Fragile Networks: Identifying Vulnerabilities and Synergies in an Uncertain World

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Pre-Conference Tutorial

Outline of Tutorial

- Module I: Network Fundamentals, Efficiency Measurement, and Vulnerability Analysis

- Module II: Applications and Extensions

- ModuIe III: Mergers and Acquisitions, Network Integration, and Synergies

Module I

Why Study Fragile Networks?

Networks provide the foundations for transportation and logistics, for communication, energy provision, social interactions, financing, and economic trade.

Today, the subject has garnered great interest due to a spectrum of catastrophic events that have drawn attention to network vulnerability and fragility.

Since many networks that underlie our societies and economies are large-scale and complex in nature, they are liable to be faced with disruptions.

Recent disasters demonstrate the importance and the vulnerability of network systems

- 9/11 Terrorist Attacks, September 11, 2001;
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 20o4;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Mediterranean cable destruction, January 30, 2008;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010.

Earthquake Damage

prcs.org.pk

Tsunami

letthesunshinein.wordpress.com

Storm Damage

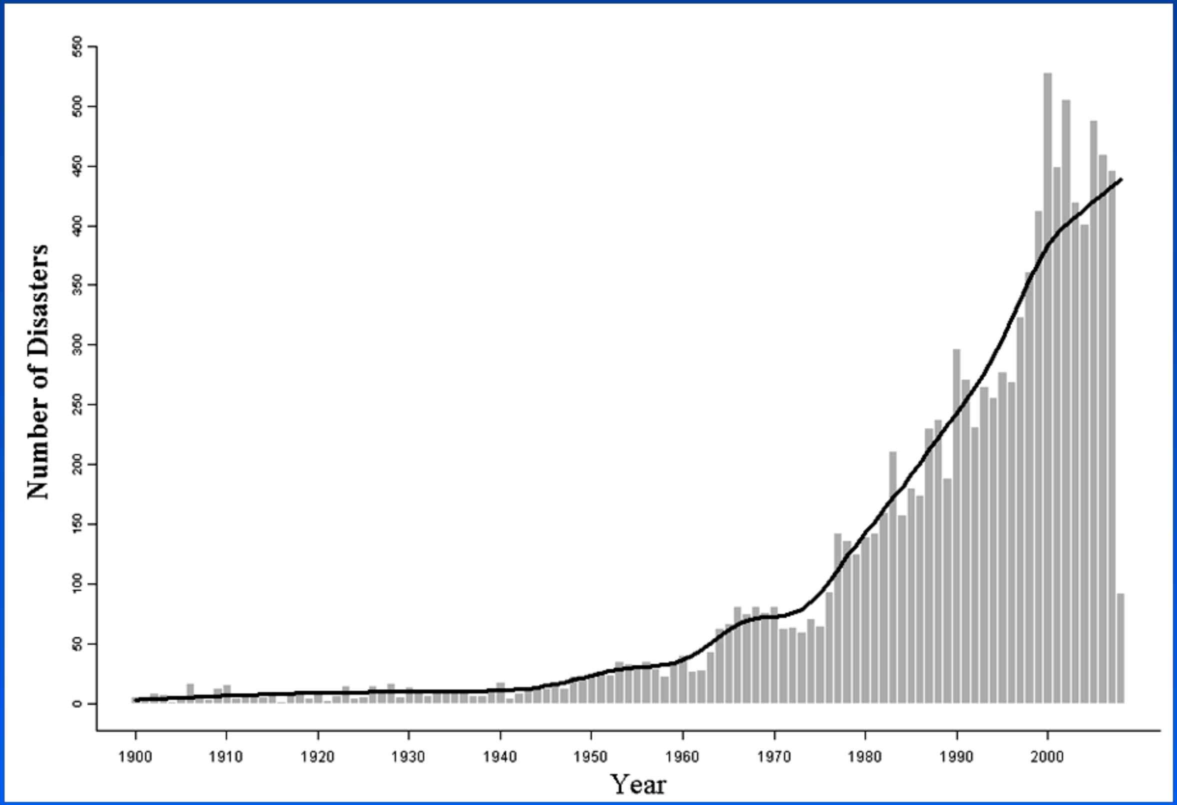
www.srh.noaa.gov

Infrastructure Collapse

www.10-7.com

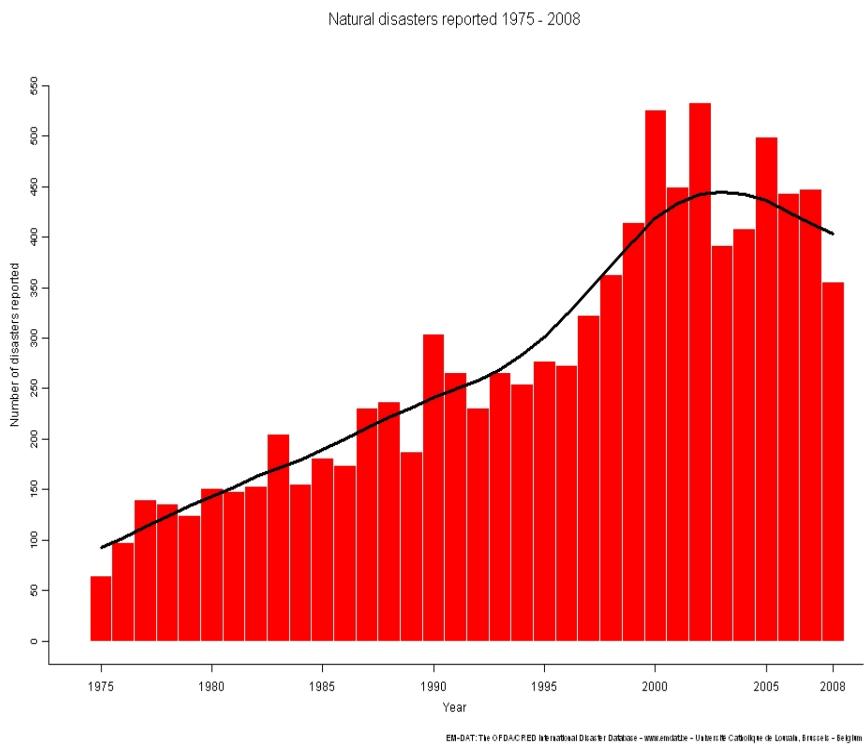
The Haitian and Chilean Earthquakes

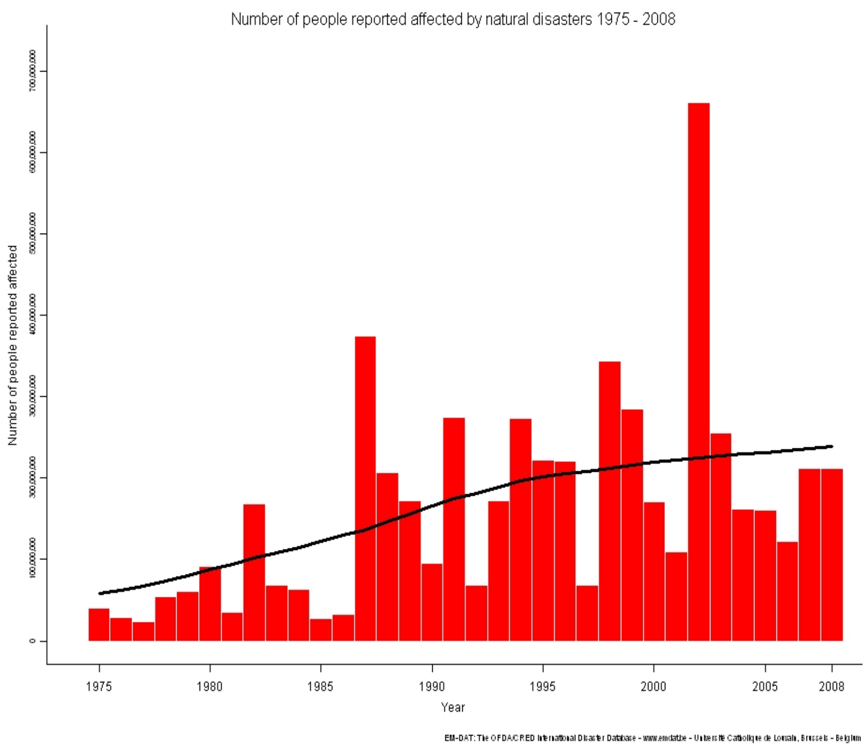
Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing.



Frequency of disasters [Source: Emergency Events Database (2008)]

Natural Disaster Trend and Number of People Affected (1975 – 2008)





Natural Disaster Trend

Source: EM-DAT

Number of People Affected

Source: EM-DAT

We are also in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- alternative and at times *conflicting criteria* used in decision-making;
- *constraints on resources*: natural, human, financial, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- *increasing risk and uncertainty*, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

This era is ideal for applying the tools of Fragile Networks.

Network problems are their own class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.

Network problems will be the focus of this tutorial with fragility as the major theme.

In this tutorial we will:

- provide you with rigorous, computer-based tools to identify the importance of nodes and links in network systems (and their rankings) under alternative user behaviors;
- quantify the effects on network robustness when the link capacities are degraded, and
- capture the environmental impacts when the network link capacities are under stress.

In this tutorial we will also:

• present a measure to quantify the synergy resulting from the integration of network systems.

The synergy measure may be used to assess *a priori* whether (or not) supply chains should be integrated or not); whether specific mergers and acquisitions should take place, and even to assess the potential benefits of the integration of organizations (and teams) in the case of humanitarian (especially logistics) operations.

This tutorial will emphasize the interdisciplinary nature of Fragile Networks and Networks, in general.

Interdisciplinary Impact of Networks

Models and Algorithms

Economics and Finance

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

OR/MS and Engineering

Energy

Manufacturing

Telecommunications

Transportation

Supply Chains

Networks

Biology

DNA Sequencing

Targeted Cancer Therapy

Sociology

Social Networks

Organizational Theory

Computer Science

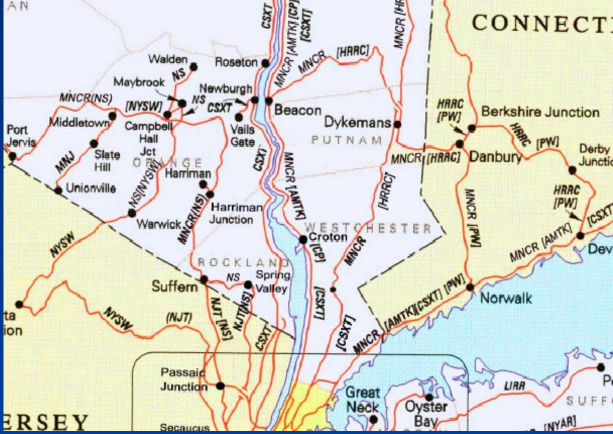
Routing Algorithms

Price of Anarchy

Background and Network Fundamentals

Transportation, Communication, and Energy Networks

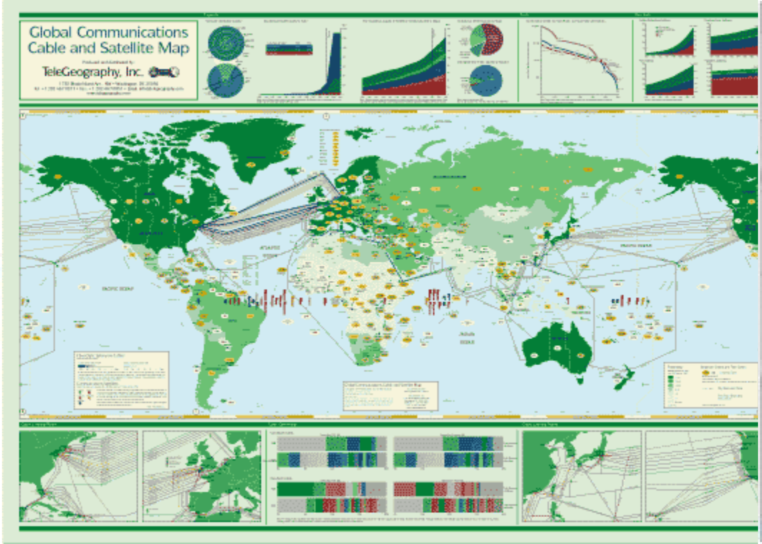
Subway Network



Railroad Network

Iridium Satellite Constellation Network

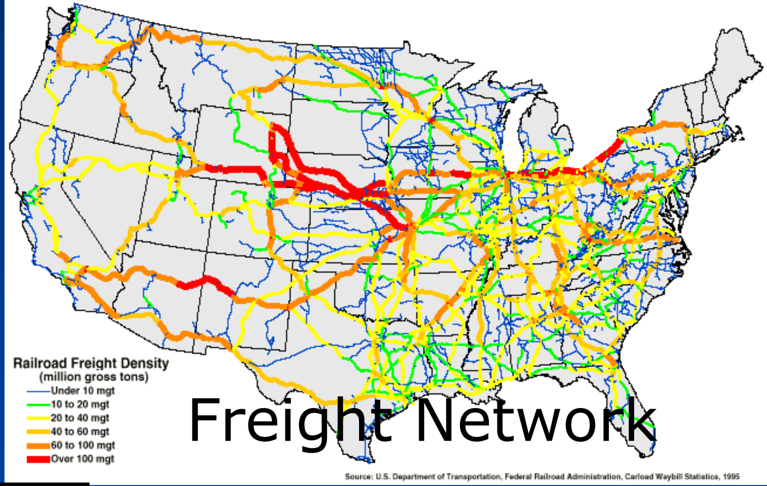
Satellite and Undersea Cable Networks



Duke Energy Gas Pipeline Network

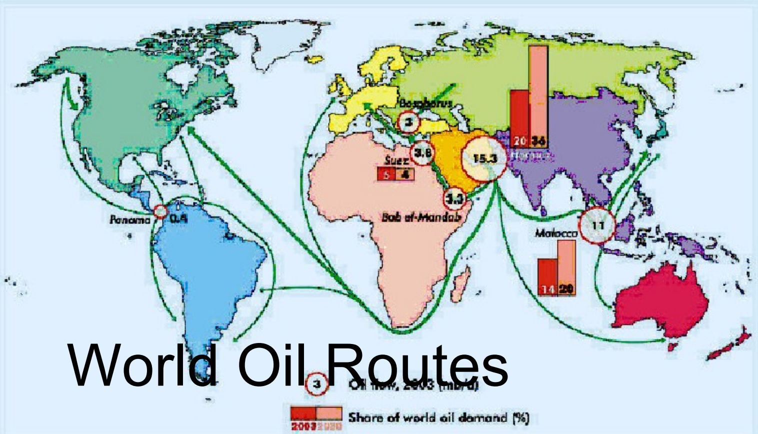
Components of Common Physical Networks

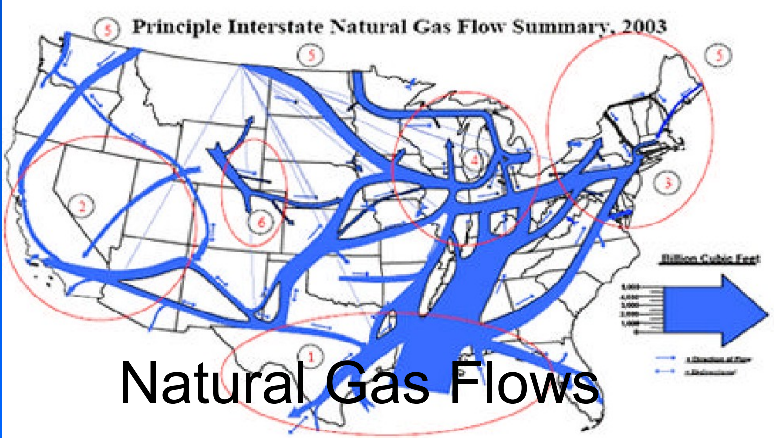
Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity



Network

Systems



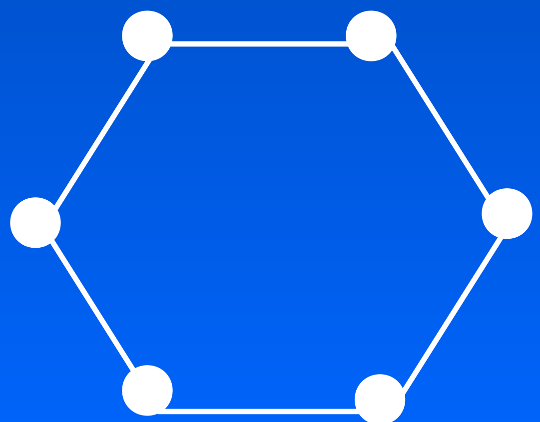


The study of the efficient operation of transportation networks dates to *ancient Rome* with a classical example being the publicly provided Roman road network and the *time of day chariot policy*, whereby chariots were banned from the ancient city of Rome at particular times of day.

Brief Early History of the Science of Networks

1736 - Euler - the earliest paper on graph theory - Konigsberg bridges problem.

1758 - Quesnay in his *Tableau Economique* introduced a graph to depict the circular flow of financial funds in an economy.



1781 - Monge, who had worked under Napoleon Bonaparte, publishes what is probably the first paper on transportation in minimizing cost.

1838 - Cournot states that a competitive price is determined by the intersection of supply and demand curves in the context of spatially separate markets in which transportation costs are included.

1841 - Kohl considered a two node, two route transportation network problem.

1845 - Kirchhoff wrote Laws of Closed Electric Circuits.

1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

1936 - Konig published the first book on graph theory.

1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

1952 - Copeland in his book asked, *Does money flow like water or electricity?*

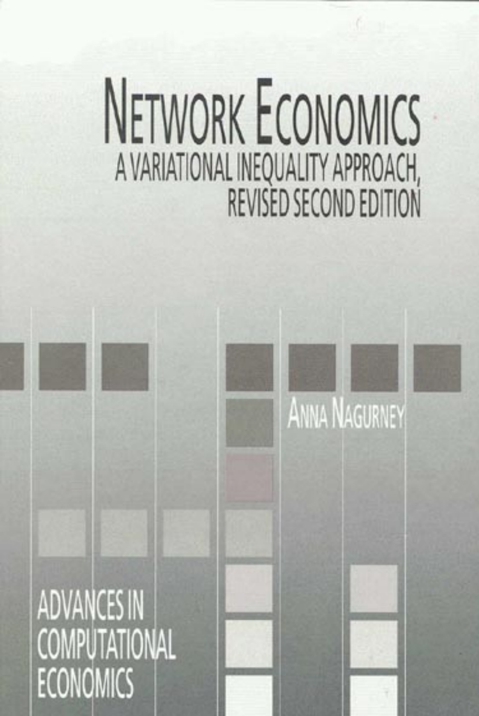
1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

1962 - Ford and Fulkerson write *Flows in Networks*.

1969 - Dafermos and Sparrow coined the terms *user-optimization* and *system-optimization* and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

The need to model and solve a spectrum of challenging network problems has given rise to new computational methodologies.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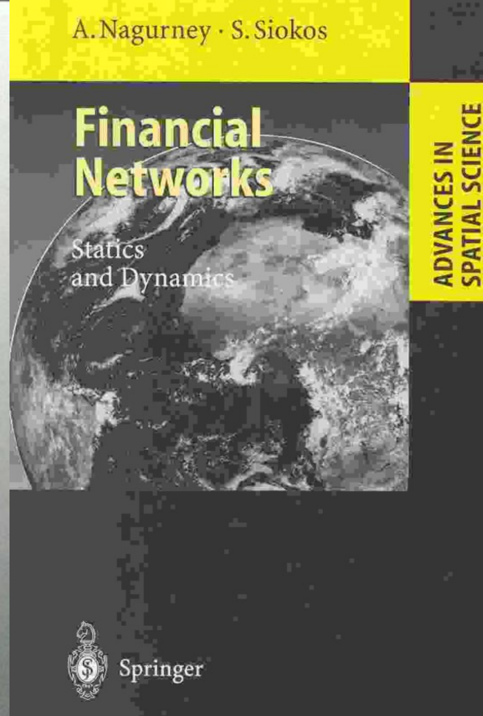
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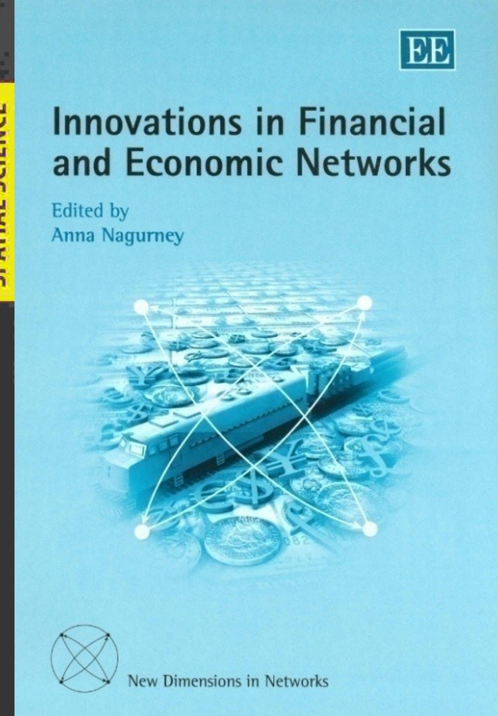
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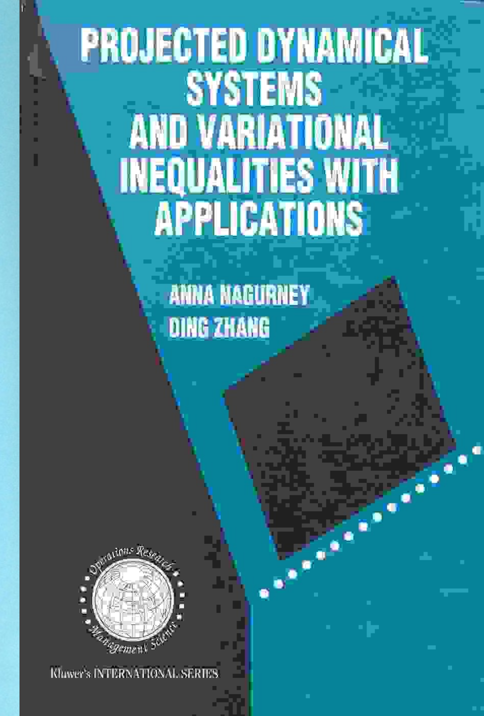
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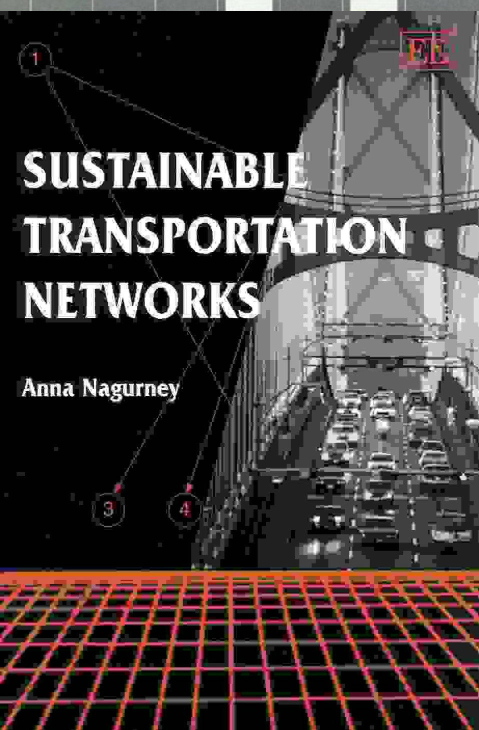
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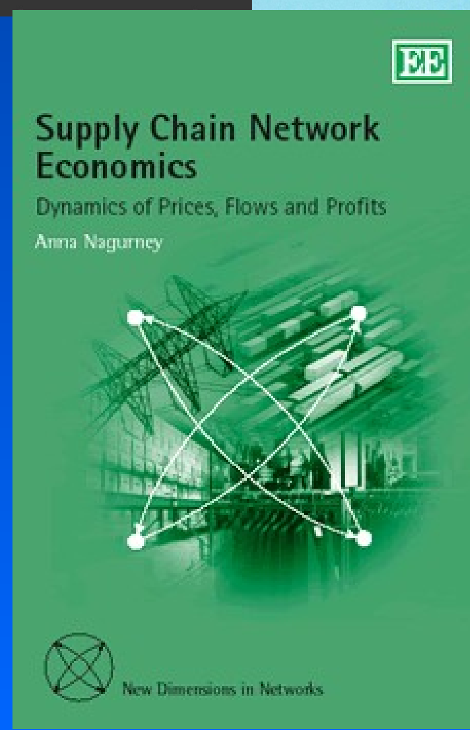
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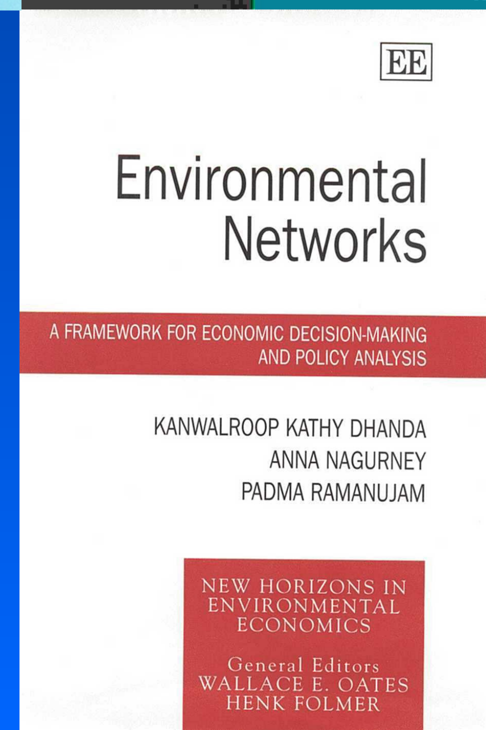
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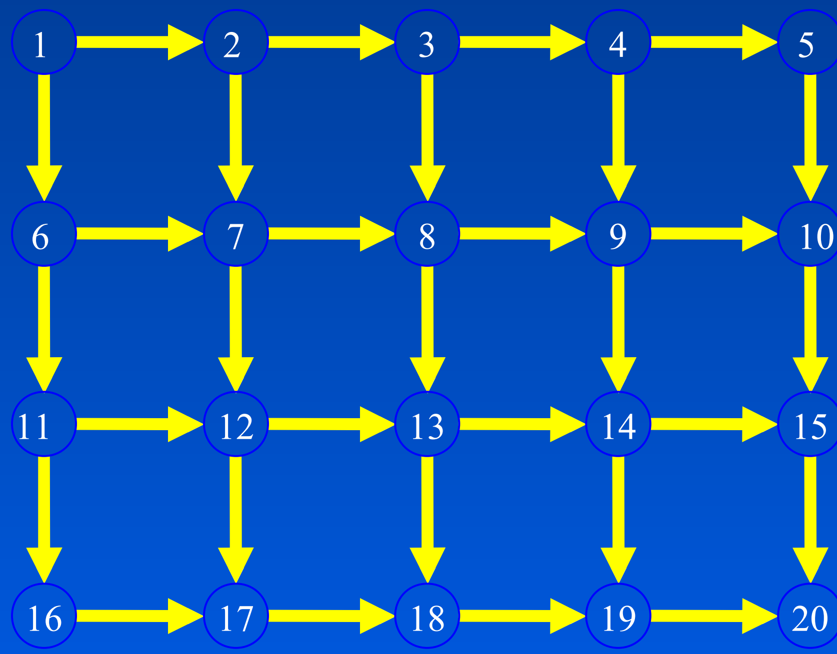
The scientific study of networks involves:

- how to ***model*** such applications as ***mathematical entities*****;**
- how to ***analyze the models*** qualitatively;
- how to ***design algorithms*** to solve the resulting models.

The Basic Components of Networks

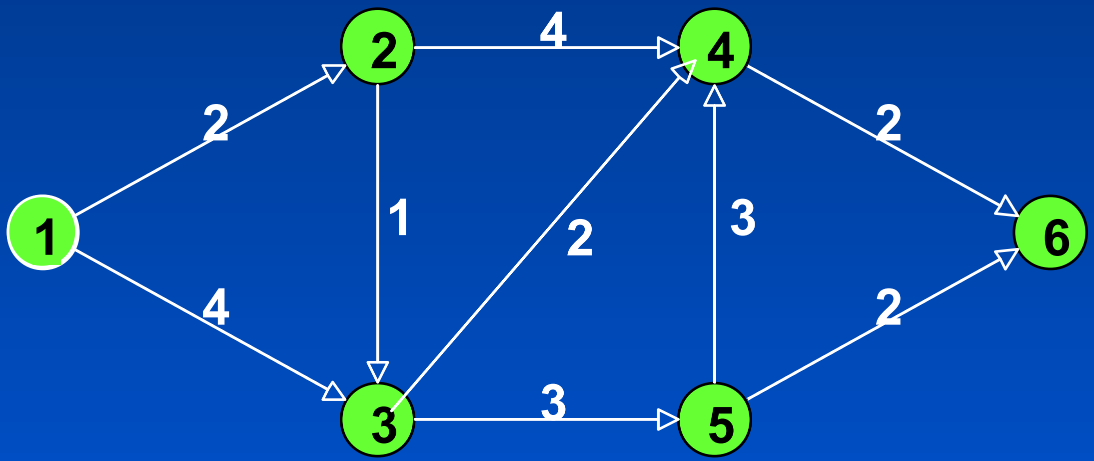
Nodes **Links** **Flows**



Many of the Classic Examples of Network Problems were Linear in Nature

- The Shortest Path Problem
- The Maximum Flow Problem
- The Minimum Cost Flow Problem.

The Shortest Path Problem



What is the shortest path from 1 to 6?

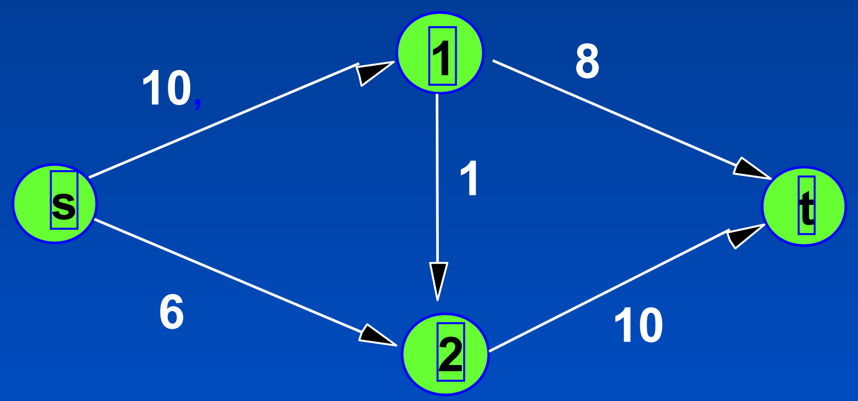
Applications of the Shortest Path Problem

arise in transportation and telecommunications.

Other applications include:

- simple building evacuation models
- DNA sequence alignment
- assembly line balancing
- compact book storage in libraries.

The Maximum Flow Problem



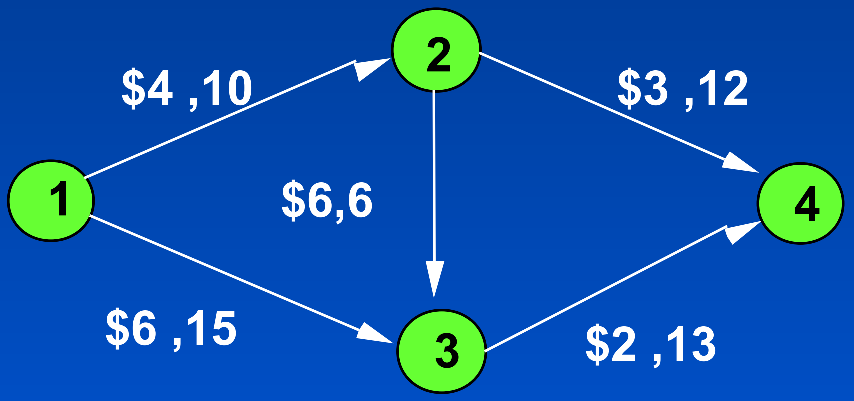
Each link has a maximum capacity.

How does one maximize the flow from s to t, subject to the link capacities?

Applications of the Maximum Flow Problem

- machine scheduling
- network reliability testing
- building evacuation.

The Minimum Cost Flow Problem



Each link has a linear cost and a maximum capacity.

How does one minimize cost for a given flow from 1 to 4?

The Optimization Formulation

Flow out of node i - Flow into node i = b(i)

Minimize $\Sigma_{i,j}\, c_{ij}\, x_{ij}$

s.t. $\Sigma_j\, x_{ij} - \Sigma_j\, x_{ji} = b(i)$ for each node i

$0 \le x_{ij} \le u_{ij}$ for all i,j

$\Sigma_i\, b(i) = 0$

Applications of the Minimum Cost Flow Problem

- warehousing and distribution
- vehicle fleet planning
- cash management
- automatic chromosome classification
- satellite scheduling.

We need to capture not only network topology (how nodes are connected with the links) but also

the behavior of users of the networks and the induced flows!

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion (leading to nonlinearities)*;
- alternative behavior of users of the network, which may lead to *paradoxical phenomena*;
- the *interactions among networks* themselves such as in transportation versus telecommunications;
- *policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.

Networks in Action

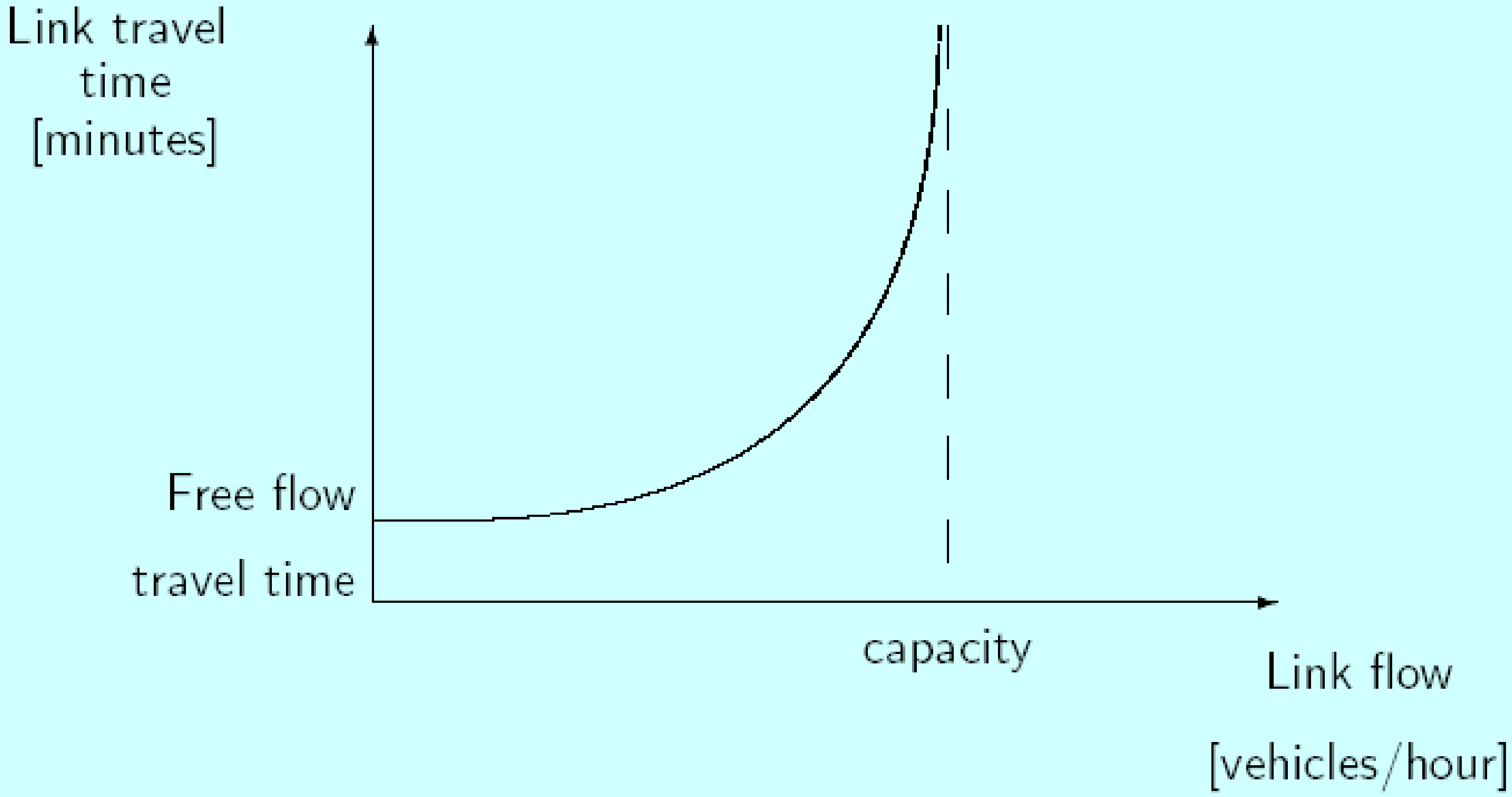
- Some social network websites, such as facebook.com and myspace.com, have over *300 million* users.
- Internet traffic is approximately *doubling* each year.
- In the US, the annual traveler delay per peak period (rush hour) has grown from 16 hours to *47 hours* since 1982.
- The total amount of delay reached *3.7 billion hours* in 2003.
- The wasted fuel amounted to *2.3 billion gallons* due to engines idling in traffic jams (Texas Transportation Institute 2005 Urban Mobility Report).

Hence, many of the network problems today are flow-dependent and increasingly nonlinear, as opposed to linear.

Therefore, the underlying functions must capture, for example, congestion!


TRAFFIC

Capturing Link Congestion



For a typical user link travel time function, where the free flow travel time refers to the travel time to traverse a link when there is zero flow on the link (or zero vehicles).

The importance of capturing user behavior on networks will now be illustrated through a famous paradox known as the *Braess paradox* in which travelers are assumed to behave in a *user-optimizing (U-O) manner*, as opposed to a *system-optimizing (S-O) one*.

Under U-O behavior, decision-makers act independently and selfishly with no concern of the impact of their travel choices on others.

Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.

User-Optimized

Decentralized	Selfish	U - O
vs. ⟺	vs. ⟺	vs.
Centralized	Unselfish	S - O

System-Optimized

Flows are routed so as to minimize the total cost to society.

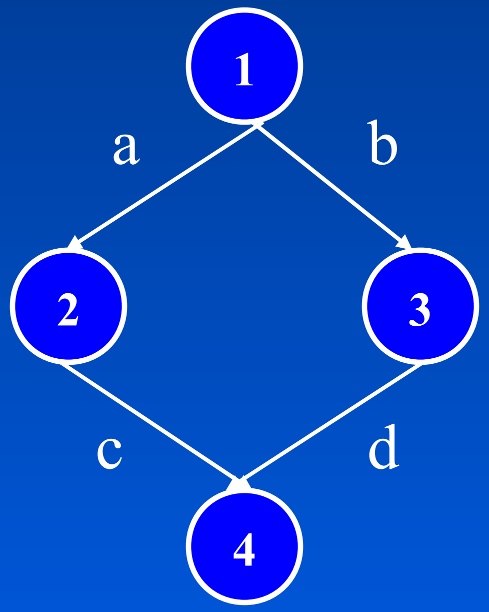
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}}^* = \mathbf{x_{p_2}}^* = \mathbf{3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83}$.



$$\mathbf{c_a(f_a)=10\,f_a \quad c_b(f_b) = f_b+50}$$

$$\mathbf{c_c(f_c) = f_c+50 \quad c_d(f_d) = 10\,f_d}$$

Adding a Link Increases Travel Cost for All!

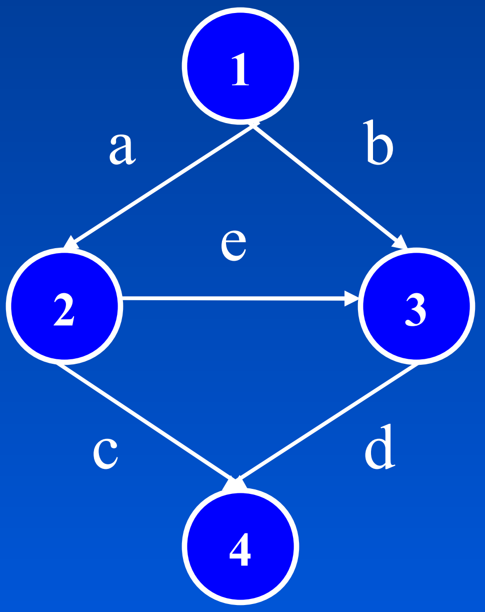
Adding a new link creates a new path **p_3=(a,e,d).**

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.

The equilibrium path travel costs: **$C_{p_1} = C_{p_2} = C_{p_3} = 92$.**

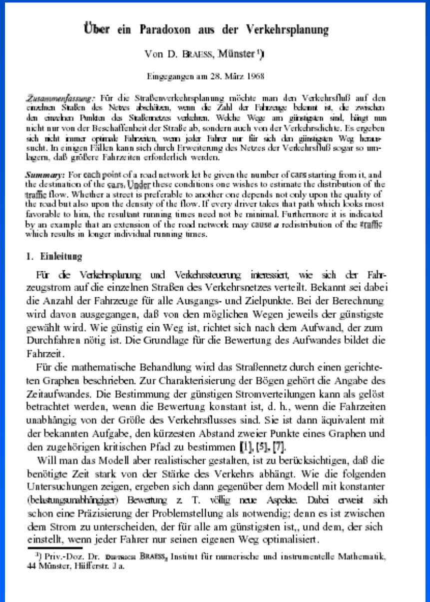


$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as:

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger in the November 2005 issue of *Transportation Science.*



Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

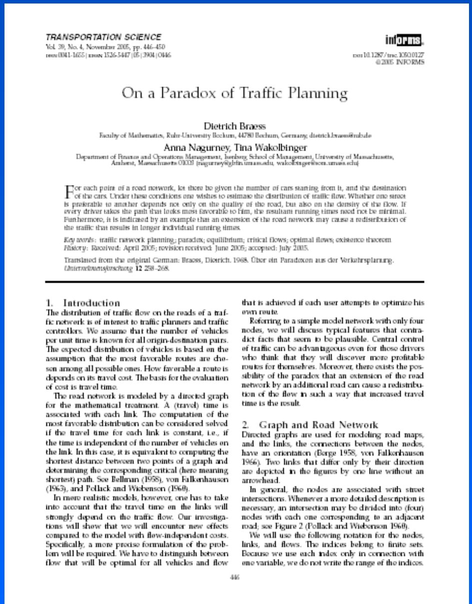
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

The Braess Paradox Around the World

1969 - Stuggart, Germany - Traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42nd Street was closed and traffic flow improved.

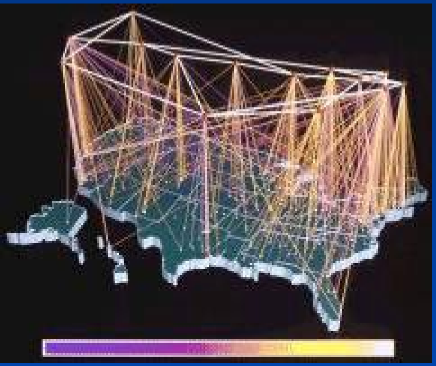
2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.

Braess

on

BROADWAY

Other Networks that Behave like Traffic Networks

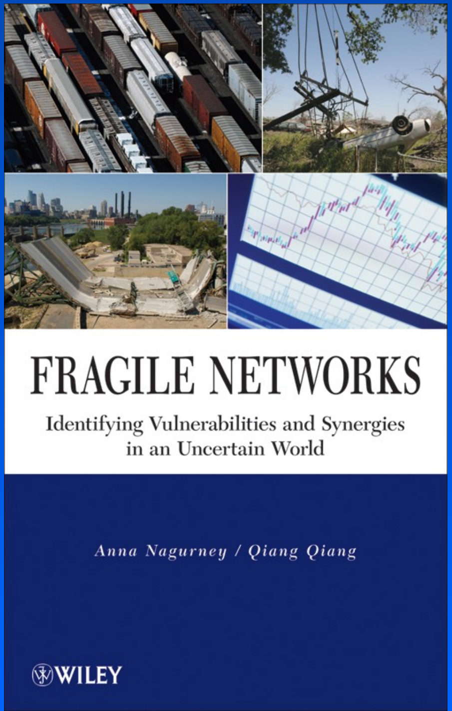


The Internet

Supply Chain Networks

Electric Power Generation/Distribution Networks

Financial Networks



This *paradox is relevant* not only to congested transportation networks but also to the Internet and electric power networks.

Hence, there are *huge implications* also for network design.

There are *two fundamental principles of travel behavior*, due to Wardrop (1952), which are referred to as user-optimal (U-O or network equilibrium) and system-optimal (S-O).

In a *user-optimized (network equilibrium) problem*, each user of a network system seeks to determine his/her cost-minimizing route of travel between an origin/destination pair, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action.

In a *system-optimized network problem*, users are allocated among the routes so as to minimize the total cost in the system.

Both classes of problems, under certain imposed assumptions, possess optimization formulations.

Bureau of Public Roads (BPR) Link Cost Function

$$c_a = c_a^0 \left[1 + \alpha (\frac{f_a}{t'_a})^{\beta} \right] ,$$

where, c_a and f_a are the travel time and link flow, respectively, on link a, c_a^0 is the free-flow travel time, and t'_a is the "practical capacity" of link a. The quantities α and β are model parameters, for which the values $\alpha = 0.15$ minutes and $\beta = 4$ are typical values. For example, these values imply that the practical capacity of a link is the flow at which the travel time is 15% greater than the free-flow travel time.

The User-Optimization (U-O) Problem Transportation Network Equilibrium

Consider a general network $G = [N, L]$, where N denotes the set of nodes, and L the set of directed links. Let a denote a link of the network connecting a pair of nodes, and let p denote an acyclic path consisting of a sequence of links connecting an origin/destination (O/D) pair of nodes. P_w denotes the set of paths connecting the O/D pair of nodes w and P the set of all paths.

Let x_p represent the flow on path p and let f_a denote the flow on link a. The following conservation of flow equations must hold:

$$f_a = \sum_{p \in P} x_p \delta_{ap},$$

where $\delta_{ap} = 1$, if link a is contained in path p, and 0, otherwise. This expression states that the flow on a link a is equal to the sum of all the path flows on paths p that contain (traverse) link a.

Moreover, if we let d_w denote the demand associated with O/D pair w, then we must have that

$$d_w = \sum_{p \in P_w} x_p,$$

where $x_p \geq 0$, $\forall p$, that is, the sum of all the path flows between an origin/destination pair w must be equal to the given demand d_w.

Let c_a denote the user cost associated with traversing link a, and C_p the user cost associated with traversing the path p. Then

$$C_p = \sum_{a \in L} c_a \delta_{ap}.$$

In other words, the cost of a path is equal to the sum of the costs on the links comprising the path. In the classical model, $c_a = c_a(f_a)$, $\forall a \in L$. In the most general case, $c_a = c_a(f), \forall a \in L$, where f is the vector of link flows.

Transportation Network Equilibrium Conditions

The network equilibrium conditions are then given by: For each path $p \in P_w$ and every O/D pair w:

$$C_p \begin{cases} = \lambda_w, & \text{if} \quad x_p^* > 0 \\ \geq \lambda_w, & \text{if} \quad x_p^* = 0 \end{cases}$$

where λ_w is an indicator, whose value is not known a priori. The equilibrium conditions state that the user costs on all used paths connecting a given O/D pair will be minimal and equalized. This is Wardrop's first principle of travel behavior.

As shown by Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969), if the user link cost functions satisfy the symmetry property that $[\frac{\partial c_a}{\partial c_b} = \frac{\partial c_b}{\partial c_a}]$, for all links a, b in the network then the solution to the above U-O problem can be reformulated as the solution to an associated optimization problem. For example, if we have that $c_a = c_a(f_a)$, for all links $a \in L$, then the solution to the U-O problem can be obtained by solving:

$$\text{Minimize} \quad \sum_{a \in L} \int_0^{f_a} c_a(y) dy$$

subject to:

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w \in W,$$

$$f_a = \sum_{p \in P} x_p, \quad \forall a \in L,$$

$$x_p \geq 0, \quad \forall p \in P.$$

The System-Optimization (S-O) Problem

The above discussion focused on the user-optimized (U-O) problem. We now turn to the system-optimized (S-O) problem in which a central controller, say, seeks to minimize the total cost in the network system, where the total cost is expressed as

$$\sum_{a \in L} \hat{c}_a(f_a)$$

where it is assumed that the total cost function on a link a is defined as:

$$\hat{c}_a(f_a) \equiv c_a(f_a) \times f_a,$$

subject to the conservation of flow constraints, and the nonnegativity assumption on the path flows. Here separable link costs have been assumed, for simplicity, and other total cost expressions may be used, as mandated by the particular application.

The S-O Optimality Conditions

Under the assumption of strictly increasing user link cost functions, the optimality conditions are: For each path $p \in P_w$, and every O/D pair w:

$$\hat{C}'_p \begin{cases} = \mu_w, & \text{if} \quad x_p > 0 \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p$ denotes the marginal total cost on path p, given by:

$$\hat{C}'_p = \sum_{a \in L} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}.$$

The above conditions correspond to Wardrop's second principle of travel behavior.

What is the S-O solution for the two Braess networks (before and after the addition of a new link e)?

Before the addition of the link e, we may write:

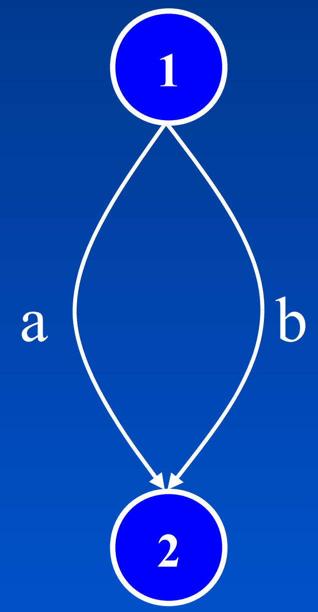
$$\hat{c}'_a = 20f_a, \quad \hat{c}'_b = 2f_b + 50,$$

$$\hat{c}'_c = 2f_c + 50, \quad \hat{c}'_d = 20f_d.$$

It is easy to see that, in this case, the S-O solution is identical to the U-O solution with $x_{p_1} = x_{p_2} = 3$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2} = 116$. Furthermore, after the addition of link e, we have that $\hat{c}'_e = 2f_e + 10$. The new path p_3 is not used in the S-O solution, since with zero flow on path p_3, we have that $\hat{C}'_{p_3} = 170$ and $\hat{C}'_{p_1} = \hat{C}'_{p_2}$ remains at 116.

Another Example

Assume a network with a single O/D pair (1,2). There are 2 paths available to travelers: $\mathbf{p_1 = a}$ and $\mathbf{p_2 = b}$.



For a travel demand of **1,** the U-O path flows are: $\mathbf{x_{p_1}^* = 1;\ x_{p_2}^* = 0}$ and

the total cost under U-O behavior is $TC_{u\text{-}o} = 1$.

$$c_a(f_a) = f_a$$

$$c_b(f_b) = f_b + 1$$

The S-O path flows are: $\mathbf{x_{p_1} = 3/4;\ x_{p_2} = 1/4}$ and

the total cost under S-O behavior is $TC_{S\text{-}o} = 7/8$.

The Price of Anarchy

The price of anarchy is defined as the ratio of the TC under U-O behavior to the TC under S-O behavior:

$$\rho = TC_{U\text{-}O} / TC_{S\text{-}O}$$

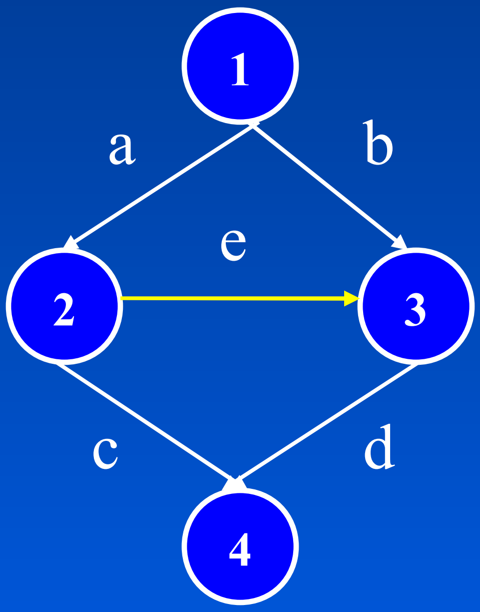
See Roughgarden (2005), *Selfish Routing and the Price of Anarchy*.

Question: When does the U-O solution coincide with the S-O solution?

Answer: In a general network, with user link cost functions given by: $c_a(f_a)=c_a^0 f_a^{\beta}$, *for all links, with* $c_a^0 \geq 0$ *and* $\beta \geq 0$.

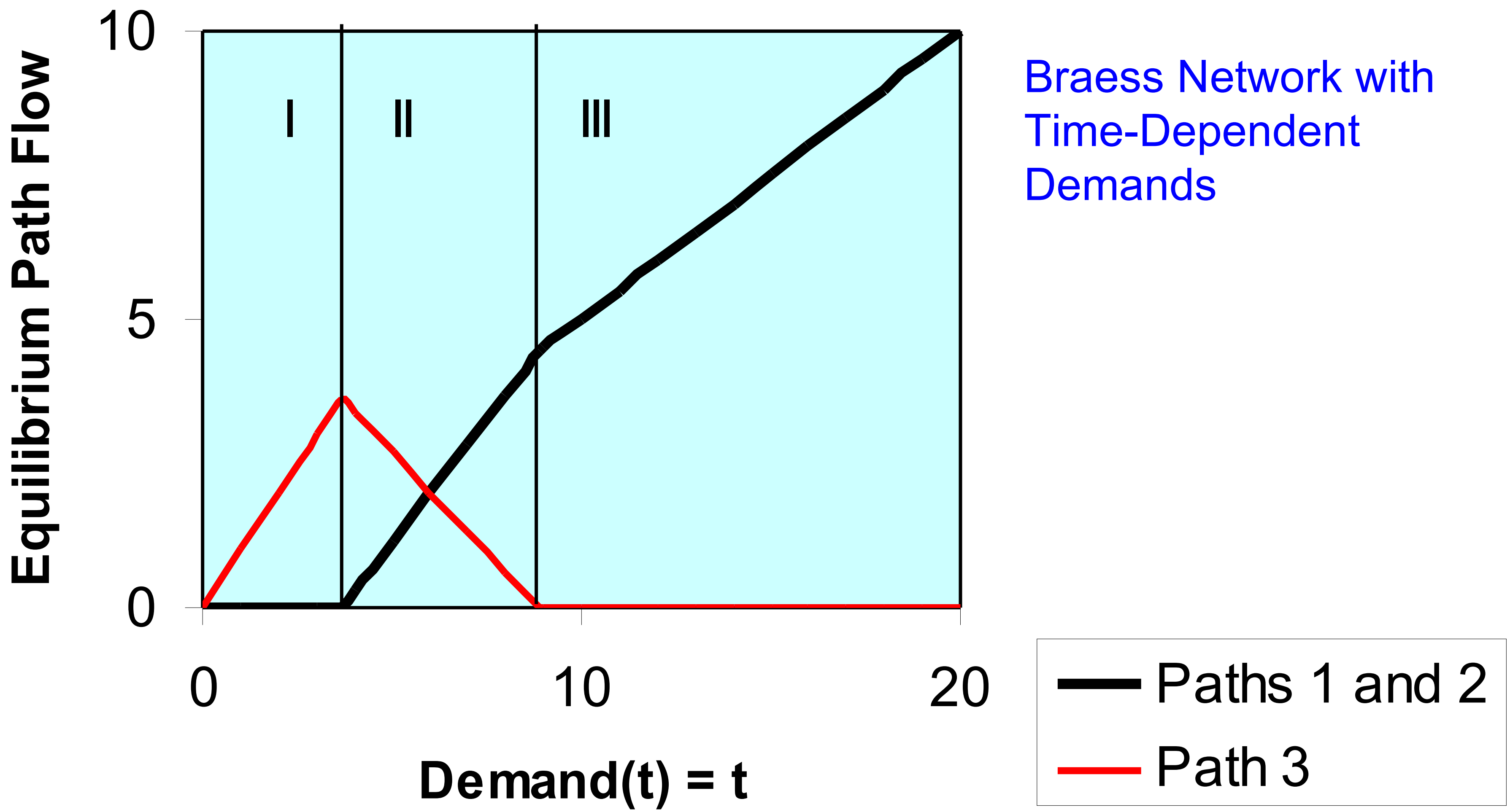
Note that for $c_a(f_a)=c_a^0$, *that is, in the case of uncongested networks, this result always holds.*

Recall again the Braess Network where we add the link e.

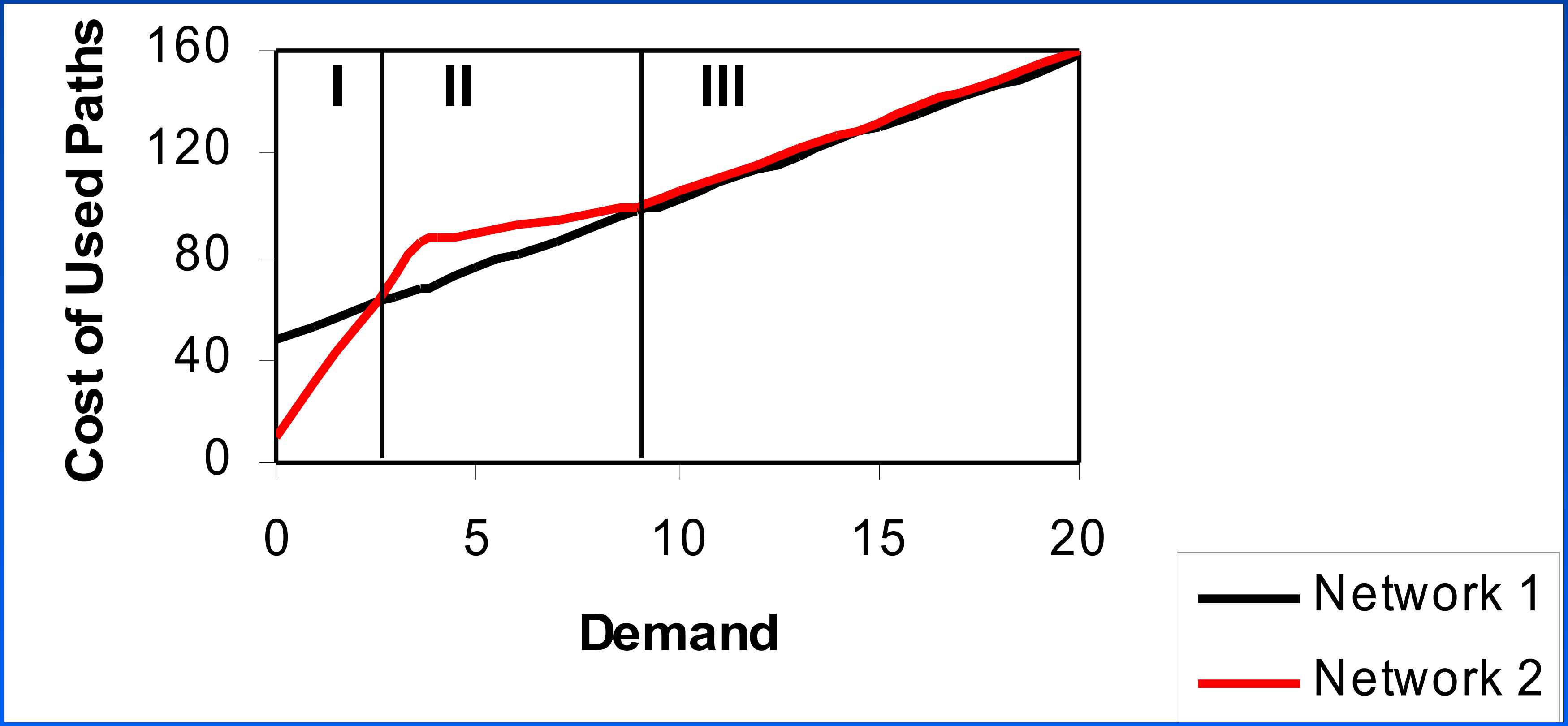


What happens if the demand varies over time?

The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands



In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

If the symmetry assumption does not hold for the user link costs functions, which is always satisfied by separable user link cost functions, then the (U-O) equilibrium conditions can *no longer* be reformulated as an associated optimization problem and the equilibrium conditions are formulated and solved as a *variational inequality (VI) problem!*

Smith (1979), Dafermos (1980)

VI Formulation of Transportation Network Equilibrium (Dafermos (1980), Smith (1979))

A traffic path flow pattern satisfies the above equilibrium conditions if and only if it satisfies the variational inequlity problem: determine $x^* \in K$, such that

$$\sum_p C_p(x^*) \times (x_p - x_p^*) \geq 0, \quad \forall x \in K.$$

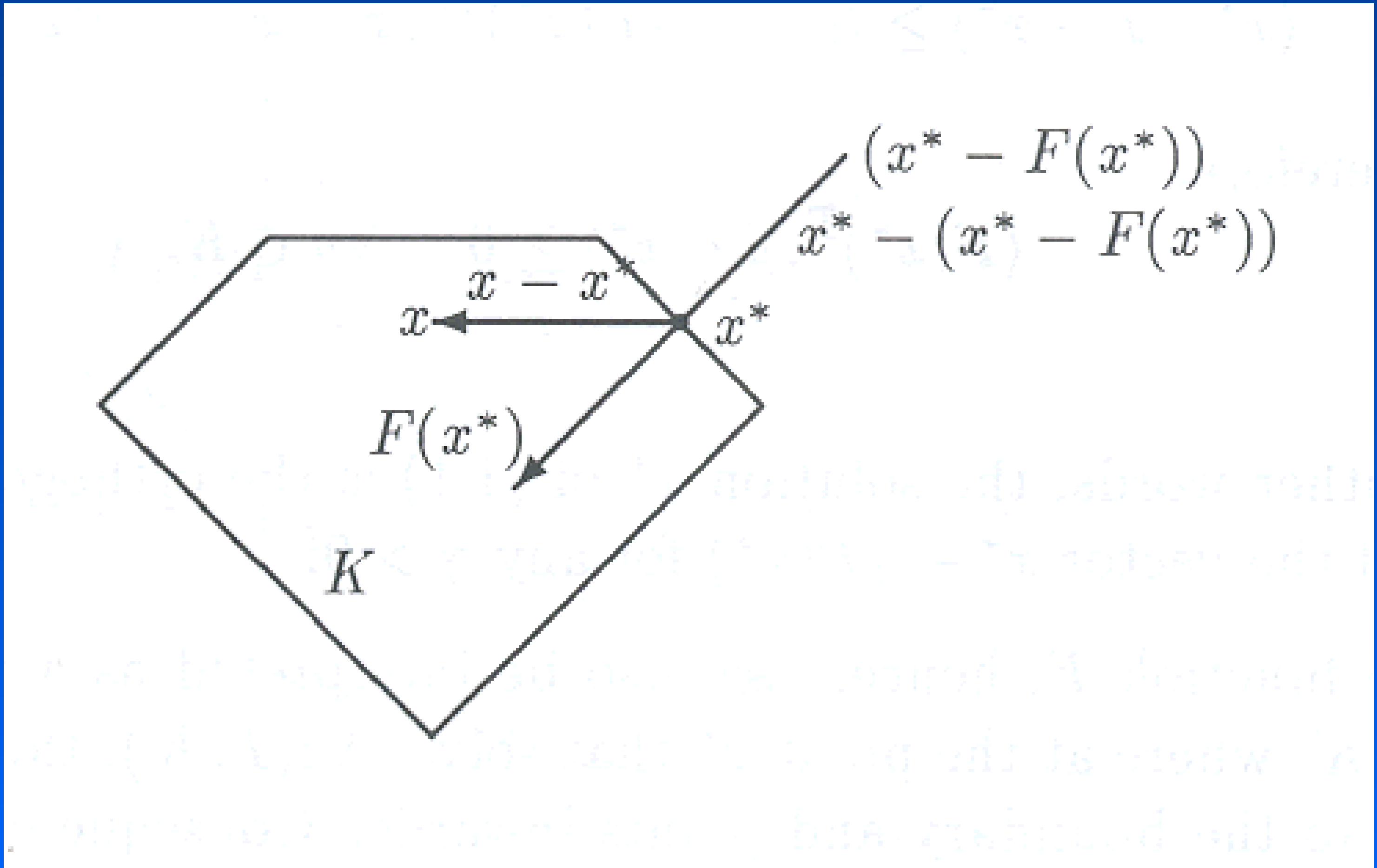
Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above.

In particular, the finite-dimensional variational inequality problem is to determine $x^* \in K \subset R^n$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

where $\langle \cdot, \cdot \rangle$ denoted the inner product in R^n and K is closed and convex.

A Geometric Interpretation of a Variational Inequality



The variational inequality problem, contains, as special cases, such classical problems as:

- systems of equations
- optimization problems
- complementarity problems

and is also closely related to fixed point problems.

Hence, it is a unifying mathematical formulation for a variety of mathematical programming problems.

Transportation and Other Network Systems

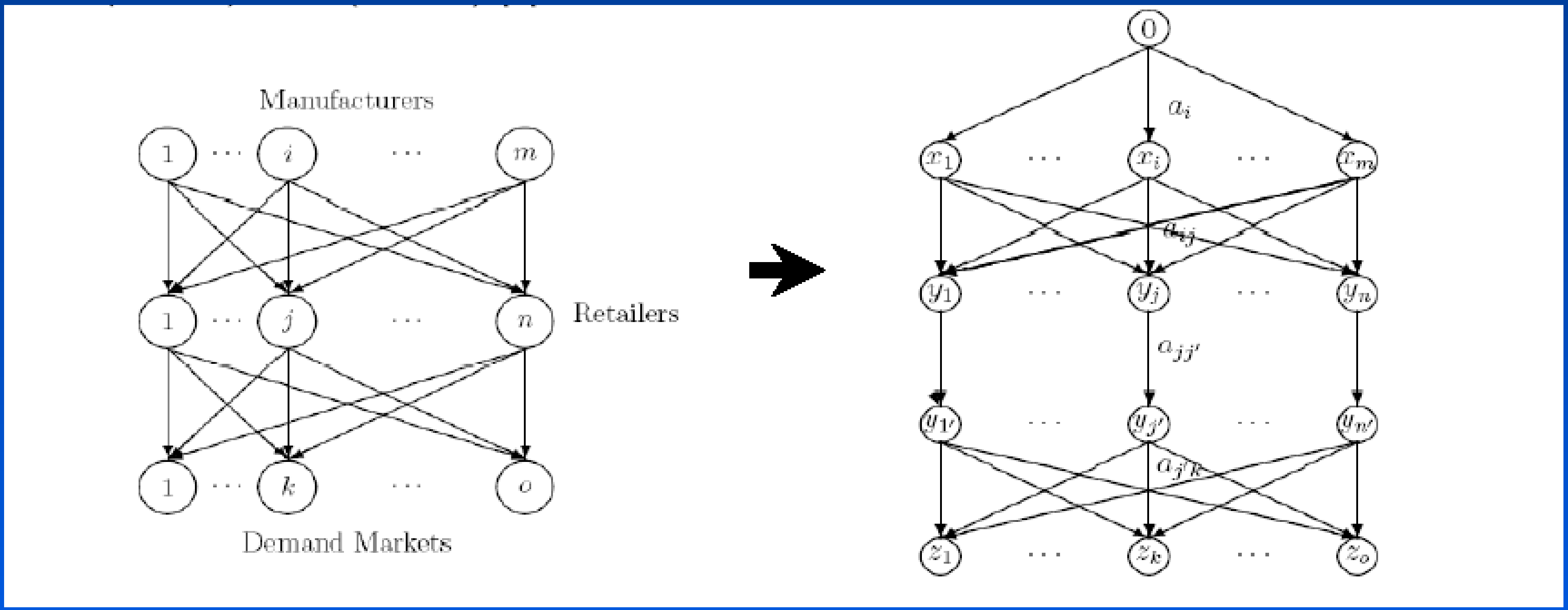
The TNE Paradigm is the Unifying Paradigm for a Variety of Network Systems:

- Transportation Networks
- the Internet
- Financial Networks
- Supply Chains
- Electric Power Networks

Other Related Applications

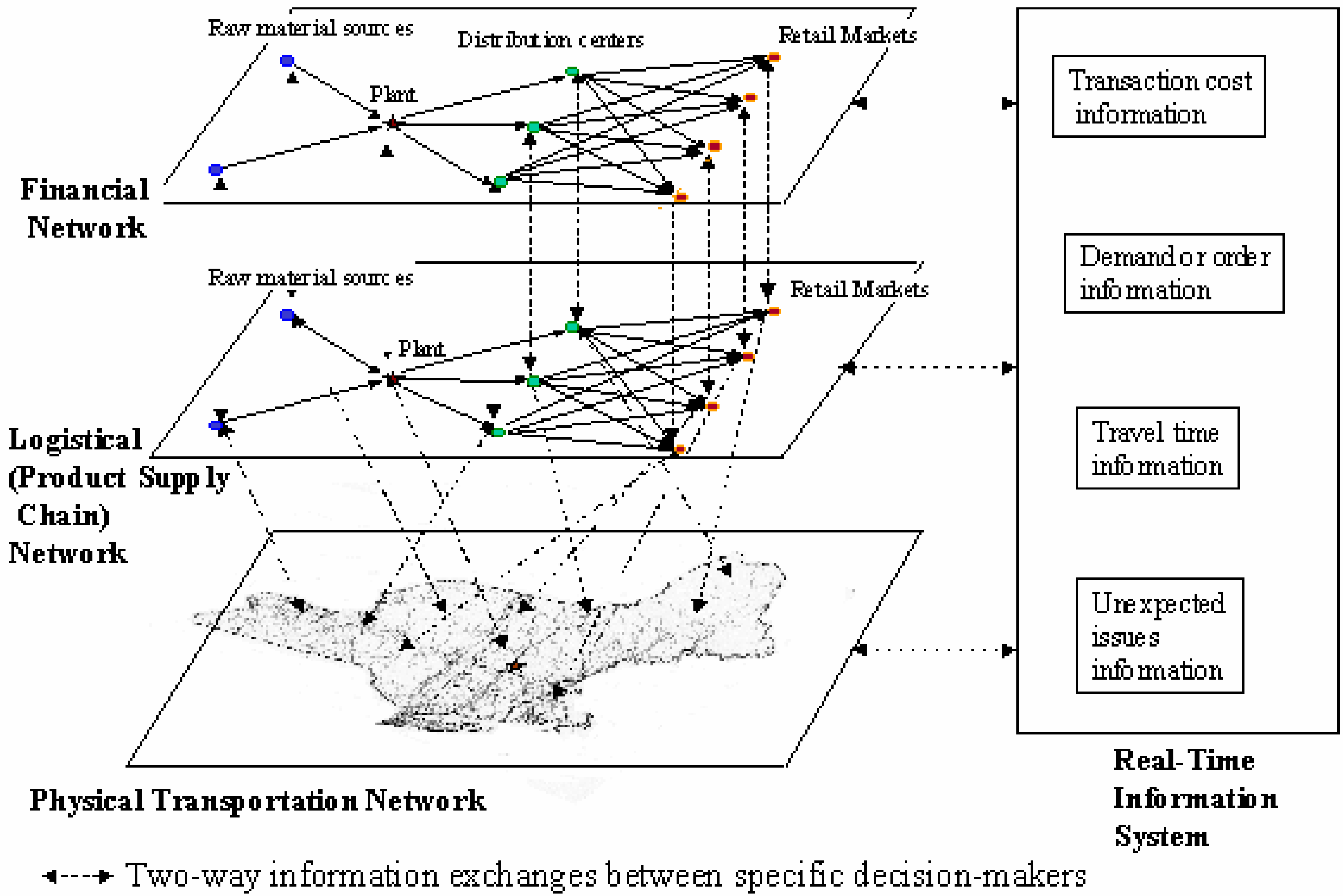
- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Social Networks integrated with Economic Networks (Supply Chains and Financial Networks)

The Equivalence of Supply Chains and Transportation Networks



Nagurney, *Transportation Research E* 42 (2006), pp 293-316.

Supply Chain -Transportation Supernetwork Representation



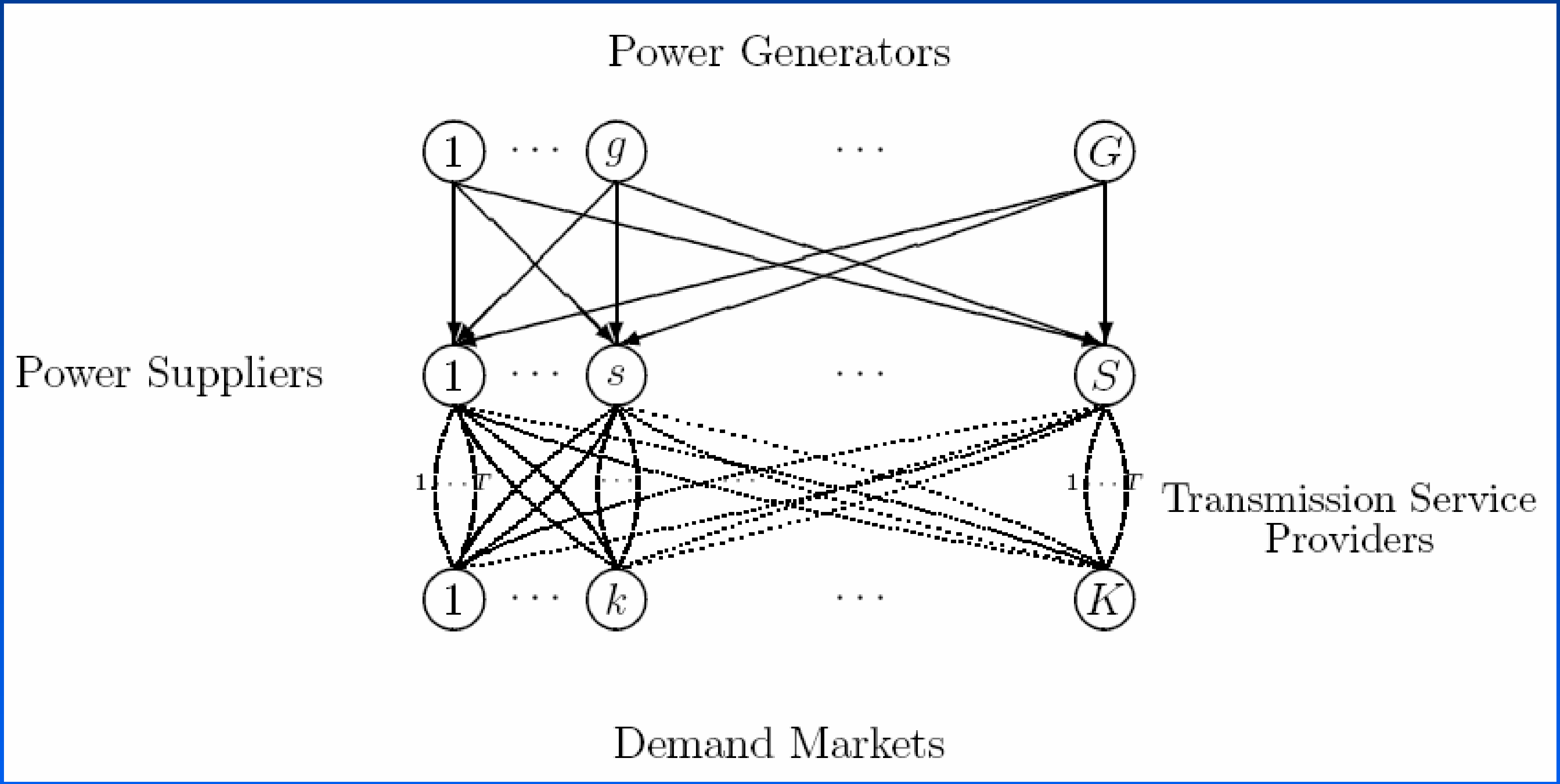
Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* 29 (2002), 795-818.

The fifth chapter of Beckmann, McGuire, and Winsten's book, ***Studies in the Economics of Transportation*** (1956) describes some *unsolved problems* including a single commodity network equilibrium problem that the authors imply could be generalized to capture electric power networks.

Specifically, they asked whether electric power generation and distribution networks can be reformulated as transportation network equilibrium problems.

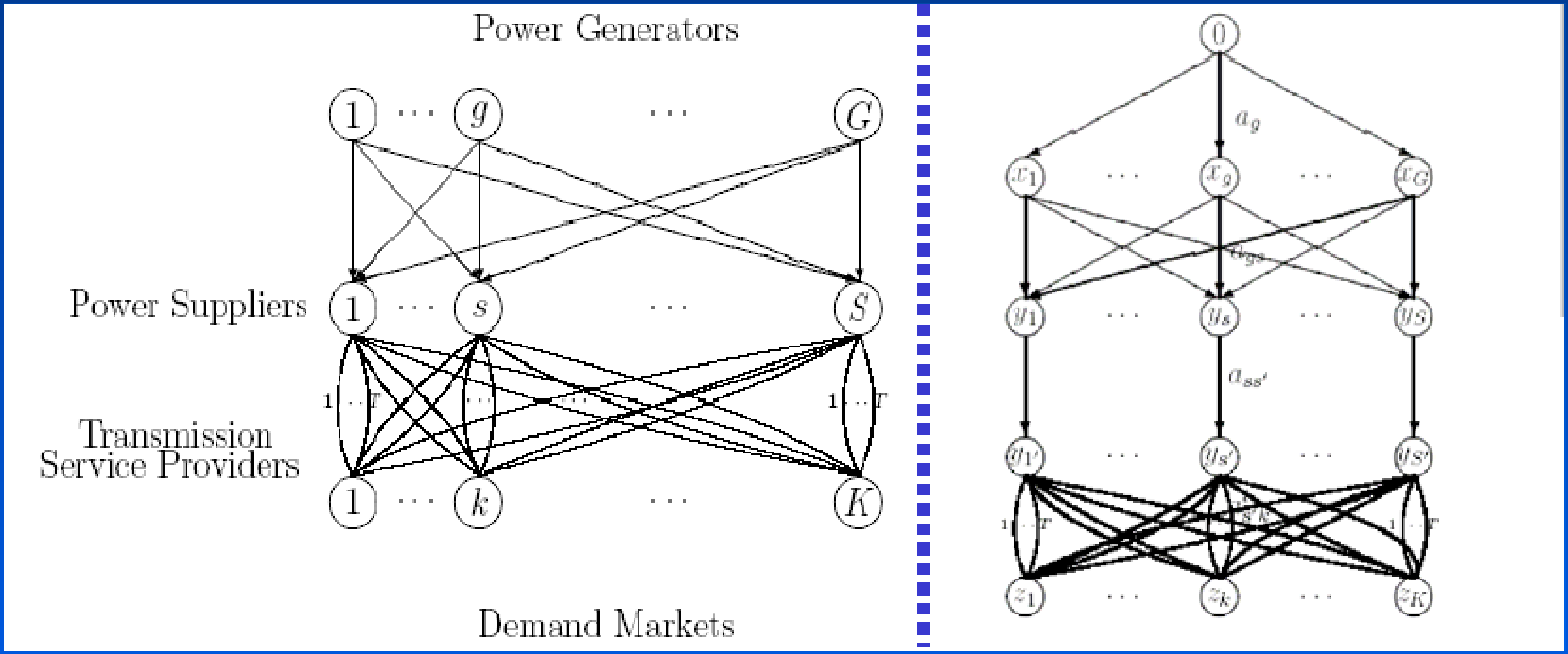
Electric Power Supply Chains

The Electric Power Supply Chain Network



Nagurney and Matsypura, *Proceedings of the CCCT* (2004).

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks



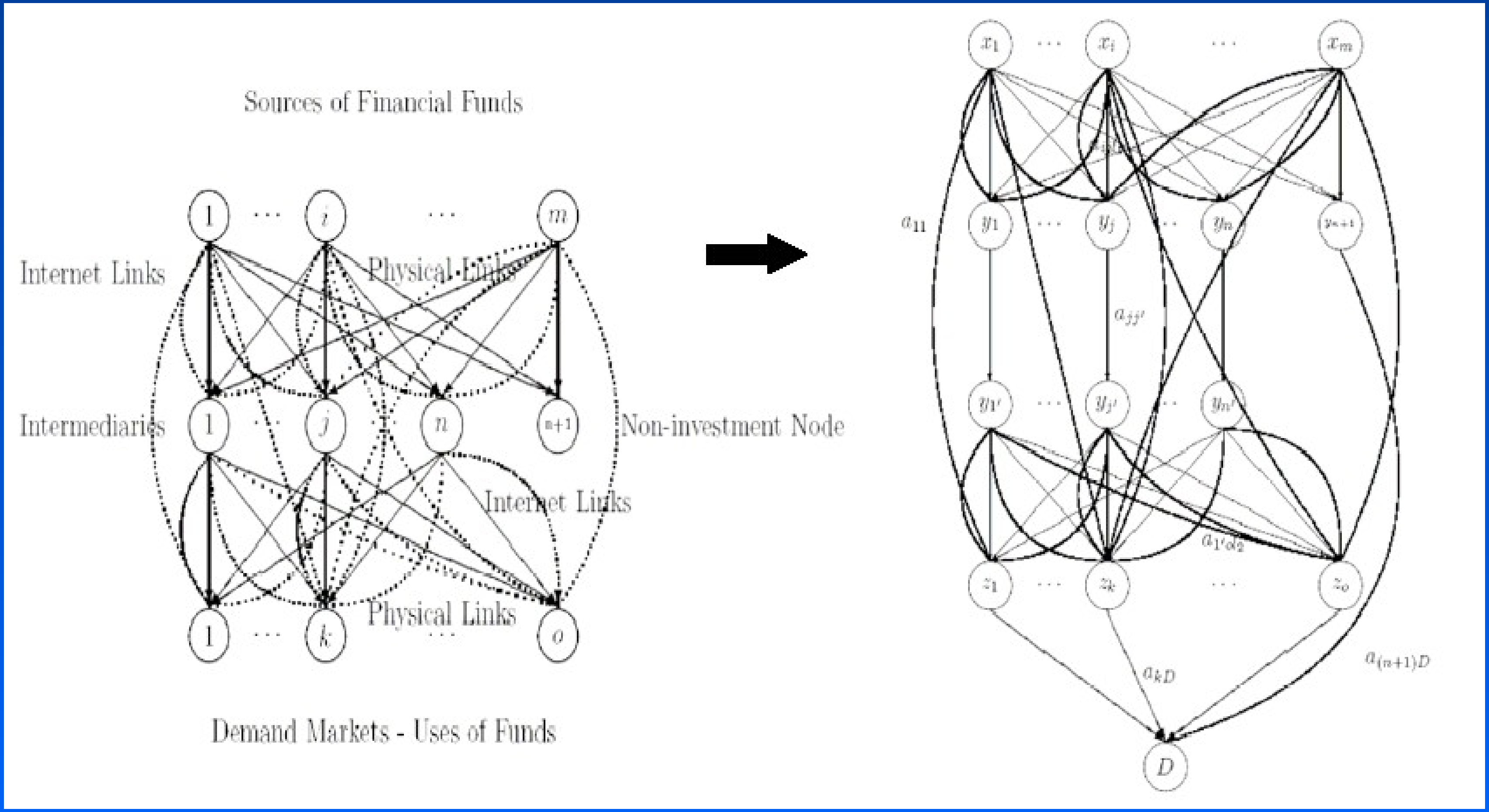
Electric Power Supply Network

Transportation Chain Network

Nagurney, Liu, Cojocaru, and Daniele, *Transportation Research E* 43 (2007).

In 1952, Copeland wondered whether money flows like water or electricity.

The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



Liu and Nagurney, ***Computational Management Science*** **4 (2007), pp 243-281.**

We have shown that *money* as well as *electricity* flow like *transportation* and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!

Recent Literature on Network Vulnerability

- Latora and Marchiori (2001, 2002, 2004)
- Holme, Kim, Yoon and Han (2002)
- Taylor and D’este (2004)
- Murray-Tuite and Mahmassani (2004)
- Chassin and Posse (2005)
- Barrat, Barthélemy and Vespignani (2005)
- Sheffi (2005)
- Dall’Asta, Barrat, Barthélemy and Vespignani (2006)
- Jenelius, Petersen and Mattson (2006)
- Taylor and D’Este (2007)

Network Centrality Measures

- Barrat et al. (2004, pp. 3748), *The identification of the most central nodes in the system is a major issue in network characterization.*

- Centrality Measures for Non-weighted Networks
  - ❖Degree, betweenness (node and edge), closeness (Freeman (1979), Girvan and Newman (2002))
  - ❖Eigenvector centrality (Bonacich (1972))
  - ❖Flow centrality (Freeman, Borgatti and White (1991))
  - ❖Betweenness centrality using flow (Izquierdo and Hanneman (2006))
  - ❖Random-work betweenness, Current-flow betweenness (Newman and Girvan (2004))

- Centrality Measures for Weighted Networks (Very Few)
  - ❖Weighted betweenness centrality (Dall'Asta et al. (2006))
  - ❖Network efficiency measure (Latora-Marchiori (2001))

Some of Our Research on Network Efficiency, Vulnerability, and Robustness

A Network Efficiency Measure for Congested Networks, Nagurney and Qiang, *Europhysics Letters* **79**, August (2007), p1-p5.

A Transportation Network Efficiency Measure that Captures Flows, Behavior, and Costs with Applications to Network Component Importance Identification and Vulnerability, Nagurney and Qiang, *Proceedings of the POMS 18th Annual Conference,* Dallas, Texas (2007).

A Network Efficiency Measure with Application to Critical Infrastructure Networks, Nagurney and Qiang, *Journal of Global Optimization* **40** (2008), pp 261-275.

Robustness of Transportation Networks Subject to Degradable Links, Nagurney and Qiang, *Europhysics Letters* **80**, December (2007).

A Unified Network Performance Measure with Importance Identification and the Ranking of Network Components, Qiang and Nagurney, *Optimization Letters* ***2*** (2008), pp 127-142.

Which Nodes and Links Really Matter?

A New Network Performance/Efficiency Measure with Applications to a Variety of Network Systems

The Nagurney and Qiang (N-Q) Network Efficiency Measure

The network performance/efficiency measure $\mathcal{E}(G,d)$, for a given network topology *G* and demand vector *d*, is defined as

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where n_w is the number of O/D pairs in the network and λ_w is the equilibrium disutility/price for O/D pair *w*.

Nagurney and Qiang, *Europhysics Letters* **79** (2007).

Importance of a Network Component

Definition:

The importance, *I(g),* of a network component *gεG* is measured by the relative network efficiency drop after *g* is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)},$$

where *G-g* is the resulting network after component g is removed.

The Latora and Marchiori (L-M) Network Efficiency Measure

Definition:

The network performance/efficiency measure, *E(G)* for a given network topology, *G*, is defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}$$

where n is the number of nodes in the network and d_{ij} is the shortest path length between node i and node j.

The L-M Measure vs. the N-Q Measure

Theorem: Equivalence in a Special Case

If positive demands exist for all pairs of nodes in the network, *G*, and each of demands is equal to 1, and if d_{ij} is set equal to λ_w, where $w=(i,j)$, for all $w \epsilon W$, then the N-Q and L-M network efficiency measures are one and the same.

The Approach to Identifying the Importance of Network Components

The elimination of a link is treated in the N-Q network efficiency measure by removing that link while the removal of a node is managed by removing the links entering and exiting that node.

In the case that the removal results in no path connecting an O/D pair, we simply assign the demand for that O/D pair to an abstract path with a cost of infinity.

The N-Q measure is well-defined even in the case of disconnected networks.

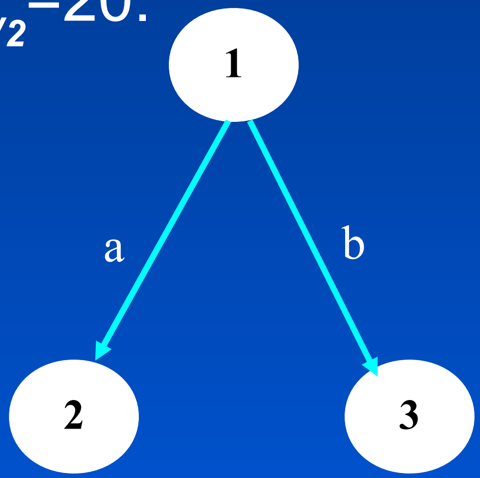
According to the European Environment Agency (2004), since 1990, the *annual number of extreme weather and climate related events has doubled*, in comparison to the previous decade. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied the Nagurney and Qiang (2007) network efficiency measure to a German highway system in order to identify the critical road elements and found that this measure provided more reasonable results than the measure of Taylor and D'Este (2007).

The N-Q measure can also be used to asses which links should be added to improve efficiency. It was used for the evaluation of the proposed North Dublin (Ireland) Metro system (October 2009 Issue of ERCIM News).

Example 1

Assume a network with two O/D pairs: $\boldsymbol{w_1}$=(1,2) and $\boldsymbol{w_2}$=(1,3) with demands: $\boldsymbol{d_{w_1}}$=100 and $\boldsymbol{d_{w_2}}$=20.



The paths are: for $\boldsymbol{w_1}$, $\boldsymbol{p_1=a}$; for $\boldsymbol{w_2}$, $\boldsymbol{p_2=b}$.

The U-O equilibrium path flows are:

$x_{p_1}^* = 100$, $x_{p_2}^* = 20$.

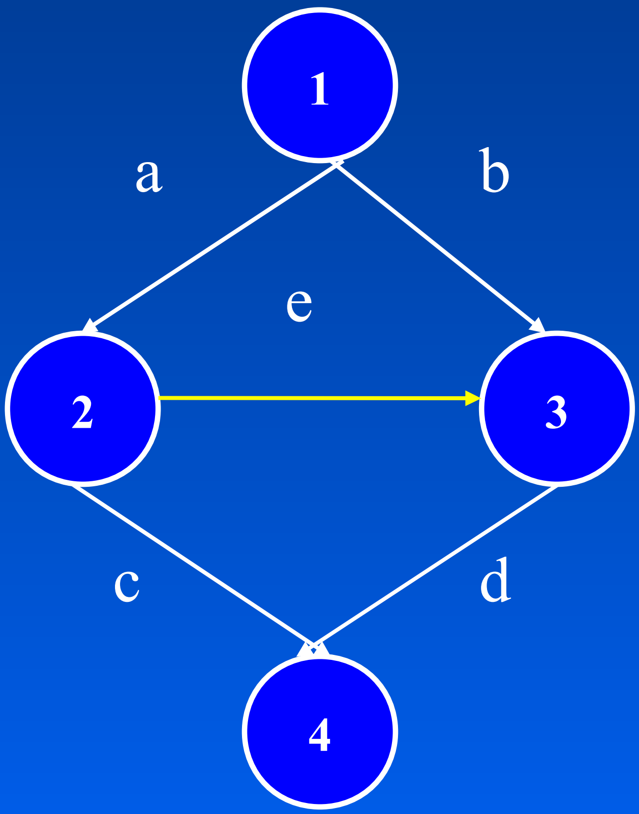
$$c_a(f_a)=0.01f_a+19$$
$$c_b(f_b)=0.05f_b+19$$

The (U-O) equilibrium path costs are: $C_{p_1}=C_{p_2}=20$.

The Importance and Ranking of Links and Nodes for Example 1

Link	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	0.8333	1	0.5000	1
Node	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.0000	1	1.0000	1
2	0.8333	2	0.5000	2
3	0.1667	3	0.5000	2

Example 2 – The Braess Network



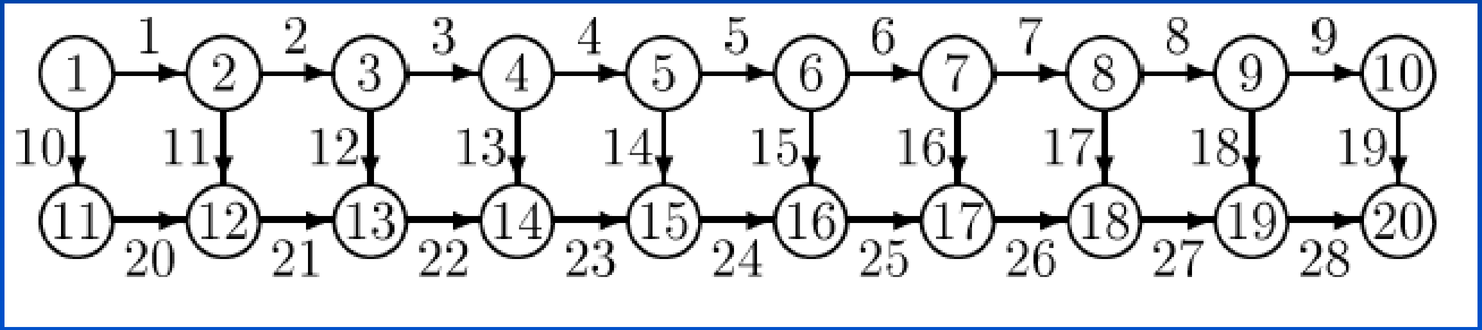
The Importance and Ranking of Links and Nodes for Example 2

Link	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	0.2096	1	0.1056	3
b	0.1794	2	0.2153	2

Node	Importance Value from Our Measure	Importance Ranking from Our Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.0000	1	N/A	N/A
2	0.2096	2	0.7635	1
3	0.2096	2	0.7635	1

Example 3

The network is given by:



w_1=(1,20) w_2=(1,19)

d_{w_1} = 100 d_{w_2} = 100

Nagurney, *Transportation Research B* (1984)

Example 3: Link Cost Functions

Link a	Link Cost Function $c_a(f_a)$
1	$.00005f_1^4 + 5f_1 + 500$
2	$.00003f_2^4 + 4f_2 + 200$
3	$.00005f_3^4 + 3f_3 + 350$
4	$.00003f_4^4 + 6f_4 + 400$
5	$.00006f_5^4 + 6f_5 + 600$
6	$7f_6 + 500$
7	$.00008f_7^4 + 8f_7 + 400$
8	$.00004f_8^4 + 5f_8 + 650$
9	$.00001f_9^4 + 6f_9 + 700$
10	$4f_{10} + 800$
11	$.00007f_{11}^4 + 7f_{11} + 650$
12	$8f_{12} + 700$
13	$.00001f_{13}^4 + 7f_{13} + 600$
14	$8f_{14} + 500$

Link a	Link Cost Function $c_a(f_a)$
15	$.00003f_{15}^4 + 9f_{15} + 200$
16	$8f_{16} + 300$
17	$.00003f_{17}^4 + 7f_{17} + 450$
18	$5f_{18} + 300$
19	$8f_{19} + 600$
20	$.00003f_{20}^4 + 6f_{20} + 300$
21	$.00004f_{21}^4 + 4f_{21} + 400$
22	$.00002f_{22}^4 + 6f_{22} + 500$
23	$.00003f_{23}^4 + 9f_{23} + 350$
24	$.00002f_{24}^4 + 8f_{24} + 400$
25	$.00003f_{25}^4 + 9f_{25} + 450$
26	$.00006f_{26}^4 + 7f_{26} + 300$
27	$.00003f_{27}^4 + 8f_{27} + 500$
28	$.00003f_{28}^4 + 7f_{28} + 650$

Algorithms for Solution

The projection method (cf. Dafermos (1980) and Nagurney (1999)) embedded with the equilibration algorithm of Dafermos and Sparrow (1969) was used for the computations.

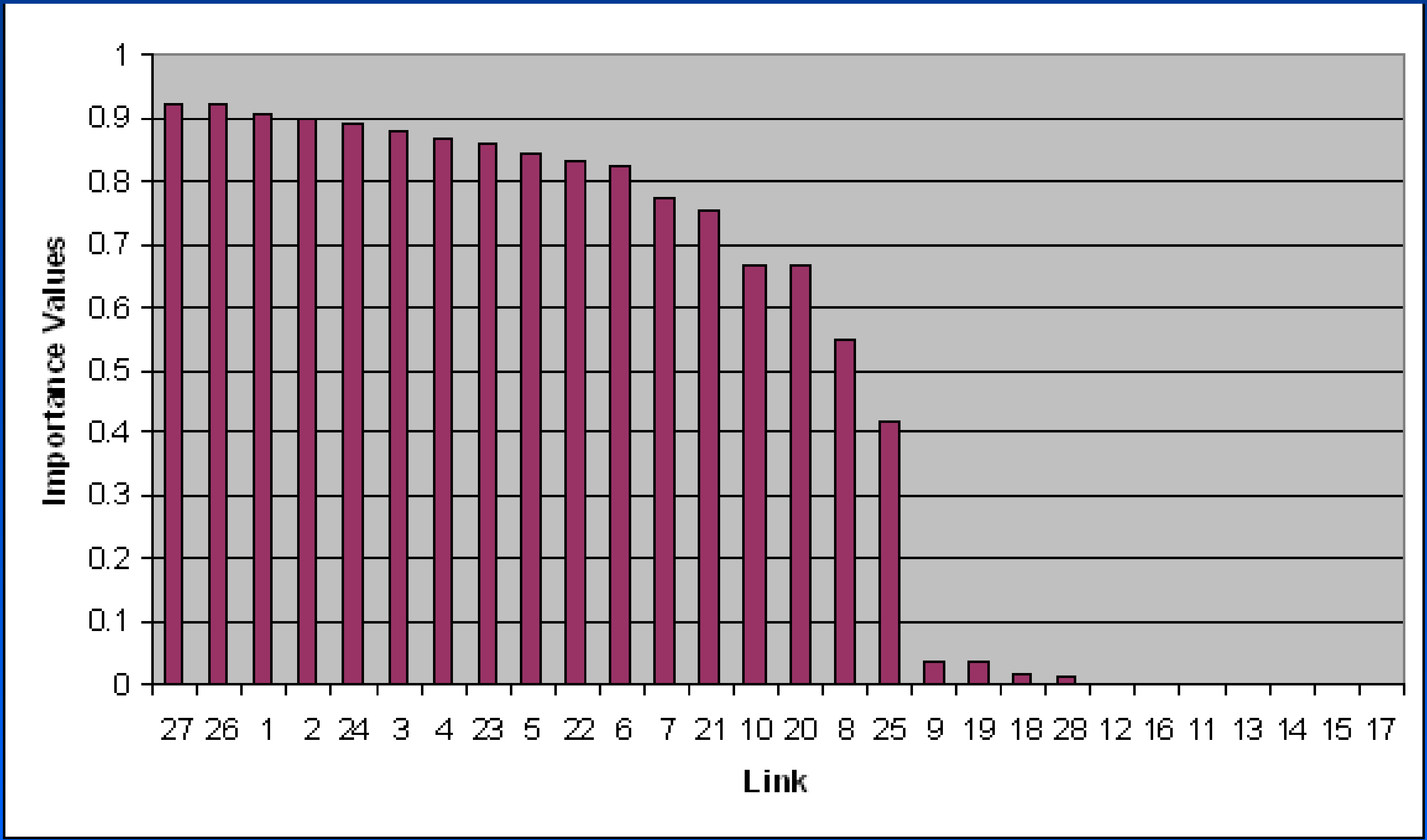
In addition, the column generation method of Leventhal, Nemhauser, and Trotter (1973) was implemented to generate paths, as needed, in the case of the large-scale Sioux Falls network example.

Example 3: The Importance and Ranking of Links

Link a	Importance Value	Importance Ranking
1	0.9086	3
2	0.8984	4
3	0.8791	6
4	0.8672	7
5	0.8430	9
6	0.8226	11
7	0.7750	12
8	0.5483	15
9	0.0362	17
10	0.6641	14
11	0.0000	22
12	0.0006	20
13	0.0000	22
14	0.0000	22

Link a	Importance Value	Importance Ranking
15	0.0000	22
16	0.0001	21
17	0.0000	22
18	0.0175	18
19	0.0362	17
20	0.6641	14
21	0.7537	13
22	0.8333	10
23	0.8598	8
24	0.8939	5
25	0.4162	16
26	0.9203	2
27	0.9213	1
28	0.0155	19

Example 3: Link Importance Rankings

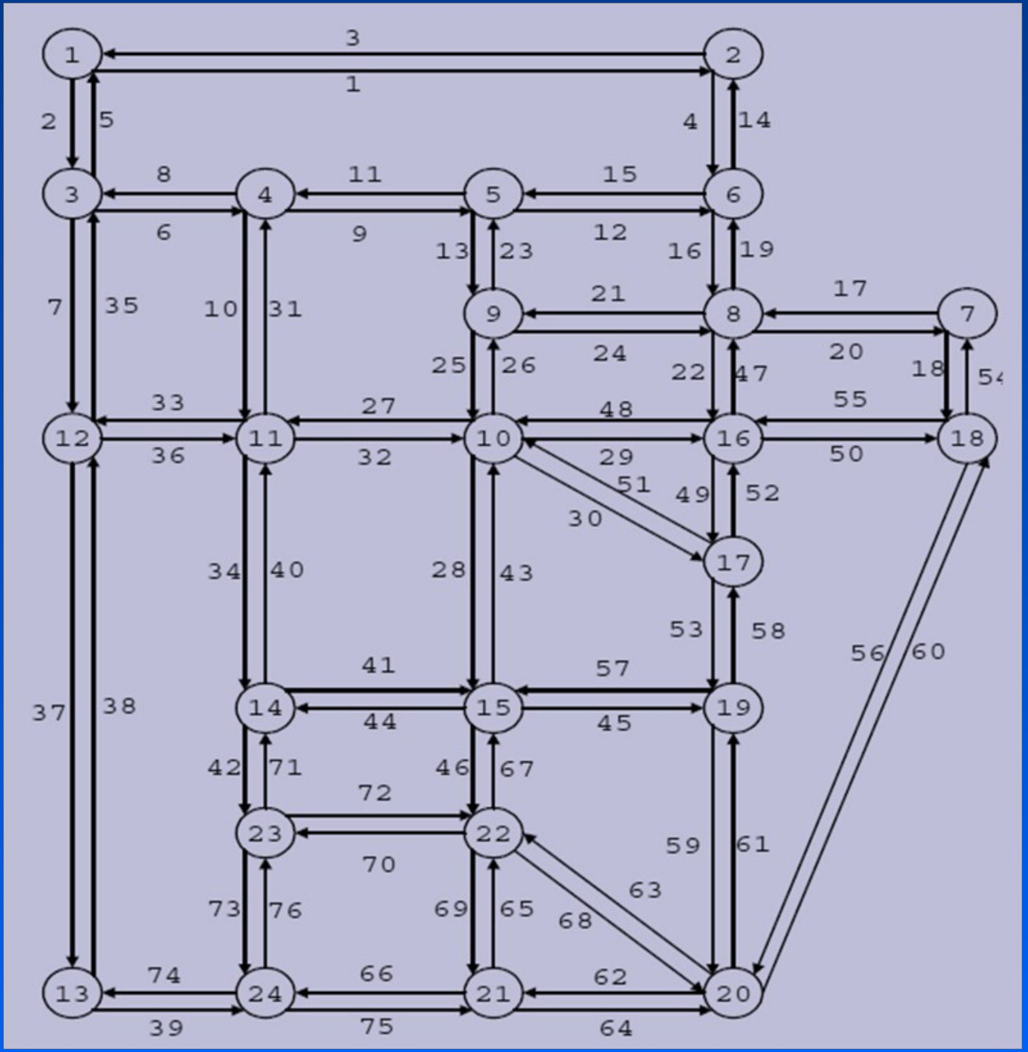


Example 4 - Sioux Falls Network

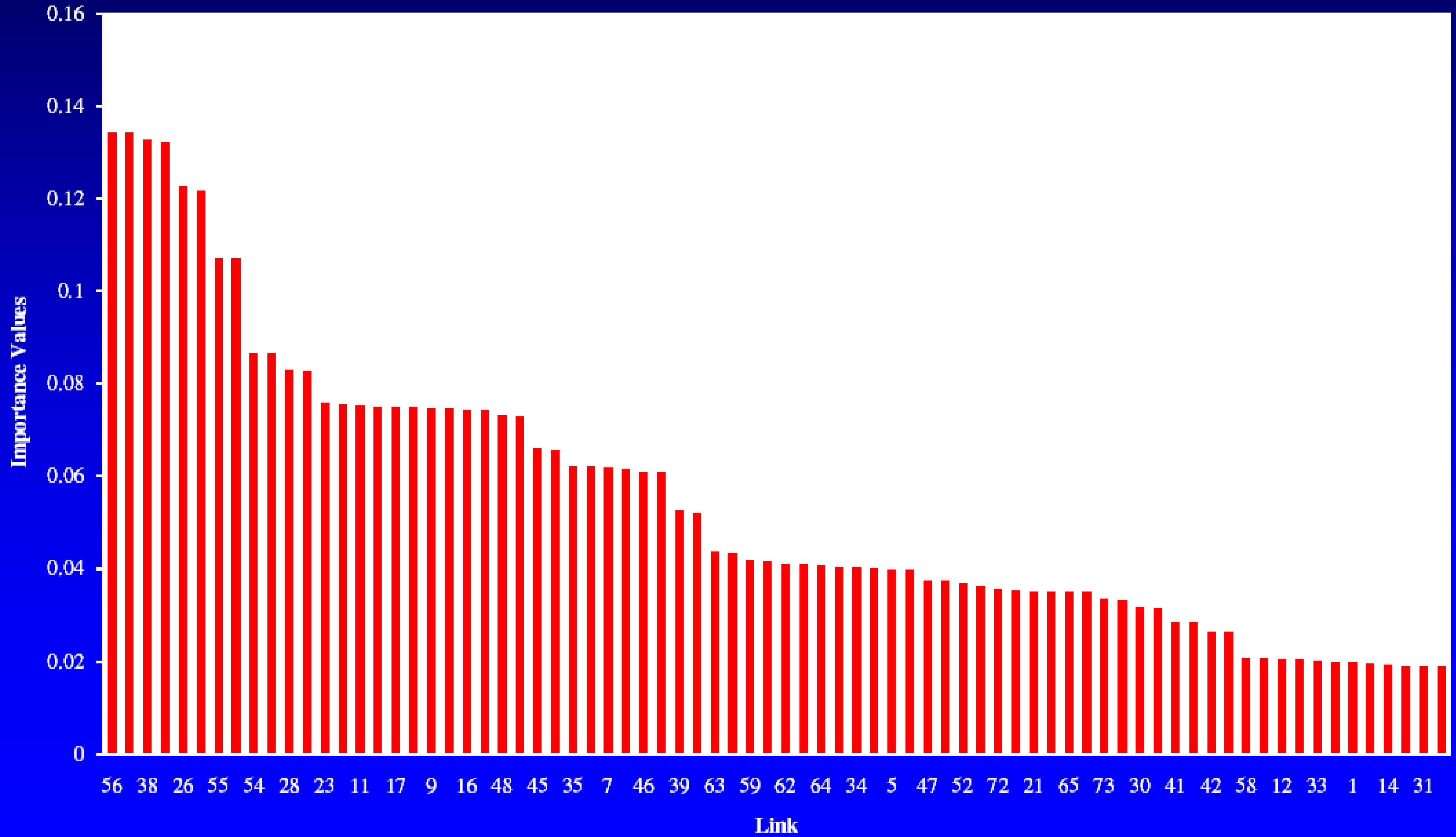
The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of BPR form.



Example 4 - Sioux Falls Network Link Importance Rankings



The Network Efficiency Measure for Dynamic Networks

A network like the Internet is volatile. Its traffic patterns can change quickly and dramatically... The assumption of a static model is therefore particularly suspect in such networks. (Roughgarden (2005)*).*

An Efficiency Measure for Dynamic Networks with Application to the Internet and Vulnerability Analysis (Nagurney and Qiang), *Netnomics* 9 (2008), pp 1-20.

The Network Efficiency Measure for Dynamic Networks – Continuous Time

The network efficiency for the network G with time-varying demand d for $t \in [0, T]$, denoted by $\mathcal{E}(G, d, T)$, is defined as follows:

$$\mathcal{E}(G, d, T) = \frac{\int_0^T [\sum_{w \in W} \frac{d_w(t)}{\lambda_w(t)}]/n_W \, dt}{T}.$$

Note that the above measure is the average network performance over time of the dynamic network.

The Network Efficiency Measure for Dynamic Networks – Discrete Time

Let $d_w^1, d_w^2, \dots, d_w^H$ denote demands for O/D pair w in H discrete time intervals, given, respectively, by: $[t_0, t_1], (t_1, t_2], \dots, (t_{H-1}, t_H]$, where $t_H \equiv T$. Assume that the demand is constant in each such time interval for each O/D pair. Denote the corresponding minimal costs for each O/D pair w at the H different time intervals by: $\lambda_w^1, \lambda_w^2, \dots, \lambda_w^H$. The demand vector d, in this special discrete case, is a vector in $R^{n_W \times H}$.

Dynamic Network Efficiency: Discrete Time Version

The network efficiency for the network (G, d) over H discrete time intervals: $[t_0, t_1], (t_1, t_2], \dots, (t_{H-1}, t_H]$, where $t_H \equiv T$, and with the respective constant demands: $d_w^1, d_w^2, \dots, d_w^H$ for all $w \in W$ is defined as follows:

$$\mathcal{E}(G, d, t_H = T) = \frac{\sum_{i=1}^{H} [(\sum_{w \in W} \frac{d_w^i}{\lambda_w^i})(t_i - t_{i-1})/n_W]}{t_H}.$$

Importance of Nodes and Links in the Dynamic Braess Network Using the N-Q Measure when T=10

Link	Importance Value	Importance Ranking
a	0.2604	1
b	0.1784	2
c	0.1784	2
d	0.2604	1
e	-0.1341	3

Node	Importance Value	Importance Ranking
1	1.0000	1
2	0.2604	2
3	0.2604	2
4	1.0000	1

Link e is never used after $t = 8.89$ and in the range $t \in [2.58, 8.89]$, it increases the cost, so the fact that link e has a negative importance value makes sense; over time, its removal would, on the average, improve the network efficiency!

The Advantages of the N-Q Network Efficiency Measure

- The measure captures demands, flows, costs, and behavior of users, in addition to network topology.
- The resulting importance definition of network components is applicable and well-defined even in the case of disconnected networks.
- It can be used to identify the importance (and ranking) of either nodes, or links, or both.
- It can be applied to assess the efficiency/performance of a wide range of network systems.
- It is applicable also to elastic demand networks (Qiang and Nagurney, *Optimization Letters* (2008)).
- It is applicab to dynamic networks (Nagurney and Qiang, *Netnomics* **(2008)**).