# *Dynamic Networks with Applications*


***Anna Nagurney***

**John F. Smith Memorial Professor**
**University of Massachusetts - Amherst**


**Radcliffe Institute for Advanced Study**
**November 9, 2005**

Eugene M. Isenberg School of Management

**The Virtual Center for Supernetworks**
**http://supernet.som.umass.edu**

## *Funding for research provided by:*

National Science Foundation

AT&T Foundation

John F. Smith Memorial Fund - University of Massachusetts at Amherst

THE ROCKEFELLER FOUNDATION

RADCLIFFE INSTITUTE FOR ADVANCED STUDY
HARVARD UNIVERSITY

# *Outline of Presentation:*

- **Background**
- **Scientific Study of Networks**
- **Interdisciplinary Impact of Networks**
- **Characteristics of Networks Today**
- **The Braess Paradox and a New Discovery**
- **New Challenges and Opportunities: Unification of EVIs and PDSs**
- **Novel Applications**

***Networks are pervasive in our daily lives and essential to the functioning of our societies and economies.***

**Everywhere we look:**

**in business, science,**

**social systems, technology,**

**and education,**

**networks provide the infrastructure for communication, production, and transportation.**

***Throughout history, networks have evolved to serve as the foundation for connecting humans to one another and their activities.***

**Roads were laid, bridges built, and waterways crossed so that humans, be they on foot, on animal, or vehicle could traverse physical distance through *transportation*.**

***Communications*** **were conducted using the available means of the period, from smoke signals, drum beats, and pigeons, to the telegraph, telephone, and computer networks of today.**

The study of the efficient operation on transportation networks dates to *ancient Rome* with a classical example being the publicly provided Roman road network and the *time of day chariot policy,* whereby chariots were banned from the ancient city of Rome at particular times of day.

# *We are in a New Era of Decision-Making Characterized by:*

- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- global reach of many decisions;
- high impact of many decisions;
- increasing risk and uncertainty, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

***The New Era is Network-Based*** **with the Internet providing critical infrastructure along with transportation/logistical networks as well as other telecommunication networks and energy networks.**

***No longer are networks independent of one another but critically linked*** **with major questions arising regarding decision-making tools.**

**Events such as the 2005 Katrina Hurricane disaster, the 2003 blackout in North America, 9/11/2001, and the computer viruses and worms demonstrate irrevocably that *we must harness the best and most powerful methodologies for the modeling, analysis, and solution of complex decision-making problems.***

# Examples of Physical Networks

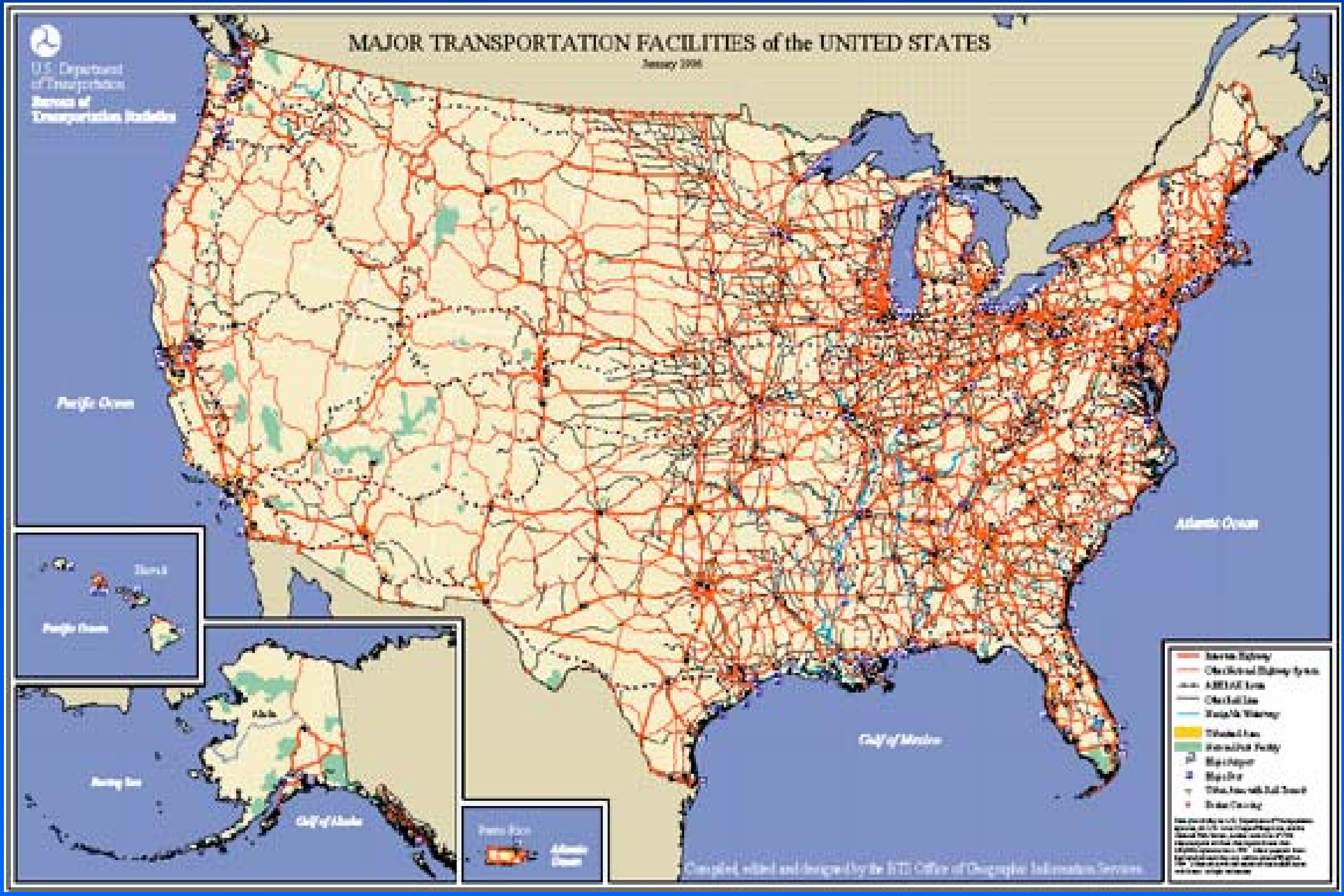
# *Transportation Networks*

provide us with the means to cross distance in order to conduct our work, and to see our colleagues, students, friends, and family members.

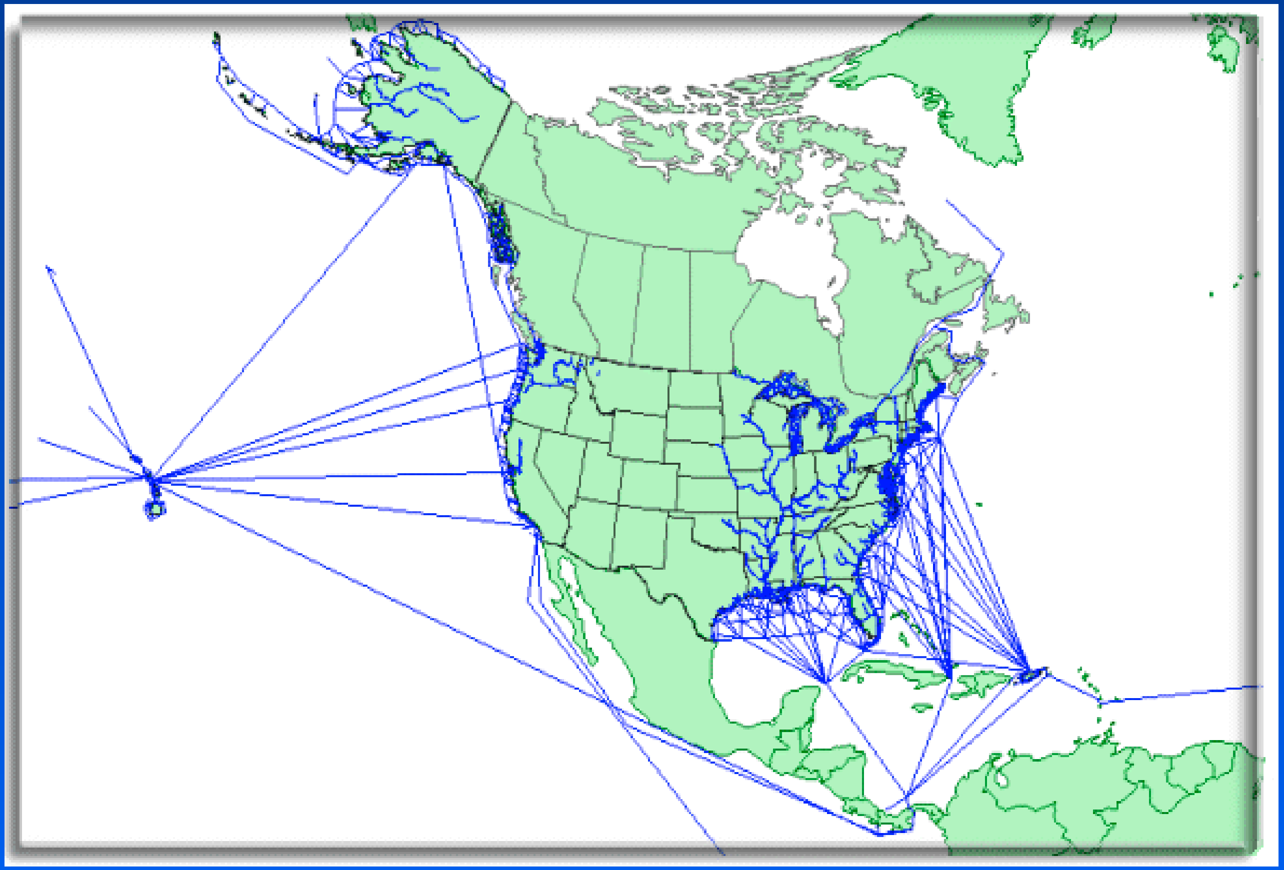
They provide us with access to food and consumer products.

# MBTA Network

# Major Highway and Railroad Networks in the US



# Water Freight Transport Routes for the US



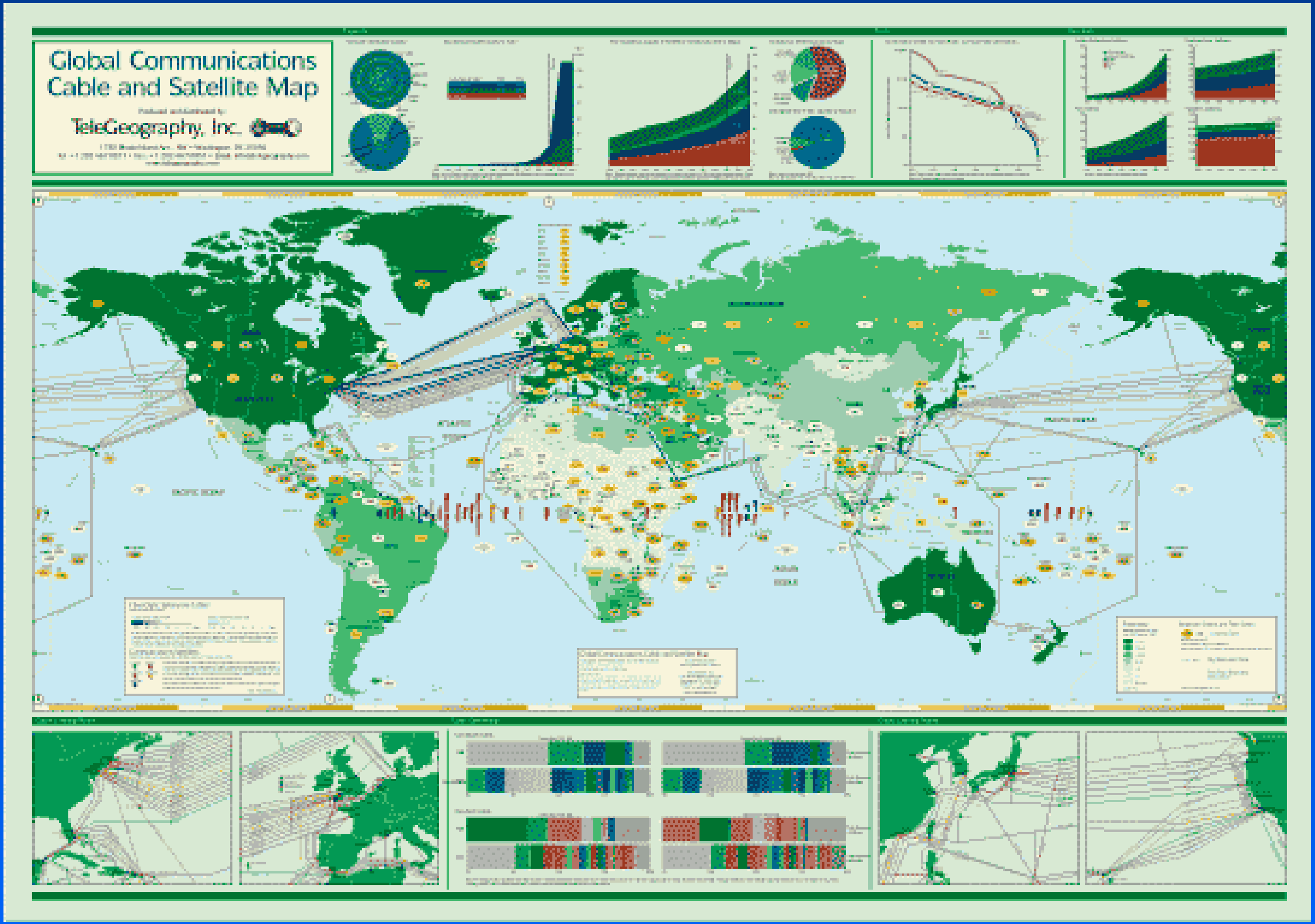
# *Communication Networks*

allow us to communicate within our own communities and across regions and national boundaries,

and have transformed the way we live, work, and conduct business.

# Iridium Satellite Constellation Network

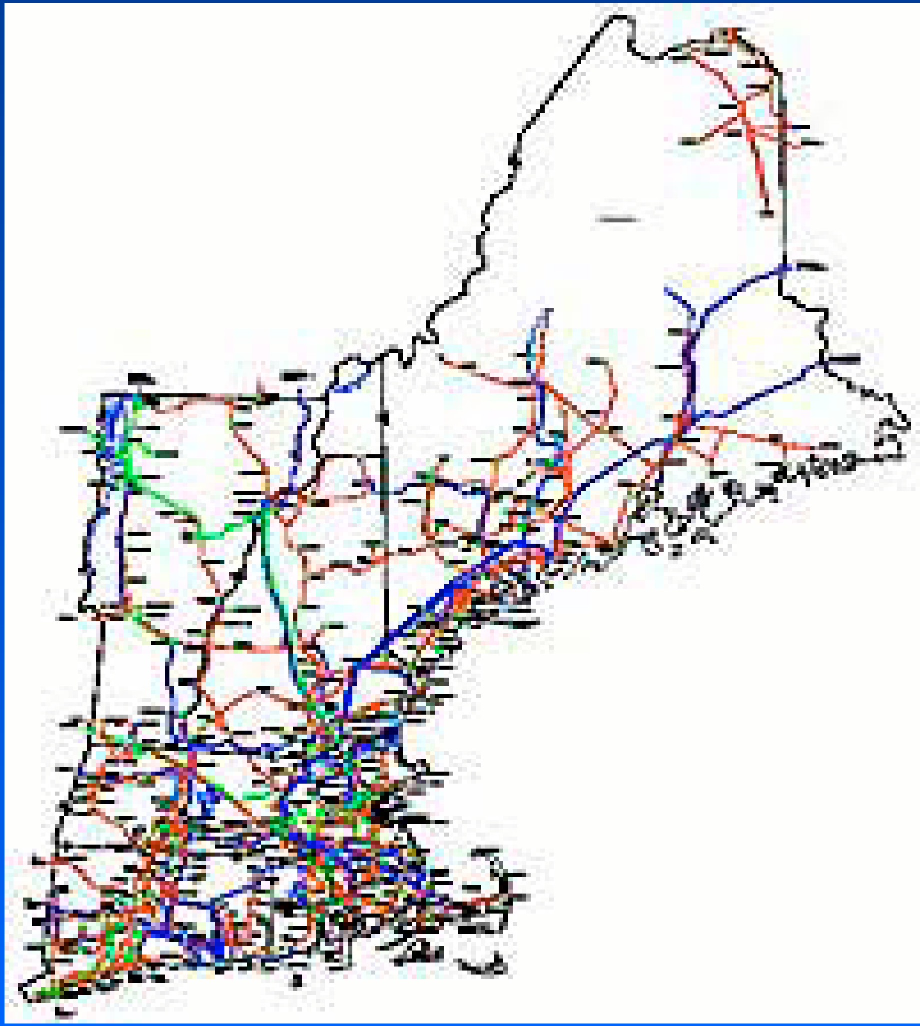
# Satellite and Undersea Cable Networks



# *Energy Networks*

provide the energy for our homes, schools, and businesses, and to run our vehicles.

# New England Electric Power Network

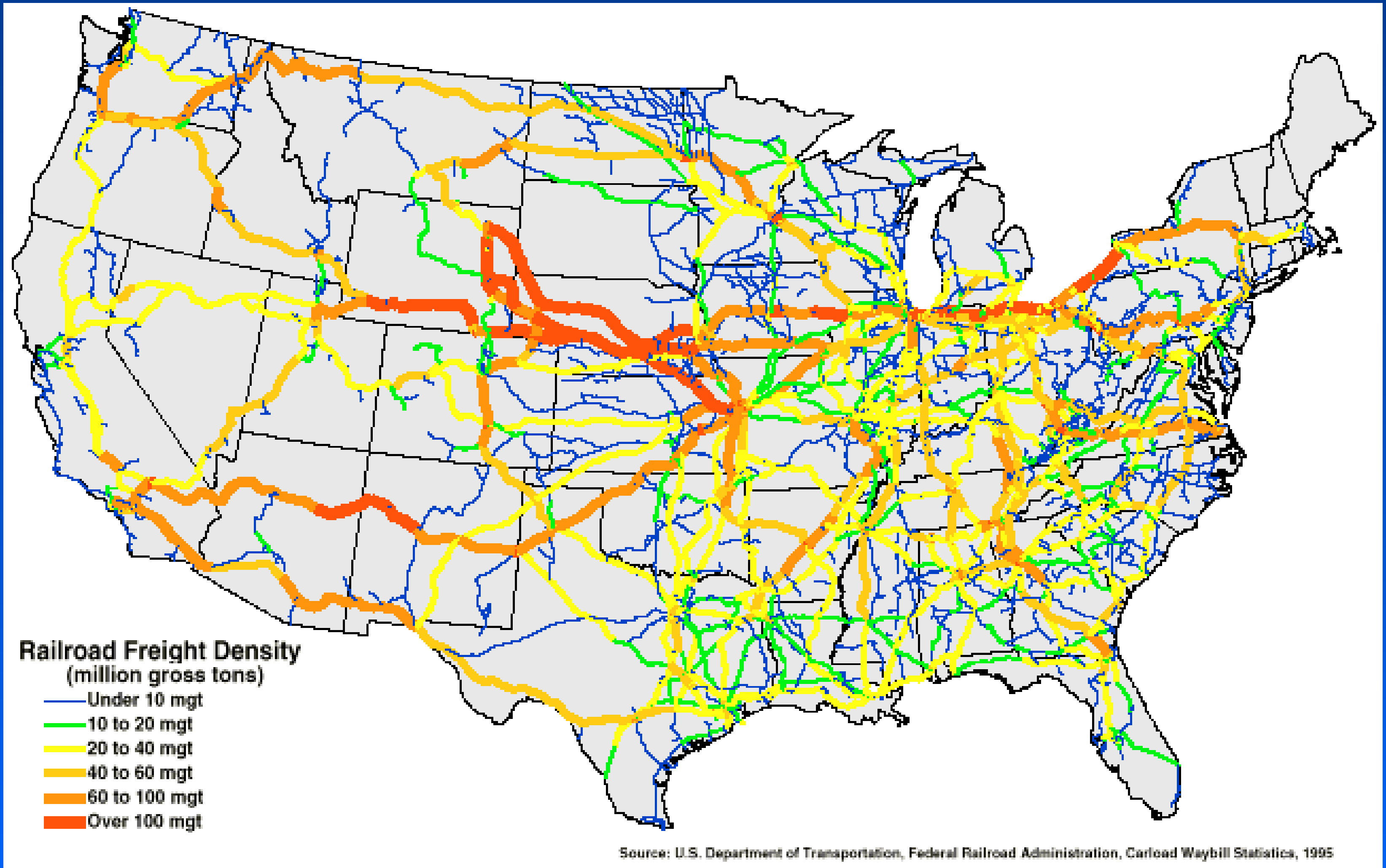


# Duke Energy Gas Pipeline Network

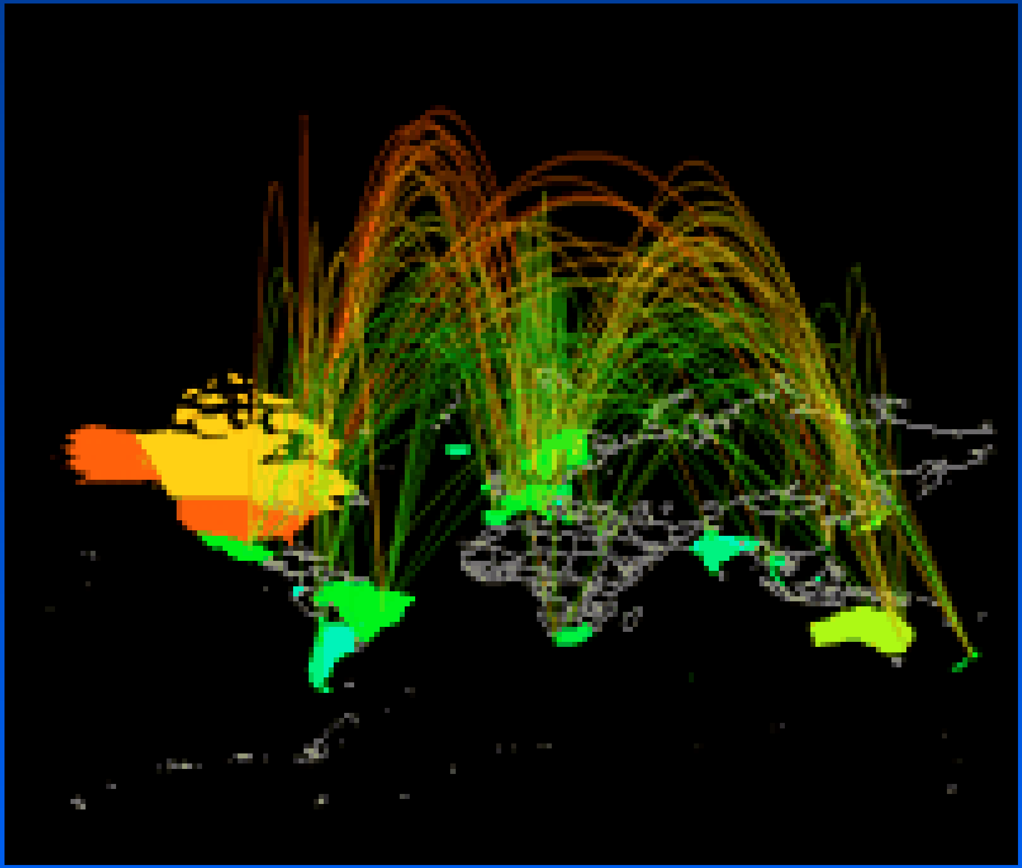
# Components of Some Common Physical Networks

| Network System | Nodes | Links | Flows |
|---|---|---|---|
| Transportation | Intersections, Homes, Workplaces, Airports, Railyards | Roads, Airline Routes, Railroad Track | Automobiles, Trains, and Planes, |
| Manufacturing and logistics | Workstations, Distribution Points | Processing, Shipment | Components, Finished Goods |
| Communication | Computers, Satellites, Telephone Exchanges | Fiber Optic Cables Radio Links | Voice, Data, Video |
| Energy | Pumping Stations, Plants | Pipelines, Transmission Lines | Water, Gas, Oil |

# US Railroad Freight Flows



# Internet Traffic Flows Over One 2 Hour Period



from Stephen Eick, Visual Insights

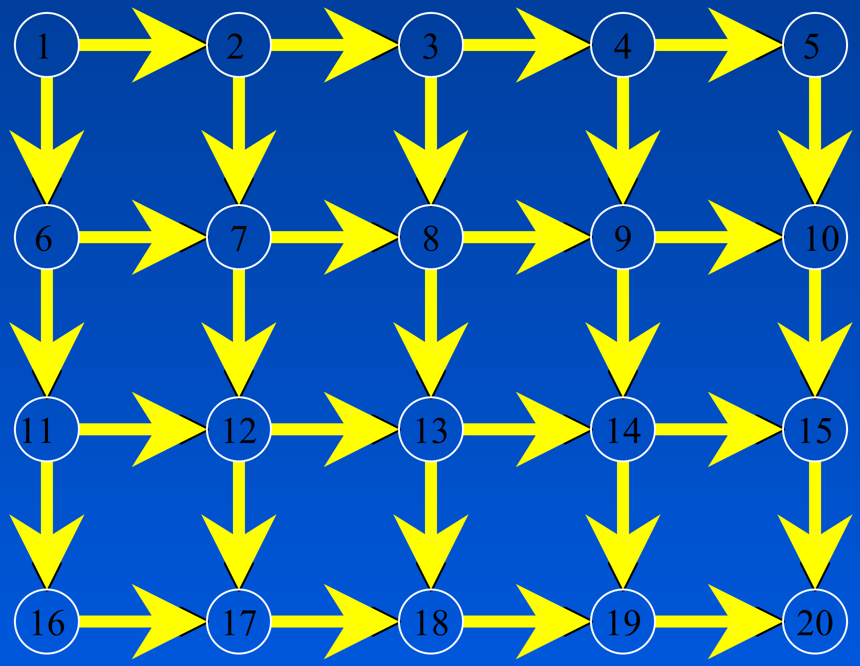
# *The scientific study of networks involves:*

- how to **model** such applications as **mathematical entities**,
- how to **study the models** qualitatively,
- how to design **algorithms** to solve the resulting models.

# *The basic components of networks are:*

- Nodes
- Links or arcs
- Flows

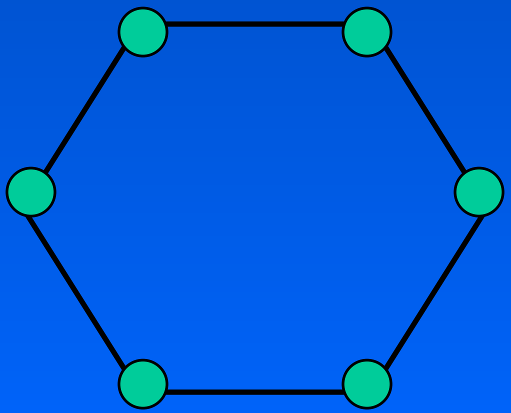
## Nodes Links Flows



# *Brief History of the Science of Networks*

**1736 - Euler credited with the earliest paper on graph theory - Konigsberg bridges problem.**

**1758 - Quesnay in his *Tableau Economique* introduced an abstract network in the form of a graph to depict the circular flow of financial funds in an economy.**



**1781 - Monge, who had worked under Napoleon Bonaparte in providing infrastructure support for his army, publishes what is probably the first paper on transportation in minimizing the cost associated with backfilling n places from m other places with surplus brash with cost being proportional to distance.**

**1838 - Cournot not only states that a competitive price is determined by the intersection of supply and demand curves but does it in the context of spatially separate markets in which transportation costs are included.**

**1841 - Kohl considered a two node, two route transportation network problem.**

**1845 - Kirchhoff wrote *Laws of Closed Electric Circuits.***

**1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.**

**1936 - Konig published the first book on graph theory.**

**1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.**

**1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.**

**1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits**

**1952 - Copeland in his book asked "Does money flow like water or electricity?"**

**1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.**

**1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).**

**1969 - Dafermos and Sparrow coined the terms *user-optimization* and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.**

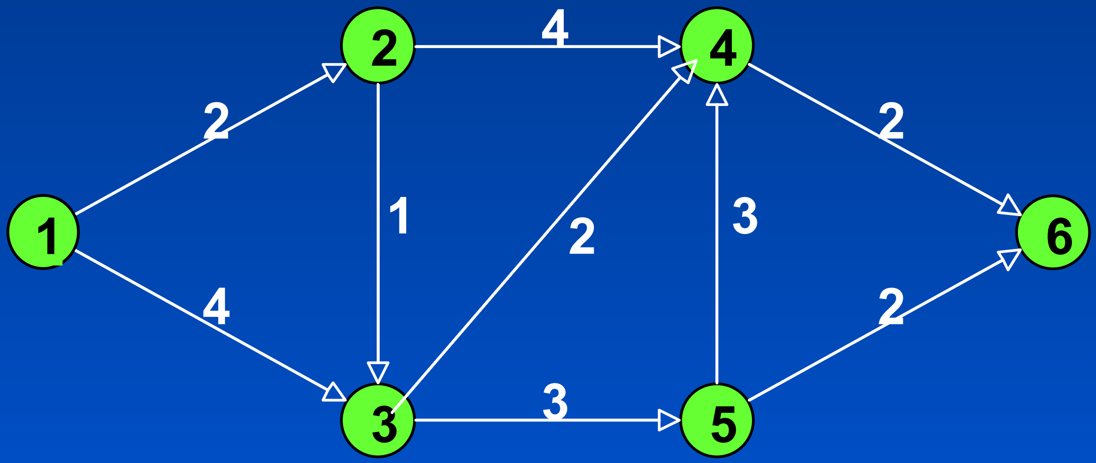
# *In a basic network problem domain:*

**one wishes to move the flow from one node to another in a way that is as efficient as possible.**

# *Classic Examples of Network Problems Are:*

- **The Shortest Path Problem**
- **The Maximum Flow Problem**
- **The Minimum Cost Flow Problem.**

# The Shortest Path Problem




**Consider a network with Origin Node 1 and a Destination Node 6.**

***What is the shortest path from 1 to 6?***

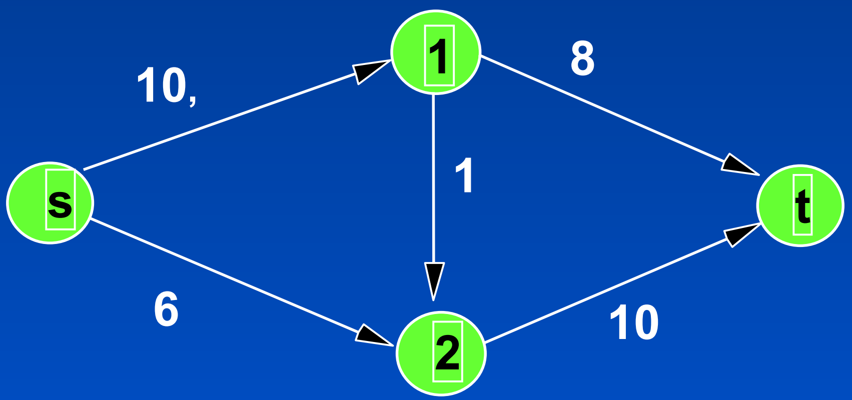
# *Applications of the Shortest Path Problem*

Arise in **transportation** and **telecommunications.**

**Other applications include:**

- simple building evacuation models
- DNA sequence alignment
- dynamic lot-sizing with backorders
- assembly line balancing
- compact book storage in libraries.

# The Maximum Flow Problem



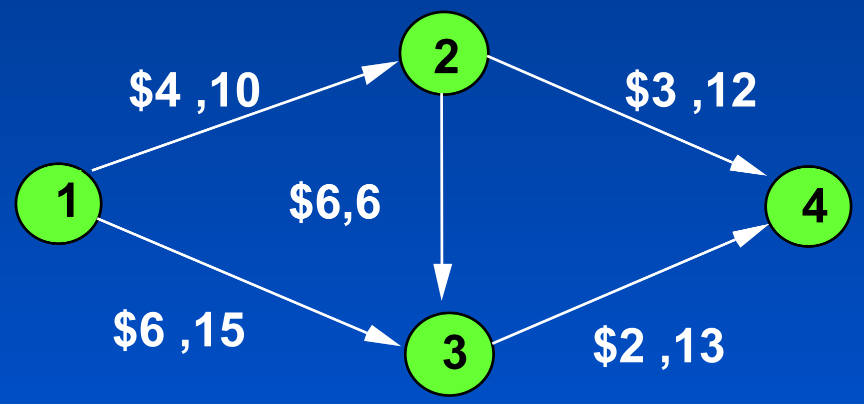

**Each link has a maximum capacity.**

***How does one Maximize the flow from s to t, subject to the link capacities?***

# *Applications of the Maximum Flow Problem*

- **machine scheduling**
- **network reliability testing**
- **building evacuation.**

# *The Minimum Cost Flow Problem*




**Each link has a linear cost and a maximum capacity.**

***How does one Minimize Cost for a given flow from 1 to 4?***

# *Applications of the Minimum Cost Flow Problem*

- **warehousing and distribution**
- **vehicle fleet planning**
- **cash management**
- **automatic chromosome classification**
- **satellite scheduling.**

**Network problems arise in other *surprising and fascinating* ways for problems, which at first glance and on the surface, may not appear to involve networks at all.**

***The study of networks is not limited to only physical networks* but also to *abstract* networks in which nodes do not coincide to locations in space.**

## *The advantages of a scientific network formalism:*

- **many present-day problems are concerned with flows (material, human, capital, informational, etc.) over space and time and, hence, ideally suited as an application domain for network theory;**

- **provides a graphical or visual depiction of different problems;**

- **helps to identify similarities and differences in distinct problems through their underlying network structure;**

- **enables the application of efficient network algorithms;**

- **allows for the study of disparate problems through a unifying methodology.**

**One of the primary purposes of scholarly and scientific investigation is to *structure* the world around us and to *discover patterns* that cut across boundaries and, hence, help to *unify diverse applications*.**

**Network theory provides us with a *powerful methodology* to establish connections with different disciplines and to *break down boundaries*.**

# Interdisciplinary Impact of Networks

Mathematics

**Networks**

### *Economics*

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

### *Engineering*

Energy

Manufacturing

Telecommunications

Transportation

### *Biology*

DNA Sequencing

Targeted Cancer Therapy

### *Sociology*

Social Networks

Organizational Theory

### *Computer Science*

Routing Algorithms

# *Characteristics of Networks Today*

- ***large-scale nature*** **and complexity of network topology;**
- ***congestion*;**
- **alternative behavior of users of the network, which may lead to *paradoxical phenomena*;**
- **the *interactions among networks themselves such as in transportation* versus telecommunications networks;**
- ***policies*** **surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.**

- ***large-scale nature and complexity of network topology***
  - **In Chicago’s Regional Network, there are 12,982 nodes, 39,018 links, and 2,297,945 O/D pairs.**
  - **AT&T’s domestic network has 100,000 O/D pairs. In their call detail graph applications (nodes are phone numbers, edges are calls) - 300 million nodes and 4 billion edges**

- ***congestion is playing an increasing role in transportation networks:***

**In the United States alone, congestion results in $100 billion in lost productivity annually. In Europe congestion is estimated to be $150 billion.**

**The number of cars is expected to increase by 50% by 2010 and to double by 2030.**

# Congestion

# *Wasting Away in Traffic*

| | Annual delay hours/driver |
|---|---|
| Los Angeles | 93 |
| San Francisco - Oakland | 72 |
| Washington, DC | 69 |
| Atlanta | 67 |
| Houston | 63 |
| Dallas | 60 |
| Chicago | 58 |
| Detroit | 57 |
| Boston | 51 |
| Miami - Hialeah | 51 |

Source: Texas Transportation Institute 2003 Data

- **alternative behaviors of the users of the network**
  - **system-optimized versus**
  - **user-optimized (network equilibrium),**

  **which may lead to**

  **paradoxical phenomena.**

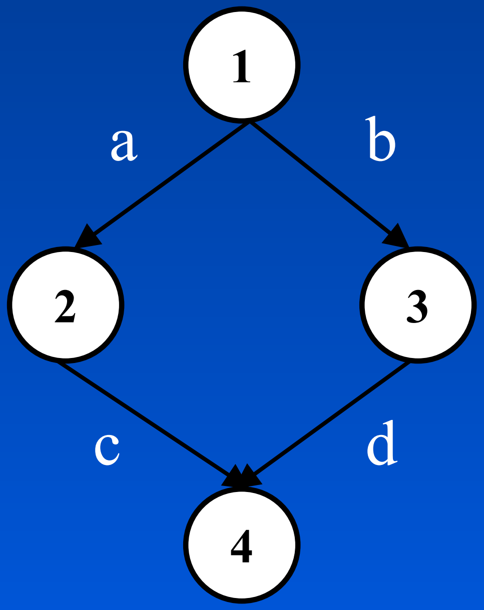
# *The Braess' Paradox*

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1$=(a,c) and $p_2$=(b,d).

For a travel demand of 6, the equilibrium path flows are $x_{p_1}{}^* = x_{p_2}{}^* = 3$ and

The equilibrium path travel cost is

$C_{p_1} = C_{p_2} = 83$.




$c_a(f_a)=10\, f_a \quad c_b(f_b) = f_b+50$

$c_c(f_c) = f_c+50 \quad c_d(f_d) = 10\, f_d$

# Adding a Link
# Increased Travel Cost for All!

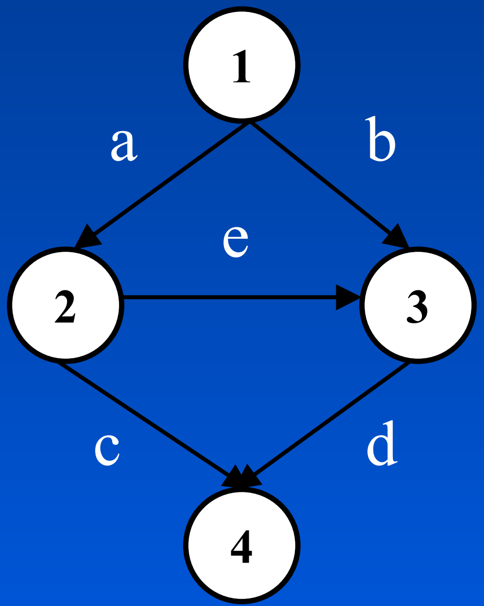
**Adding a new link creates a new path $p_3$=(a,e,d).**

**The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path $p_3$, $C_{p_3}$=70.**

**The new equilibrium flow pattern network is**

$$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2.$$

**The equilibrium path travel costs: $C_{p_1} = C_{p_2} = C_{p_3} = 92$.**




$$c_e(f_e) = f_e + 10$$

**This phenomenon is relevant to telecommunications networks and the Internet which is another example of a**

***noncooperative network.***

**The Price of Anarchy!!!**

The 1968 Braess article has been translated from German to English and appears as

*On a Paradox of Traffic Planning*

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science.*

**Transportation science has historically been the discipline that has *pushed the frontiers* in terms of methodological developments for network problems beginning with the work of Beckmann, McGuire, and Winsten (1956).**

**Dafermos (1980) showed that the traffic network equilibrium (also referred to as user-optimization) conditions as formulated by Smith (1979) were a finite-dimensional variational inequality.**

**In 1993, Dupuis and Nagurney proved that the set of solutions to a variational inequality problem coincided with the set of solutions to a projected dynamical system in $R^n$.**

**In 2002, Cojocaru proved this result for Hilbert Spaces.**

# *EQUILIBRIA of PDS and VARIATIONAL INEQUALITIES*

**An important feature of any PDS is that it is intimately related to a variational inequality problem (VI).**

**Theorem**

*The equilibria of a PDS:*

$$\frac{\partial}{\partial t}(x(t)) = \Pi_K(x(t), -F(x(t)))$$

$$= \lim_{\delta \to 0} \frac{P_K(x(t) - \delta F(x(t))) - x(t)}{\delta}, \quad x(0) = x_0,$$

*that is,* $x^* \in K$ *such that*

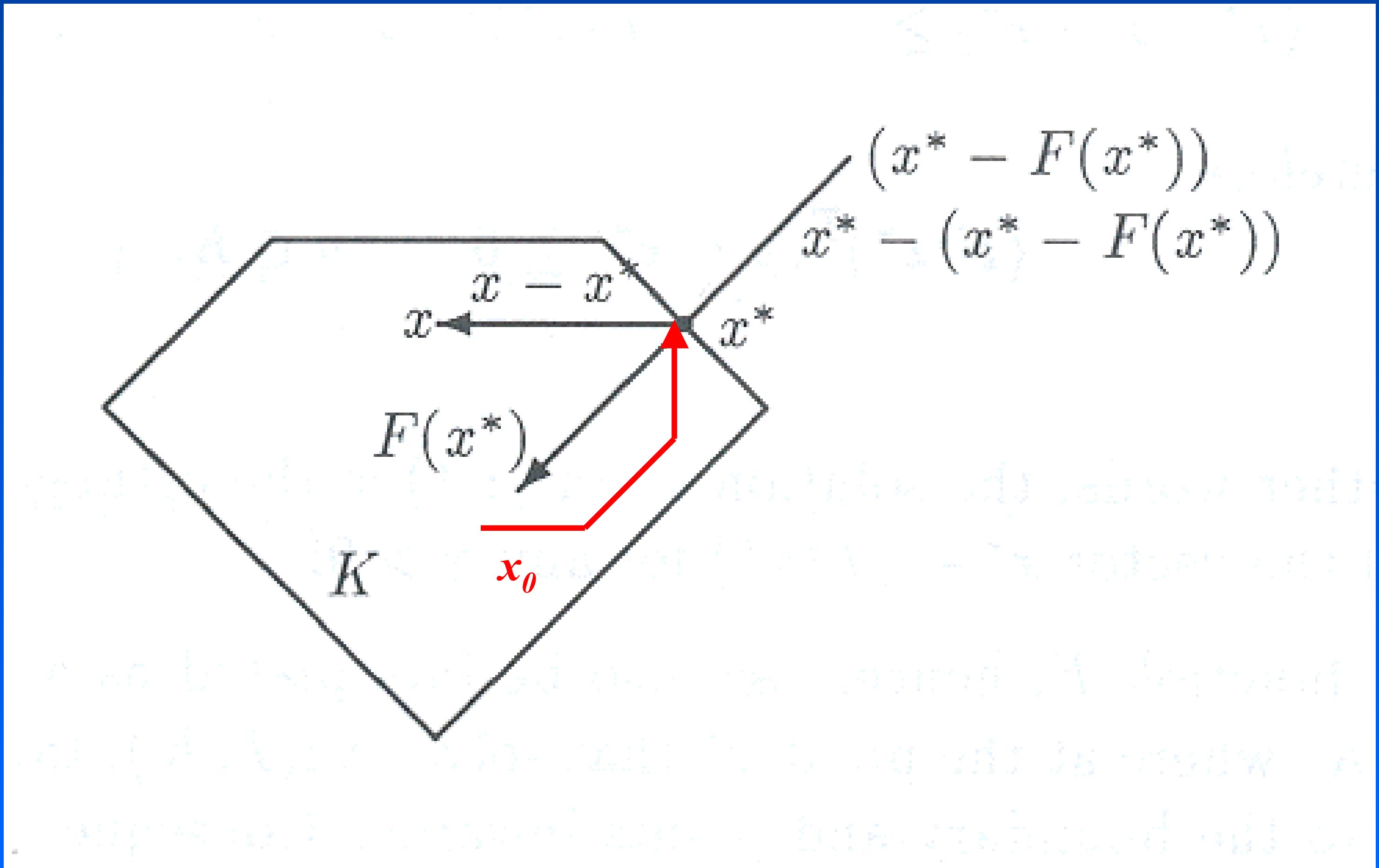
$$\Pi_K(x^*, -F(x^*)) = 0$$

*are solutions to the* $\mathrm{VI}(F, K)$*: find* $x^* \in K$ *such that*

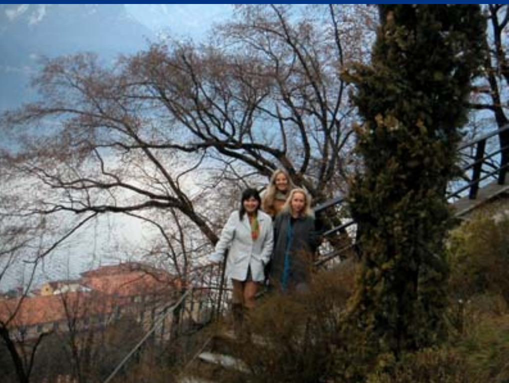
$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

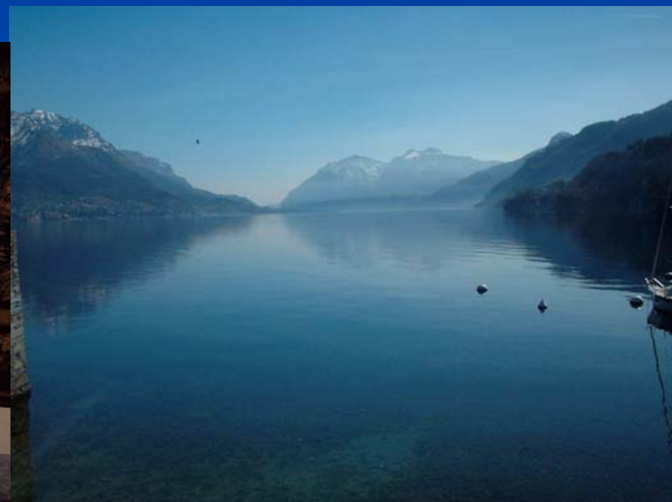
*and vice-versa, where* $\langle \cdot, \cdot \rangle$ *denotes the inner product on* $X$*, where* $X$ *is a Hilbert space.*

# A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System



# Bellagio Research Team Residency March 2004






**We are working with Professors Cojocaru and Daniele on infinite-dimensional projected dynamical systems and evolutionary variational inequalities and their relationships and unification.**

**This allows us to model dynamic networks with:**

- ***dynamic (time-dependent)*** **supplies and demands**
- ***dynamic (time-dependent)*** **capacities**
- ***structural changes*** **in the networks themselves.**

# *A New Discovery through the Investigation of Network Dynamics in the Form of Increasing Travel Demand*

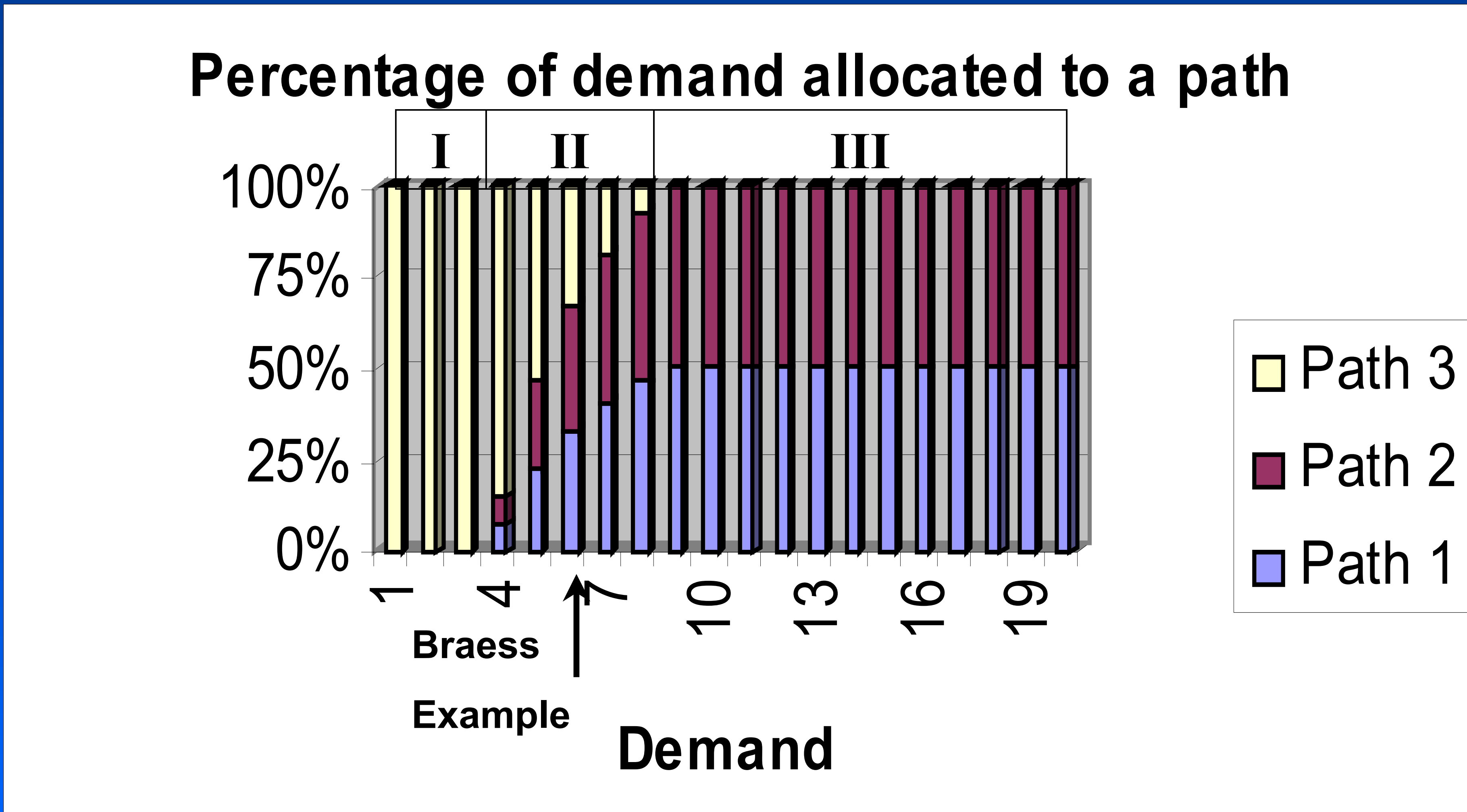
***What happens if the demand is varied in the Braess Network?***

**The answer lies in the solution of an Evolutionary (Time-Dependent) Variational Inequality.**

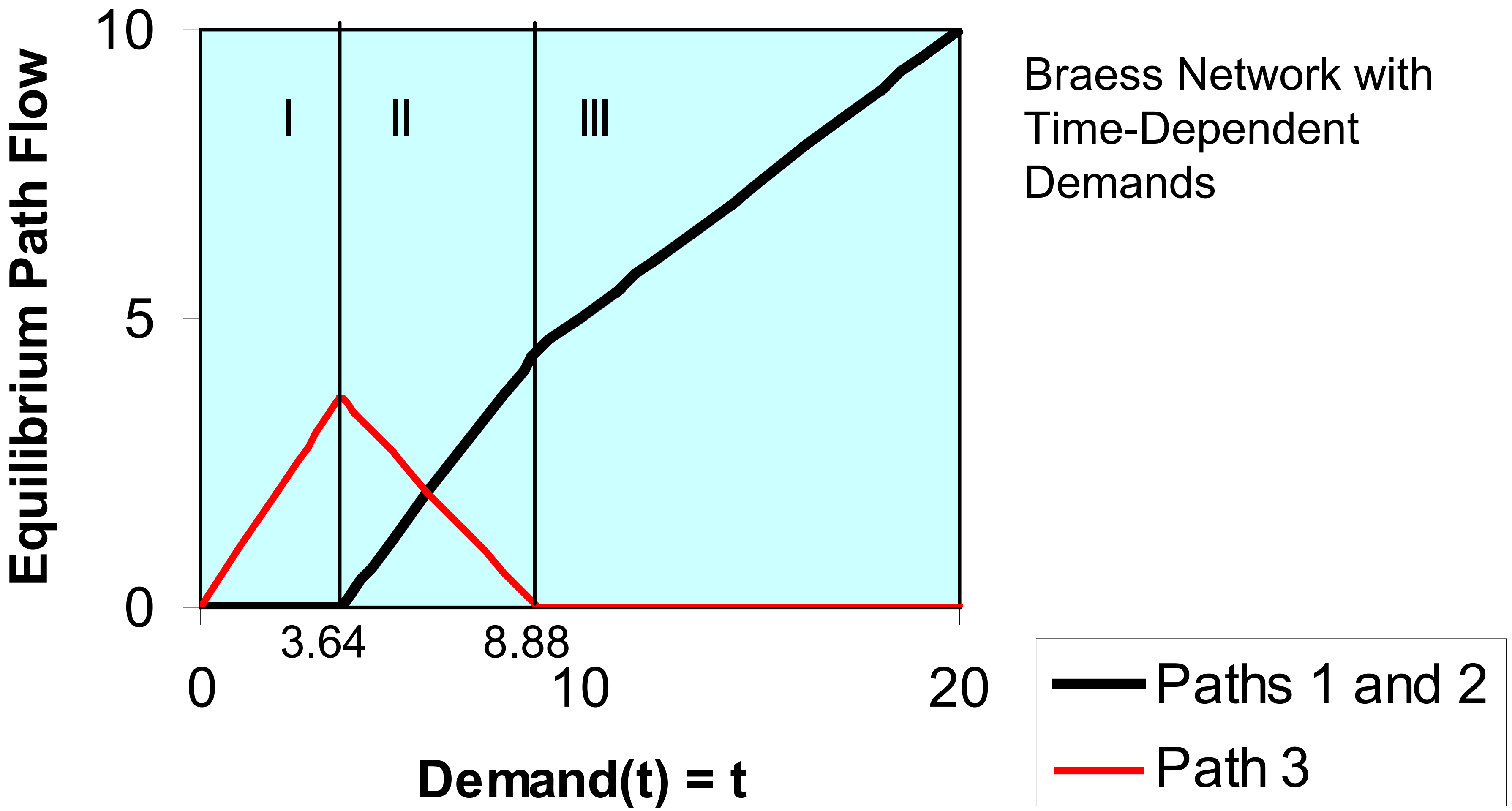
Find $x^* \in K$, such that

$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle \, dt \geq 0 \qquad \forall x \in K$$

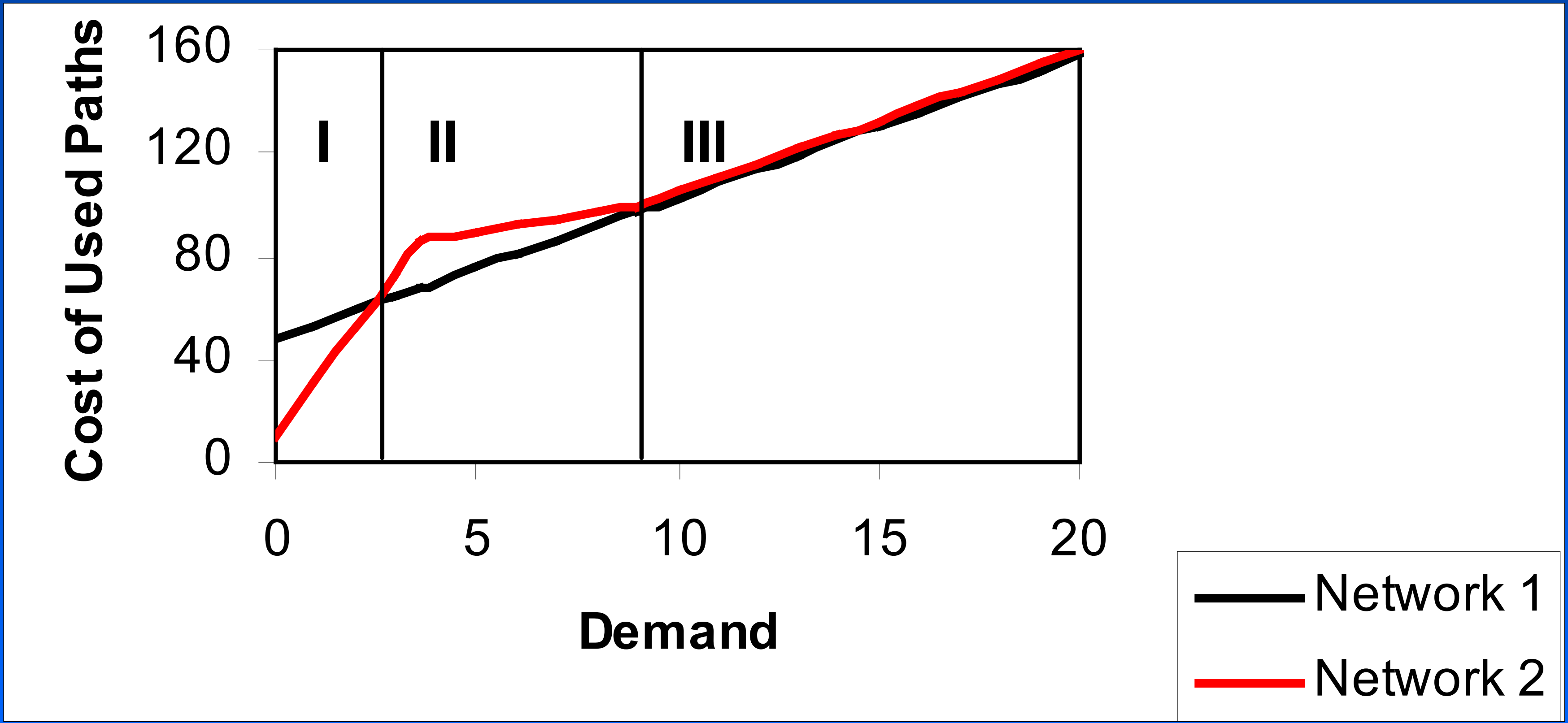
# What happens if the demand changes



# The Solution of an Evolutionary (Time Dependent) Variational Inequality

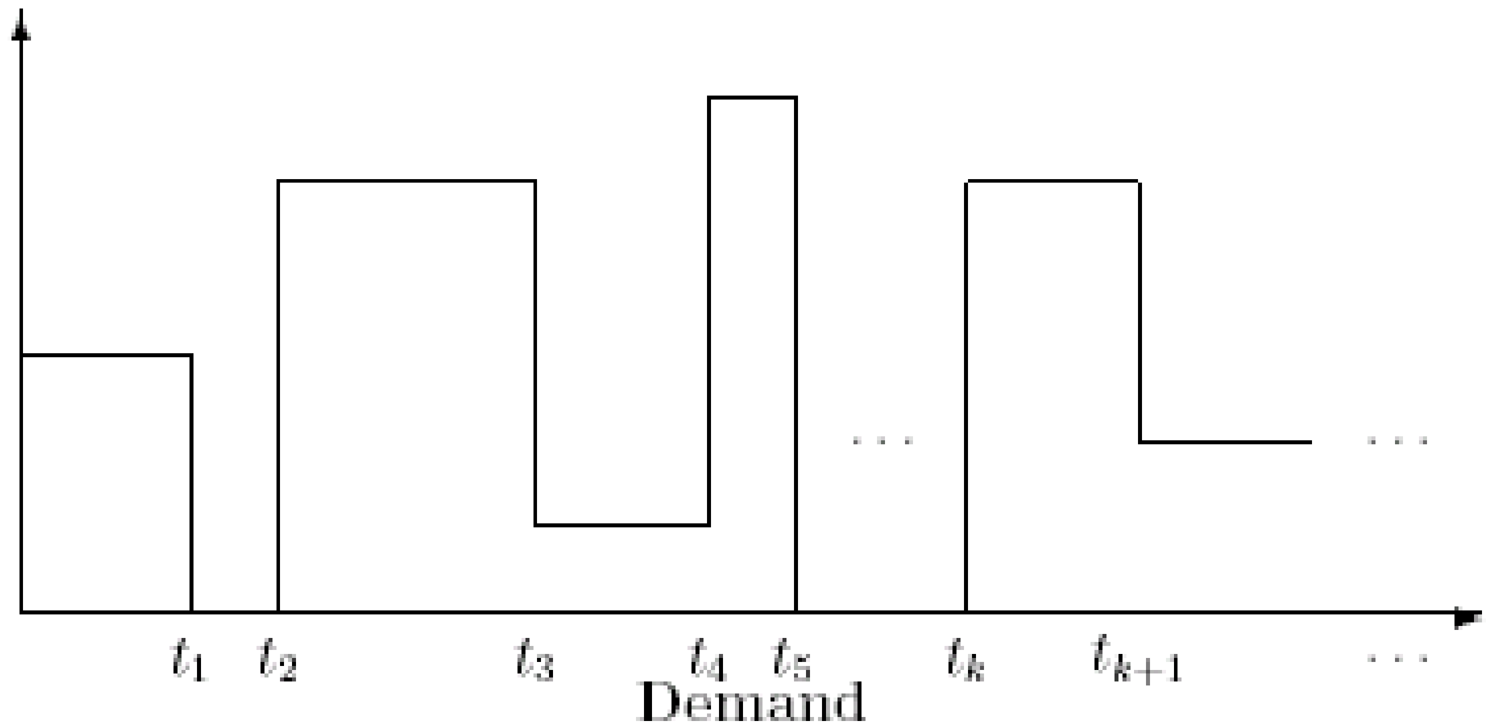


# In Regime II, the Addition of a New Road Makes Everyone Worse Off!



*The new road is NEVER used after a certain demand is reached even if the travel demand approaches infinity.*

*Hence, in general, except for a limited range of travel demand, building the new road is a complete waste!*




**If the demand is a step function, the solution to the EVI has the structure:**

$$x^*(t) = \begin{cases} x_1^* & \text{if } 0 \le t \le t_1 \\ x_2^* & \text{if } t_1 < t \le t_2 \\ \vdots & \vdots \\ x_{k+1}^* & \text{if } t_k < t \le t_{k+1} \\ \vdots & \vdots \end{cases}$$

# *New Challenges and Opportunities: The Unification of EVIs and PDSs*

# *Double-Layered Dynamics*

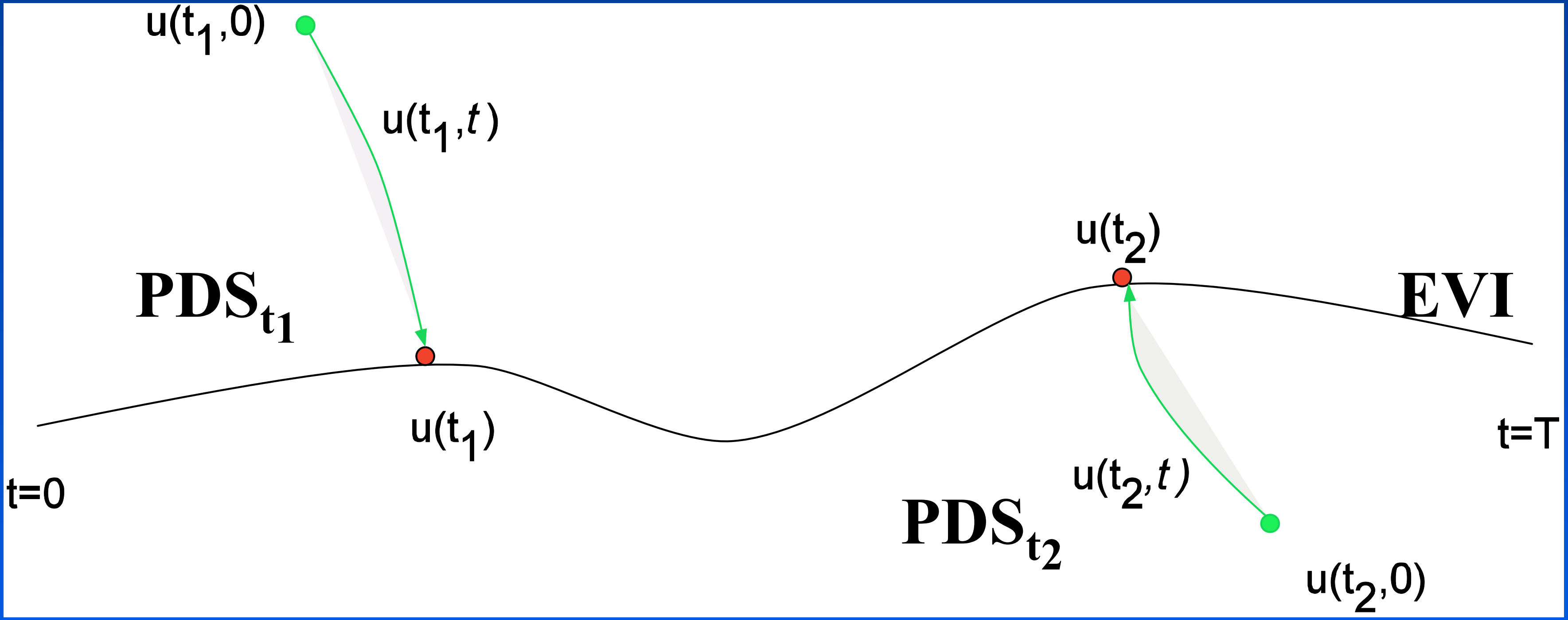
**The unification of EVIs and PDSs allows the modeling of dynamic networks over different time scales.**

## Papers:

**Projected Dynamical Systems and Evolutionary Variational Inequalities via Hilbert Spaces with Applications (Cojocaru, Daniele, and Nagurney), *Journal of Optimization Theory and Applications*, vol. 127, no. 3, pp. 1-15, December 2005.**

**Double-Layered Dynamics: A Unified Theory of Projected Dynamical Systems and Evolutionary Variational Inequalities (Cojocaru, Daniele, and Nagurney), *European Journal of Operational Research,* in press.**

# A Pictorial of the Double-Layered Dynamics



**Theorem (Cojocaru, Daniele, and Nagurney (2005))**

*The solutions to the EVI problem are the same as the critical points of the PDS and vice versa, that is, the critical points of the PDS are the solutions to the EVI.*

*Hence, by choosing the Hilbert space to be* $L^2([0,T], R^q)$, *we find that the solutions to the evolutionary variational inequality: find* $x^* \in K$ *such that*

$$\int_0^T \langle F(x^*(t)), x(t) - x^*(t)\rangle dt \geq 0, \quad \forall x \in K$$

*are the same as the critical points of the equation:*

$$\frac{\partial x(t,\tau)}{\partial \tau} = \Pi_K(x(t,\tau), -F(x(t,\tau))),$$

*that is, the points such that*

$$\Pi_K(x(t,\tau), -F(x(t,\tau))) \equiv 0 \quad a.e. \ in \ [0,T],$$

*which are obviously stationary with respect to* $\tau$.

**Theorem (Cojocaru, Daniele, and Nagurney (2005))**

*Consider the above EVI with $F$ Lipschitz continuous and strongly pseudo-monotone with degree $\alpha < 2$ on $K$, for almost all fixed $t \in [0, T]$, there exists $l_t > 0$, finite, such that the unique equilibrium $x^* := x^*(t)$ of the $PDS_t$ is reached by the (unique) solution $x(t, \tau)$ of the $PDS_t$, starting at the initial point $x_0^t \in K_t$. The time $l_t$ depends upon $\eta, \alpha$ and $||x_0^t - x^*||$.*

We have proved that for each $x_0^* \in K_t$, there exists $l_t < \infty$, depending on $\eta, \alpha, ||x_0^t - x^*||$, given by

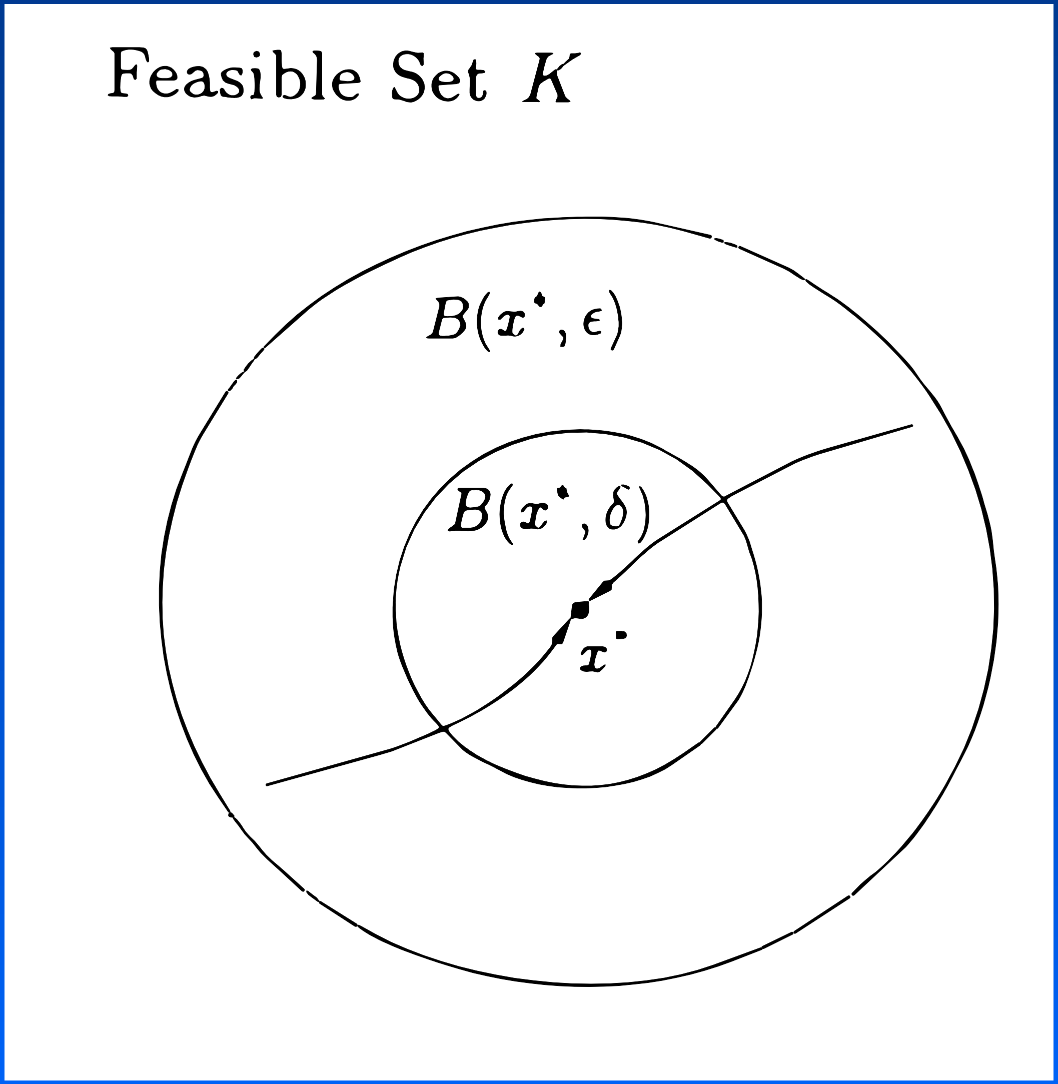
$$l_t := \frac{||x_0^t - x^*||^{2-\alpha}}{(2-\alpha)\eta},$$

such that whenever $\alpha < 2$,

$$D(\tau) > 0 \text{ when } \tau < l_t \text{ and } D(\tau) = 0 \text{ when } \tau \geq l_t.$$

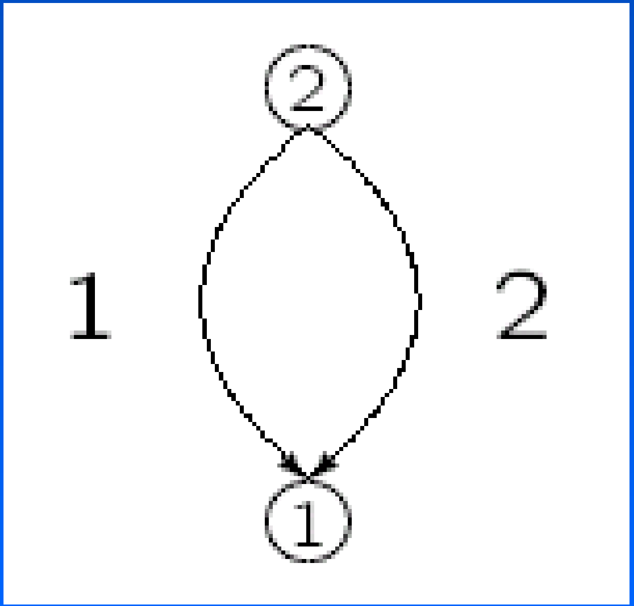
**In other words, *x** is a globally finite-time attractor for the unique solution of $PDS_t$ starting at $x_0^t$ and it will be reached in $l_t$ units of time.**

# *A Globally Finite-Time Attractor x**



# *A Dynamic Network Example with Time-Varying Demand and Capacities*

**We consider a network consisting of a single origin/destination pair of nodes and two paths connecting these nodes.**



**Let cost on path 1 be: $2x_1(t)-1.5$ and cost on path 2 be: $x_2(t)-1$.**

**The demand is t in the interval [0,2].**

**Suppose that we also have capacities:**
**$(0,0) \leq (x_1(t), x_2(t)) \leq (t, 3/2\, t)$.**

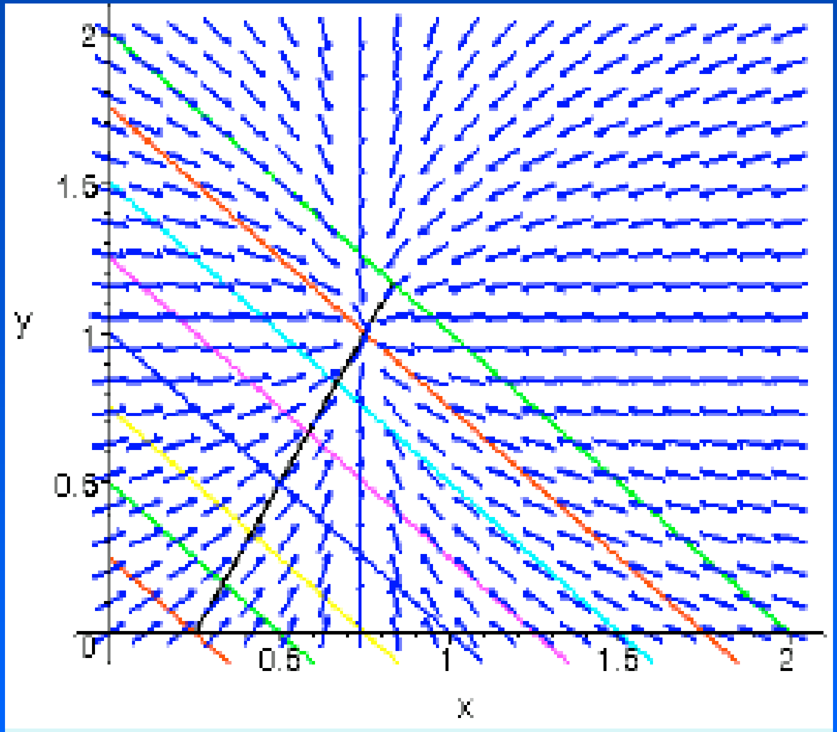
**With the help of PDS theory, we can compute an approximate curve of equilibrium by choosing**

$$t_0 \in \left\{ \frac{k}{4} | k \in \{0, \ldots, 8\} \right\}.$$

**Using a simple MAPLE computation, we obtain that the equilibria are the points:**

$$\left\{(0,0), \left(\frac{1}{4},0\right), \left(\frac{1}{3},\frac{1}{6}\right), \left(\frac{5}{12},\frac{1}{3}\right), \left(\frac{1}{2},\frac{1}{2}\right), \left(\frac{7}{12},\frac{2}{3}\right), \left(\frac{2}{3},\frac{5}{6}\right), \left(\frac{3}{4},1\right), \left(\frac{5}{6},\frac{7}{6}\right)\right\}.$$

**Interpolating these points, we obtain the approximate curve of network equilibria:**

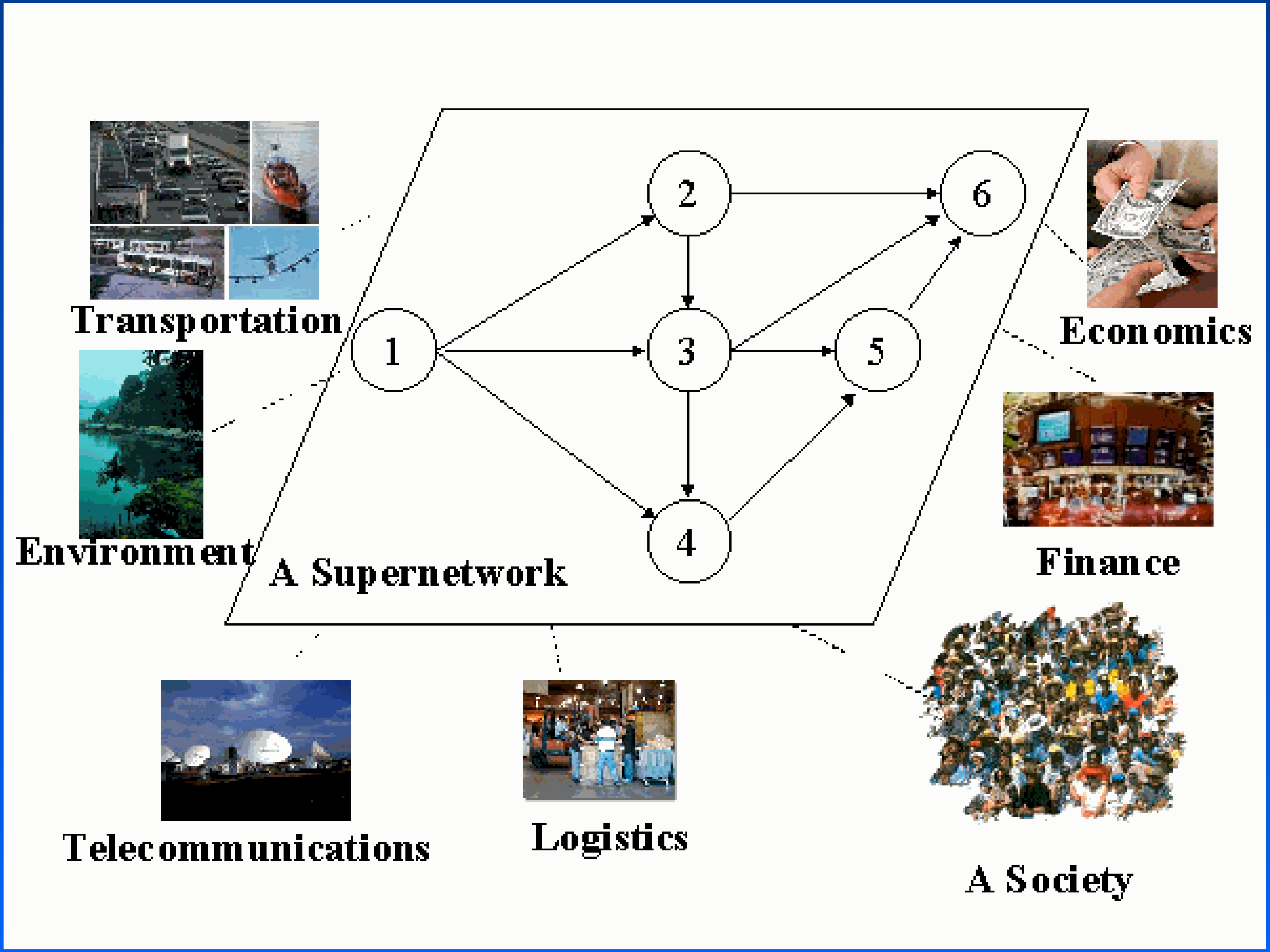


## *The tools that we are using in our dynamic network research include:*

- **network theory**
- **optimization theory**
- **game theory**
- **evolutionary variational inequality theory**
- **projected dynamical systems theory**
- **double layered dynamics theory**
- **network visualization tools.**

# Novel Applications
# of
# Dynamic Networks

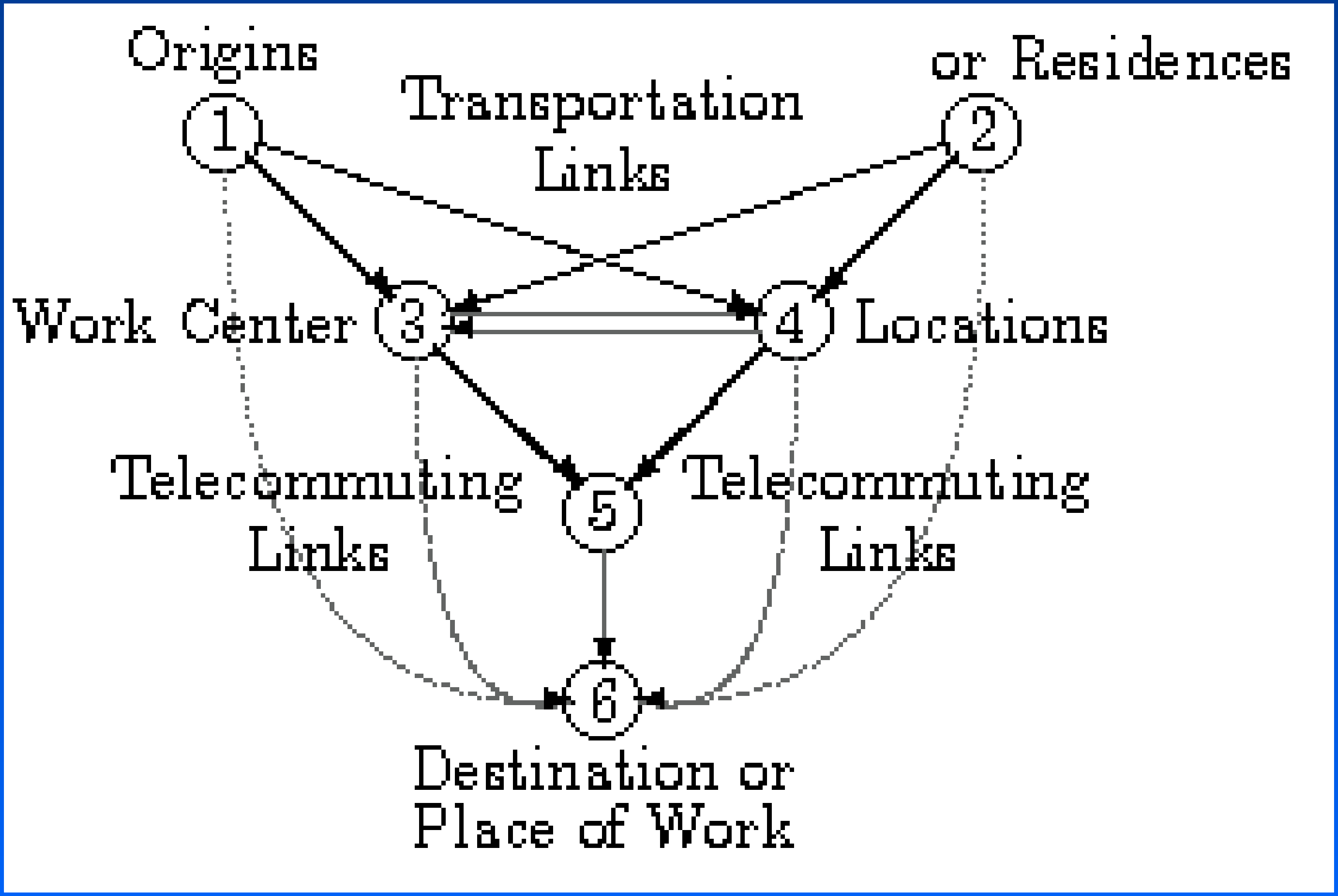
# Supernetworks: A New Paradigm



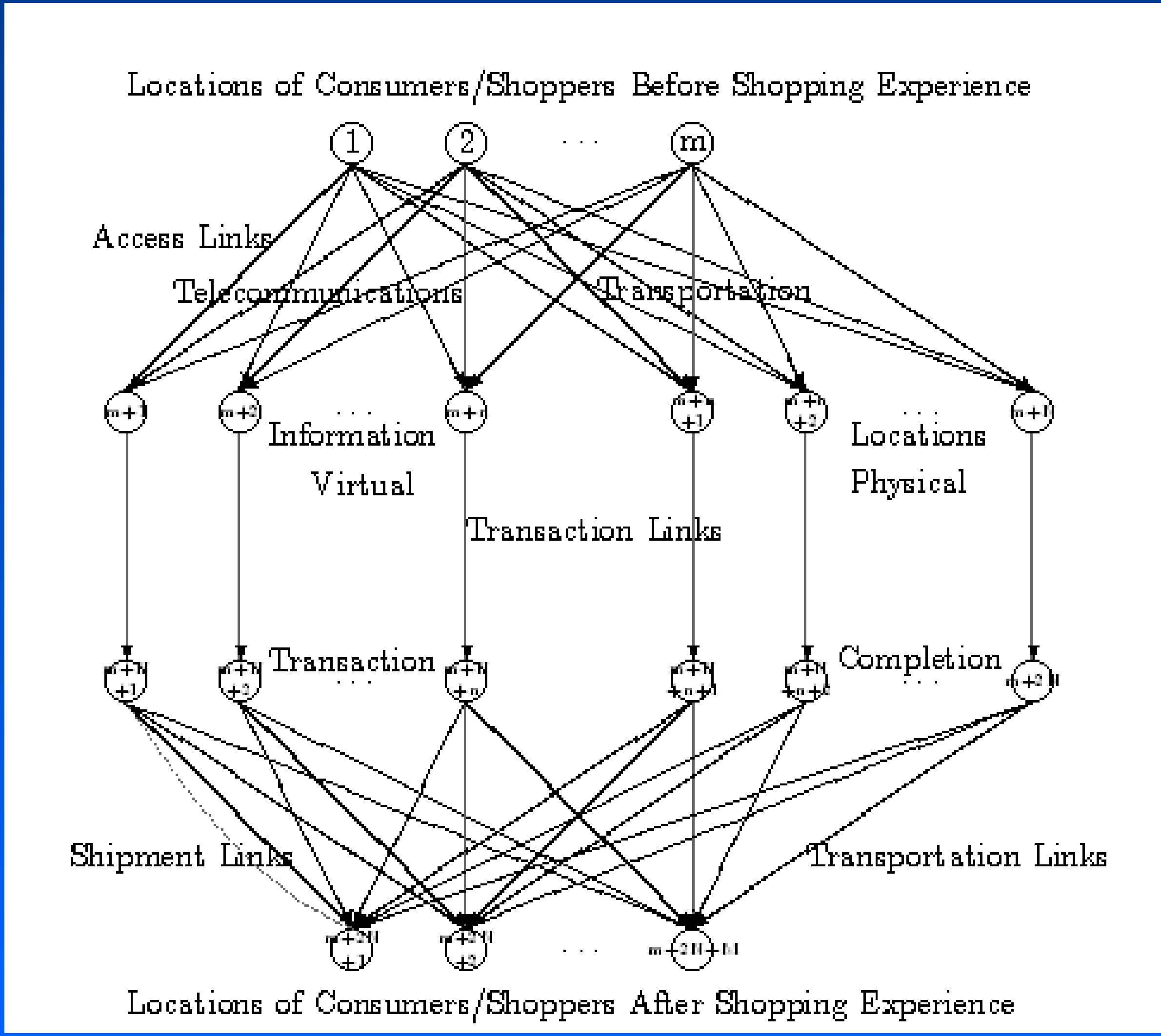
# *Applications of Supernetworks*

- **Telecommuting/Commuting Decision-Making**
- **Teleshopping/Shopping Decision-Making**
- **Supply Chain Networks with Electronic Commerce**
- **Financial Networks with Electronic Transactions**
- **Reverse Supply Chains with E-Cycling**
- **Knowledge Networks**
- **Energy Networks/Power Grids**

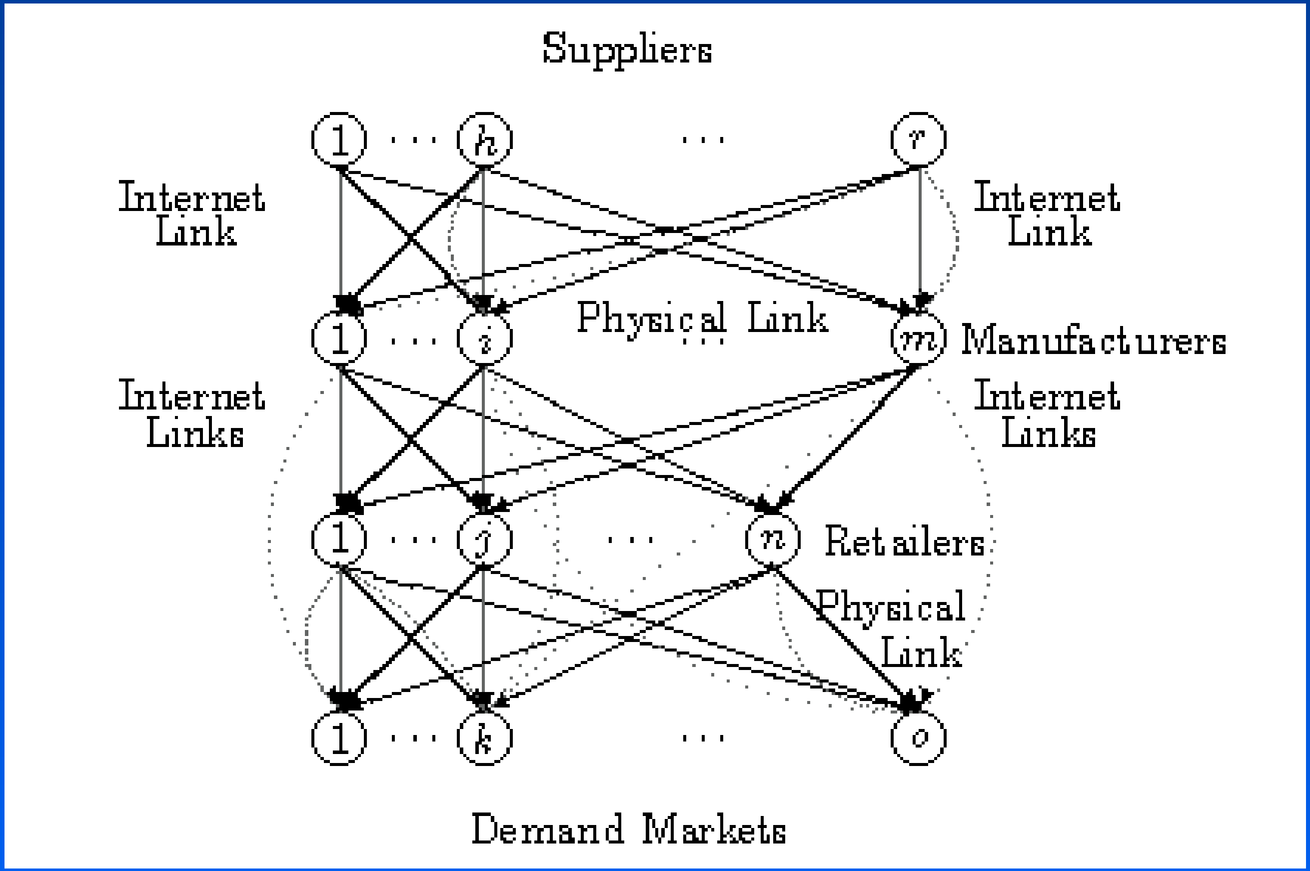
# A Supernetwork Conceptualization of Commuting versus Telecommuting



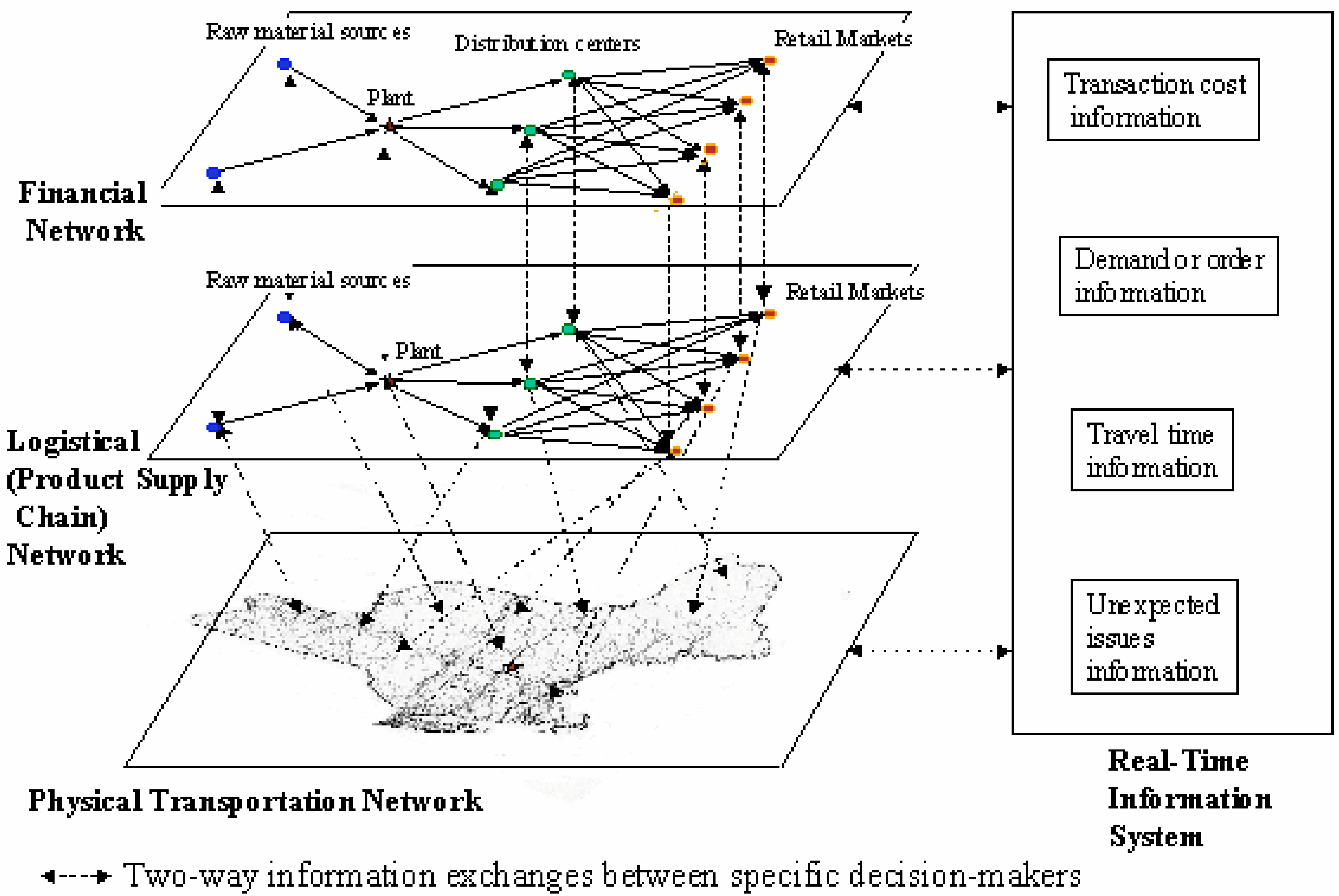
# A Supernetwork Framework for Teleshopping versus Shopping



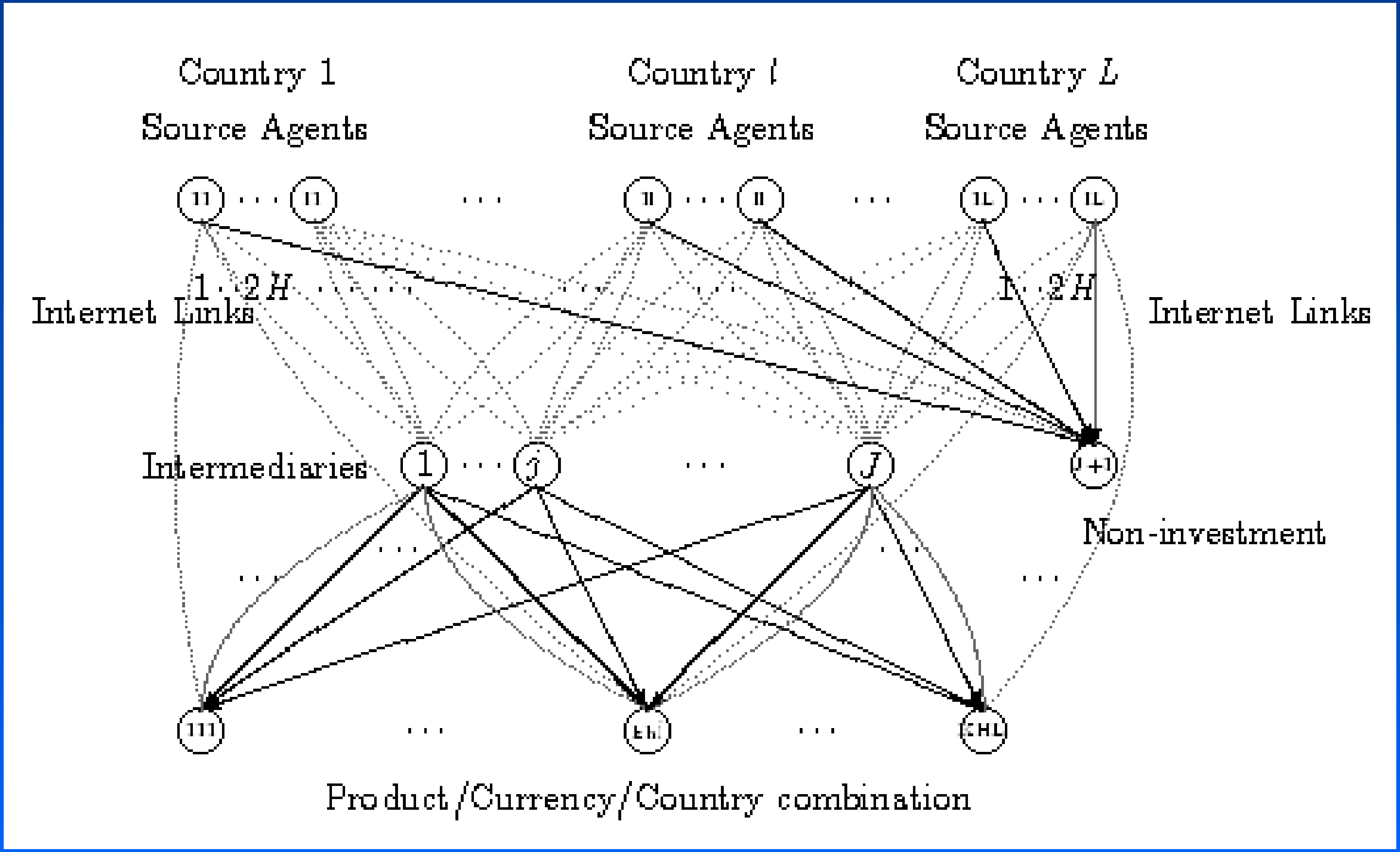
# The Supernetwork Structure of a Supply Chain Network



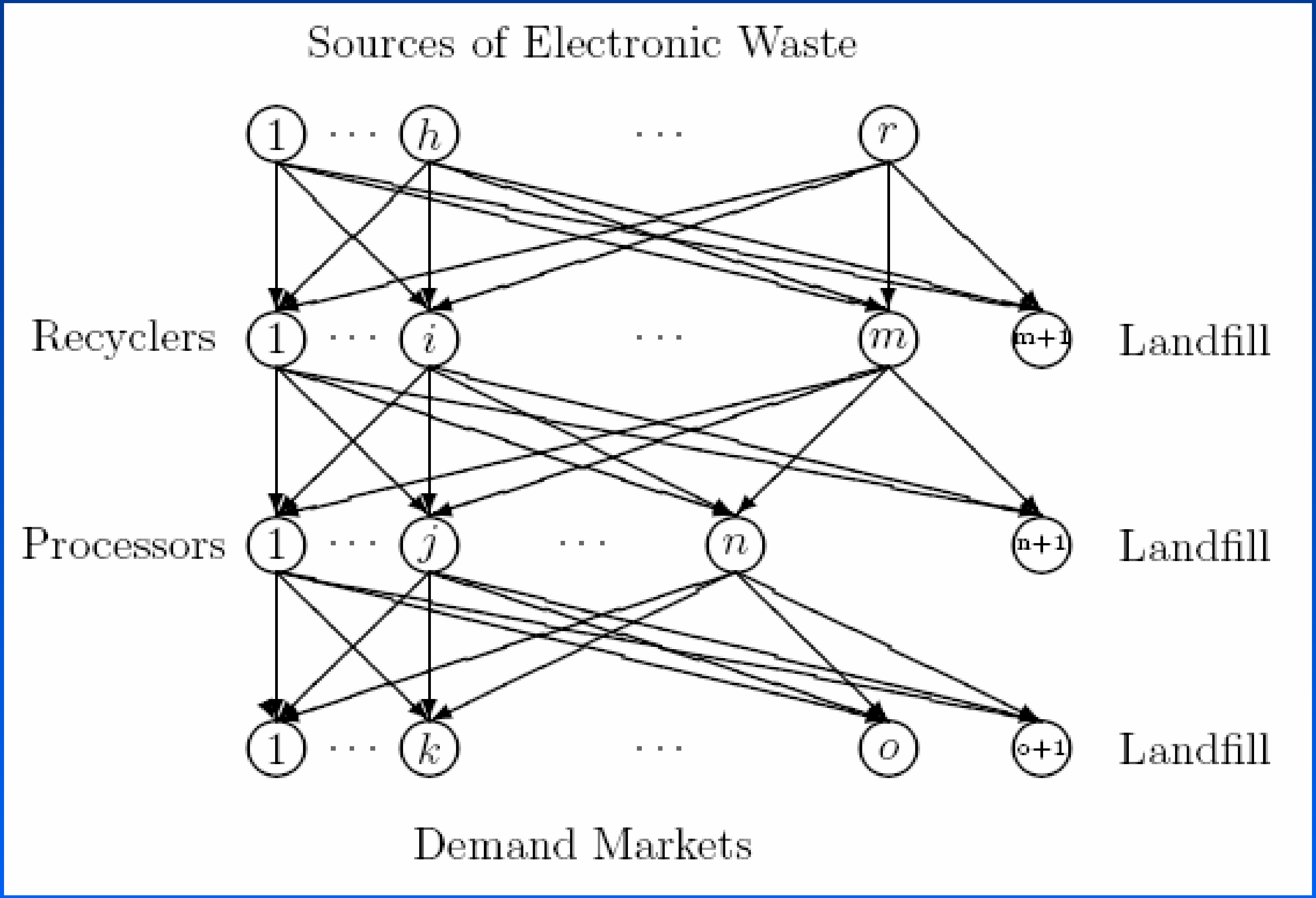
# Supply Chain -Transportation Supernetwork Representation



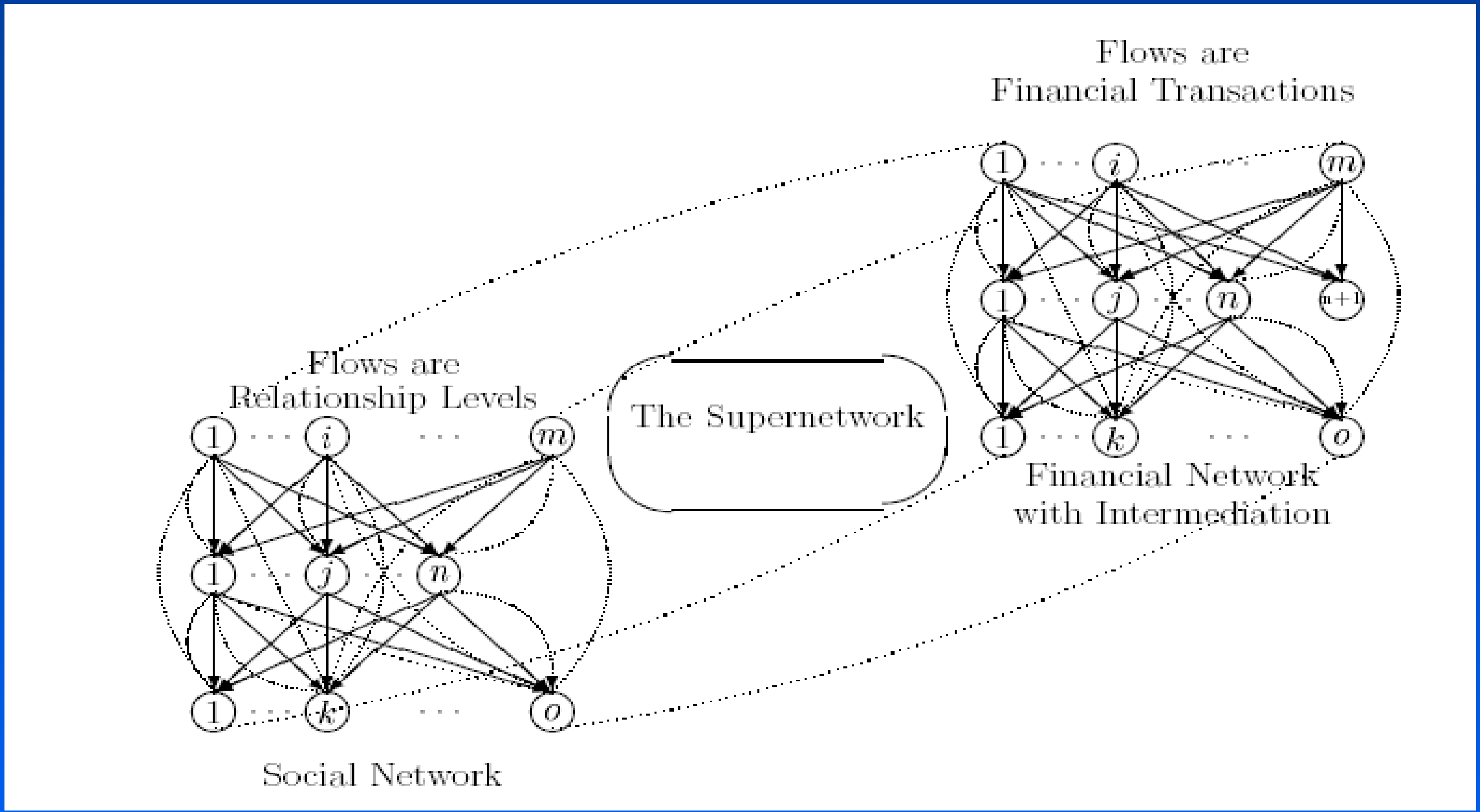
# International Financial Networks with Electronic Transactions



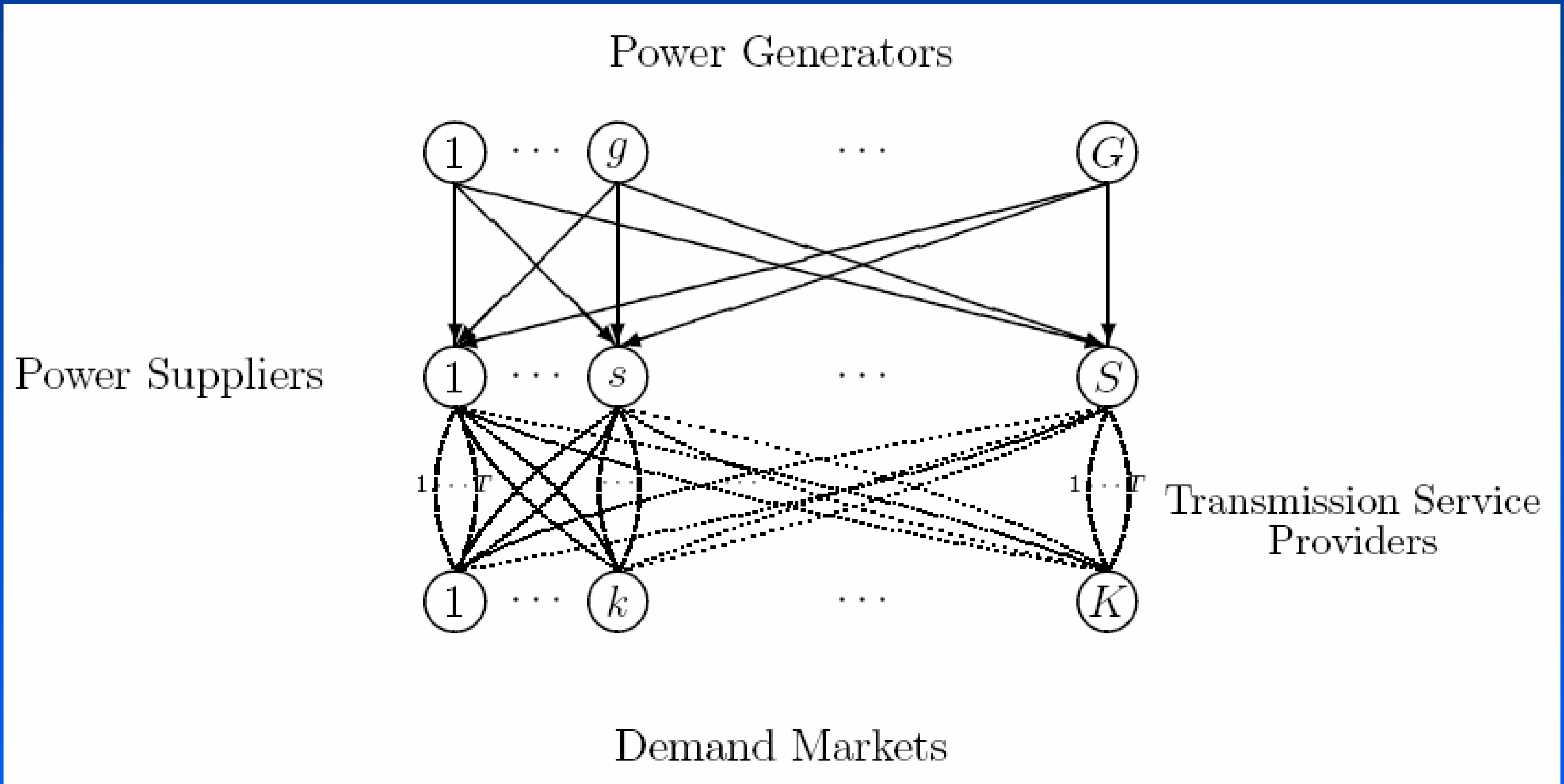
# The 4-Tiered E-Cycling Network



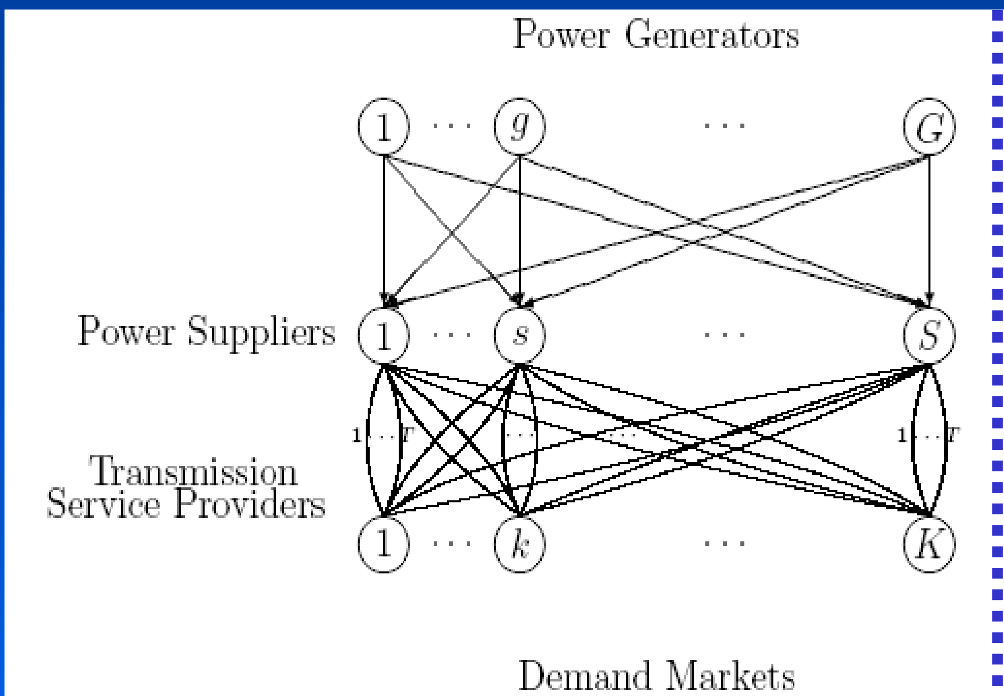
# Supernetwork Structure: Integrated Financial/Social Network System

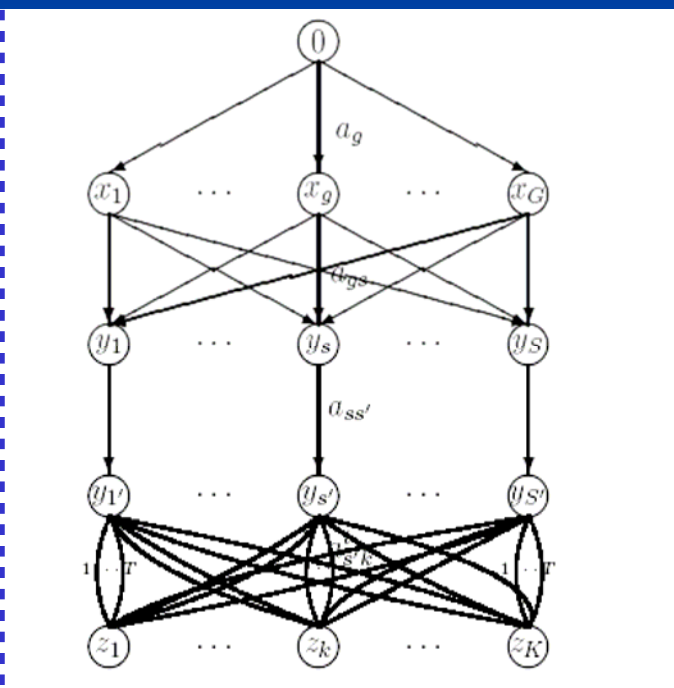


# The Electric Power Supply Chain Network



# *The Transportation Network Equilibrium Reformulation*

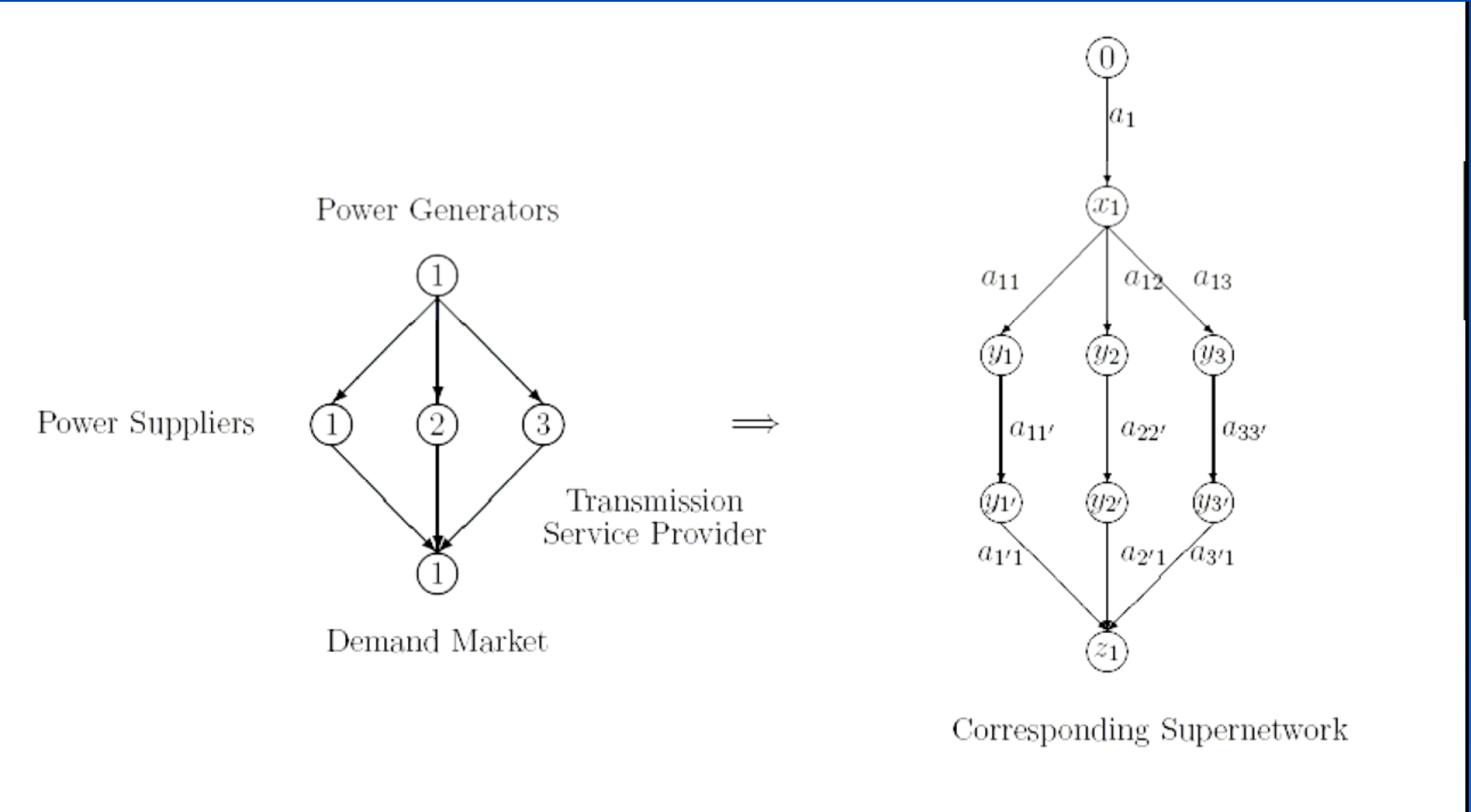


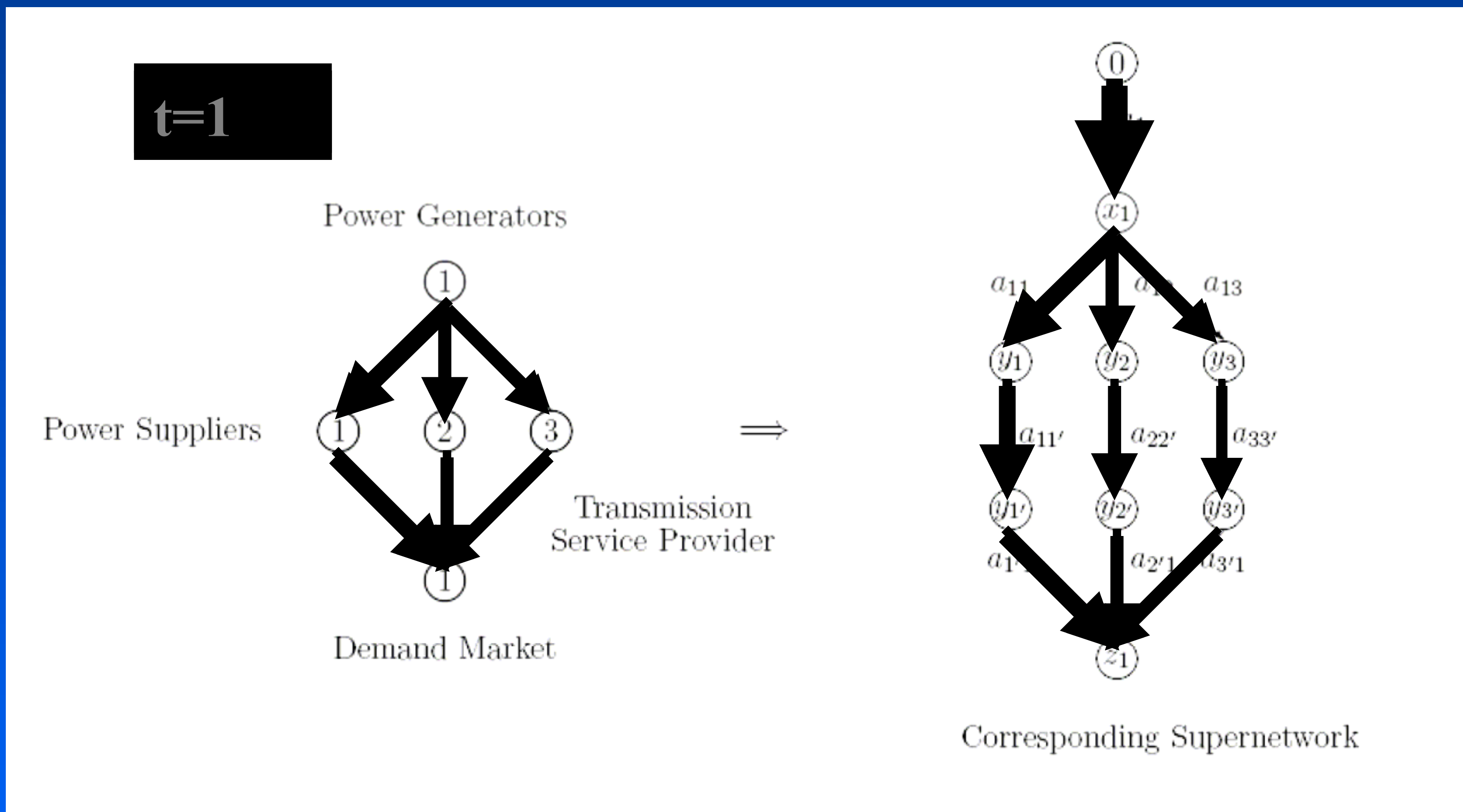

**Electric Power Supply Chain Network**

**Transportation Network**

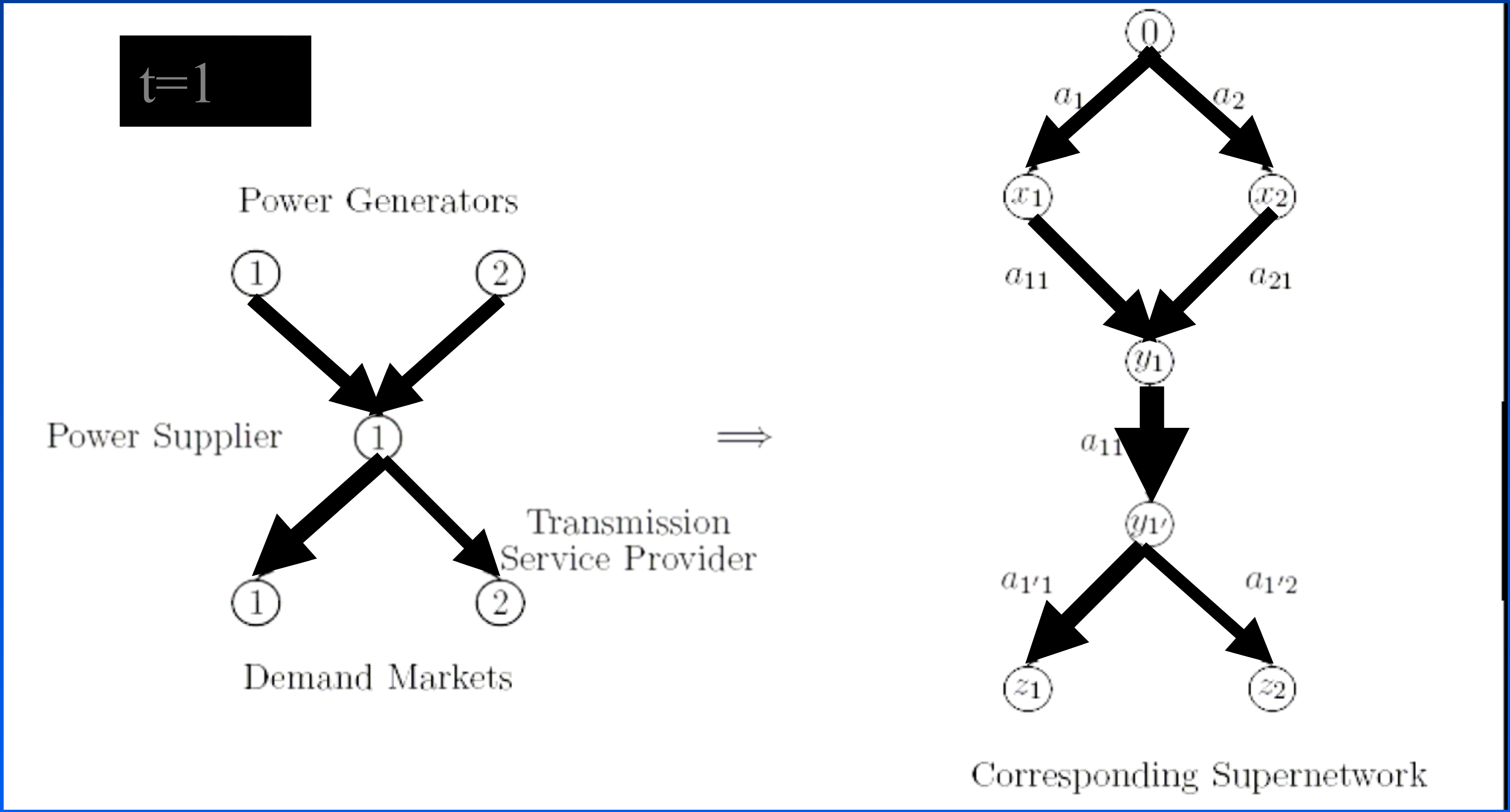
# Preliminary Results on Solving Time-Dependent Electric Power Supply Chain Networks as Dynamic Transportation Problems



# Flow Results



# More Flow Results



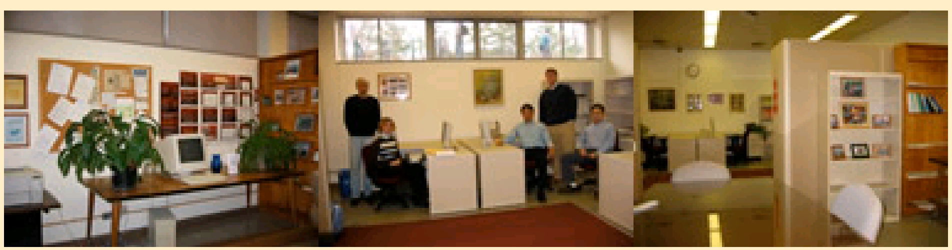
# The Virtual Center for Supernetworks

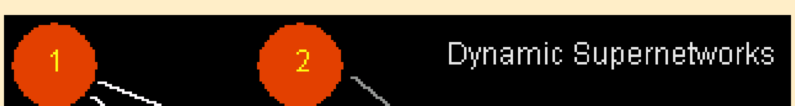

***Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life***



**The Virtual Center for Supernetworks** at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

Supernetworks Lab Page and Virtual Tour






Anna Nagurney will deliver her Fellowship Lecture, *Dynamic Networks with Applications,* at the Radcliffe Institute for Advanced Studies, Harvard University, November 9, 2005

**NEW!**

Special Sessions on the 50th Anniversary of Beckmann, McGuire, and Winsten's *Studies in the Economics of Transporation* at the November 2005 INFORMS Meeting in San Francisco

**NEW!**

Fall 2005 INFORMS Student Chapter Speaker Series Announced!!!

# *Thank you!*

For more information, see
http://supernet.som.umass.edu

Eugene M. Isenberg School of Management

**The Virtual Center for Supernetworks**

RADCLIFFE INSTITUTE FOR ADVANCED STUDY
HARVARD UNIVERSITY