A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption

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Motivation

The transformation of the electric power industry from a regulated to a competitive industry

- In the US,
- In the EU,
- And in many other countries

The dramatic increase in the number of market participants

- In the US alone as of December 1, 2003, there were 1310 companies eligible to sell wholesale power at market-based rates
  (Statistics available at http://www.eia.doe.gov).

Changes in electricity trading patterns

The Importance of New Models

"[In recent years] the adequacy of the bulk power transmission system has been challenged to support the movement of power in unprecedented amounts and in unexpected directions" (North American Electric Reliability Council (1998)).

"There is a critical need to be sure that reliability is not taken for granted as the industry restructures, and thus does not fall through the cracks" (Secretary of Energy Advisory Board's (SEAB) Task Force on Electric System Reliability (1998)).

These Concerns have Stimulated Much Research Activity:

Schweppe et al., 1988
Hogan, 1992
Chao and Peck, 1996
Wu et al., 1996
Kahn, 1998
Singh, 1999
Jing-Yuan and Smeers, 1999
Hobbs et al., 2000
Day et al., 2002
…

References

Nagurney, A. (1999), **Network Economics: A Variational Inequality Approach**, Second and Revised Edition, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A. and J. Dong (2002), **Supernetworks: Decision-Making for the Information Age**, Edward Elgar Publishing, Cheltenham, England.

Nagurney, A., Dong, J. and D. Zhang (2002), "A Supply Chain Network Equilibrium Model," *Transportation Research E* 38, 281-303.

Despite all the Analytical Efforts…

August 14, 2003 - The biggest power outage in US history occurred

- Approximately 50 million people were left without electricity.
- 61,800 megawatts of electric load were affected.

September 2003

A major outage occurred in England

Another significant outage initiated in Switzerland and cascaded over much of Italy

A Novel Approach

Based on a supply chain network framework

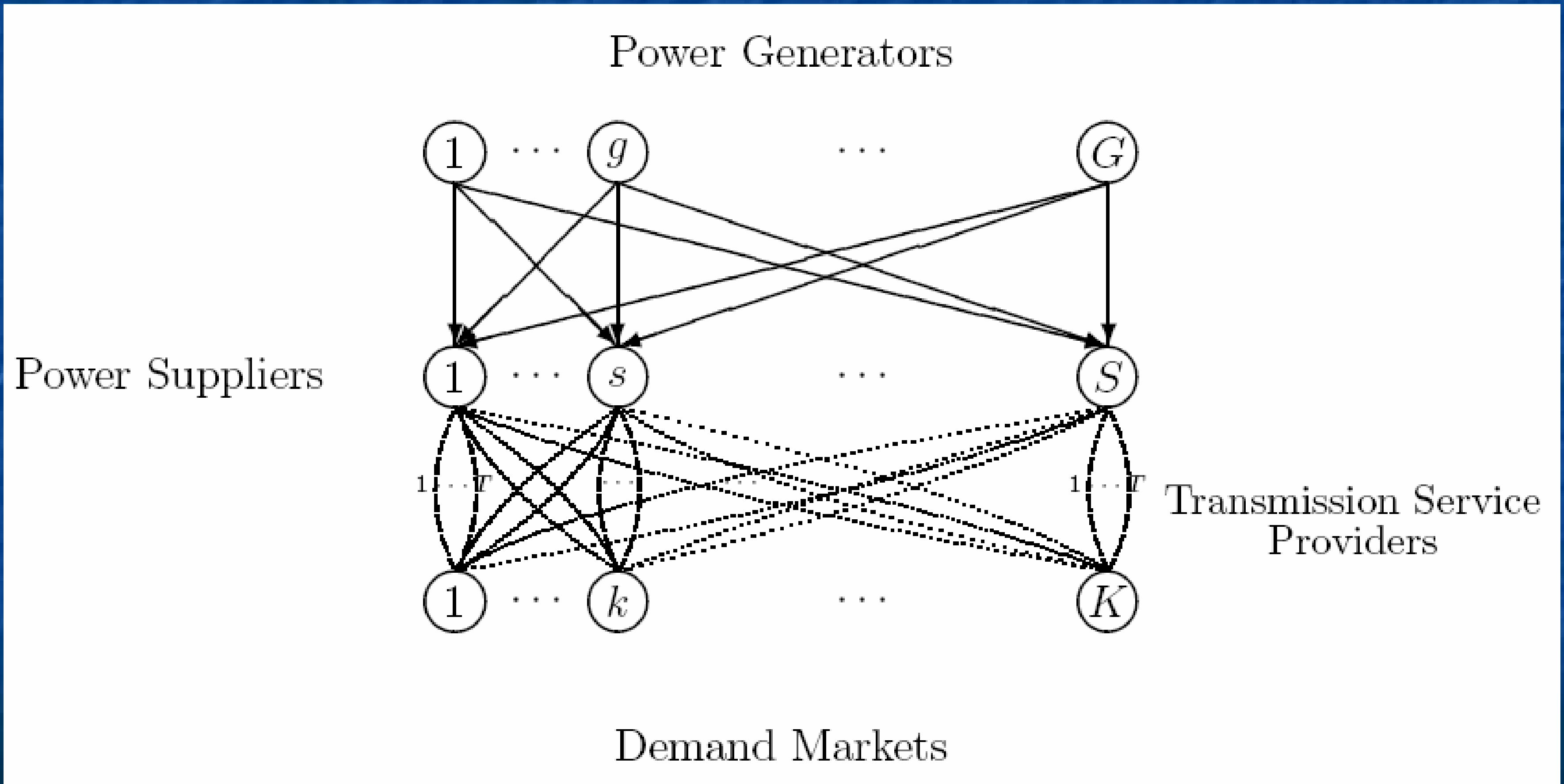
Several different types of decision-makers:

- Power Generators
- Power Suppliers
- Consumers
- with Multiple Transmission Service Providers.

Explicit modeling of the behavior of decision-makers

Computation of equilibrium electric power flows, as well as the prices associated with various transactions

The Electric Power Supply Chain Network



The Behavior of Power Generators and their Optimality Conditions

Power generator's optimization problem

$$\text{Maximize} \quad \sum_{s=1}^{S} \rho_{1gs}^{*} q_{gs} - f_g(Q^1) - \sum_{s=1}^{S} c_{gs}(Q^1)$$

$$\text{subject to:} \qquad q_{gs} \geq 0, \qquad \forall s.$$

Optimality conditions of the power generators

$$\sum_{g=1}^{G} \sum_{s=1}^{S} \left[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}} - \rho_{1gs}^{*} \right] \times [q_{gs} - q_{gs}^{*}] \geq 0, \quad \forall Q^1 \in R_{+}^{GS}.$$

A Power Supplier's Optimization Problem

$$\text{Maximize} \quad \sum_{k=1}^{K}\sum_{t=1}^{T}\rho_{2sk}^{t*}q_{sk}^{t} - c_s(Q^1, Q^2) - \sum_{g=1}^{G}\rho_{1gs}^{*}q_{gs} - \sum_{g=1}^{G}\hat{c}_{gs}(Q^1) - \sum_{k=1}^{K}\sum_{t=1}^{T}c_{sk}^{t}(Q^2)$$

subject to:

$$\sum_{k=1}^{K}\sum_{t=1}^{T}q_{sk}^{t} \le \sum_{g=1}^{G}q_{gs}$$

$$q_{gs} \ge 0, \quad \forall g$$

$$q_{sk}^{t} \ge 0, \quad \forall k, \forall t.$$

Power Suppliers' Optimality Conditions

$$\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q^t_{sk}}+\frac{\partial c^t_{sk}(Q^{2*})}{\partial q^t_{sk}}-\rho^{t*}_{2sk}+\gamma^*_s\right]\times[q^t_{sk}-q^{t*}_{sk}]$$

$$+\sum_{s=1}^{S}\sum_{g=1}^{G}\left[\frac{\partial c_s(Q^{1*},Q^{2*})}{\partial q_{gs}}+\frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}}+\rho^*_{1gs}-\gamma^*_s\right]\times[q_{gs}-q^*_{gs}]$$

$$+\sum_{s=1}^{S}\left[\sum_{g=1}^{G}q^*_{gs}-\sum_{k=1}^{K}\sum_{t=1}^{T}q^{t*}_{sk}\right]\times[\gamma_s-\gamma^*_s]\geq 0,\qquad \forall(Q^1,Q^2,\gamma)\in R_+^{S(G+KT+1)}.$$

The Equilibrium Conditions for the Demand Markets

We say that vector (Q^{2*}, ρ^*_3) is an equilibrium vector if for each *s, k, t*:

$$\rho^{t*}_{2sk} + \hat{c}^t_{sk}(Q^{2*}) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^{t*}_{sk} \geq 0, \\ \geq \rho^*_{3k}, & \text{if} \quad q^{t*}_{sk} = 0. \end{cases}$$

$$d_k(\rho^*_3) \begin{cases} = \sum_{s=1}^{S} \sum_{t=1}^{T} q^{t*}_{sk}, & \text{if} \quad \rho^*_{3k} \geq 0, \\ \leq \sum_{s=1}^{S} \sum_{t=1}^{T} q^{t*}_{sk}, & \text{if} \quad \rho^*_{3k} = 0. \end{cases}$$

Electric Power Supply Chain Network Equilibrium

Definition:

The equilibrium state of the electric power supply chain network is one where the electric power flows between the tiers of the network coincide and the electric power flows and prices satisfy the sum of equilibrium conditions presented above.

Variational Inequality Formulation

Determine $(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in \mathcal{K}$ satisfying:

$$\sum_{g=1}^{G}\sum_{s=1}^{S}\left[\frac{\partial f_g(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_s(Q^{1*}, Q^{2*})}{\partial q_{gs}} + \frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}} - \gamma_s^*\right] \times [q_{gs} - q_{gs}^*]$$

$$+\sum_{s=1}^{S}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_s(Q^{1*}, Q^{2*})}{\partial q_{sk}^t} + \frac{\partial c_{sk}^t(Q^{2*})}{\partial q_{sk}^t} + \hat{c}_{sk}^t(Q^{2*}) + \gamma_s^* - \rho_{3k}^*\right] \times [q_{sk}^t - q_{sk}^{t*}]$$

$$+\sum_{s=1}^{S}\left[\sum_{g=1}^{G} q_{gs}^* - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{sk}^{t*}\right] \times [\gamma_s - \gamma_s^*] + \sum_{k=1}^{K}\left[\sum_{s=1}^{S}\sum_{t=1}^{T} q_{sk}^{t*} - d_k(\rho_3^*)\right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, \rho_3) \in \mathcal{K},$$

where $\mathcal{K} \equiv \{(Q^1, Q^2, \gamma, \rho_3) | (Q^1, Q^2, \gamma, \rho_3) \in R_+^{GS+TSK+S+K}\}$.

Standard Variational Inequality Form

Determine $X^* \in \mathcal{K}$ satisfying:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $X \equiv (Q^1, Q^2, \gamma, \rho_3)$,

and $F(X) \equiv (F_{gs}, F_{sk}^t, F_s, F_k)_{g=1,\dots,G;s=1,\dots,S;t=1,\dots,T;k=1,\dots,K}$

Additional Theoretical Results

We have established:

- Existence of the solution
- Uniqueness of the solution
- Convergence of the algorithm

The Algorithm

The algorithm that we propose is the modified projection method of Korpelevich (1977).

The algorithm is guaranteed to converge provided that:

- F(X) is monotone
- F(X) is Lipschitz continuous.

Modified Projection Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and let a be a scalar such that $0 < a \leq \frac{1}{L}$, where L is the Lipschitz continuity constant.

Step 1: Computation

Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle \bar{X}^{\mathcal{T}} + aF(X^{\mathcal{T}-1})^T - X^{\mathcal{T}-1}, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 2: Adaptation

Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + aF(\bar{X}^{\mathcal{T}})^T - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 3: Convergence Verification

If $\max |X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} =: \mathcal{T} + 1$, and go to Step 1.

Computation of the Prices

Prices associated with the first tier of nodes – generators:

$$\rho^*_{1gs} = \frac{\partial f_g(Q^{1*})}{\partial q_{gs}} + \frac{\partial c_{gs}(Q^{1*})}{\partial q_{gs}} \text{ for any } g, s \text{ such that } q^*_{gs} > 0,$$

or

$$\rho^*_{1gs} = \gamma^*_s - \frac{\partial c_s(Q^{1*}, Q^{2*})}{\partial q_{gs}} - \frac{\partial \hat{c}_{gs}(Q^{1*})}{\partial q_{gs}} \text{ for any } g, s \text{ such that } q^*_{gs} > 0.$$

Prices associated with the second tier of nodes – suppliers:

$$\rho^{t*}_{2sk} = \rho^*_{3k} - \hat{c}^t_{sk}(Q^{2*}) \text{ for any } s, t, k \text{ such that } q^{t*}_{sk} > 0,$$

or

$$\rho^{t*}_{2sk} = \frac{\partial c_s(Q^{1*}, Q^{2*})}{\partial q^t_{sk}} + \frac{\partial c^t_{sk}(Q^{2*})}{\partial q^t_{sk}} + \gamma^*_s \text{ for any } s, t, k \text{ such that } q^{t*}_{sk} > 0.$$

Numerical Examples: Example 1

Generating and transaction cost functions of the power generators:

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1,$$

$$f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2,$$

$$f_3(q) = .5q_3^2 + .5q_1q_3 + 2q_3,$$

$$c_{11}(Q^1) = .5q_{11}^2 + 3.5q_{11},$$

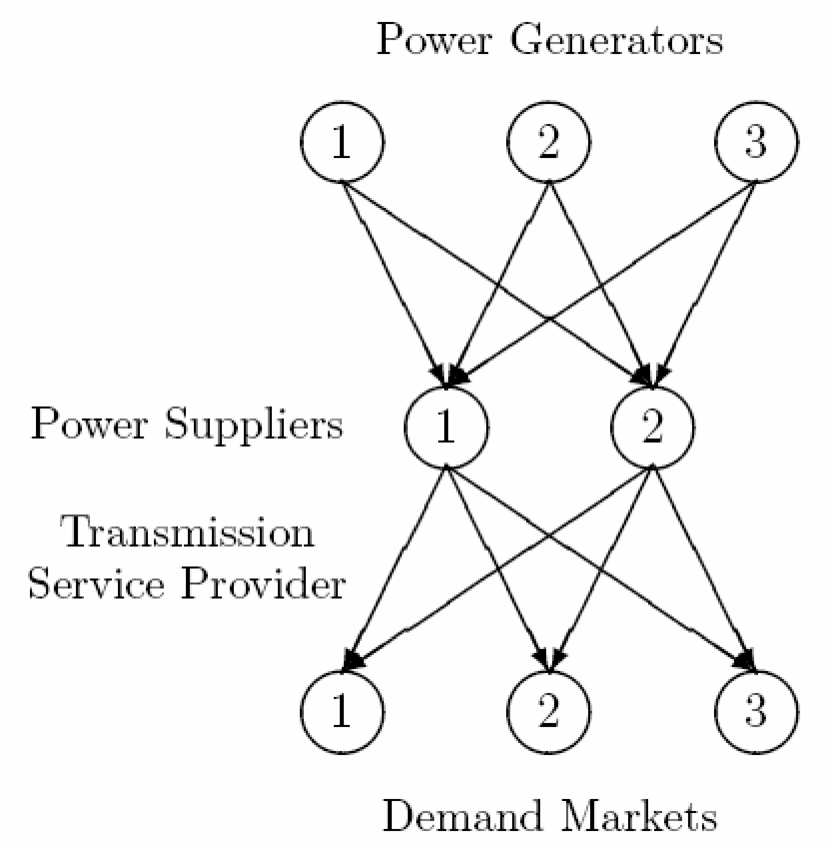
$$c_{12}(Q^1) = .5q_{12}^2 + 3.5q_{12},$$

$$c_{21}(Q^1) = .5q_{21}^2 + 3.5q_{21},$$

$$c_{22}(Q^1) = .5q_{22}^2 + 3.5q_{22},$$

$$c_{31}(Q^1) = .5q_{31}^2 + 2q_{31},$$

$$c_{32}(Q^1) = .5q_{32}^2 + 2q_{32}.$$

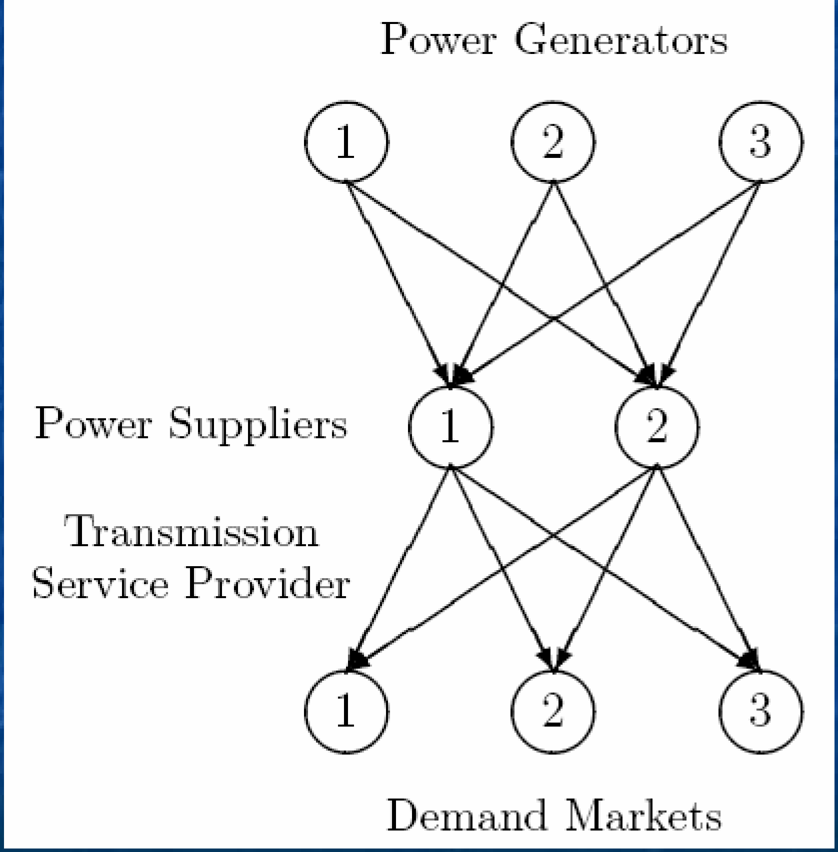


Example 1

The operating costs of the power suppliers:

$$c_1(Q^1, Q^2) = .5(\sum_{i=1}^{2} q_{i1})^2,$$

$$c_2(Q^1, Q^2) = .5(\sum_{i=1}^{2} q_{i2})^2.$$



Example 1

The transaction costs between the power suppliers and the consumers:

$$\hat{c}^1_{11}(Q^2) = q^1_{11} + 5,$$

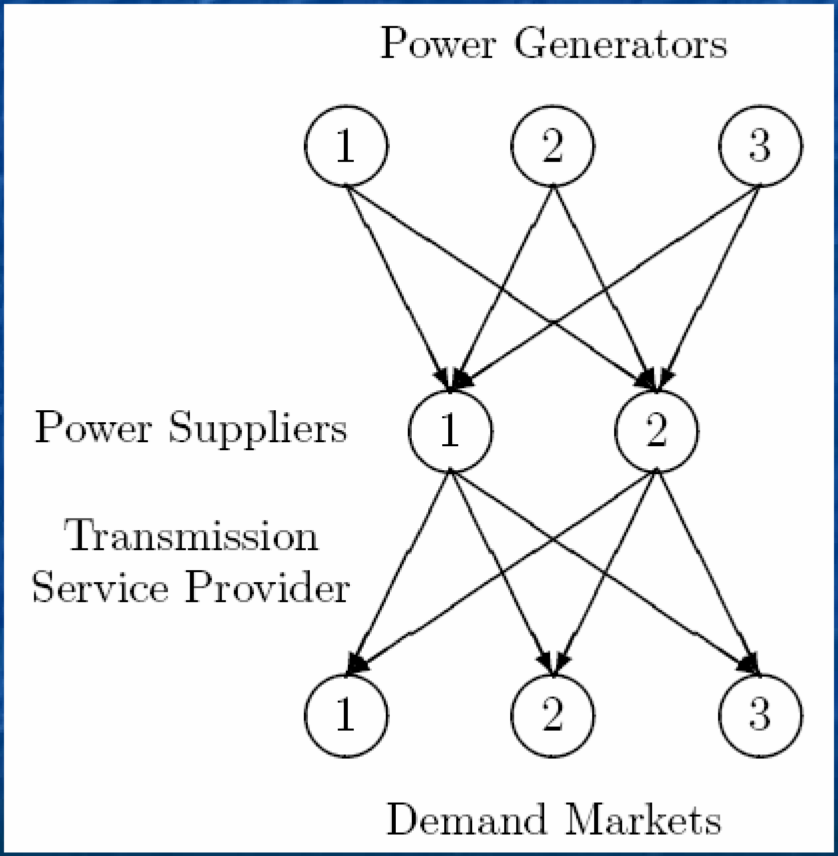
$$\hat{c}^1_{12}(Q^2) = q^1_{12} + 5,$$

$$\hat{c}^1_{13}(Q^2) = q^1_{13} + 5,$$

$$\hat{c}^1_{21}(Q^2) = q^1_{21} + 5,$$

$$\hat{c}^1_{22}(Q^2) = q^1_{22} + 5,$$

$$\hat{c}^1_{23}(Q^2) = q^1_{23} + 5.$$



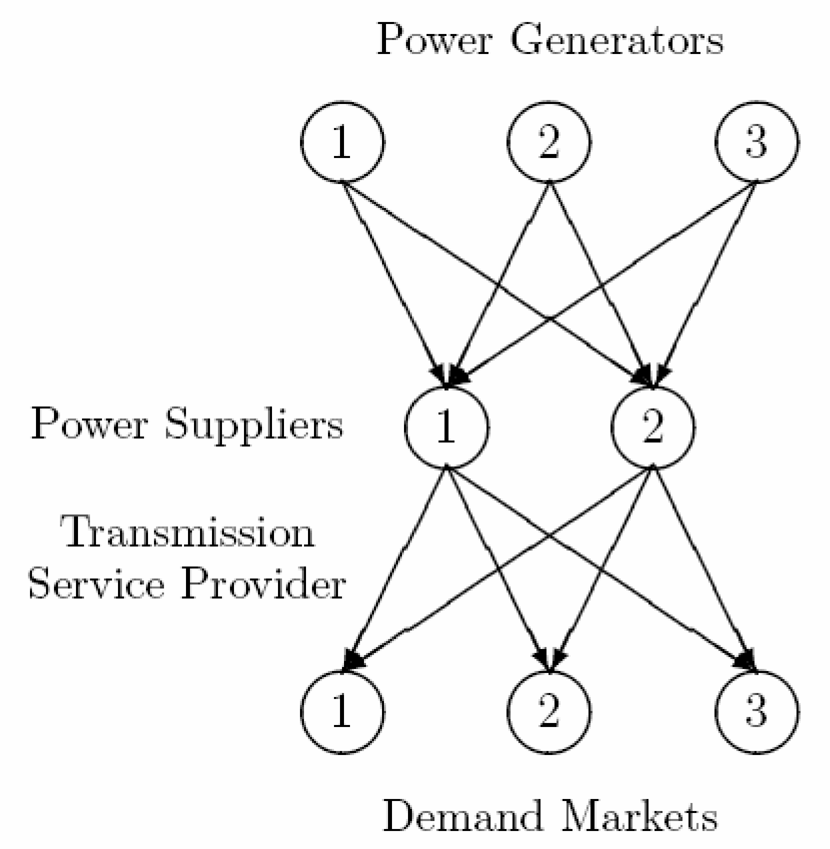
Example 1

The demand functions:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1100,$$

$$d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1100,$$

$$d_3(\rho_3) = -2\rho_{33} - 1.5\rho_{31} + 1200.$$



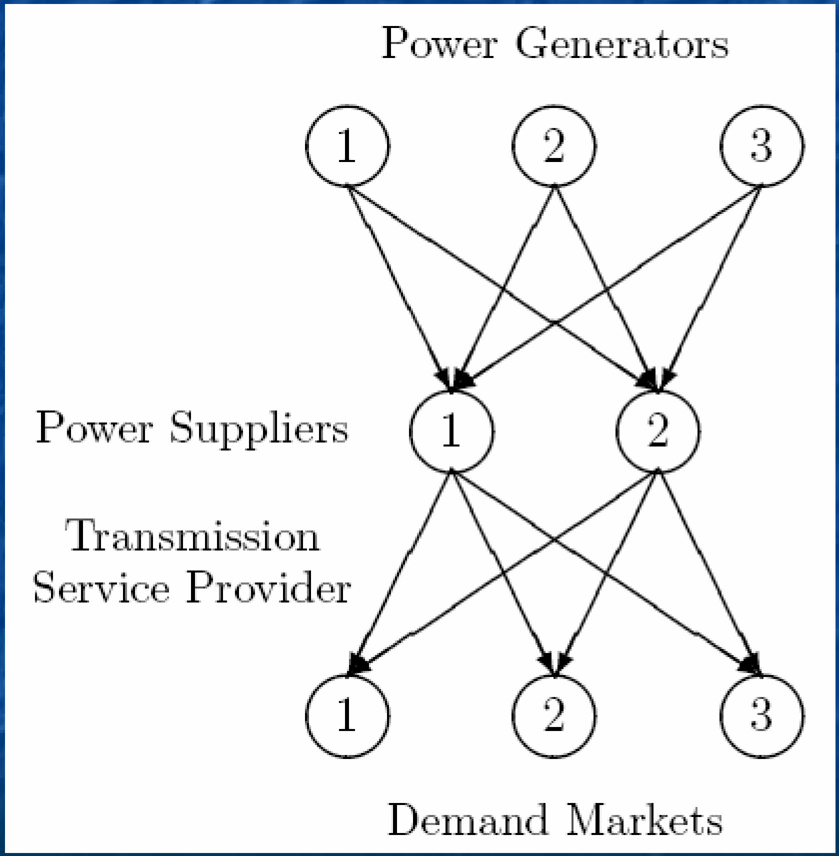
The Results

	Example 1	Example 2	Example 3	Example 4
$q^*_{11} = q^*_{12}$	14.2762			
$q^*_{21} = q^*_{22}$	14.2762			
$q^*_{31} = q^*_{32}$	57.6051			
$q^{1*}_{11} = q^{1*}_{21}$	20.3861			
$q^{1*}_{12} = q^{1*}_{22}$	20.3861			
$q^{1*}_{32} = q^{1*}_{32}$	20.3861			
$\gamma_1^* = \gamma_2^*$	277.2487			
ρ_{31}^*	302.6367			
ρ_{32}^*	302.6367			
ρ_{33}^*	327.6367			

Example 2

The first demand function is changed to:

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1500$$



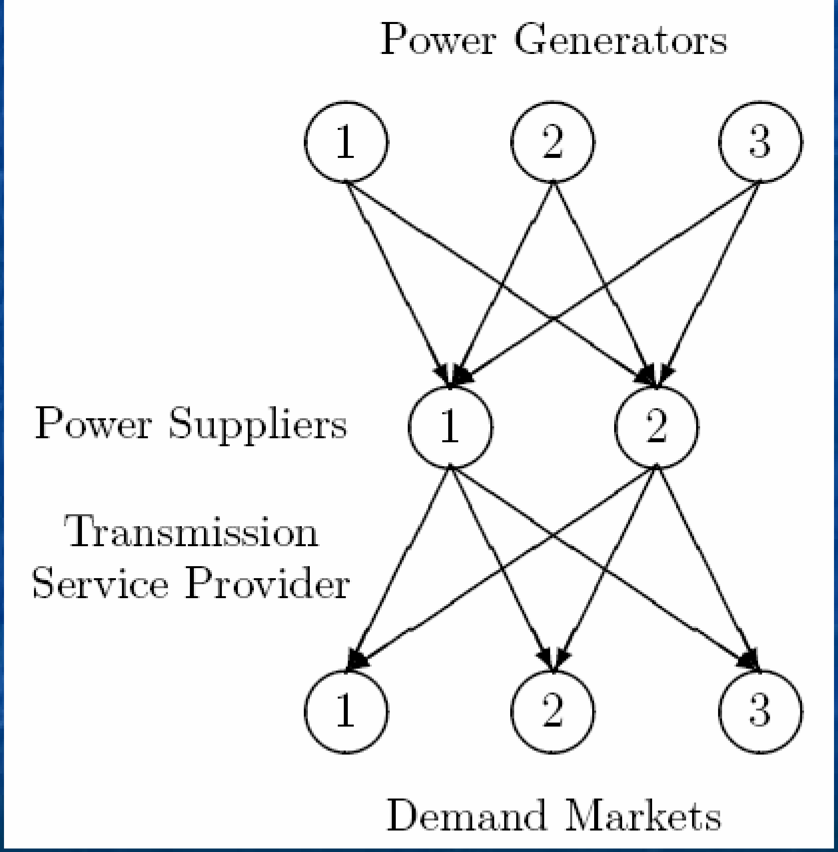
The Results

	Example 1	Example 2	Example 3	Example 4
$q^*_{11} = q^*_{12}$	14.2762	19.5994		
$q^*_{21} = q^*_{22}$	14.2762	19.5994		
$q^*_{31} = q^*_{32}$	57.6051	78.8967		
$q^{1*}_{11} = q^{1*}_{21}$	20.3861	118.0985		
$q^{1*}_{12} = q^{1*}_{22}$	20.3861	0.00		
$q^{1*}_{32} = q^{1*}_{32}$	20.3861	0.00		
$\gamma_1^* = \gamma_2^*$	277.2487	378.3891		
ρ_{31}^*	302.6367	501.4873		
ρ_{32}^*	302.6367	173.8850		
ρ_{33}^*	327.6367	223.8850		

Example 3

The generating function of the first power generator is changed to:

$$f_1(q) = 5q_1^2 + q_1 q_2 + 2q_1$$



The Results

	Example 1	Example 2	Example 3	Example 4
$q^*_{11} = q^*_{12}$	14.2762	19.5994	10.3716	
$q^*_{21} = q^*_{22}$	14.2762	19.5994	21.8956	
$q^*_{31} = q^*_{32}$	57.6051	78.8967	84.2407	
$q^{1*}_{11} = q^{1*}_{21}$	20.3861	118.0985	116.5115	
$q^{1*}_{12} = q^{1*}_{22}$	20.3861	0.00	0.00	
$q^{1*}_{32} = q^{1*}_{32}$	20.3861	0.00	0.00	
$\gamma_1^* = \gamma_2^*$	277.2487	378.3891	383.6027	
ρ_{31}^*	302.6367	501.4873	505.1135	
ρ_{32}^*	302.6367	173.8850	171.1657	
ρ_{33}^*	327.6367	223.8850	221.1657	

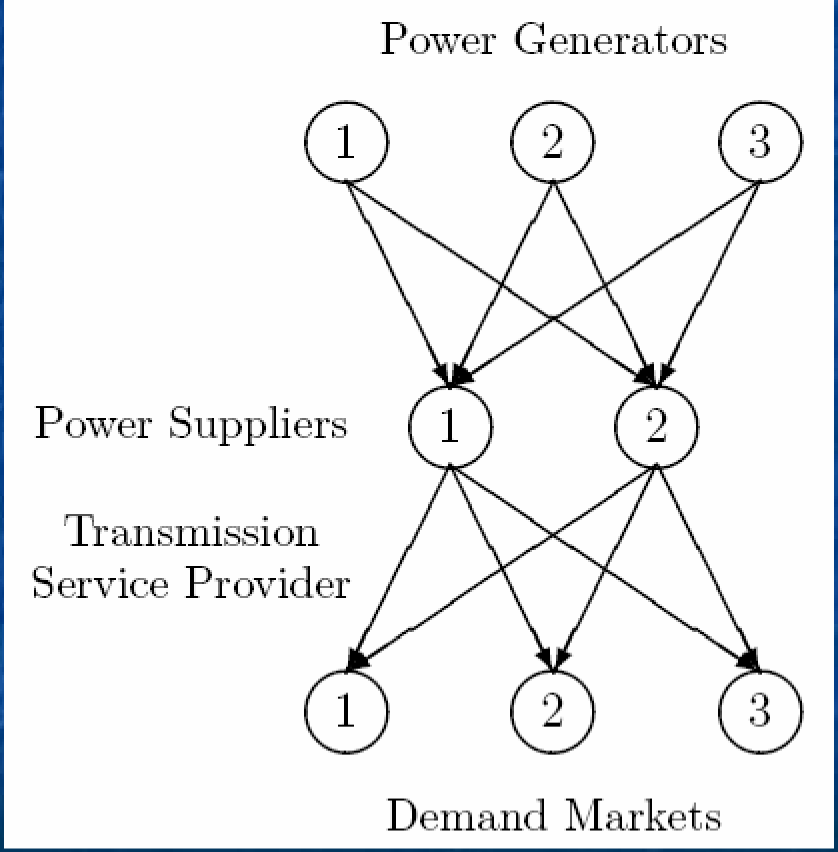
Example 4

The demand functions were changed to:

$$d_1(\rho_3) = -2\rho_{31} + 1500,$$

$$d_2(\rho_3) = -2\rho_{32} + 1100,$$

$$d_3(\rho_3) = -2\rho_{33} + 1200.$$



The Results

	Example 1	Example 2	Example 3	Example 4
$q^*_{11} = q^*_{12}$	14.2762	19.5994	10.3716	14.1801
$q^*_{21} = q^*_{22}$	14.2762	19.5994	21.8956	29.9358
$q^*_{31} = q^*_{32}$	57.6051	78.8967	84.2407	114.9917
$q^{1*}_{11} = q^{1*}_{21}$	20.3861	118.0985	116.5115	111.3682
$q^{1*}_{12} = q^{1*}_{22}$	20.3861	0.00	0.00	11.3683
$q^{1*}_{32} = q^{1*}_{32}$	20.3861	0.00	0.00	36.3682
$\gamma_1^* = \gamma_2^*$	277.2487	378.3891	383.6027	522.2619
ρ_{31}^*	302.6367	501.4873	505.1135	638.6319
ρ_{32}^*	302.6367	173.8850	171.1657	538.6319
ρ_{33}^*	327.6367	223.8850	221.1657	563.6319

Conclusions

We proposed a novel, supply chain network framework for rigorous:

Modeling,

Qualitative analysis, and

Computation of solutions of equilibrium electric power market flows and prices.

Thank You!

For more information see:

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