

Modeling Generator Power Plant Portfolios and Pollution Taxes in Electric Power Supply Chain Networks

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Electricity is Modernity



Electricity is Big Business

- In US: half a trillion dollars worth of net assets
- Over \$220 billion of annual sales
- Consumes almost 40% of domestic primary energy (Edison Electric Institute (2000), Energy Information Administration (2000, 2005))
- Deregulation: from vertically integrated to competitive markets (Casazza and Delea (2003) and the volumes edited by Singh (1999) and Zaccour (1998))



Environmental Impacts

- Heavy user of fossil fuels:
Electric power industry
generates over a third of the total
 CO_2 and NO_x emissions.
- Climate change poses immense
risks, thus the electricity
industry needs to be addressed.
(Poterba (1993) and Cline
(1999))



Tax the Bad, Credit the Good

- Market failure: externalities need to be internalized.
- Renewables are long-term solution (e.g., solar power and wind power (Painuly (2001))).
- Tax on bad emissions is one solution (e.g. CO₂ tax (Baranzini, Goldemberg, and Sperk (2000))).
- Credit for clean energy (e.g. tradable green certificates (RECS (1999) and Schaeffer et al (1999))).

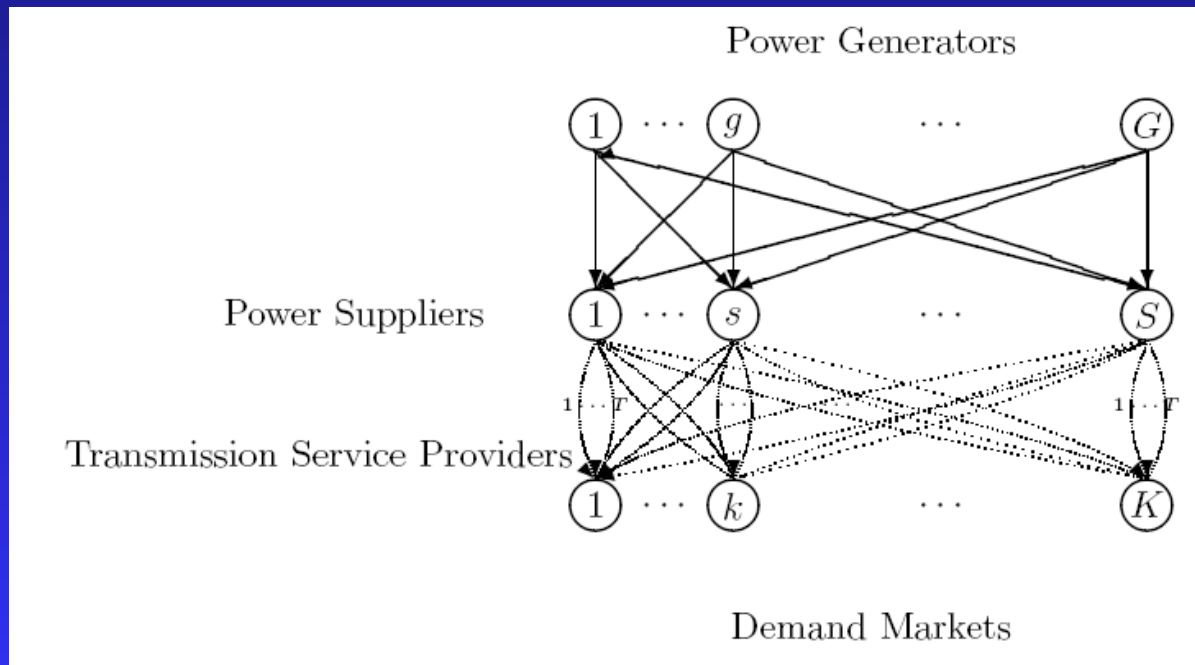


References

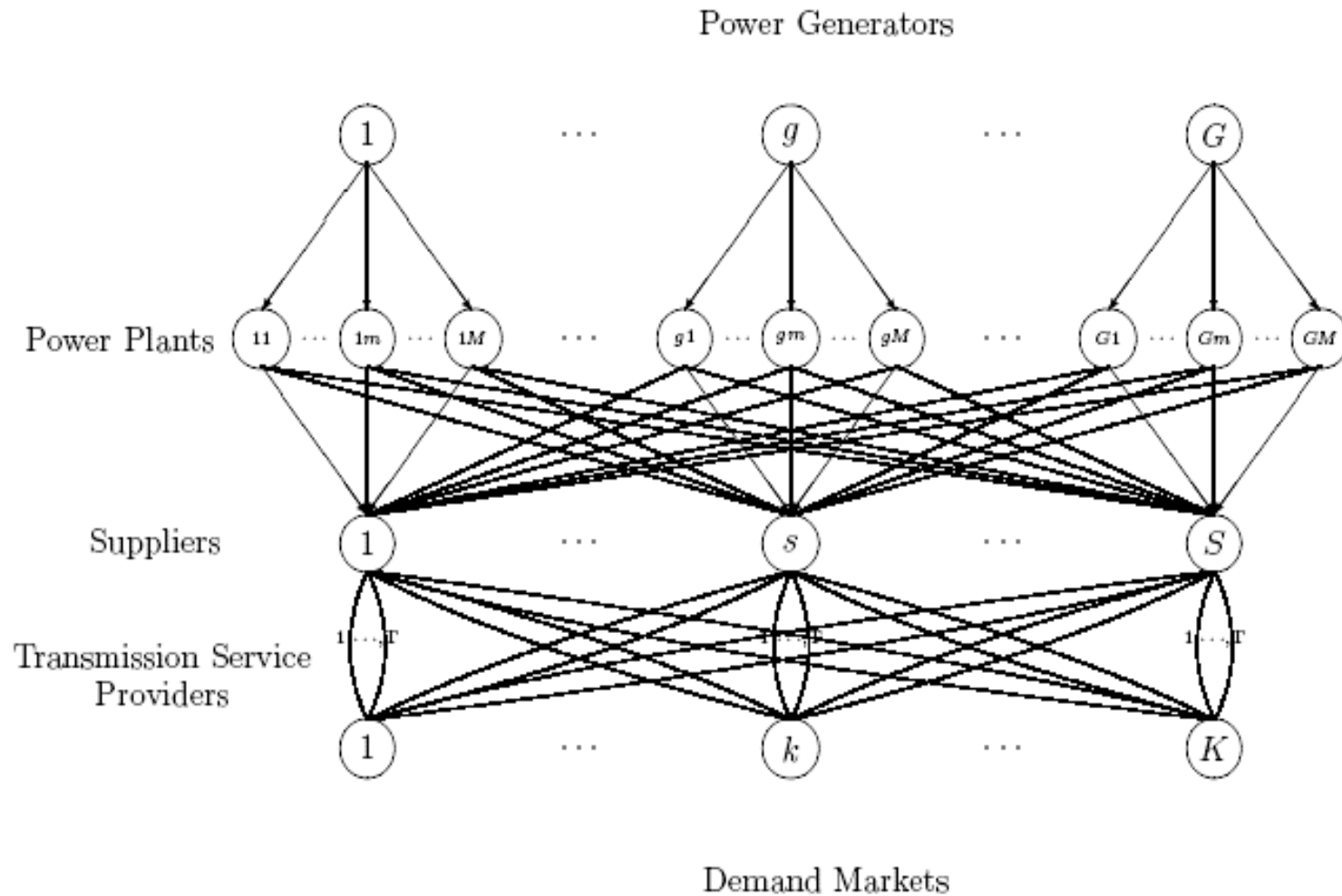
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Overview of the Electric Power Supply Chain Network Equilibrium Model

- Nagurney, A. and Matsypura, D. (2004), A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption, (*Proceedings of the International Conference on Computing, Communications and Control Technologies*, Austin, Texas, Volume VI: (2004) pp 127-134.))



The Extended Model



Modeling Energy Taxes and Credits: The Genco's Choice

- Each genco has a portfolio of power plants.
- Each power plant can have different generating costs and transaction costs.
- Generating costs can reflect capital costs, O&M and fuel costs.
- Transaction costs reflect possible taxes or credits.
 - ◆ e.g. carbon taxes
- We utilized this model to demonstrate the impact of the carbon taxes on the gencos' decisions as well as the equilibrium pattern of the entire electric power supply chain network.

The Behavior of the Gencos

- Each individual genco is a profit maximizer.

$$\text{Maximize} \quad \sum_{m=1}^M \sum_{s=1}^S \rho_{1gms}^* q_{gms} - \sum_{m=1}^M f_{gm}(q_m) - \sum_{m=1}^M \sum_{s=1}^S c_{gms}(q_{gms}) - \sum_{m=1}^M \tau_{gm} e_{gm} q_{gm} \quad (1)$$

subject to:

$$\sum_{s=1}^S q_{gms} = q_{gm}, \quad m = 1, \dots, M, \quad (2)$$

$$q_{gms} \geq 0, \quad m = 1, \dots, M; s = 1, \dots, S. \quad (3)$$

The Optimality Condition of the Gencos

- The optimality conditions for all power generators simultaneously, under the above assumptions, coincide with the solution of the following variational inequality:

$$\sum_{g=1}^G \sum_{m=1}^M \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm} e_{gm} \right] \times [q_{gm} - q_{gm}^*] \\ + \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} - \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*] \geq 0, \quad \forall (q, Q^1) \in \mathcal{K}^1,$$

where $\mathcal{K}^1 \equiv \{(q, Q^1) | (q, Q^1) \in R_+^{GM+GMS} \text{ and (2) holds}\}.$

The Behavior of Power Suppliers

- The optimization problem faced by supplier s may be expressed as follows:

$$\text{Maximize } \sum_{k=1}^K \sum_{t=1}^T \rho_{2sk}^t q_{sk}^t - c_s(Q^1) - \sum_{g=1}^G \sum_{m=1}^M \rho_{1gms}^* q_{gms} - \sum_{g=1}^G \sum_{m=1}^M \hat{c}_{gms}(q_{gms}) - \sum_{k=1}^K \sum_{t=1}^T c_{sk}^t(q_{sk}^t) \quad (6)$$

subject to:

$$\sum_{k=1}^K \sum_{t=1}^T q_{sk}^t = \sum_{g=1}^G \sum_{m=1}^M q_{gms} \quad (7)$$

$$q_{gms} \geq 0, \quad g = 1, \dots, G, \quad m = 1, \dots, M, \quad (8)$$

$$q_{sk}^t \geq 0, \quad k = 1, \dots, K; t = 1, \dots, T. \quad (9)$$

The Optimality Condition of the Suppliers

- The optimality conditions for all suppliers, simultaneously, can be expressed as the following variational inequality:

determine $(Q^{2*}, Q^{1*}) \in \mathcal{K}^2$ such that

$$\sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[\frac{\partial c_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} - \rho_{2sk}^{t*} \right] \times [q_{sk}^t - q_{sk}^{t*}]$$

$$+ \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[\frac{\partial c_s(Q^{1*})}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} + \rho_{1gms}^* \right] \times [q_{gms} - q_{gms}^*] \geq 0, \quad \forall (Q^2, Q^1) \in \mathcal{K}^2,$$

where $\mathcal{K}^2 \equiv \{(Q^2, Q^1) | (Q^2, Q^1) \in R_+^{STK+GMS} \text{ and (7) holds}\}.$

The Equilibrium Conditions at the Demand Markets

- At each demand market k the following conservation of flow equation must be satisfied:

$$d_k = \sum_{s=1}^S \sum_{t=1}^T q_{sk}^t, \quad k = 1, \dots, K.$$

- The market equilibrium conditions at demand market k take the form: for each power supplier s ; $s = 1, \dots, S$ and transaction mode t ; $t = 1, \dots, T$:

$$\rho_{2sk}^{t*} + \hat{c}_{sk}^t(Q^{2*}) \begin{cases} = \rho_{3k}(d^*), & \text{if } q_{sk}^{t*} > 0, \\ \geq \rho_{3k}(d^*), & \text{if } q_{sk}^{t*} = 0. \end{cases}$$

Electric Power Supply Chain Network Equilibrium

Definition: The equilibrium state of the electric power supply chain network is one where the electric power flows between the tiers of the network coincide and the electric power flows satisfy the conservation of flow equations, the sum of optimality conditions of the power generators and the power suppliers, and the equilibrium conditions at the demand markets.

Variational Inequality Formulation

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(q^*, h^*, Q^{1*}, Q^{2*}, d^*) \in \mathcal{K}^5$ satisfying:

$$\begin{aligned}
 & \sum_{g=1}^G \sum_{m=1}^M \left[\frac{\partial f_{gm}(q_m^*)}{\partial q_{gm}} + \tau_{gm} \right] \times [q_{gm} - q_{gm}^*] + \sum_{s=1}^S \frac{\partial c_s(h^*)}{\partial h_s} \times [h_s - h_s^*] \\
 & + \sum_{g=1}^G \sum_{m=1}^M \sum_{s=1}^S \left[\frac{\partial c_{gms}(q_{gms}^*)}{\partial q_{gms}} + \frac{\partial \hat{c}_{gms}(q_{gms}^*)}{\partial q_{gms}} \right] \times [q_{gms} - q_{gms}^*] \\
 & + \sum_{s=1}^S \sum_{k=1}^K \sum_{t=1}^T \left[\frac{\partial \hat{c}_{sk}^t(q_{sk}^{t*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) \right] \times [q_{sk}^t - q_{sk}^{t*}] - \sum_{k=1}^K \rho_{3k}(d^*) \times [d_k - d_k^*] \geq 0, \\
 & \forall (q, h, Q^1, Q^2, d) \in \mathcal{K}^5,
 \end{aligned} \tag{18}$$

Transportation Network Equilibrium Isomorphisms

- Nagurney, A. (2006), On the Relationship Between Supply Chain and Transportation Network Equilibria: A Supernetwork Equivalence with Computations, (*Transportation Research E* (2006) 42: (2006) pp 293-316)
- Liu, Z. and Nagurney, A. (2006), Financial Networks with Intermediation and Transportation Network Equilibria: A Supernetwork Equivalence and Reinterpretation of the Equilibrium Conditions with Computations, (*To appear in Computational Management Science*)
- Nagurney, A and Liu, Z. (2005), Transportation Network Equilibrium Reformulations of Electric Power Networks with Computations

Overview of the Transportation Network Equilibrium Model with Elastic Demands

- Dafermos, S. (1982), The general multimodal network equilibrium problem with elastic demand. *Networks* 12, 57-72.
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Transportation Network Equilibrium Conditions

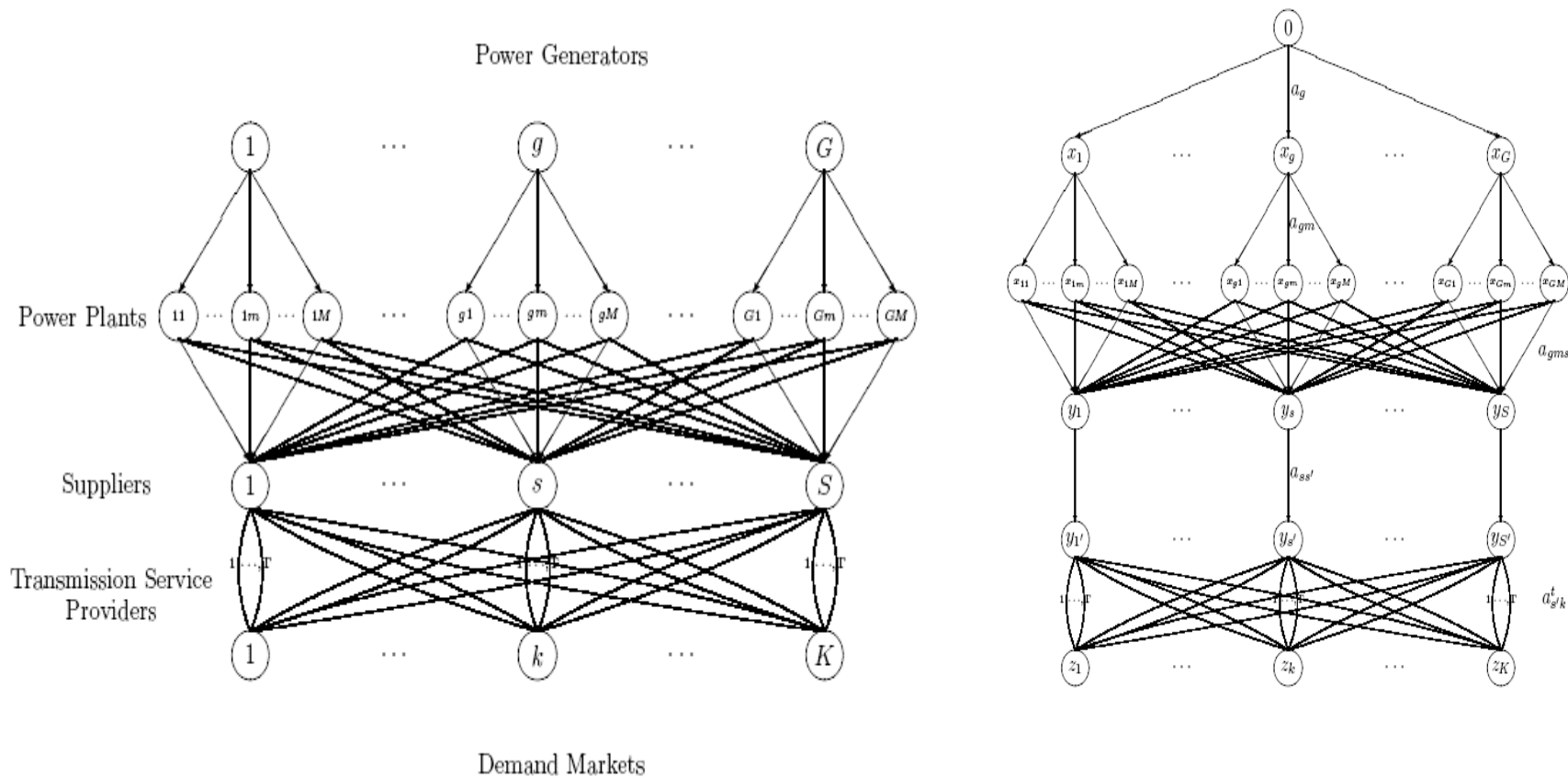
- In equilibrium, the following conditions must hold for each O/D pair and each path

$$C_p(x^*) - \lambda_w(d^*) \begin{cases} = 0, & \text{if } x_p^* > 0, \\ \geq 0, & \text{if } x_p^* = 0. \end{cases}$$

- A path flow pattern and associated travel demand pattern is a transportation network equilibrium if and only if it satisfies the variational inequality:

$$\sum_{w \in W} \sum_{p \in P_w} C_p(x^*) \times [x_p - x_p^*] - \sum_{w \in W} \lambda_w(d^*) \times [d_w - d_w^*] \geq 0, \quad \forall (x, d) \in \mathcal{K}^6, \quad (27)$$

Supernetwork Equivalence of the Electric Power Supply Chain Network with a Transportation Network



The Importance of the Transportation Network Reformulation

- This equivalence demonstrates the generality of the concepts of transportation network equilibrium.
 - ◆ One of the unsolved problems originally proposed in the seminal book of Beckmann, McGuire, and Winsten (1956).
- Theoretical results obtained for elastic demand transportation network equilibrium models can now be transferred to electric power networks
 - ◆ Existence and uniqueness (Dafermos (1982))
 - ◆ Stability and sensitivity analysis (see also Nagurney and Zhang (1996))

The Importance of the Transportation Network Reformulation

- This equivalence also allows for the transfer of methodological tools developed for transportation network equilibrium problems to the electric power network application.
- This equivalence provides new interpretations of electric power supply chain network equilibria in terms of paths and path flows.
- This equivalence also unveils opportunities for further modeling enhancements.
 - ◆ e.g. one may construct network representations of actual power grids and substitute these for the corresponding transmission links in the supernetwork.

Computations

- First, construct the equivalent transportation network equilibrium model.
- Solve the transportation network equilibrium model using the Euler method.
- Convert the solution of the transportation network to the electric power supply chain network equilibrium model.

The Euler Method

- The Euler method is induced by the general iterative scheme of Dupuis and Nagurney (1993) and has been applied by Nagurney and Zhang (1996) to solve variational inequality (27) in path flows (see also, e.g., Zhang and Nagurney (1997)). Convergence results can be found in the above references.

The Euler Method

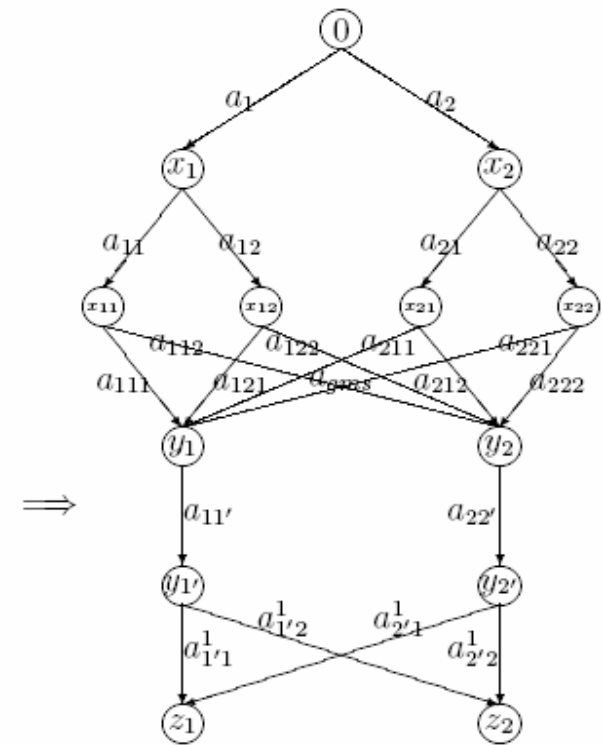
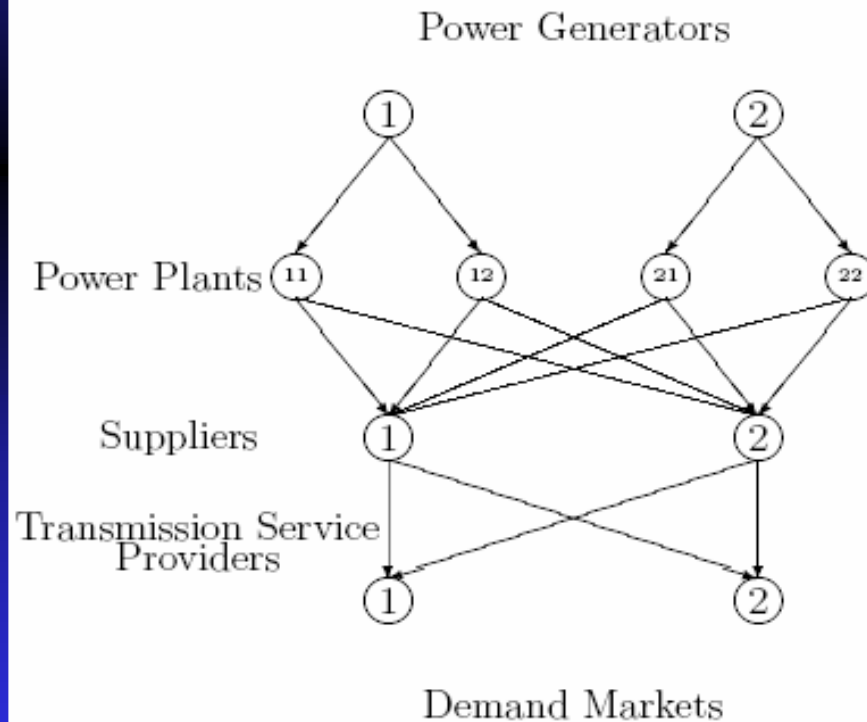
- For the solution of the equivalent transportation equilibrium model, the Euler method takes the form: at iteration τ compute the path flows for paths $p \in P$ (and the travel demands) according to:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + \alpha_\tau(\lambda_w(d^\tau) - C_p(x^\tau))\}. \quad (53)$$

- The simplicity of (53) lies in the explicit formula that allows for the computation of the path flows in closed form at each iteration.
- The demands at each iteration simply satisfy conservation of flow equations and this expression can be substituted into the $\lambda_w(\cdot)$ functions.

Numerical Examples

We used numerical examples to demonstrate the impact of the carbon taxes on the electric power supply chain network.



Example 1 (Benchmark)

- In example 1, we assumed that none of the gencos' power plants were polluting and that, hence, all the terms: e_{gm} ; $g = 1, 2$; $m = 1, 2$ were equal to zero. Hence, we did not impose any taxes.
- Generating cost functions

$$f_{11}(q_1) = 2.5q_{11}^2 + q_{11}q_{21} + 2q_{11}, \quad f_{12}(q_2) = 2.5q_{12}^2 + q_{11}q_{12} + 2q_{22}, \quad f_{21}(q_1) = .5q_{21}^2 + .5q_{11}q_{21} + 2q_{21}, \\ f_{22}(q_2) = .5q_{22}^2 + q_{12}q_{22} + 2q_{22}.$$

- Transaction cost between power generators and the suppliers

$$c_{111}(q_{111}) = .5q_{111}^2 + 3.5q_{111}, \quad c_{112}(q_{112}) = .5q_{112}^2 + 3.5q_{112}, \quad c_{121}(q_{121}) = .5q_{121}^2 + 3.5q_{121}, \\ c_{122}(q_{122}) = .5q_{122}^2 + 3.5q_{122}, \\ c_{211}(q_{211}) = .5q_{211}^2 + 2q_{211}, \quad c_{212}(q_{212}) = .5q_{212}^2 + 2q_{212}, \quad c_{221}(q_{221}) = .5q_{221}^2 + 2q_{221},$$

Example 1 (Benchmark)

- The operating cost of the power suppliers

$$c_1(Q^1) = .5\left(\sum_{i=1}^2 q_{i1}\right)^2, \quad c_2(Q^1) = .5\left(\sum_{i=1}^2 q_{i2}\right)^2.$$

- The demand market price function

$$\rho_{31}(d) = -1.33d_1 + 366.6, \quad \rho_{32} = -1.33d_2 + 366.6,$$

- The transaction cost between the suppliers and the customers

$$\hat{c}_{sk}^1(q_{sk}^1) = q_{sk}^1 + 5, \quad s = 1, 2; k = 1, 2.$$

The Solution of Example 1

- The equilibrium pattern for the transportation network

$$\begin{aligned}f_{a_1}^* &= 32.53, & f_{a_2}^* &= 115.22, \\f_{a_{11}}^* &= 22.57, & f_{a_{12}}^* &= 9.96, & f_{a_{21}}^* &= 22.90, & f_{a_{22}}^* &= 92.32, \\f_{a_{11'}}^* &= f_{a_{22'}}^* & &= 73.87, \\f_{a_{111}}^* &= 11.29, & f_{a_{112}}^* &= 11.29, & f_{a_{121}}^* &= 4.98, & f_{a_{122}}^* &= 4.98, \\f_{a_{211}}^* &= 11.45, & f_{a_{212}}^* &= 11.45, & f_{a_{221}}^* &= 46.16, & f_{a_{222}}^* &= 46.16,\end{aligned}$$

The Solution of Example 1

- The equilibrium pattern for the electrical power supply chain network

$$q_1^* = 32.53, \quad q_2^* = 115.22,$$

$$q_{11}^* = 22.57, \quad q_{12}^* = 9.96, \quad q_{21}^* = 22.90, \quad q_{22}^* = 9.96,$$

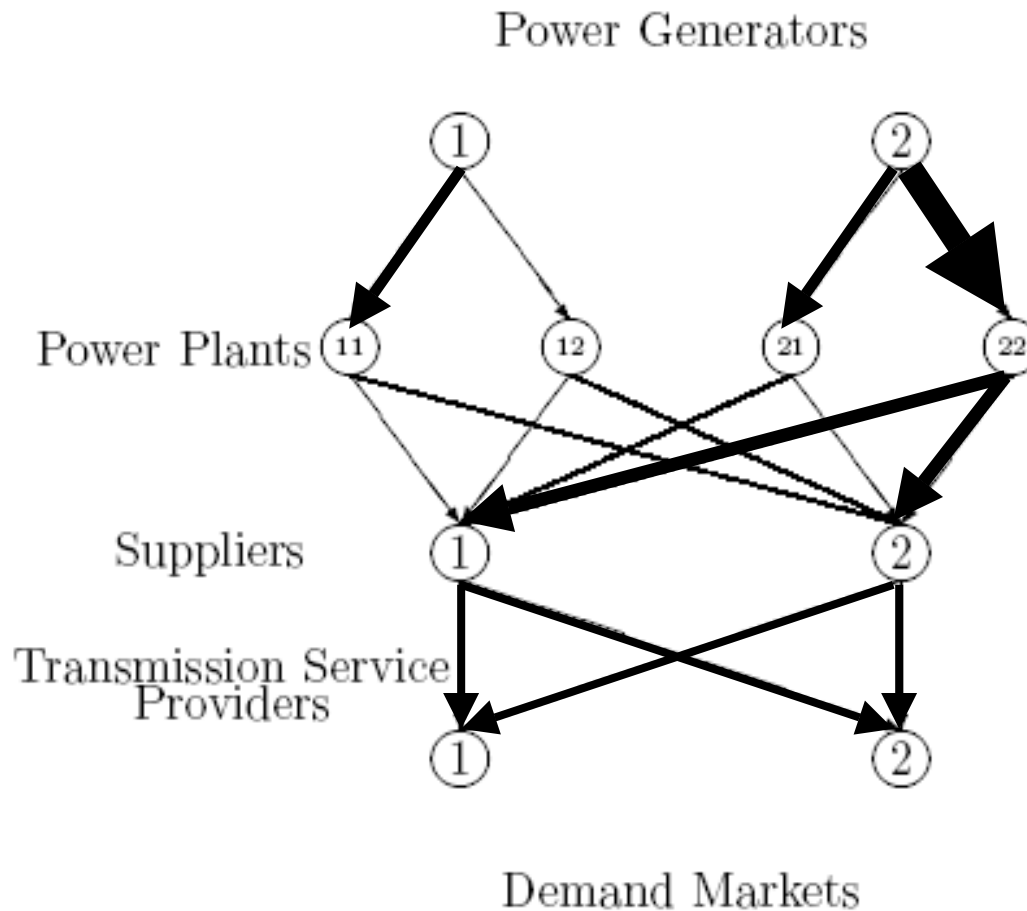
$$h_1^* = h_2^* = 73.87,$$

$$q_{111}^* = 11.29, \quad q_{112}^* = 11.29, \quad q_{121}^* = 4.98, \quad q_{222}^* = 4.98,$$

$$q_{211}^* = 11.45, \quad q_{212}^* = 11.45, \quad q_{221}^* = 46.16, \quad q_{222}^* = 46.16,$$

$$q_{1'1}^{1*} = q_{1'2}^{1*} = q_{2'1}^{1*} = q_{2'2}^{1*} = 36.94.$$

The Solution of Example 1



Example 2

- We kept the data identical to that in Example 1 except that now we considered power plant 1 of genco 1 to be polluting with $e_{11} = 1$.
- Our goal was to identify a tax high enough so that the polluting power plant would not produce at all, which means that the corresponding equilibrium link flow would be zero.
- By setting $\tau_{11} = 133$ (determined through simulations) we obtained that the flow on link a_{11} is equal to zero, which means that this pollution tax was sufficiently high enough that the genco did not use the polluting plant at all.

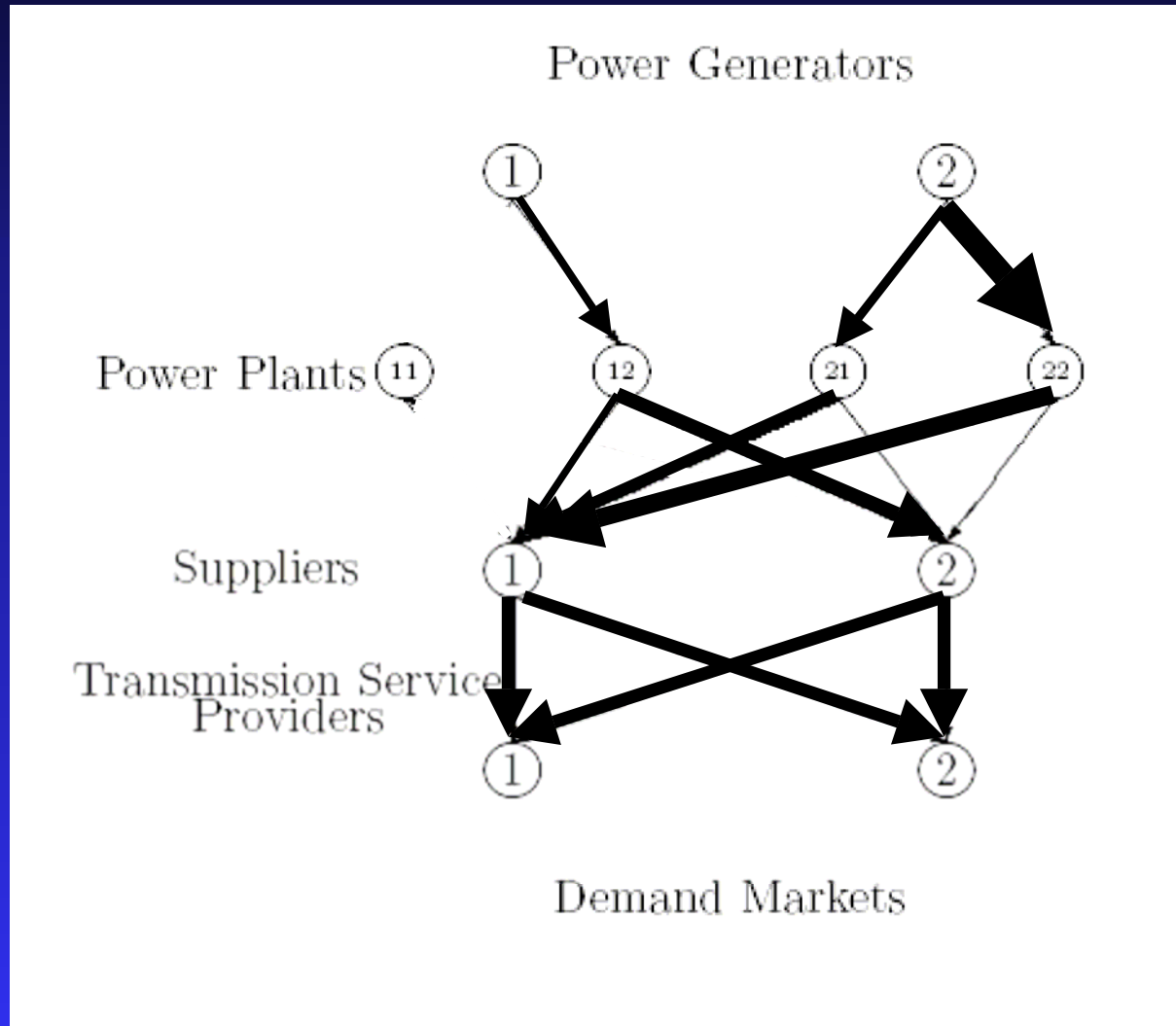
The Solution of Example 2

■ Solution

$$\begin{aligned}q_1^* &= 10.77, & q_2^* &= 128.71, \\q_{11}^* &= 0.00, & q_{12}^* &= 10.77, & q_{21}^* &= 29.14, & q_{22}^* &= 99.58, \\h_1^* &= h_2^* &= 69.74, \\q_{111}^* &= 0.00, & q_{112}^* &= 0.00, & q_{121}^* &= 5.38, & q_{222}^* &= 5.38, \\q_{211}^* &= 11.45, & q_{212}^* &= 14.57, & q_{221}^* &= 49.79, & q_{222}^* &= 49.79,\end{aligned}$$

- Power plant 1 of genco 1 has no emissions now and the total emissions have been reduced by 22.57 by imposing $\tau_{11} = 133$.

The Solution of Example 2



Example 3

- Example 3 was constructed as follows from Example 2. The data were identical to the data in Example 2, except that we now assumed that the first power plant of genco 2 was also polluting with $e_{12} = 1$.
- We imposed the same tax on the first power plant of the second genco as we had for power plant 1 of genco 1.
- Hence, in this example, all taxes were equal to zero except that $\tau_{11} = \tau_{12} = 133$.

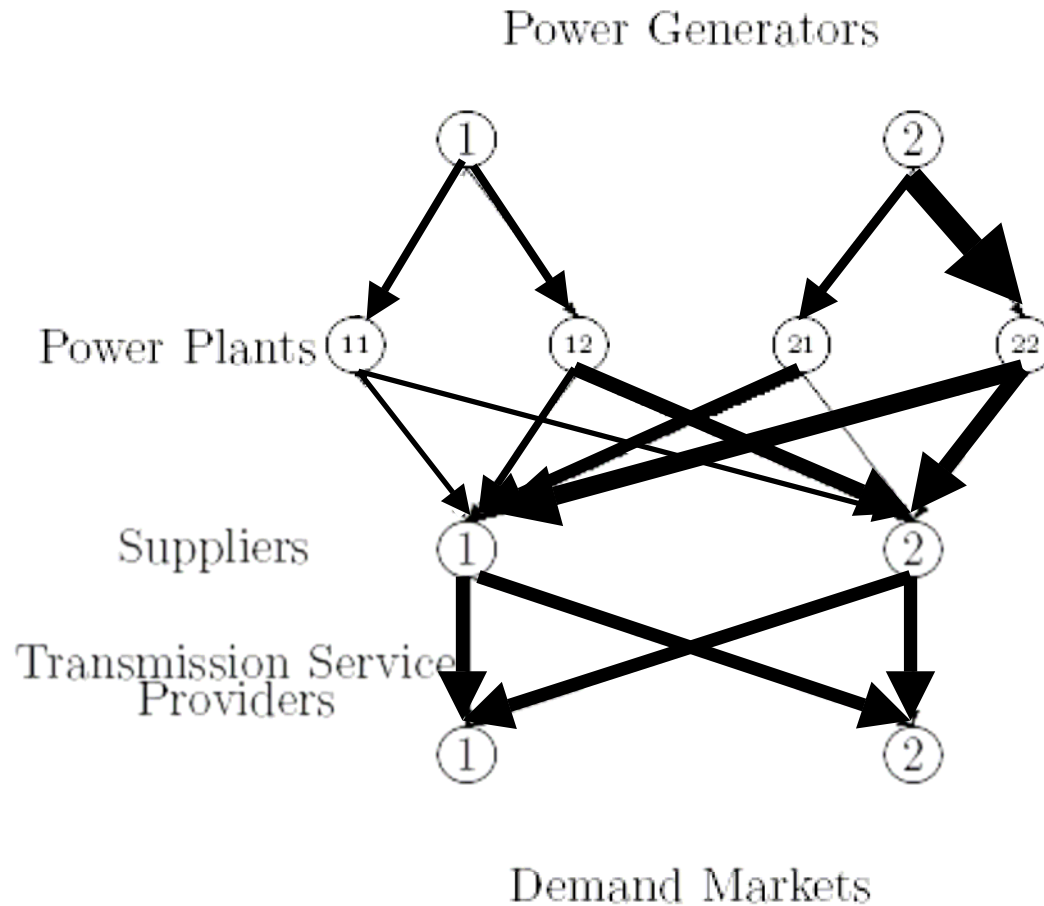
The Solution of Example 3

■ Solution

$$\begin{aligned}q_1^* &= 17.42, & q_2^* &= 113.36, \\q_{11}^* &= 5.80, & q_{12}^* &= 11.61, & q_{21}^* &= 6.14, & q_{22}^* &= 107.22, \\h_1^* &= h_2^* &= 65.39, \\q_{111}^* &= 2.90, & q_{112}^* &= 2.90, & q_{121}^* &= 5.81, & q_{122}^* &= 5.81, \\q_{211}^* &= 3.07, & q_{212}^* &= 3.07, & q_{221}^* &= 53.61, & q_{222}^* &= 53.61, \\q_{1'1}^{1*} &= q_{1'2}^{1*} = q_{2'1}^{1*} = q_{2'2}^{1*} &= 32.69. \\d_1^* &= d_2^* &= 65.39. \\ \rho_{31} &= \rho_{32} &= 279.63\end{aligned}$$

- Total pollution = 11.94
- Total pollution has been decreased by 50% by imposing $\tau_{11} = \tau_{12} = 133$.

The Solution of Example 3



Example 4

- In Example 3, though we imposed taxes: $\tau_{11} = \tau_{12} = 133$, the first power plant of each genco was still producing.
- In Example 4, our goal was to identify how high the taxes should be on the first (assumed to be polluting) power plants of each generator so that neither high-polluting power plant would be used.
- We conducted the following simulation: we increased the taxes from 133 for both those power plants (thus, we used as the baseline Example 3) until we achieved zero production at those power plants.
- Taxes of $\tau_{11} = \tau_{12} = 188$ yielded the desired policy result that there was zero production at the noxious power plants.

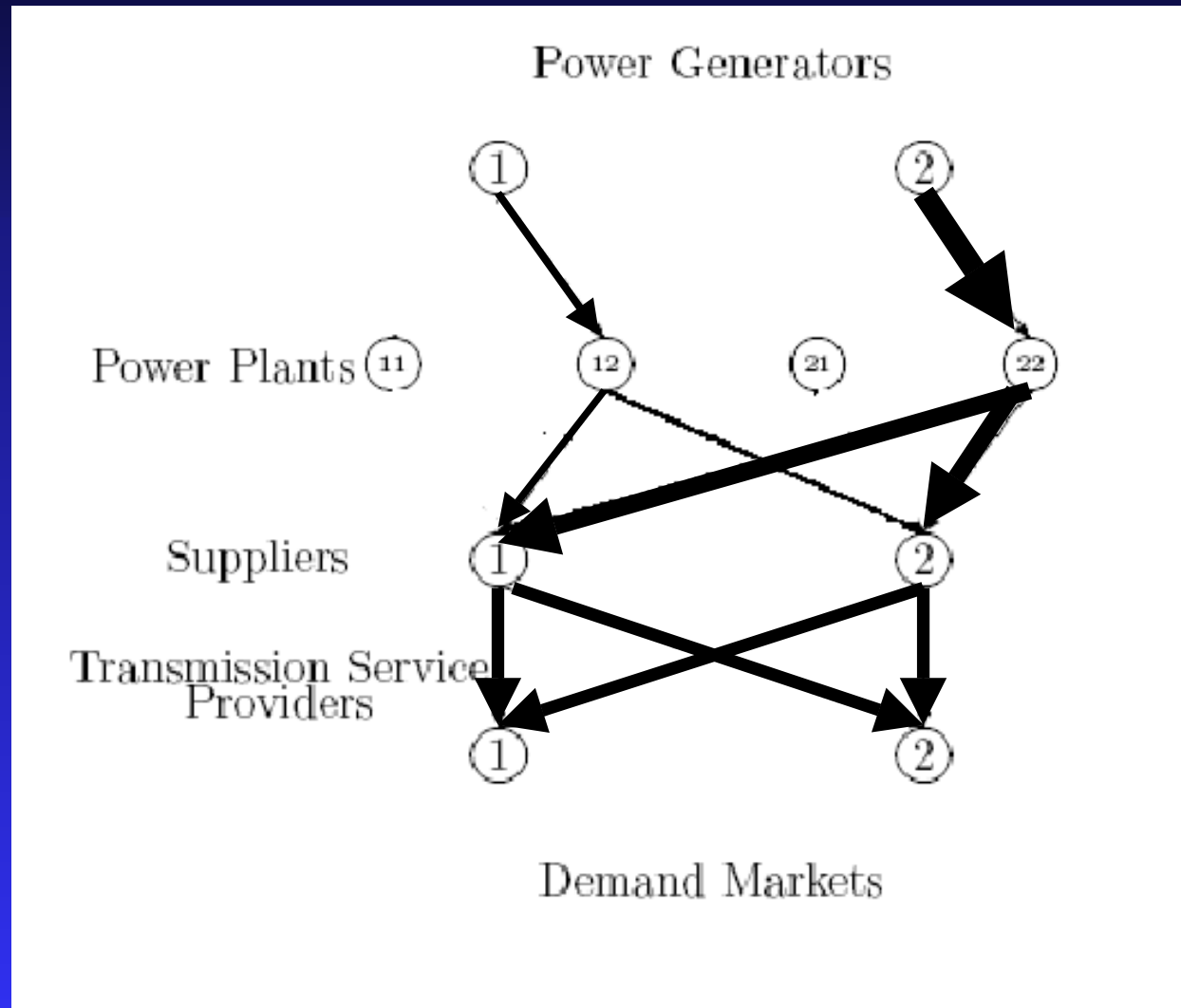
The Solution of Example 4

■ Solution

$$\begin{aligned}q_1^* &= 12.20, & q_2^* &= 112.53, \\q_{11}^* &= 0.00, & q_{12}^* &= 12.20, & q_{21}^* &= 0.00, & q_{22}^* &= 112.53, \\h_1^* &= h_2^* &= 62.37, \\q_{111}^* &= 0.00, & q_{112}^* &= 0.00, & q_{121}^* &= 6.10, & q_{122}^* &= 6.10, \\q_{211}^* &= 0.00, & q_{212}^* &= 0.00, & q_{221}^* &= 56.26, & q_{222}^* &= 56.26, \\q_{1'1}^{1*} &= q_{1'2}^{1*} = q_{2'1}^{1*} &= q_{2'2}^{1*} &= 31.18. \\d_1^* &= d_2^* &= 46.37. \\ \rho_{31} &= \rho_{32} &= 304.93,\end{aligned}$$

- Total pollution = 0
- Compared with Example 3, the prices increased, the demands decreased, and the pollution was reduced to zero.

The Solution of Example 4



Conclusions

- We developed a new model of electric power supply chain networks with distinct power plants, which allows for distinct fuels.
- The model also included carbon pollution taxes that can be imposed on the various power generator/power plant combinations.
- We established the supernetwork equivalence of the extended electric power supply chain networks with the transportation networks.

Conclusions

- The theoretical results and methodological tools developed for elastic demand transportation network equilibrium models can now be transferred to electric power supply chain networks.
- The numerical examples illustrated some of the types of simulations that can be conducted in order to investigate the ramification of the imposition of pollution taxes.

Conclusions

- We are also investigating the modeling and computational framework which can help policy makers determine the optimal carbon taxes on the electric power generation industry.
 - ◆ Nagurney, A., Liu, Z. and Woolley, T. (2006), Optimal Endogenous Carbon Taxes for Electric Power Supply Chains with Power Plants (To appear in *Mathematical and Computer Modelling*.)

Thank You!

For more information, see:

The Virtual Center for Supernetworks

<http://supernet.som.umass.edu>



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