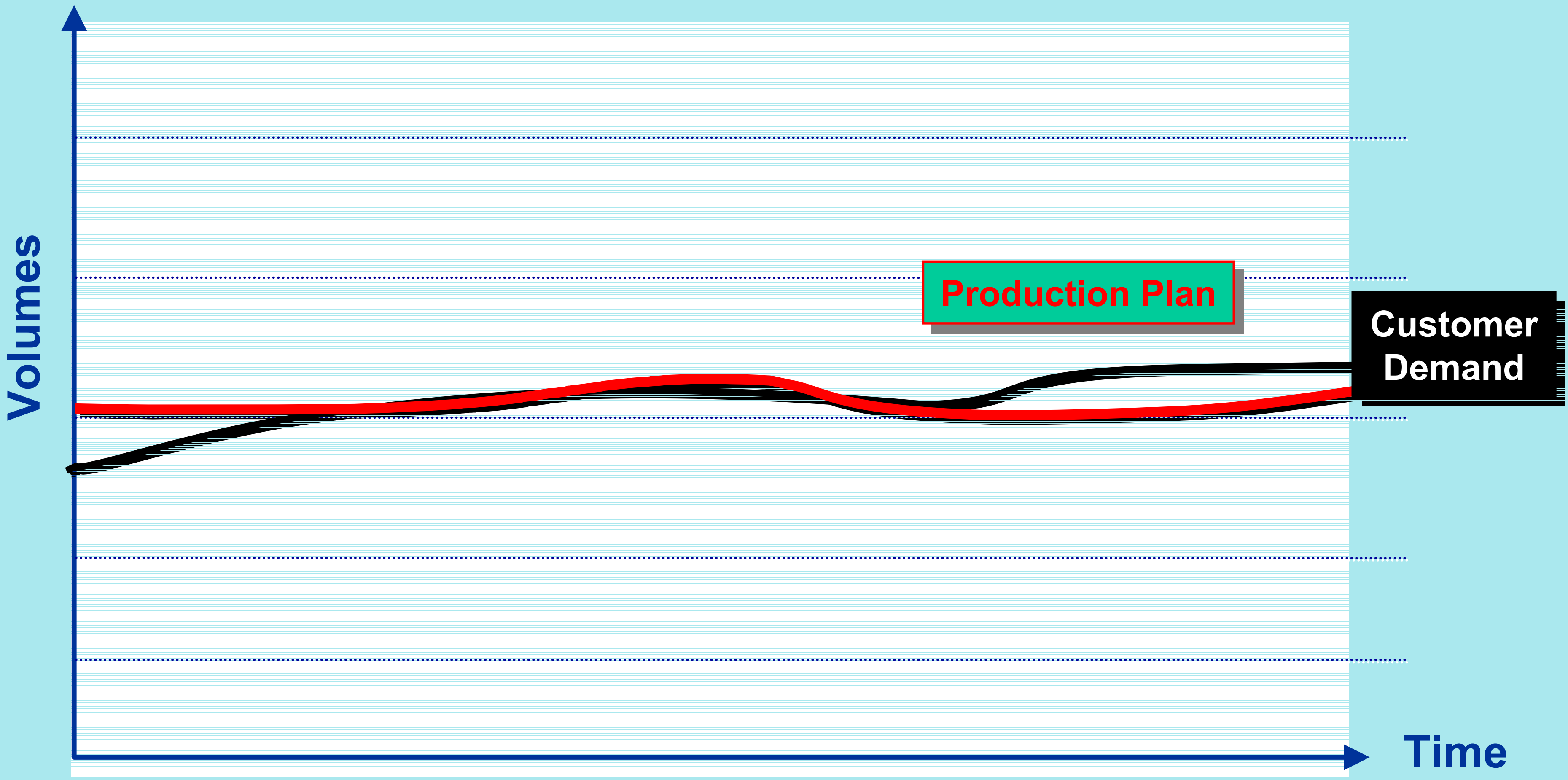
Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

What Management Gets...



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

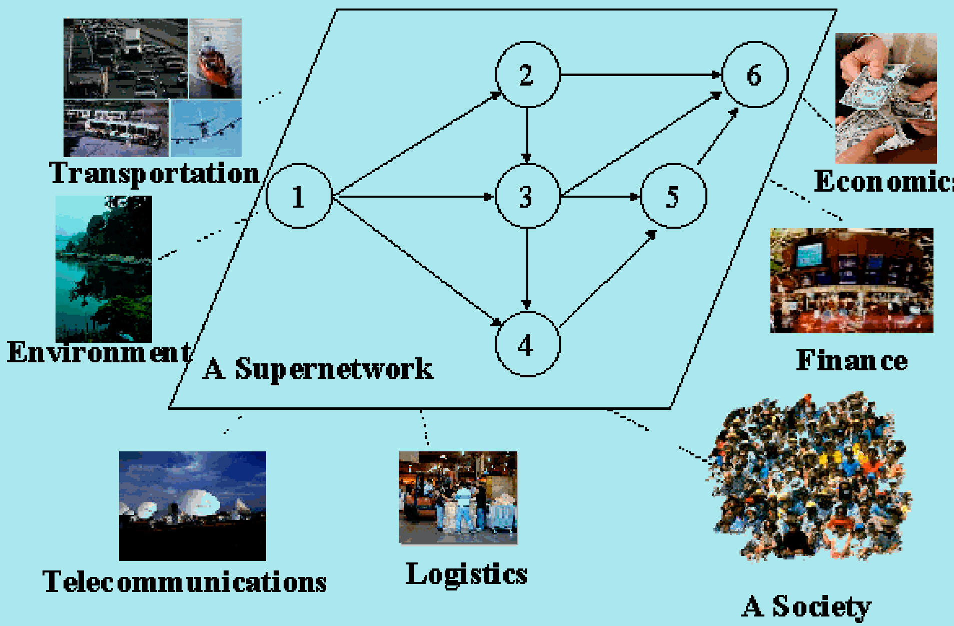
What Management Wants…

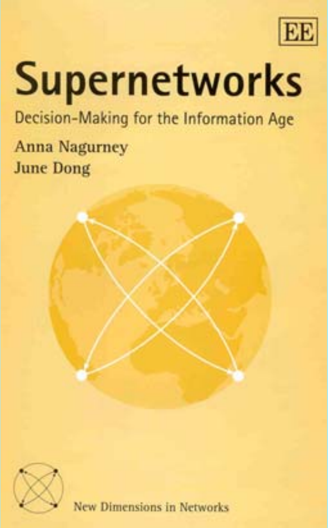


Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

What is a Supernetwork?

A Supernetwork **is a network, consisting of nodes, links, and flows, that is** *over* ***and*** *above* **existing networks.**





Applications of Supernetworks

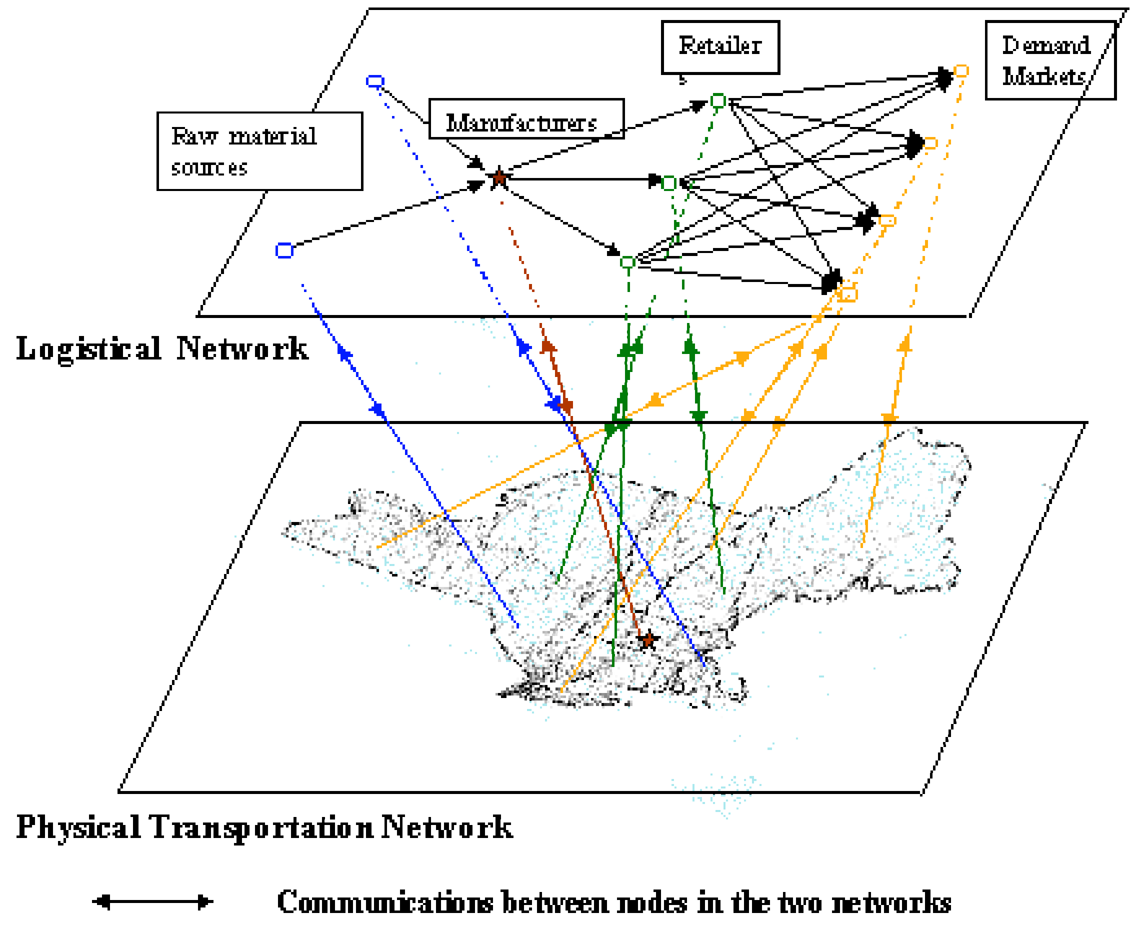
Telecommuting/Commuting Decision-Making

Teleshopping/Shopping Decision-Making

Supply Chain Networks with Electronic Commerce

Financial Networks with Electronic Transactions.

A Multitiered Supply Chain Supernetwork Example



Dong, Zhang, Nagurney, "A Supply Chain Network Equilibrium Model with Random Demands," European Journal of Operations Research, 2004.

- **Assume general random demands facing retailers**
- **Study Noncooperative behaviors**
- **Formulate the optimality conditions as VIP**
- **Give conditions of the distribution functions for the existence and uniqueness of the solution**
- **Algorithms**

The Supernetwork Supply Chain Model with Random Demands

Captures

- multi-tiers
  - manufacturers
  - distributors
  - retailers
  - demand markets
- competition at the same tier
- Coordination among different tiers

Determines

- production quantities
- shipments
- prices
- expected demands

Includes

- physical transactions
- electronic transactions

Supernetwork Structure

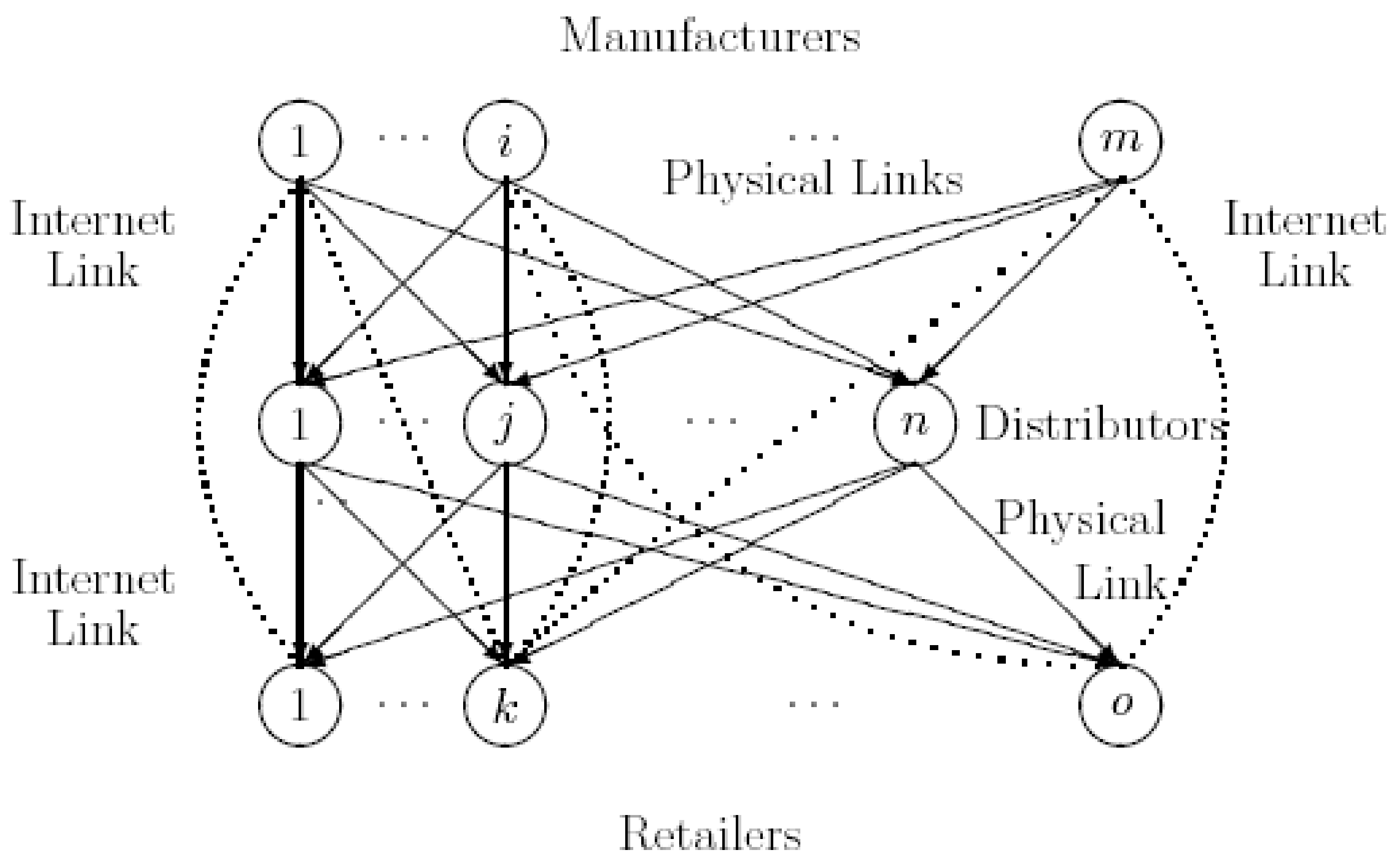


Figure 1: The Supernetwork Structure of the Supply Chain

Behavior of the Manufacturers

Profit maximization problem for manufacturer ***i***

$$\text{Maximize} \sum_{j=1}^{n} \rho_{1ij}^{*} q_{ij} + \sum_{k=1}^{o} \rho_{1ik}^{*} q_{ik} - f_i(Q^1, Q^2) - \sum_{j=1}^{n} c_{ij}(q_{ij}) - \sum_{k=1}^{o} c_{ik}(q_{ik}),$$

subject to $q_{ij} \geq 0$, for all j, and $q_{ik} \geq 0$, for all k.

q_i : production output

q_{ij}: shipment from i to j

q_{ik} : shipment from i to j via e-link

ρ_{1ij}^{*} : price charged by manufacturer i to distributor j

ρ_{1ik}^{*} : price charged by manufacturer i to retailer k

The Optimality Condition for Manufacturers

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}}-\rho_{1ij}^*\right]\times\left[q_{ij}-q_{ij}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}}-\rho_{1ik}^*\right]\times[q_{ik}-q_{ik}^*]\geq 0,\quad \forall (Q^1,Q^2)\in R_+^{mn+mo}. \quad (5)$$

Behavior of the Distributors

Profit maximization problem for distributor ***j***

$$\text{Maximize} \quad \gamma_j^* \sum_{k=1}^{o} q_{jk} - c_j(Q^1, Q^3) - \sum_{i=1}^{m} \rho_{1ij}^* q_{ij}$$

subject to:

$$\sum_{k=1}^{o} q_{jk} \leq \sum_{i=1}^{m} q_{ij},$$

q_{ij}: shipment from *i* to *j*

q_{jk} : shipment from *j* to *j*

γ_j^* : price charged by distributor *j*

Optimality Conditions for the Distributors

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_j(Q^{1*},Q^{3*})}{\partial q_{ij}}+\rho_{1ij}^*-\rho_{2j}^*\right]\times\left[q_{ij}-q_{ij}^*\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[-\gamma_j^*+\frac{\partial c_j(Q^{1*},Q^{3*})}{\partial q_{jk}}+\rho_{2j}^*\right]\times\left[q_{jk}-q_{jk}^*\right]+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}q_{ij}^*-\sum_{k=1}^{o}q_{jk}^*\right]\times\left[\rho_{2j}-\rho_{2j}^*\right]\geq 0$$

$$\forall Q^1\in R_+^{mn},\forall Q^3\in R_+^{no},\forall\rho_2\in R_+^n, \quad (9)$$

The Retailers

$$s_k = \sum_{i=1}^{m} q_{ik} + \sum_{j=1}^{n} q_{jk}$$

ρ_{3k} : demand price at retailer k

$\hat{d}_k(\rho_{3k})$: random demand at retailer k

$\square_k(x, \rho_{3k})$: density function

$P_k(x, \rho_{3k})$: probability function of $\hat{d}_k(\rho_{3k})$

$$P_k(x, \rho_{3k}) = P_k(\hat{d}_k \le x) = \int_0^x \square_k(x, \rho_{3k})dx$$

$min\{s_k, \hat{d}_k\}$: the actual sale of k cannot exceed this amount

$$\Delta_k^+ \equiv \max\{0, s_k - \hat{d}_k\} \tag{11}$$

and

$$\Delta_k^- \equiv \max\{0, \hat{d}_k - s_k\}, \tag{12}$$

where Δ_k^+ is a random variable representing the excess supply (inventory), whereas Δ_k^- is a random variable representing the excess demand (shortage).

Note that the expected values of excess supply and excess demand of retailer k are scalar functions of s_k and ρ_{3k}. In particular, let e_k^+ and e_k^- denote, respectively, the expected values: $E(\Delta_k^+)$ and $E(\Delta_k^-)$, that is,

$$e_k^+(s_k, \rho_{3k}) \equiv E(\Delta_k^+) = \int_0^{s_k} (s_k - x)\mathcal{F}_k(x, \rho_{3k})dx, \tag{13}$$

$$e_k^-(s_k, \rho_{3k}) \equiv E(\Delta_k^-) = \int_{s_k}^{\infty} (x - s_k)\mathcal{F}_k(x, \rho_{3k})dx. \tag{14}$$

λ_k^+ : unit penalty of having excess supply at retailer k

λ_k^- : unit penalty of having excess demand at retailer k

The expected total penalty of retailer k is given by

$$\boxed{E(\lambda_k^+\Delta_k^+ + \lambda_k^-\Delta_k^-) = \lambda_k^+ e_k^+(s_k, \rho_{3k}) + \lambda_k^- e_k^-(s_k, \rho_{3k}).}$$

Behavior of the Retailers

Profit maximization problem for retailer *k*

$$\text{Maximize } E(\rho_{3k}\min\{s_k, \hat{d}_k\}) - E(\lambda_k^+\Delta_k^+ + \lambda_k^-\Delta_k^-) - c_k(Q^2, Q^3) - \sum_{i=1}^m \rho_{1ik}^* q_{ik} - \sum_{j=1}^n \gamma_j^* q_{jk}$$

subject to:

$$q_{ik} \geq 0, \quad q_{jk} \geq 0, \text{ for all } i \text{ and } j.$$

Applying the definitions of Δ_k^+, Δ_k^-,

$$\text{Maximize} \quad \rho_{3k} d_k(\rho_{3k}) - (\rho_{3k} + \lambda_k^-) e_k^-(s_k, \rho_{3k}) - \lambda_k^+ e_k^+(s_k, \rho_{3k}) - c_k(Q^2, Q^3) - \sum_{i=1}^m \rho_{1ik}^* q_{ik} - \sum_{j=1}^n \gamma_j^* q_{jk}$$

where $d_j(\rho_{3k}) \equiv E(\hat{d}_k)$ is a scalar function of ρ_{3k}.

Optimality Conditions for the Retailers

Assuming that the handling cost for each retailer is continuous and convex, then the optimality conditions for all the retailers satisfy the variational inequality: determine $(Q^{2^*}, Q^{3^*}) \in R_+^{mo+no}$, satisfying:

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\lambda_k^+ P_k(s_k^*, \rho_{3k}) - (\lambda_k^- + \rho_{3k})(1 - P_k(s_k^*, \rho_{3k})) + \frac{\partial c_k(Q^{2^*}, Q^{3^*})}{\partial q_{ik}} + \rho_{1ik}^*\right] \times [q_{ik} - q_{ik}^*]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\lambda_k^+ P_k(s_k^*, \rho_{3k}) - (\lambda_k^- + \rho_{3k})(1 - P_k(s_k^*, \rho_{3k})) + \frac{\partial c_k(Q^{2^*}, Q^{3^*})}{\partial q_{jk}} + \gamma_j^*\right] \times \left[q_{jk} - q_{jk}^*\right] \geq 0,$$

$$\forall (Q^2, Q^3) \in R_+^{mo+no}. \qquad (20)$$

The Stochastic Market Equilibrium Conditions

For any retailer *k*

$$\hat{d}_k(\rho_{3k}^*) \begin{cases} \leq \sum_{i=1}^{m} q_{ik}^* + \sum_{j=1}^{o} q_{jk}^* & \text{a.e., if} \quad \rho_{3k}^* = 0 \\ = \sum_{i=1}^{m} q_{ik}^* + \sum_{j=1}^{o} q_{jk}^* & \text{a.e., if} \quad \rho_{3k}^* > 0, \end{cases}$$

where a.e. means that the corresponding equality or inequality holds almost everywhere.

$$\sum_{k=1}^{o} \left(\sum_{i=1}^{m} q_{ik}^* + \sum_{j=1}^{n} q_{jk}^* - d_j(\rho_{3k}^*) \right) \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \qquad \forall \rho_3 \in R_+^o, \qquad (22)$$

where ρ_3 is the o-dimensional column vector with components: $\{\rho_{31}, \ldots, \rho_{3o}\}$.

Supply Chain Network Equilibrium with Random Demands

Definition:

The equilibrium state of the supply chain with random demands is one where the product flows between the tiers of the decision-makers coincide and the product shipments and prices satisfy the sum of the optimality conditions (5), (9), and (20), and the conditions (22).

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation

A product shipment and price pattern $(Q^{1^*}, Q^{2^*}, Q^{3^*}, \rho_2^*, \rho_3^*) \in \mathcal{K}$ *is an equilibrium pattern of the supply chain model according to Definition 1 if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1^*}, Q^{2^*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} + \frac{\partial c_j(Q^{1^*}, Q^{3^*})}{\partial q_{ij}} - \rho_{2j}^*\right] \times \left[q_{ij} - q_{ij}^*\right]$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1^*}, Q^{2^*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}} + \frac{\partial c_k(Q^{2^*}, Q^{3^*})}{\partial q_{ik}}\right.$$

$$\left. + \lambda_k^+ P_k(s_k^*, \rho_{3k}^*) - (\lambda_k^- + \rho_{3k}^*)(1 - P_k(s_k^*, \rho_{3k}^*))\right] \times \left[q_{ik} - q_{ik}^*\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\lambda_k^+ P_k(s_k^*, \rho_{3k}^*) - (\lambda_k^- + \rho_{3k}^*)(1 - P_k(s_k^*, \rho_{3k}^*)) + \frac{\partial c_j(Q^{1^*}, Q^{3^*})}{\partial q_{jk}}\right.$$

$$\left. + \frac{\partial c_k(Q^{2^*}, Q^{3^*})}{\partial q_{jk}} + \rho_{2j}^*\right] \times \left[q_{jk} - q_{jk}^*\right] + \sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} q_{jk}^*\right] \times \left[\rho_{2j} - \rho_{2j}^*\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{j=1}^{n} q_{jk}^* + \sum_{i=1}^{m} q_{ik}^* - d_k(\rho_3^*)\right] \times \left[\rho_{3k} - \rho_{3k}^*\right] \geq 0, \quad \forall (Q^1, Q^2, Q^3, \rho_2, \rho_3) \in \mathcal{K}, \qquad (23)$$

Qualitative Properties

Under certain conditions we proved the existence of the solution and the uniqueness of the solution to the VIP.

Theorem 3: Existence

Suppose that there exist positive constants M, N, R with $R > 0$, such that:

$$\frac{\partial f_i(Q^1,Q^2)}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij})}{\partial q_{ij}} + \frac{\partial c_j(Q^1,Q^3)}{\partial q_{ij}} \geq M, \quad \forall Q^1 \text{ with } q_{ij} \geq N, \quad \forall i,j \tag{28a}$$

$$\frac{\partial f_i(Q^1,Q^2)}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}} + \frac{\partial c_k(Q^2,Q^3)}{\partial q_{jk}} + \lambda_k^+ P_k(s_k,\rho_{3k}) - (\lambda_k^- + \rho_{3k})(1 - P_k(s_k,\rho_{3k})) \geq M,$$
$$\forall Q^2 \text{ with } q_{ik} \geq N, \quad \forall i,k, \tag{28b}$$

$$\lambda_k^+ P_k(s_k,\rho_{3k}) - (\lambda_k^- + \rho_{3k})(1 - P_k(s_k,\rho_{3k})) + \frac{\partial c_j(Q^1,Q^3)}{\partial q_{jk}} + \frac{\partial c_k(Q^2,Q^3)}{\partial q_{jk}} \geq M,$$
$$\forall Q^3 \text{ with } q_{jk} \geq N, \quad \forall j,k, \tag{28c}$$

and

$$d_k(\rho_{3k}) \leq N, \quad \forall \rho_3 \quad \text{with} \quad \rho_{3k} \geq R, \quad \forall k. \tag{29}$$

Then, variational inequality (24) admits at least one solution.

Theorem 4: Monotonicity

The function that enters the variational inequality problem (24) is monotone, if the condition assumed in Lemma 1 is satisfied for each k; $k = 1, \cdots, o$, and if the following conditions are also satisfied.

Suppose that the production cost functions $f_i; i = 1, ..., m$, are additive, as defined in Definition 2, and that the $f_i^1; i = 1, ..., m$, are convex functions. If the c_{ij}, c_{ik}, c_k and c_j functions are convex, for all i, j, k, then the vector function F that enters the variational inequality (24) is monotone, that is,

$$\langle F(X') - F(X''), X' - X'' \rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}. \tag{36}$$

Theorem 5: Strict Monotonicity

The function that enters the variational inequality problem (24) is strictly monotone, if the conditions mentioned in Lemma 1 for $g_k(s_k, \rho_{3k})$ are satisfied strictly for all k and if the following conditions are also satisfied.

Suppose that the production cost functions $f_i; i = 1, ..., m$, are additive, as defined in Definition 2, and that the $f_i^1; i = 1, ..., m$, are strictly convex functions. If the c_{ij}, c_{ik}, c_k and c_j functions are strictly convex, for all i, j, k, then the vector function F that enters the variational inequality (24) is strictly monotone, that is,

$$\langle F(X') - F(X''), X' - X'' \rangle > 0, \quad \forall X', X'' \in \mathcal{K}. \tag{46}$$

Theorem 6: Uniqueness

Under the conditions indicated in Theorem 5, the function that enters the variational inequality (24) has a unique solution in $\mathcal{K}$.

From Theorem 6 it follows that, under the above conditions, the equilibrium product shipment pattern between the manufacturers and the retailers, as well as the equilibrium price pattern at the retailers, is unique.

Theorem 7: Lipschitz Continuity

The function that enters the variational inequality problem (24) is Lipschitz continuous, that is,

$$\|F(X') - F(X'')\| \leq L\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \text{ with } L > 0, \tag{47}$$

under the following conditions:

(i). Each f_i; $i = 1, ..., m$, is additive and has a bounded second order derivative;

(ii). The c_{ij}, c_{ik}, c_k, and c_j have bounded second order derivatives, for all i, j, k;

Algorithm

Modified Projection Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and let a be a scalar such that $0 < a \leq \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. Korpelevich (1977)) (see (47)).

Step 1: Computation

Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle \bar{X}^{\mathcal{T}} + aF(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{49}$$

Step 2: Adaptation

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + aF(\bar{X}^{\mathcal{T}}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{50}$$

Step 3: Convergence Verification

If $\max |X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} =: \mathcal{T} + 1$, and go to Step 1.

We now state the convergence result for the modified projection method for this model.

Theorem 8: Convergence

Assume that the function that enters the variational inequality (23) (or (24)) has at least one solution and satisfies the conditions in Theorem 4 and in Theorem 7. Then the modified projection method described above converges to the solution of the variational inequality (23) or (24).

Numerical Examples

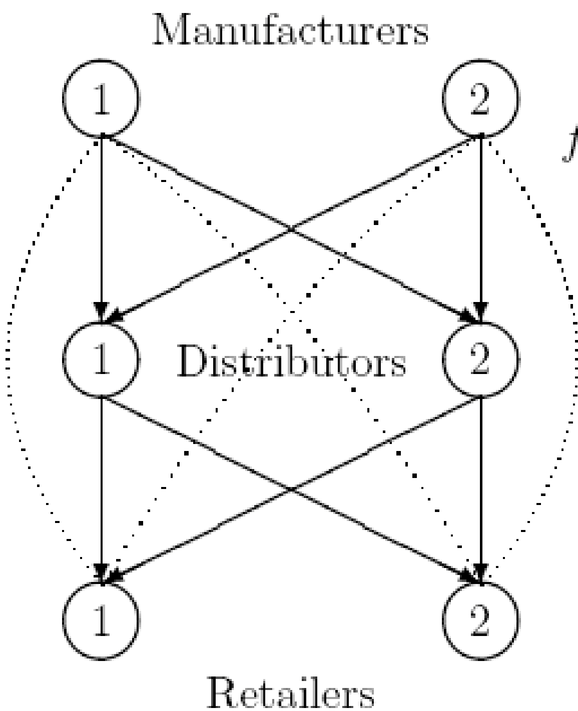
In all the examples, we assumed that the demands associated with the retail outlets followed a uniform distribution. Hence, we assumed that the random demand, $\hat{d}_k(\rho_{3k})$, of retailer k, is uniformly distributed in $[0, \frac{b_k}{\rho_{3k}}]$, $b_k > 0$; $k = 1, \ldots, o$. Therefore,

$$P_k(x, \rho_{3k}) = \frac{x\rho_{3k}}{b_k}, \tag{51}$$

$$\mathcal{F}_j(x, \rho_{3k}) = \frac{\rho_{3k}}{b_k}, \tag{52}$$

$$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{b_k}{\rho_{3k}}; \quad k = 1, \ldots, o. \tag{53}$$

It is easy to verify that the expected demand function $d_k(\rho_{3k})$ associated with retailer k is a decreasing function of the price at the demand market.



$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2.$$

$$c_{ij}(q_{ij}) = .5q_{ij}^2 + 3.5q_{ij}, \quad \text{for } i = 1, 2;\ j = 1, 2.$$

$$c_{ik}(q_{ik}) = .5q_{ik}^2 + 5q_{ik}, \quad \text{for } i = 1, 2;\ k = 1, 2.$$

$$c_j(Q^1, Q^3) = .5(\sum_{i=1}^{2} q_{ij})^2, \quad \text{for } j = 1, 2,$$

$$c_k(Q^2, Q^3) = .5(\sum_{j=1} q_{jk})^2, \quad \text{for } k = 1, 2.$$

Example 1

$$\lambda_k^+ = \lambda_k^- = 1, k = 1,2$$

$$b_k = 100, k = 1,2.$$

$$q_{ij}^* = .3697, i = 1,2; j = 1,2.$$

$$q_{ik}^* = .3487, i = 1,2; k = 1,2.$$

$$q_{jk}^* = .3697, j = 1,2; k = 1,2.$$

$$\rho_{2j}^* = 15.2301, j = 1,2.$$

$$\rho_{3k}^* = 34.5573, k = 1,2.$$

$$d_1(\rho_{31}^*) = d_2(\rho_{32}^*) = 1.4469.$$

Example 2

$$\lambda_k^+ = \lambda_k^- = 1, k = 1,2$$

$$b_k = 1000, k = 1,2.$$

$$q_{ij}^* = .6974, i = 1,2; j = 1,2.$$

$$q_{ik}^* = 1.9870, i = 1,2; k = 1,2.$$

$$q_{jk}^* = .6973, j = 1,2; k = 1,2.$$

$$\rho_{2j}^* = 39.8051, j = 1,2.$$

$$\rho_{3k}^* = 92.9553, k = 1,2.$$

$$d_1(\rho_{31}^*) = d_2(\rho_{32}^*) = 5.3789.$$

Questions?

More information about Supernetworks can be found at

http://supernet.som.umass.edu

Contact me at dong@oswego.edu