The Co-Evolution and Emergence of Integrated International Financial Networks and Social Networks: Theory, Analysis, and Computations

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Outline of Presentation

- International financial networks
- The role of social networks in economic transactions
- The framework of supernetworks
- Related financial network literature
- Supernetworks consisting of international financial networks and social networks

International Financial Networks

- Advances in telecommunications, including the adoption of the Internet by businesses, consumers, and financial institutions have had an enormous effect on financial services and options available for financial transactions.
- Distribution channels have been transformed; New types of services and products introduced.
- Electronic commerce (e-commerce) through the internet has allowed for new connections not previously possible.

Motivation (E-Finance)

Electronic financial transactions in 2001:

- 15 million Americans paid their bills online with up to 46 million expected by 2005.
- $160 billion in mortgages were taken out online in the US (cf. Mullaney and Little (2002)).

Contributions

First international network models to include:

- Multicriteria decision-makers:
  - Net revenue maximization
  - Risk minimization
- Electronic transactions
- Dynamic adjustment processes

The models can handle as many
  - Countries, currencies
  - Source agents, intermediaries, demand markets

as needed

Definition of Social Networks

"A social network is a set of actors that may have relationships with one another. Networks can have few or many actors (nodes), and one or more kinds of relations (edges) between pairs of actors." (Hannemann, 2001)

Roles of Social Networks in Economic Transactions

- Examples from Sociology
  - Embeddedness theory
    - Granovetter (1985)
    - Uzzi (1996)

- Examples from Economics
  - Williamson (1983)
  - Joskow (1988)
  - Crawford (1990)
  - Vickers and Waterson (1991)
  - Muthoo (1998)

Roles of Social Networks in Economic Transactions

- Examples from Marketing
  - Relationship marketing
    - Ganesan (1994)
    - Bagozzi (1995)

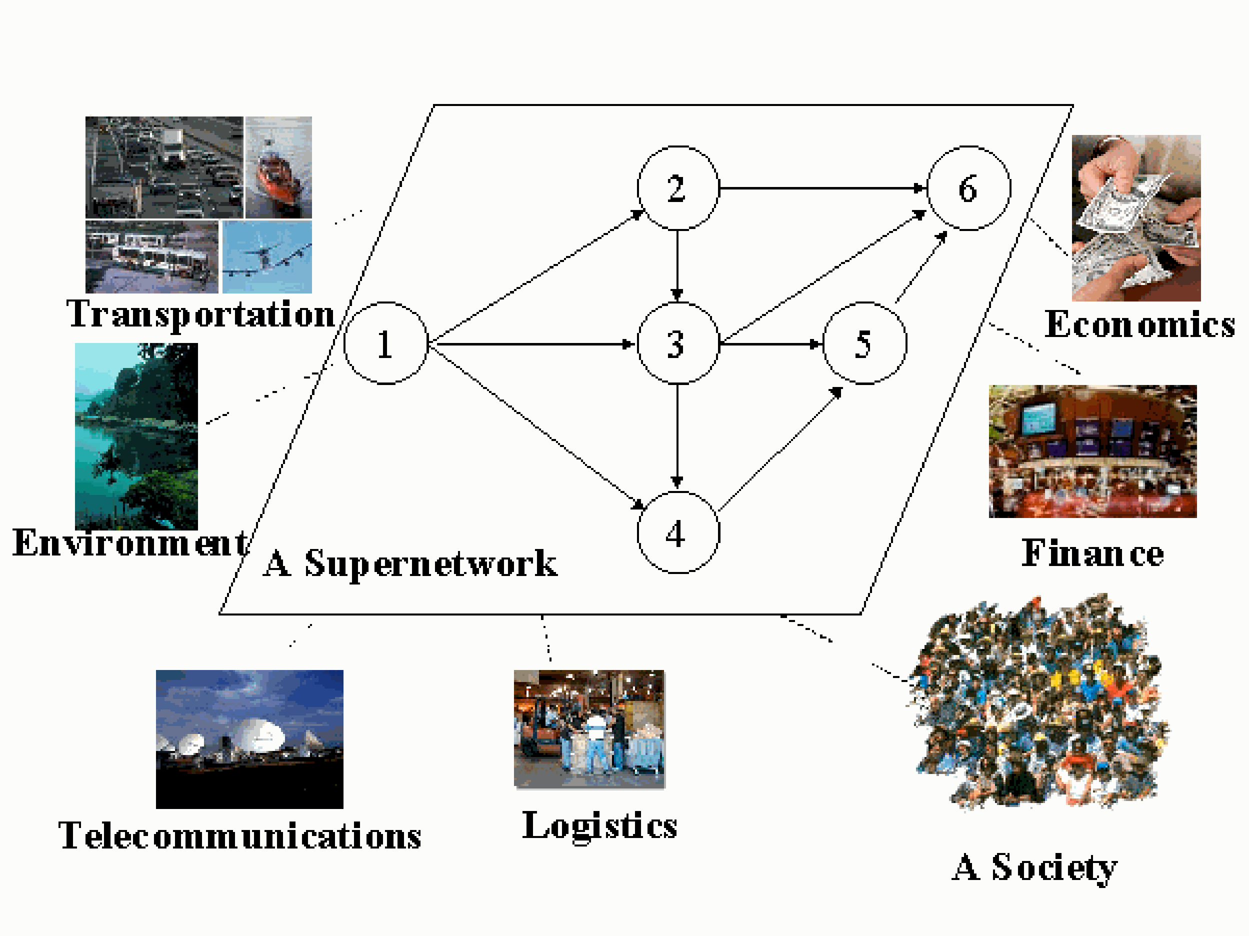
Roles of Social Networks in Financial Transactions

- Examples in the context of micro-financing
  - Ghatak (2002), Anthony (1997)
- Examples in the context of lending
  - Sharpe (1990), Petersen and Rajan (1994, 1995), Berger and Udell (1995), Uzzi (1997, 1999), DiMaggio and Louch (1998), Arrow (1998), Wilner (2000), Burt (2000), Boot and Thakor (2000)

Novelty of Our Research

- Supernetworks show the dynamic co-evolution of financial (financial product, price and even informational) flows and the social network structure
- Financial flows and social network structure are interrelated
- Network of relationships with a measurable economic value

Supernetworks



Tools That We Have Been Using

- Network theory
- Optimization theory
- Game theory
- Variational inequality theory
- Projected dynamical systems theory (which we have been instrumental in developing)
- Network visualization tools

Applications of Supernetworks

- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Energy Networks/Power Grids
- Knowledge Networks

Some of the Related Financial Network Literature

- Nagurney A. and Siokos S. (1997), **Financial Networks: Statics and Dynamics**, Springer-Verlag, Heidelberg, Germany.
- Nagurney, A. and Ke, K. (2001), "Financial Networks with Intermediation," *Quantitative Finance* **1**, 441-451.
- Nagurney, A. and Ke, K. (2003), "Financial Networks with Electronic Transactions: Modeling, Analysis, and Computations," *Quantitative Finance* ***3***, 71-87.
- Nagurney, A. and Cruz, J. (2003), "International Financial Networks with Electronic Transactions," in *Innovations in Financial and Economic Networks*, A. Nagurney, Editor, Edward Elgar Publishers, Cheltenham, England, pp. 136-168.
- Nagurney, A. and Cruz, J. (2004), "Dynamics of International Financial Networks with Risk Management," *Quantitative Finance* **4**, 276-291

Supernetworks Integrating Social Networks with Supply Chains

- Wakolbinger, T., and Nagurney, A. (2004), “Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations,” to appear in *Netnomics*

Supernetworks Integrating Social Networks with International Financial Networks

- Decision-makers in the network can decide about the relationship levels [0,1] that they want to establish.
- Establishing relationship levels incurs some costs.
- Higher relationship levels
  - Reduce transaction costs
  - Reduce risk
  - Have some additional value (“relationship value”)

Supernetworks Integrating Social Networks with International Financial Networks

Dynamic evolution of

- Financial flows and associated prices on the financial network with intermediation
- Relationship levels on the social network

Features of the Model

- Models the interaction of global financial and social networks
- Captures interactions among individual sectors
- Includes electronic transactions
- Allows for non-investment
- Incorporates transaction costs and risk

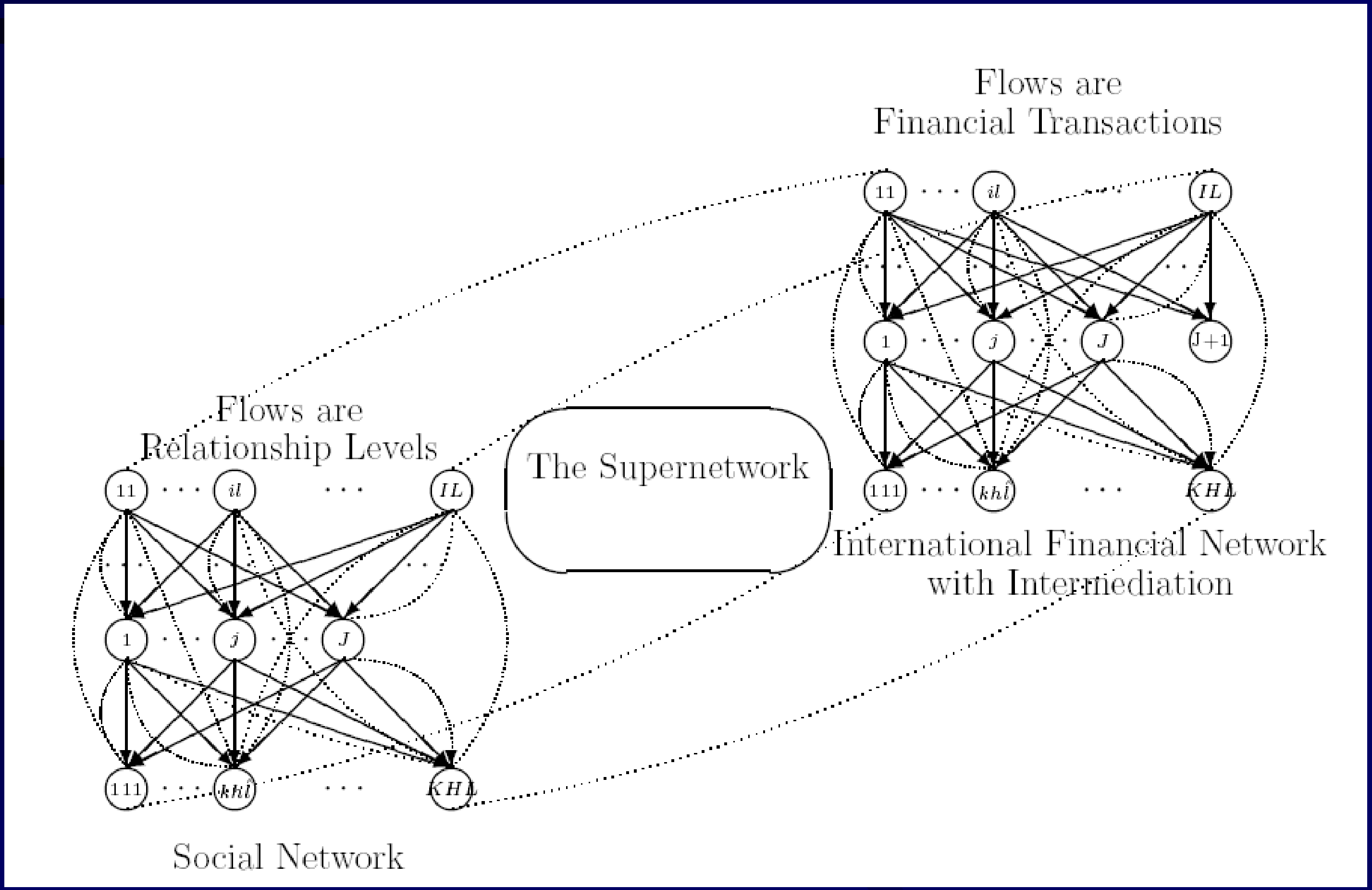
Assumptions of the Model

- Source agents can transact either physically or electronically with the intermediaries in different currencies.
- Source agents can transact directly with the demand markets via Internet links.
- Demand in a country can be associated with a particular currency.
- All transaction costs are measured in a base currency (US Dollar).

Multicriteria Decision-Makers

- Source agents and intermediaries
  - Maximize net revenue
  - Minimize risk
  - Maximize relationship value
  - Individual weights assigned to the different criteria

Supernetwork Structure: Integrated International Financial/ Social Network System



Some Model Notation

X^{il}_{jhm}	flow on link joining node *il* with node *j* in currency *h* and via mode *m*
y^{j}_{khlm}	flow on link joining node *j* with node *khl via mode m*
X^{il}_{khl}	flow on internet link joining node *il* with node *khl*
ρ^{il}_{1jhm}	price associated with the instrument in currency *h* transacted between source *il* and intermediary *j* via mode *m*
ρ^{j}_{2khlm}	price associated with intermediary *j* and demand market *k* in currency *h* and country *l* and mode *m*
ρ^{il}_{1khl}	price associated with the product in currency *h* transacted between source *il* and demand market *khl*
ρ_{3khl}	price of the instrument at demand market *k* in currency *h* and in country *l*
e_h	rate of appreciation of currency *h* against the basic currency

Some Model Notation

η^{il}_{jhm}	relationship level between source agent *il* and intermediary *j* associated with currency *h* and mode of transaction *m*
η^{il}_{khl}	relationship level associated with the virtual mode of transaction between source agent *il* and demand market *khl*
η^{j}_{khlm}	relationship level between intermediary *j* and demand market *khl* transacting through mode *m*

A Source Agent's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad U^{il} = \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho_{1jhm}^{il*} + e_h^*)x_{jhm}^{il} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho_{1khl}^{il*} + e_h^*)x_{kh\hat{l}}^{il}$$

$$-\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} c_{jhm}^{il}(x_{jhm}^{il}, \eta_{jhm}^{il}) - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} c_{kh\hat{l}}^{il}(x_{kh\hat{l}}^{il}, \eta_{kh\hat{l}}^{il}) - \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} b_{jhm}^{il}(\eta_{jhm}^{il})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} b_{kh\hat{l}}^{il}(\eta_{kh\hat{l}}^{il}) - \alpha^{il}(\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} r_{jhm}^{il}(x_{jhm}^{il}, \eta_{jhm}^{il}) + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} r_{kh\hat{l}}^{il}(x_{kh\hat{l}}^{il}, \eta_{kh\hat{l}}^{il}))$$

$$+\beta^{il}(\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} v_{jhm}^{il}(\eta_{jhm}^{il}) + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} v_{kh\hat{l}}^{il}(\eta_{kh\hat{l}}^{il})) \tag{18}$$

subject to:

$$x_{jhm}^{il} \geq 0, \quad x_{kh\hat{l}}^{il} \geq 0, \quad \forall j, h, m, k, \hat{l}, \tag{19}$$

$$0 \leq \eta_{jhm}^{il} \leq 1, \quad 0 \leq \eta_{kh\hat{l}}^{il} \leq 1, \quad \forall j, h, m, k, \hat{l}, \tag{20}$$

and the constraint (1) for source agent il.

$$\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} x_{jhm}^{il} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} x_{kh\hat{l}}^{il} \leq S^{il}, \quad \forall i, l. \tag{1}$$

Optimality Condition of Source Agents

determine $(x^{1*}, x^{2*}, \eta^{1*}, \eta^{2*}) \in \mathcal{K}^1$, satisfying

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\alpha^{il}\frac{\partial r^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial x^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial x^{il}_{jhm}} - \rho^{il*}_{1jhm} - e^*_h\right] \times \left[x^{il}_{jhm} - x^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\alpha^{il}\frac{\partial r^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}}, \eta^{il*}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}}, \eta^{il*}_{kh\hat{l}})}{\partial x^{il}_{kh\hat{l}}} - \rho^{il*}_{1kh\hat{l}} - e^*_h\right] \times \left[x^{il}_{kh\hat{l}} - x^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} + \frac{\partial b^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} - \beta^{il}\frac{\partial v^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}\right.$$

$$\left. +\alpha^{il}\frac{\partial r^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}\right] \times \left[\eta^{il}_{jhm} - \eta^{il*}_{jhm}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial c^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}}, \eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} + \frac{\partial b^{il}_{kh\hat{l}}(\eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \beta^{il}\frac{\partial v^{il}_{kh\hat{l}}(\eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} + \alpha^{il}\frac{\partial r^{il}_{kh\hat{l}}(x^{il*}_{kh\hat{l}}, \eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}}\right]$$

$$\times \left[\eta^{il}_{kh\hat{l}} - \eta^{il*}_{kh\hat{l}}\right] \geq 0, \quad \forall (x^1, x^2, \eta^1, \eta^2) \in \mathcal{K}^1, \tag{21}$$

where

$$\mathcal{K}^1 \equiv \left[(x^1, x^2, \eta^1, \eta^2) | x^{il}_{jhm} \geq 0,\ x^{il}_{kh\hat{l}} \geq 0,\ 0 \leq \eta^{il}_{jhm} \leq 1,\ 0 \leq \eta^{il}_{kh\hat{l}} \leq 1,\right.$$

$$\left. \forall i, l, j, h, m, k, \hat{l},\ \textit{and}\ (1)\ \textit{holds}\right] \tag{22}$$

A Financial Intermediary's Multicriteria Decision-Making Problem

$$
\begin{aligned}
\text{Maximize} \quad U^j = & \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}(\rho^{j*}_{2kh\hat{l}m}+e^*_h)y^j_{kh\hat{l}m} - c_j(x^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{c}^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm}) \\
& - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m},\eta^j_{kh\hat{l}m}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{b}^{il}_{jhm}(\eta^{il}_{jhm}) \\
& - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}b^j_{kh\hat{l}m}(\eta^j_{kh\hat{l}m}) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho^{il*}_{1jhm}+e^*_h)x^{il}_{jhm} \\
& -\delta^j\Big(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{r}^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm}) + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}r^j_{kh\hat{l}m}(y^j_{kh\hat{l}m},\eta^j_{kh\hat{l}m})\Big) \\
& +\gamma^j\Big(\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{v}^{il}_{jhm}(\eta^{il}_{jhm}) + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}v^j_{kh\hat{l}m}(\eta^j_{kh\hat{l}m})\Big) \qquad (41)
\end{aligned}
$$

subject to:

$$\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}y^j_{kh\hat{l}m} \le \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}x^{il}_{jhm}, \qquad (42)$$

$$x^{il}_{jhm} \ge 0, \quad y^j_{kh\hat{l}m} \ge 0, \quad \forall i,l,k,h,\hat{l},m, \qquad (43)$$

$$0 \le \eta^{il}_{jhm} \le 1, \quad 0 \le \eta^j_{kh\hat{l}m} \le 1, \quad \forall i,l,h,m,k,\hat{l}. \qquad (44)$$

Optimality Condition of Intermediaries

determine $(x^{1*}, y^*, \eta^{1*}, \eta^{3*}, \lambda^*) \in \mathcal{K}^2$, such that

$$
\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\delta^j \frac{\partial \hat{r}^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial x^{il}_{jhm}} + \frac{\partial c_j(x^{1*})}{\partial x^{il}_{jhm}} + \rho^{il*}_{1jhm} + e^*_h + \frac{\partial \hat{c}^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial x^{il}_{jhm}} - \lambda^*_j\right]
$$

$$
\times \left[x^{il}_{jhm} - x^{il*}_{jhm}\right]
$$

$$
+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\delta^j \frac{\partial r^j_{kh\hat{l}m}(y^{j*}_{kh\hat{l}m}, \eta^{j*}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(y^{j*}_{kh\hat{l}m}, \eta^{j*}_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} - \rho^{j*}_{2kh\hat{l}m} - e^*_h + \lambda^*_j\right]
$$

$$
\times \left[y^j_{kh\hat{l}m} - y^{j*}_{kh\hat{l}m}\right]
$$

$$
+\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\delta^j \frac{\partial \hat{r}^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(x^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} - \gamma^j \frac{\partial \hat{v}^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}\right.
$$

$$
\left. + \frac{\partial \hat{b}^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}\right] \times \left[\eta^{il}_{jhm} - \eta^{il*}_{jhm}\right]
$$

$$
+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\delta^j \frac{\partial r^j_{kh\hat{l}m}(y^{j*}_{kh\hat{l}m}, \eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(y^{j*}_{kh\hat{l}m}, \eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} - \gamma^j \frac{\partial v^j_{kh\hat{l}m}(\eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}\right.
$$

$$
\left. + \frac{\partial b^j_{khl}(\eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}\right] \times \left[\eta^j_{kh\hat{l}m} - \eta^{j*}_{kh\hat{l}m}\right]
$$

$$
+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} x^{il*}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} y^{j*}_{kh\hat{l}m}\right] \times \left[\lambda_j - \lambda^*_j\right] \geq 0, \quad \forall (x^1, y, \eta^1, \eta^3, \lambda) \in \mathcal{K}^2, \tag{45}
$$

where

$$
\mathcal{K}^2 \equiv \left[(x^1, y, \eta^1, \eta^3, \lambda) | x^{il}_{jhm} \geq 0,\ y^j_{kh\hat{l}m} \geq 0,\ 0 \leq \eta^{il}_{jhm} \leq 1,\ 0 \leq \eta^j_{kh\hat{l}m} \leq 1,\ \lambda_j \geq 0,\right.
$$

$$
\left. \forall i, l, j, h, m, k, \hat{l}\right]. \tag{46}
$$

Equilibrium Conditions for the Demand Markets

for all intermediaries: $j = 1, \ldots, J$

and all mode m; $m = 1, 2$:

$$\rho^{j*}_{2kh\hat{l}m} + e^*_h + \hat{c}^j_{kh\hat{l}m}(x^{2*}, y^*, \eta^{2*}, \eta^{3*}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad y^{j*}_{kh\hat{l}m} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad y^{j*}_{kh\hat{l}m} = 0, \end{cases} \tag{50}$$

and for all source agents il; $i = 1, \ldots, I$ and $l = 1, \ldots, L$:

$$\rho^{il*}_{1kh\hat{l}} + e^*_h + \hat{c}^{il}_{kh\hat{l}}(x^{2*}, y^*, \eta^{2*}, \eta^{3*}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad x^{il*}_{kh\hat{l}} > 0 \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad x^{il*}_{kh\hat{l}} = 0. \end{cases} \tag{51}$$

In addition, we must have that

$$d_{kh\hat{l}}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} > 0 \\ \leq \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} = 0. \end{cases} \tag{52}$$

VI Formulation of the Equilibrium Conditions for the Demand Markets

determine $(x^{2*}, y^*, \rho_3^*) \in R_+^{(IL+2J+1)KHL}$, such that

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\rho_{2kh\hat{l}m}^{j*} + e_h^* + \hat{c}_{kh\hat{l}m}^{j}(x^{2*}, y^*, \eta^{2*}, \eta^{3*}) - \rho_{3kh\hat{l}}^*\right] \times \left[y_{kh\hat{l}m}^{j} - y_{kh\hat{l}m}^{j*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\rho_{1kh\hat{l}}^{il*} + e_h^* + \hat{c}_{kh\hat{l}}^{il}(x^{2*}, y^*, \eta^{2*}, \eta^{3*}) - \rho_{3kh\hat{l}}^*\right] \times \left[x_{kh\hat{l}}^{il} - x_{kh\hat{l}}^{il*}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2} y_{kh\hat{l}m}^{j*} + \sum_{i=1}^{I}\sum_{l=1}^{L} x_{kh\hat{l}}^{il*} - d_{kh\hat{l}}(\rho_3^*)\right] \times \left[\rho_{3kh\hat{l}} - \rho_{3kh\hat{l}}^*\right] \geq 0,$$

$$\forall (x^2, y, \rho_3) \in R_+^{(IL+2J+1)KHL}. \qquad (53)$$

The Equilibrium State

Definition 1: The equilibrium state of the supernetwork integrating the international financial network with the social network is one where the financial flows and relationship levels between the tiers of the network coincide and the financial flows, relationship levels, and prices satisfy the sum of conditions (21), (45), and (53).

The equilibrium state is equivalent to a VI of the form:

$$\text{determine } X^* \in \mathcal{K} \text{ satisfying}$$

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

Dynamics of the Financial Products

- Between intermediaries and demand markets

$$\dot{y}^j_{kh\hat{l}m} = \begin{cases} \rho_{3kh\hat{l}} - \delta^j \frac{\partial r^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} - \frac{\partial c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} & \\ -\hat{c}^j_{kh\hat{l}m}(x^2, y, \eta^2, \eta^3) - \lambda_j, & \text{if} \quad y^j_{kh\hat{l}m} > 0 \\ \max\{0, \rho_{3kh\hat{l}} - \delta^j \frac{\partial r^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} - \frac{\partial c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial y^j_{kh\hat{l}m}} & \\ -\hat{c}^j_{kh\hat{l}m}(x^2, y, \eta^2, \eta^3) - \lambda_j\}, & \text{if} \quad y^j_{kh\hat{l}m} = 0. \end{cases}$$

Dynamics of the Relationship Levels

- Between the source agents and the financial intermediaries

$$\dot{\eta}^{il}_{jhm} = \begin{cases} \beta^{il}\frac{\partial v^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} + \gamma^{j}\frac{\partial \hat{v}^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} \\ -\frac{\partial \hat{c}^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \alpha^{il}\frac{\partial r^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \delta^{j}\frac{\partial \hat{r}^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} \\ -\frac{\partial b^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial \hat{b}^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}, & \text{if} \quad 0 < \eta^{il}_{jhm} < 1 \\ \\ \min\{1, \max\{0, \beta^{il}\frac{\partial v^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} + \gamma^{j}\frac{\partial \hat{v}^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} \\ -\frac{\partial \hat{c}^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \alpha^{il}\frac{\partial r^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \delta^{j}\frac{\partial \hat{r}^{il}_{jhm}(x^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} \\ -\frac{\partial b^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial \hat{b}^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}}\}\}, & \text{otherwise}, \end{cases}$$

Dynamics of the Relationship Levels

- Between the source agents and the demand markets

$$\dot{\eta}^{il}_{kh\hat{l}} = \begin{cases} \beta^{il}\frac{\partial v^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial b^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \alpha^{il}\frac{\partial r^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}}, & \text{if} \quad 0 < \eta^{il}_{kh\hat{l}} < 1, \\ \min\{1, \max\{0, \beta^{il}\frac{\partial v^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial b^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} \\ -\alpha^{il}\frac{\partial r^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}}\}\}, & \text{otherwise} \end{cases}$$

Dynamics of the Relationship Levels

- Between the financial intermediaries and the demand markets

$$\dot{\eta}^j_{kh\hat{l}m} = \begin{cases} \gamma^j \frac{\partial v^j_{kh\hat{l}m}(\eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} - \delta^j \frac{\partial r^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} - \frac{\partial c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} \\ -\frac{\partial b^j_{khl}(\eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}, & \text{if} \quad 0 < \eta^j_{kh\hat{l}m} < 1, \\ \\ \min\{1, \max\{0, \gamma^j \frac{\partial v^j_{kh\hat{l}m}(\eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} - \delta^j \frac{\partial r^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} \\ -\frac{\partial c^j_{kh\hat{l}m}(y^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} - \frac{\partial b^j_{khl}(\eta^j_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}\}\}, & \text{otherwise,} \end{cases}$$

Dynamics of the Prices

- At demand markets

$$\dot{\rho}_{3k h\hat{l}} = \begin{cases} d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j}_{kh\hat{l}m} - \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il}_{kh\hat{l}}, & \text{if} \quad \rho_{3kh\hat{l}} > 0 \\ \max\{0, d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J} \sum_{m=1}^{2} y^{j}_{kh\hat{l}m} - \sum_{i=1}^{I} \sum_{l=1}^{L} x^{il}_{kh\hat{l}}\}, & \text{if} \quad \rho_{3kh\hat{l}} = 0. \end{cases}$$

- At intermediaries

$$\dot{\lambda}_j = \begin{cases} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} y^{j}_{kh\hat{l}m} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} x^{il}_{jhm}, & \text{if} \quad \lambda_j > 0 \\ \max\{0, \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} y^{j}_{kh\hat{l}m} - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} x^{il}_{jhm}\}, & \text{if} \quad \lambda_j = 0. \end{cases}$$

Projected Dynamical System

- The dynamic models can be rewritten as a projected dynamical system (Nagurney and Zhang (1996a)) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{66}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X (cf. (58)) and $X_0 = (x^{10}, x^{20}, y^0, \eta^{10}, \eta^{20}, \eta^{30}, \lambda^0, \rho_3^0)$ is the initial point corresponding to the initial financial flow and price pattern.

- The set of stationary points of the projected dynamical system (66) coincides with the set of solutions of the variational inequality problem (54) and, thus, with the set of equilibrium points as defined in Definition 1.

Computational Procedure

We use the Euler Method to solve the Variational Inequality (VI) problem and to track the dynamic trajectories associated with the projected dynamical systems. The VI is in standard form:

determine $X^* \in \mathcal{K}$ satisfying

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \to 0$, as $T \to \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

$$y = P_{\mathcal{K}} X = \arg\min_{z \in \mathcal{K}} \|X - z\|.$$

Step 2: Convergence Verification

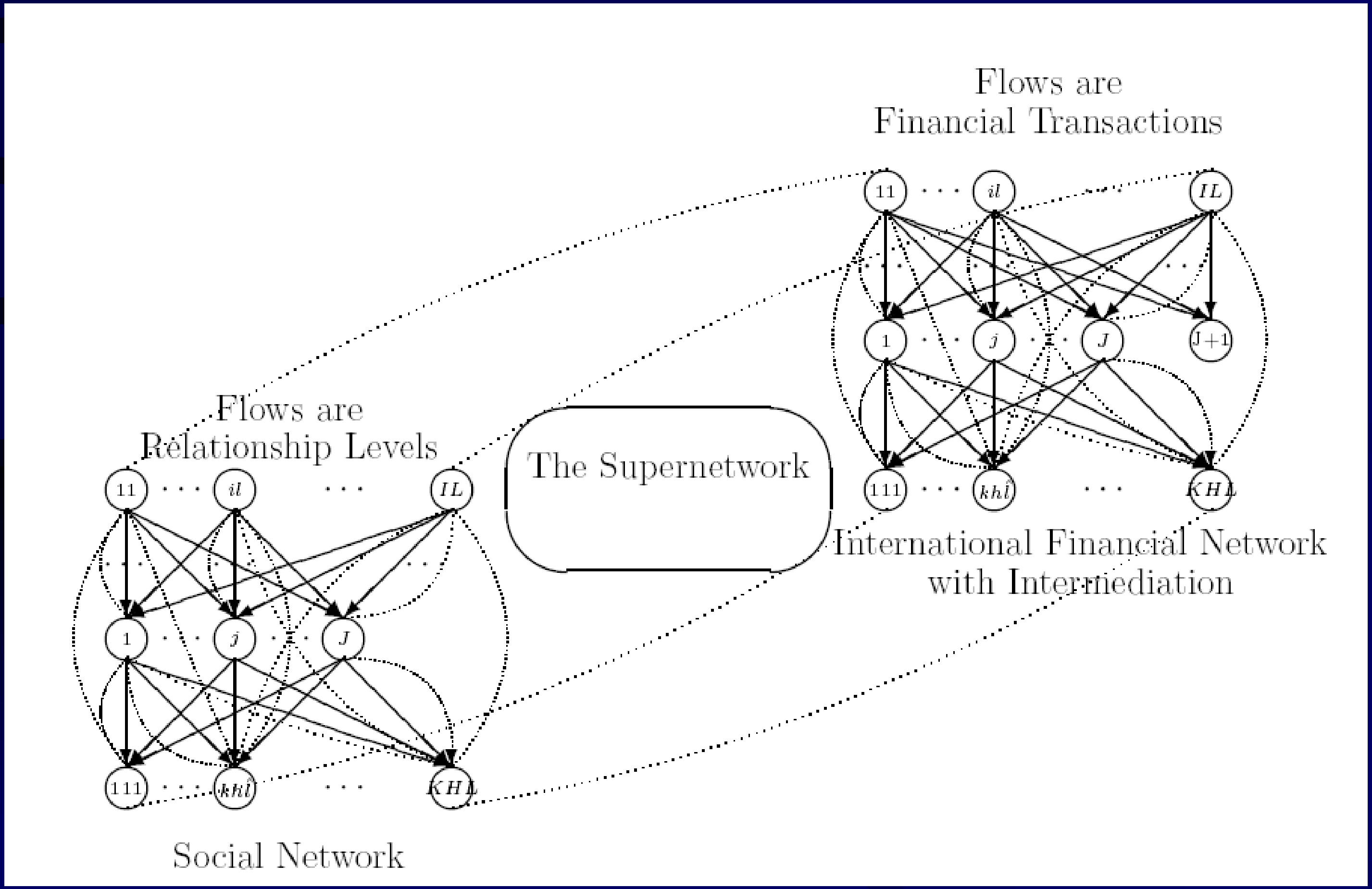
If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1,

Qualitative Properties

We have established

- Existence of a solution to the VI
- Uniqueness of a solution to the VI
- Conditions for the existence of a unique trajectory to the projected dynamical system
- Convergence of the Euler method

Supernetwork Structure: Integrated International Financial/ Social Network System



Numerical Examples 1 and 2

- 1 country, 2 currencies
- 2 source agents, 2 intermediaries, 2 financial products
- Electronic transactions only between source agents and demand markets
- Relationship levels only between source agents and intermediaries and between intermediaries and demand markets
- Financial holdings of each source agent are 20
- Variance-covariance matrices are equal to identity matrices

Numerical Examples: Data for Source Agents

– Transaction cost functions

$$c^{il}_{jhm}(x^{il}_{jhm}, \eta^{il}_{jhm}) = .5(x^{il}_{jhm})^2 + 3.5x^{il}_{jhm} - \eta^{il}_{jhm};$$

$$i = 1, 2; l = 1; j = 1, 2; h = 1, 2; m = 1.$$

$$c^{il}_{kh\hat{l}}(x^{il}_{kh\hat{l}}) = .5(x^{il}_{kh\hat{l}})^2 + x_{kh\hat{l}}; \quad \forall i, l, \hat{l}, k, h.$$

Numerical Examples: Data for Intermediaries

– Handling cost functions

$$c_j(x^1) = .5(\sum_{i=1}^{2}\sum_{h=1}^{2} x_{jh1}^{i1})^2; \quad j = 1, 2.$$

– Transaction cost functions

$$\hat{c}_{jhm}^{il}(x_{jhm}^{il}, \eta_{jhm}^{il}) = 1.5{x_{jhm}^{il}}^2 + 3x_{jhm}^{il};$$

$$i = 1, 2; l = 1; j = 1, 2; h = 1, 2; m = 1.$$

Numerical Examples: Data for Demand Markets

– Demand functions

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000,$$

$$d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000,$$

$$d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000,$$

– Transaction cost functions

$$\hat{c}^{il}_{kh\hat{l}}(x^2, y, \eta^2, \eta^3) = .1x^{il}_{kh\hat{l}} + 1, \quad \forall i, l, \hat{l}, k, h.$$

$$\hat{c}^{j}_{kh\hat{l}m}(x^2, y, \eta^2, \eta^3) = y^{j}_{kh\hat{l}m} + 5 - \eta^{j}_{kh\hat{l}m}; \;\; k = 1, 2; h = 1, 2; \hat{l} = 1; m = 1.$$

Numerical Examples 1 and 2

- Relationship value functions

$$v^{il}_{jhm}(\eta^{il}_{jhm}) = \eta^{il}_{jhm}, \quad \forall i, l, j, h, m; \quad v^{j}_{kh\hat{l}m}(\eta^{j}_{kh\hat{l}m}) = \eta^{j}_{kh\hat{l}m}, \quad \forall j, k, h, \hat{l}, m,$$

- Relationship cost functions

$$b^{il}_{jhm}(\eta^{il}_{jhm}) = 2\eta^{il}_{jhm}, \quad \forall i, l, j, h, m;$$

$$b^{j}_{kh\hat{l}m}(\eta^{j}_{kh\hat{l}m}) = \eta^{j}_{kh\hat{l}m}, \quad \forall j, k, h, \hat{l}, m.$$

Differences between Examples

- Example 1
  - The weight for relationship value is equal to 1.
- Example 2
  - The weight for relationship value for the two source agents increased from 1 to 20.
- Example 3
  - 2 countries instead of 1 - The data for the first country in Example 1 was replicated.
- Example 4
  - Like Example 3 but weight for relationship was increased from 1 to 20.

Numerical Examples: Equilibrium Financial Product Transactions

- Example 1 and 2

$$x^{1*} := x^{11*}_{111} = x^{11*}_{121} = x^{11*}_{211} = x^{11*}_{221} = x^{211}_{11*} = x^{22*}_{211} = x^{21*}_{211} = x^{21*}_{221} = .0662;$$

$$x^{2*} := x^{11*}_{111} = x^{11*}_{121} = x^{11*}_{211} = x^{11*}_{221} = x^{21*}_{111} = x^{21*}_{121} = x^{21*}_{211} = x^{21*}_{221} = 4.9938;$$

$$y^{*} := y^{1*}_{1111} = y^{1*}_{1211} = y^{1*}_{2111} = y^{1*}_{2211} = y^{2*}_{1111} = y^{2*}_{1211} = y^{2*}_{2111} = y^{2*}_{2211} = .0642.$$

- Example 3 and 4

$$x^{il*}_{kh\hat{l}} = 2.5000 \text{ for all } i, l, k, h, \hat{l}.$$

  – All other flows were equal to 0.

Numerical Examples: Equilibrium Prices

- Example 1 and 2
  - At intermediaries

$$\lambda_1^* = \lambda_2^* = 272.7246,$$

  - At demand markets

$$\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = 282.8586.$$

- Example 3 and 4
  - At intermediaries

$$\lambda_1^* = \lambda_2^* = 279.6194,$$

  - At demand markets

$$\rho_{3kh\hat{l}}^* = 282.8578, \ \forall k, h, \hat{l}.$$

Numerical Examples: Equilibrium Relationship Levels

- Example 1 and 3
  - All relationship levels are equal to 0.
- Example 2 and 4
  - Relationship levels of source agents are equal to 1.

Types of Simulations

- We can simulate
  - Changes in transaction, handling, and relationship production cost functions
  - Changes in demand and risk functions
  - Changes in weights for relationship value and risk
  - Addition and removal of actors
  - Addition and removal of multiple transaction modes
  - Addition and removal of countries and currencies

Summary

- We model the behavior of the decision-makers, their interactions, and the dynamic evolution of the associated variables.
- We study the problems qualitatively as well as computationally.
- We develop algorithms, implement them, and establish conditions for convergence.
- We have studied to-date "good behavior." Fascinating questions arise when there may be situations of instability, multiple equilibria, chaos, cycles, etc.

The full text of the related papers can be found under Downloadable Articles at:

http://supernet.som.umass.edu

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Virtual Center for Supernetworks

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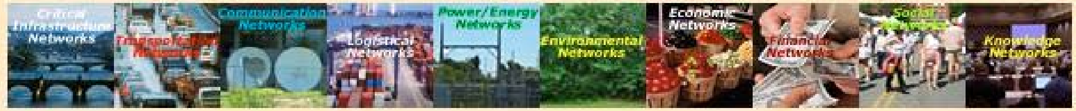


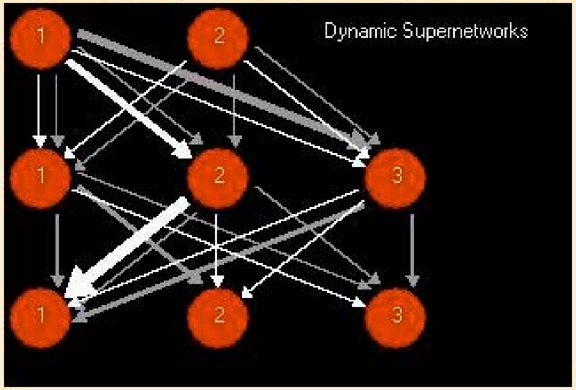
Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center.

NEW! The Supernetwork Sentinel Fall 2004 Issue

NEW! Isenberg School of Management Website

NEW! Papers on Dynamic Supernetworks



NEW! INFORMS Student Chapter

NEW! Fall 2004 Seminar Series!!!

Top site users during the past month were from UMass, Cornell, Columbia, and Purdue University. Top foreign users were from China, Russia, Australia, Italy, France, and Morocco.

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Mission: The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, logistical, telecommunication, and power networks to economic, environmental, financial, knowledge and social networks.

The applications of Supernetworks include: transportation, logistics, critical infrastructure, telecommunications, power and energy, electronic commerce, supply chain management, environment, economics, finance, knowledge and social networks, and decision-making.

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Thank you!

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