An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England

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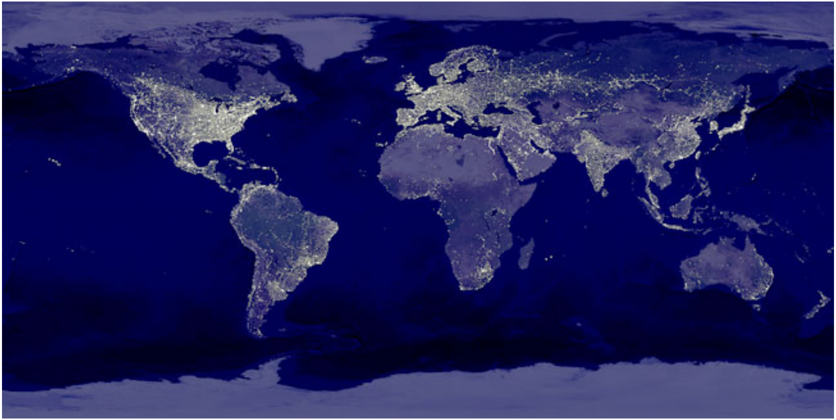

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Outline

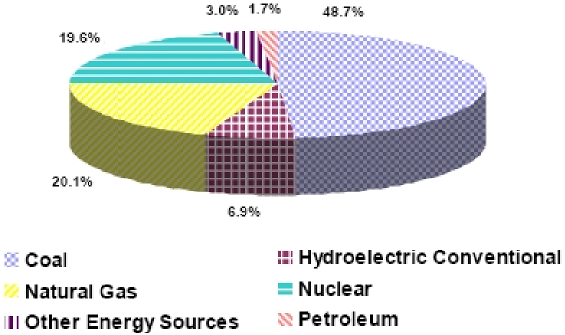
- Introduction
- Literature review
- An integrated electric power supply chain and fuel market network framework
- Empirical case study and examples
- Conclusions

Electric Power Supply Chains

Electric Power Supply Chains

- The US electric power industry: Half a trillion dollars of net assets, $220 billion annual sales, 40% of domestic primary energy (Energy Information Administration (2000, 2005))
- Deregulation
- Electric power supply chain networks
  - Various generation technologies
  - Insensitive demands
  - Transmission congestion
  - High storage costs

Sources of Electricity in the U.S. in 2007



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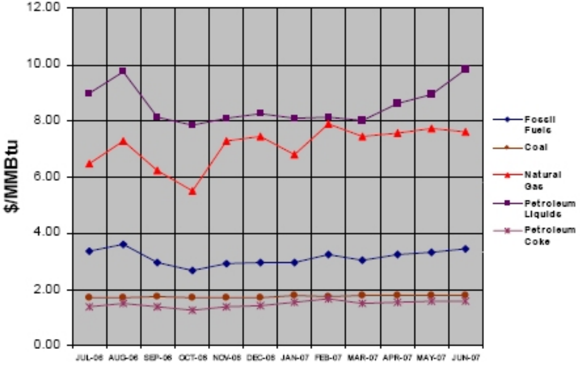
Electric Power Industry Fuel Costs, July 2006 through June 2007



Source: http://www.eia.doe.gov

Electric Power Supply Chains and Fuel Markets

- In the U.S., electric power generation accounts for 30% of the natural gas demand (over 50% in the summer), 90% of the coal demand, and over 45% of the residual fuel oil demand.
- The demand from electric power generation has been significantly and steadily increasing while demands from all other sectors (industrial, commercial, and residential) have been slightly decreasing.

Electric Power Supply Chains and Fuel Markets (Cont'd)

The interactions between electric power supply chains and fuel markets affect demands and prices of electric power and fuels.

- From December 1, 2005 to April 1, 2006, the wholesale electricity price in New England decreased by 38% mainly because the delivered natural gas price declined by 45%.
- In August, 2006, the natural gas price jumped 14% because hot weather across the US led to elevated demand for electricity. This high electricity demand also caused the crude oil price to rise by 1.6%.

Electric Power Supply Chains and Fuel Markets (Cont'd)

The availability and the reliability of diversified fuel supplies also affect national security.

- In January 2004, over 7000MW of electric power generation, which accounts for almost one fourth of the total capacity of New England, was unavailable during the electric system peak due to the limited natural gas supply.
- The American Association of Railroads has requested that the Federal Energy Regulatory Commission (FERC) investigate the reliability of the energy supply chain with a focus on electric power and coal transportation.

Literature Review

- Beckmann, McGuire, and Winsten (1956): How are electric power flows related to transportation flows?
- Deregulation
  - Smeers (1997), Hogan (1992), Chao and Peck (1996), Wu et al. (1996), Casazza and Delea (2003), Hobbs and Pang (2003), Chen et al. (2004), etc.
- Electric power wholesale and retail markets
  - Visudhiphan and Ilic (1999), Raineri, Rios, and Vasquez (2005), Ruff (2002), Borenstein and Holland (2003), Borenstein (2005a), Joskow and Tirole (2004), and Garcia, Campos, and Reitzes (2005), etc.

Literature Review (Cont'd)

- Emery and Liu (2001)
- Routledge, Seppi, and Spatt (2001), Bessembinder and Lemmon (2002)
- Deng, Johnson, and Sogomonian (2001)
- Barron and Brown (1986), Huntington and Schuler (1997), Brown and Yucel (2007)

Literature Review (Cont'd)

- A. Nagurney and D. Matsypura, "A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption," in **Optimisation, Econometric and Financial Analysis**, E. J. Kontoghiorghes and C. Gatu, Editors (2006) Springer, Berlin, Germany, pp 3-27

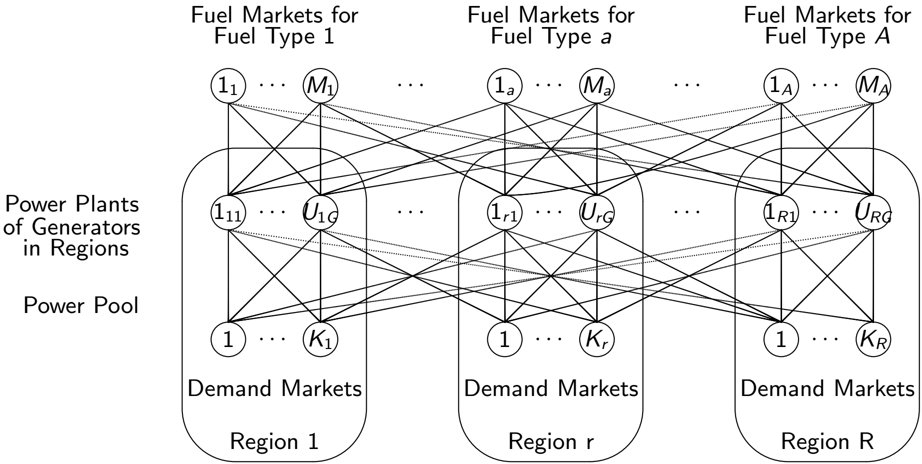
- A. Nagurney, Z. Liu, M. G. Cojocaru, and P. Daniele, "Dynamic electric power supply chains and transportation networks: An evolutionary variational inequality formulation," *Transportation Research E* 43 (2007), 624-646

- D. Matsypura, A. Nagurney, and Z. Liu, "Modeling of electric power supply chain networks with fuel suppliers via variational inequalities," *International Journal of Emerging Electric Power Systems* 8 (2007), 1, Article 5

An Integrated Electric Power Supply Chain and Fuel Market Network Framework

- Z. Liu and A. Nagurney, “An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England,” 2007

The Electric Power Supply Chain Network with Fuel Supply Markets



The Equilibrium Conditions for the Fuel Supply Markets

We assume that the following conservation of flow equations must hold for all fuel supply markets $m_a = 1, \ldots, M_A$; $a = 1, \ldots, A$:

$$\sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} q_{m_a r_1 g u} + \bar{q}_{m_a} = h_{m_a}.$$

The equilibrium conditions for suppliers at fuel supply market m_a; $m_a = 1, ..., M_a$; $a = 1, ..., A$, take the form: for each power plant u; $u = 1, ..., U_{r_1 g}$; $g = 1, ..., G$; $r_1 = 1, ..., R$:

$$\pi_{m_a}(h^*) + c_{m_a r_1 g u} \begin{cases} = \rho^*_{m_a r_1 g u}, & \text{if} \quad q^*_{m_a r_1 g u} > 0, \\ \geq \rho^*_{m_a r_1 g u}, & \text{if} \quad q^*_{m_a r_1 g u} = 0. \end{cases}$$

The Equilibrium Conditions for the Fuel Supply Markets

We can express these equilibrium conditions as the following variational inequality: determine $Q^{1*} \in \mathcal{K}^1$, such that

$$\sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left[\pi_{m_a}(Q^{1*}) + c_{m_a r_1 g u} - \rho^*_{m_a r_1 g u} \right] \times [q_{m_a r_1 g u} - q^*_{m_a r_1 g u}] \geq 0,$$

$$\forall Q^1 \in \mathcal{K}^1, \quad (4)$$

where $\mathcal{K}^1 \equiv \{Q^1 | Q^1 \in R_+^{MU}\}$.

Power Generator's Maximization Problem

$$\text{Maximize} \quad \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \rho_{wr_2 k}^{r_1 gu*} q_{wr_2 k}^{r_1 gu}$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \rho_{wr_2}^{*} y_{wr_2}^{r_1 gu} + \sum_{w=1}^{W} \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} L_w \varphi_{wr_1}^{*} z_w^{r_1 gu}$$

$$- \sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} \rho_{m_a r_1 gu}^{*} q_{m_a r_1 gu}$$

$$- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} f_{wr_1 gu}(q_w^{r_1 gu}) - \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} c_{wr_1 gur_2 k}(q_{wr_2 k}^{r_1 gu})$$

$$- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} c_{wr_1 gur_2}(y_{wr_2}^{r_1 gu}) - \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} c_{wr_1 gu}(z_w^{r_1 gu})$$

$$- \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{U_{r_1 g}} \sum_{b=1}^{B} \sum_{r_2=1}^{R} \mu_{wb}^{*} \alpha_{r_1 r_2 b} [\sum_{k=1}^{K_{r_2}} q_{wr_2 k}^{r_1 gu} + y_{wr_2}^{r_1 gu}]$$

Power Generator's Maximization Problem (Cont'd)

subject to:

$$\sum_{r_2=1}^{R}\sum_{k=1}^{K_{r_2}} q^{r_1gu}_{wr_2k} + \sum_{r_2=1}^{R} y^{r_1gu}_{wr_2} = q^{r_1gu}_{w}, \quad \forall r_1, u, w, \qquad (5)$$

$$\sum_{a=1}^{A} \beta_{r_1gua} \sum_{m_a=1}^{M_a} q_{m_a r_1 gu} + \sum_{w=1}^{W} L_w \beta_{r_1gu0} q^{r_1gu}_{w} = \sum_{w=1}^{W} L_w q^{r_1gu}_{w}, \quad \forall r_1, u, \qquad (6)$$

$$q^{r_1gu}_{w} + z^{r_1gu}_{w} \le Cap_{r_1gu}, \quad \forall r_1, u, w,$$

$$z^{r_1gu}_{w} \le OP_{r_1gu}, \quad \forall r_1, u, w,$$

$$q^{r_1gu}_{wr_2k} \ge 0, \quad \forall r_1, u, w, r_2, k,$$

$$q_{m_a r_1 gu} \ge 0, \quad \forall m_a, r_1, u,$$

$$y^{r_1gu}_{wr_2} \ge 0, \quad \forall r_1, u, w, r_2,$$

$$z^{r_1gu}_{w} \ge 0, \quad \forall r_1, u, w.$$

Power Generators' Optimization Conditions

Determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Z^*, \eta^*, \lambda^*) \in \mathcal{K}^2$ satisfying

$$\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left[\frac{\partial f_{wr_1 gu}(q_w^{r_1 gu*})}{\partial q_w^{r_1 gu}} + \eta_w^{r_1 gu*} \right] \times [q_w^{r_1 gu} - q_w^{r_1 gu*}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \left[\frac{\partial c_{wr_1 gur_2 k}(q_{wr_2 k}^{r_1 gu*})}{\partial q_{wr_2 k}^{r_1 gu}} + \sum_{b=1}^{B} \mu_{wb}^* \alpha_{r_1 r_2 b} - \rho_{wr_2 k}^{r_1 gu*} \right]$$
$$\times [q_{wr_2 k}^{r_1 gu} - q_{wr_2 k}^{r_1 gu*}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \left[\frac{\partial c_{wr_1 gur_2}(y_{wr_2}^{r_1 gu*})}{\partial y_{wr_2}^{r_1 gu}} + \sum_{b=1}^{B} \mu_{wb}^* \alpha_{r_1 r_2 b} - \rho_{wr_2}^* \right] \times [y_{wr_2}^{r_1 gu} - y_{wr_2}^{r_1 gu*}]$$

$$+ \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \rho_{m_a r_1 gu}^* \times [q_{m_a r_1 gu} - q_{m_a r_1 gu}^*]$$

Power Generators' Optimization Conditions (Cont'd)

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left[\frac{\partial c_{wr_1gu}(z_w^{r_1gu*})}{\partial z_w^{r_1gu}} + \lambda_w^{r_1gu*} + \eta_w^{r_1gu*} - \varphi_w^{r_1gu*} \right] \times [z_w^{r_1gu} - z_w^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} [Cap_{r_1gu} - q_w^{r_1gu*} - z_w^{r_1gu*}] \times [\eta_w^{r_1gu} - \eta_w^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} [OP_{r_1gu} - z_w^{r_1gu*}] \times [\lambda_w^{r_1gu} - \lambda_w^{r_1gu*}] \geq 0,$$

$$\forall (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in \mathcal{K}^2, \tag{14}$$

where $\mathcal{K}^2 \equiv \{(Q^1, q, Q^2, Y^1, Z, \eta, \lambda) |$
$(Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in R_+^{WMU+UKW+WUR+4WU}$, and (5) and (6) hold$\}$.

The ISO's Role

The following conditions must hold for each interface b and at each demand level w, where $b = 1, \ldots, B$; $w = 1, \ldots, W$:

$$\sum_{r_1=1}^{R}\sum_{r_2=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\sum_{k=1}^{K_{r_2}} q_{wr_2k}^{r_1gu*} + \sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} y_{wr_2}^{r_1gu*} + \sum_{k=1}^{K_{r_2}} y_{wr_1r_2k}^{*}]\alpha_{r_1r_2b} \begin{cases} = Cap_b, & \text{if} \quad \mu_{wb}^* > 0, \\ \leq Cap_b, & \text{if} \quad \mu_{wb}^* = 0. \end{cases}$$

The ISO ensures that the regional electricity markets $r = 1, \ldots, R$ clear at each demand level $w = 1, \ldots, W$, that is,

$$\sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} y_{wr}^{r_1gu*} \begin{cases} = \sum_{r_2=1}^{R}\sum_{k=1}^{K_r} y_{wrr_2k}^*, & \text{if} \quad \rho_{wr}^* > 0, \\ \geq \sum_{r_2=1}^{R}\sum_{k=1}^{K_r} y_{wrr_2k}^*, & \text{if} \quad \rho_{wr}^* = 0. \end{cases}$$

The ISO also ensures that the regional operating reserve markets; hence, $r_1 = 1, \ldots, R$ clear at each demand level $w = 1, \ldots, W$, that is,

$$\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} z_w^{r_1gu*} \begin{cases} = OPR_{wr_1}, & \text{if} \quad \varphi_{wr_1}^* > 0, \\ \geq OPR_{wr_1}, & \text{if} \quad \varphi_{wr_1}^* = 0. \end{cases}$$

The ISO's Role (Cont'd)

We can express these equilibrium conditions using the following variational inequality: determine $(\mu^*, \rho_3^*, \varphi^*) \in R_+^{WB+2WR}$, such that

$$\sum_{w=1}^{W} L_w \sum_{b=1}^{B} [Cap_b - \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{k=1}^{K_{r_2}} q_{wr_2 k}^{r_1 gu*} + \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} y_{wr_2}^{r_1 gu*} + \sum_{k=1}^{K_{r_2}} y^*_{wr_1 r_2 k}] \alpha_{r_1 r_2 b}]$$

$$\times [\mu_{wb} - \mu^*_{wb}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r=1}^{R} [\sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} y_{wr}^{r_1 gu*} - \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} y^*_{wr r_2 k}] \times [\rho_{wr} - \rho^*_{wr}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} z_w^{r_1 gu*} - OPR_{wr_1}] \times [\varphi_{wr_1} - \varphi^*_{wr_1}] \geq 0, \quad \forall (\mu, \rho_3, \varphi) \in R_+^{WB+2WR}. \tag{18}$$

The Equilibrium Conditions for the Demand Markets

We assume that all demand markets have fixed and known demands. and the following conservation of flow equations, hence, must hold for all demand markets $k = 1, \ldots, K_{r_2}$, all regions $r_2 = 1, \ldots, R$, and at all demand levels $w = 1, \ldots, W$:

$$\sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} q_{wr_2 k}^{r_1 gu*} + \sum_{r_1=1}^{R} y_{wr_1 r_2 k}^{*} = (1 + \kappa_{wr_2}) \bar{d}_{wr_2 k}. \tag{19}$$

The equilibrium conditions for consumers at demand market k in region r_2 take the form: for each power plant u; $u = 1, ..., U_{r_1 g}$; each generator $g = 1, ..., G$; each region $r_1 = 1, ..., R$, and each demand level w; $w = 1, \ldots, W$:

$$\rho_{wr_2 k}^{r_1 gu*} + \hat{c}_{r_2 k}^{wr_1 gu}(Q_w^{2*}) \begin{cases} = \rho_{wr_2 k}^{*}, & \text{if} \quad q_{wr_2 k}^{r_1 gu*} > 0, \\ \geq \rho_{wr_2 k}^{*}, & \text{if} \quad q_{wr_2 k}^{r_1 gu*} = 0; \end{cases}$$

and

$$\rho_{wr_1}^{*} + \sum_{b=1}^{B} \mu_{wb}^{*} \alpha_{r_1 r_2 b} + \hat{c}_{wr_1 r_2 k}(Y_w^{2*}) \begin{cases} = \rho_{wr_2 k}^{*}, & \text{if} \quad y_{wr_1 r_2 k}^{*} > 0, \\ \geq \rho_{wr_2 k}^{*}, & \text{if} \quad y_{wr_1 r_2 k}^{*} = 0. \end{cases}$$

The Equilibrium Conditions for the Demand Markets (Cont'd)

We can express these equilibrium conditions using the following variational inequality: determine $(Q^{2*}, Y^{2*}) \in \mathcal{K}^3$, such that

$$\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \left[\rho_{wr_2 k}^{r_1 gu*} + \hat{c}_{r_2 k}^{wr_1 gu}(Q_w^{2*}) \right] \times [q_{wr_2 k}^{r_1 gu} - q_{wr_2 k}^{r_1 gu*}]$$

$$+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \left[\rho_{wr_2}^{*} + \sum_{b=1}^{B} \mu_{wb}^{*} \alpha_{r_1 r_2 b} + \hat{c}_{wr_1 r_2 k}(Y_w^{2*}) \right] \times [y_{wr_1 r_2 k} - y_{wr_1 r_2 k}^{*}] \geq 0,$$

$$\forall (Q^2, Y^2) \in \mathcal{K}^3, \tag{22}$$

where $\mathcal{K}^3 \equiv \{(Q^2, Y^2) | (Q^2, Y^2) \in R_+^{WUK+WRK}$ and (19) holds$\}$.

The Equilibrium Conditions for the Electric Power Supply Chain Network

Definition 1: Electric Power Supply Chain Network Equilibrium
The equilibrium state of the electric power supply chain network with fuel supply market is one where the fuel flows and electric power flows satisfy the sum of conditions (4), (14), (18), and (22).

Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*) \in \mathcal{K}^4$ satisfying

$$\sum_{a=1}^{A}\sum_{m_a=1}^{M_a}\sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\left[\pi_{m_a}(Q^{1*}) + c_{m_ar_1gu}\right] \times [q_{m_ar_1gu} - q^*_{m_ar_1gu}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\left[\frac{\partial f_{wr_1gu}(q_w^{r_1gu*})}{\partial q_w^{r_1gu}} + \eta_w^{r_1gu*}\right] \times [q_w^{r_1gu} - q_w^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\sum_{r_2=1}^{R}\sum_{k=1}^{K_{r_2}}\left[\frac{\partial c_{wr_1gur_2k}(q_{wr_2k}^{r_1gu*})}{\partial q_{wr_2k}^{r_1gu}} + \sum_{b=1}^{B}\mu^*_{wb}\alpha_{r_1r_2b} + \hat{c}_{r_2k}^{wr_1gu}(Q_w^{2*})\right] \times [q_{wr_2k}^{r_1gu} - q_{wr_2k}^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\sum_{r_2=1}^{R}\left[\frac{\partial c_{wr_1gur_2}(y_{wr_2}^{r_1gu*})}{\partial y_{wr_2}^{r_1gu}} + \sum_{b=1}^{B}\mu^*_{wb}\alpha_{r_1r_2b} - \rho^*_{wr_2}\right] \times [y_{wr_2}^{r_1gu} - y_{wr_2}^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\left[\frac{\partial c_{wr_1gu}(z_w^{r_1gu*})}{\partial z_w^{r_1gu}} + \lambda_w^{r_1gu*} + \eta_w^{r_1gu*} - \varphi_w^{r_1gu*}\right] \times [z_w^{r_1gu} - z_w^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K_{r_2}}\left[\rho^*_{wr_1} + \hat{c}_{wr_1r_2k}(Y_w^{2*}) + \sum_{b=1}^{B}\mu^*_{wb}\alpha_{r_1r_2b}\right] \times [y_{wr_1r_2k} - y^*_{wr_1r_2k}]$$

Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium (Cont'd)

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} \left[Cap_{r_1gu} - q_w^{r_1gu*} - z_w^{r_1gu*}\right] \times [\eta_w^{r_1gu} - \eta_w^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} \left[OP_{r_1gu} - z_w^{r_1gu*}\right] \times [\lambda_w^{r_1gu} - \lambda_w^{r_1gu*}]$$

$$+\sum_{w=1}^{W} L_w \sum_{b=1}^{B}[Cap_b - \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}}\sum_{k=1}^{K_{r_2}} q_{wr_2k}^{r_1gu*} + \sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} y_{wr_2}^{r_1gu*} + \sum_{k=1}^{K_{r_2}} y^*_{wr_1r_2k}]\alpha_{r_1r_2b}] \times [\mu_{wb} - \mu^*_{wb}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}[\sum_{r_1=1}^{R}\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} y_{wr}^{r_1gu*} - \sum_{r_2=1}^{R}\sum_{k=1}^{K_{r_2}} y^*_{wrr_2k}] \times [\rho_{wr} - \rho^*_{wr}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{U_{r_1g}} z_w^{r_1gu*} - OPR_{r_1}]\times[\varphi_{wr_1} - \varphi^*_{wr_1}] \geq 0, \quad \forall (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in \mathcal{K}^4, \tag{23}$$

where $\mathcal{K}^4 \equiv \{(Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) | (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in R_+^{MU+WUK+WUR+4WU+WRK+WB+2WR}$ and (5), (6), and (19) hold$\}$.

Some Qualitative Properties

Theorem 2: Existence
Assume that the feasible set $\mathcal{K}$ is nonempty. Then variational inequality (23) admits a solution.

Theorem 3: Uniqueness
Assume the conditions in Theorem 2 and that the function $F(X)$ that enters variational inequality (23) is strictly monotone on $\mathcal{K}$, that is,

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle > 0, \quad \forall X', X'' \in \mathcal{K}, X' \neq X''$$

Then the solution X^* to variational inequality (23) is unique.

Computational Method

- We let VIP denote the original variational inequality problem, and we decompose VIP into W subproblems based on W demand levels.
- We let $\bar{\pi}_{m_a}$ denote the price of fuel a at market m_a, and group all the $\bar{\pi}_{m_a}$s into vector $\bar{\pi}$.
- Each subproblem is denoted by $SVIP_w(\bar{\pi})$.
- Note that in each $SVIP_w(\bar{\pi})$ the fuel prices $\bar{\pi}$ are fixed and given.

Computational Method (Cont'd)

We first define $X^{\mathcal{T}} \equiv (Q^{3\mathcal{T}}, q^{\mathcal{T}}, Q^{2\mathcal{T}}, Y^{1\mathcal{T}}, Y^{2\mathcal{T}}, Z^{\mathcal{T}}, \eta^{\mathcal{T}}, \lambda^{\mathcal{T}}, \mu^{\mathcal{T}}, \rho_3^{\mathcal{T}}, \varphi^{\mathcal{T}})^T$.

Step 0: Initialization
Initialize X^0. Let $\mathcal{T} = 1$.

Step 1: Compute Fuel Prices
Use $Q^{3(\mathcal{T}-1)}$ to compute $\bar{\pi}^{\mathcal{T}}_{m_a} = \pi_{m_a}(Q^{3(\mathcal{T}-1)})$, $\forall a, m_a$.

Step 2: Solve Subproblems
For each w, compute $X^{\mathcal{T}}$ by solving each $SVIP_w(\bar{\pi}^{\mathcal{T}})$ individually.

Step 3: Convergence Verification
If $||X^{\mathcal{T}} - X^{\mathcal{T}-1}||_\infty \leq \epsilon$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$ and go to Step 1. We set the tolerance $\epsilon = 10^{-3}$ for all computations in this paper.

Each subproblem $SVIP(\bar{\pi})$ can be reformulated and solved as the user-optimal transportation network model with Lagrangian multipliers.

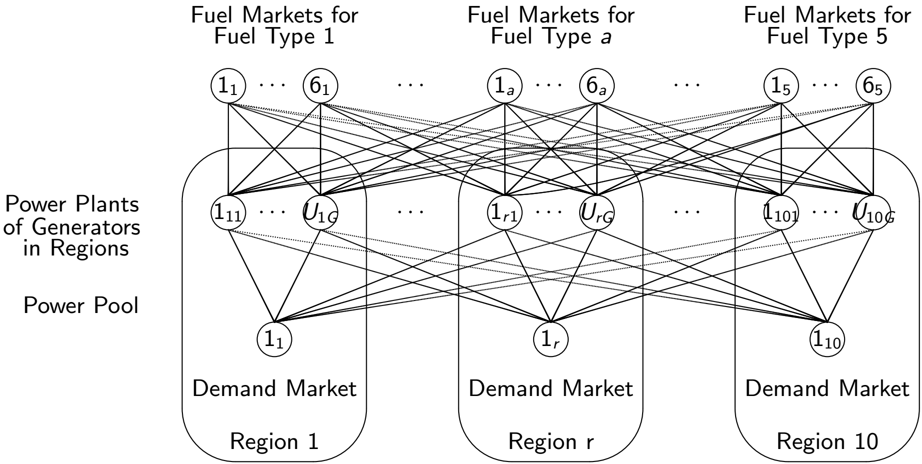
Empirical Case Study and Examples

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Ten regions (R=10): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut(excluding Southwest Connecticut), 5. Southwest Connecticut(excluding Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeast Massachusetts, 9. West and Central Massachusetts, 10. Boston/Northeast Massachusetts
- Hourly demand/price data of July 2006 ($24 \times 31 = 744$ scenarios)
- 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$)

Empirical Case Study and Examples

- Example 1: Predictions to the actual regional electric power prices
- Example 2: Sensitivity analysis for peak-hour electricity prices under natural gas and oil price variations
- Example 3: The impact of the oil price on the natural gas price through electric power markets
- Example 4: The impact of changes in the electricity demands for electricity on the electric power and fuel supply markets

The New England Electric Power Supply Chain Network with Fuel Supply Markets



Example 1: Predictions to the actual regional electric power prices

Average Regional Demands for Each Demand Level (Mwh)

Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
1	1512	1425	1384	1292	1051	889
2	1981	1868	1678	1481	1193	1005
3	774	760	717	654	560	500
4	2524	2199	2125	1976	1706	1432
5	2029	1798	1636	1485	1257	1065
6	1067	931	838	740	605	509
7	1473	1305	1223	1112	952	801
8	2787	2478	2315	2090	1736	1397
9	2672	2457	2364	2262	2448	2186
10	4383	4020	3684	3260	2744	2384
Total	21201	19241	17963	16350	14252	12168

Example 1: Predictions to the actual regional electric power prices

Actual Regional Prices ($/Mwh)

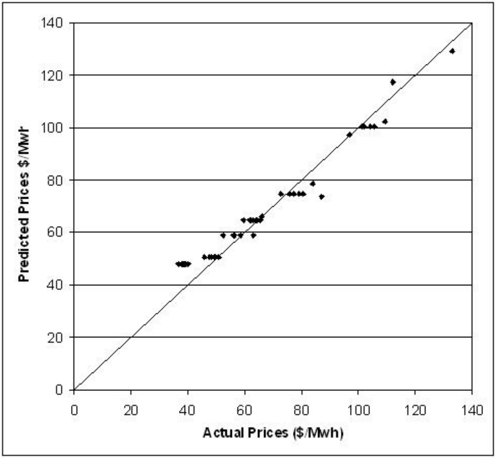
Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
ME	96.83	72.81	59.78	52.54	45.79	36.70
NH	102.16	77.17	63.07	56.31	48.20	38.35
VT	105.84	80.69	65.32	58.39	49.71	39.24
CT	133.17	112.25	86.85	65.97	50.92	39.97
RI	101.32	75.66	61.84	56.06	47.55	37.94
SE MA	101.07	75.78	62.09	56.27	47.54	38.05
WC MA	104.15	79.19	64.49	58.41	49.25	39.53
NE MA	109.29	83.96	63.93	63.02	48.11	38.22
Average	111.66	87.36	69.15	60.18	48.80	38.79

Example 1: Predictions to the actual regional electric power prices

Predicted Regional Prices ($/Mwh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
ME	92.10	74.62	64.77	58.71	50.31	48.00
NH	100.28	74.62	64.77	58.71	50.31	48.00
VT	100.28	74.62	64.77	58.71	50.31	48.00
CT	129.8	117.21	73.50	65.91	50.31	48.00
RI	100.28	74.62	64.77	58.71	50.31	48.00
SE MA	100.28	74.62	64.77	58.71	50.31	48.00
WC MA	100.28	74.62	64.77	58.71	50.31	48.00
NE MA	102.21	78.43	64.82	58.71	50.31	48.00
Average	108.28	86.34	67.01	60.56	50.31	48.00
Average (*)	100.28	76.19	65.07	58.71	50.31	48.00
(*) is the predicted weighted average electricity price without the consideration of physical transmission constraints						

Actual Prices vs. Predicted Prices ($/Mwh)



Example 2: Peak Electric Power Prices under Fuel Price Variations

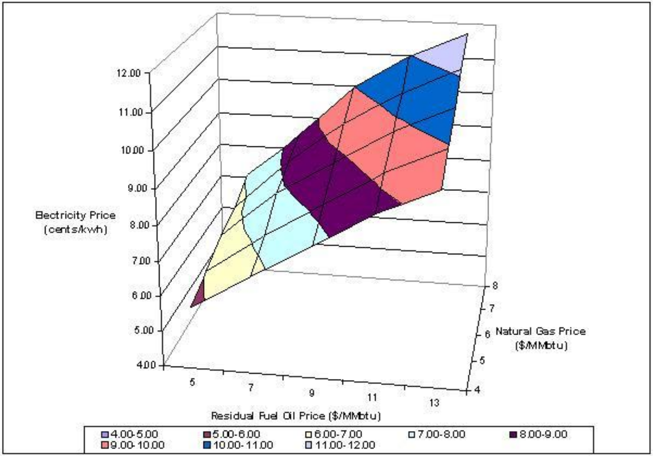
- Natural gas units and oil units generate 38% and 24% of electric power in New England, respectively
- Generating units that burn gas or oil set electric power market price 85% of the time

Example 2: Peak Electric Power Prices under Fuel Price Variations

Average Peak Electricity Prices under Fuel Price Variations

Electricity Price (cents/kwh)		Residual Fuel Oil Prices ($/MMBtu) 5.00	7.00	9.00	11.00	13.00
	4.00	5.76	6.74	7.73	8.70	9.45
	5.00	6.06	7.24	8.22	9.20	9.95
Natural Gas	6.00	6.45	7.82	8.81	9.79	10.54
($/MMBtu)	7.00	6.67	8.19	9.39	10.36	11.12
	8.00	7.08	8.44	9.97	10.96	11.71

Example 2: Peak Electric Power Prices under Fuel Price Variations



Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

- Two cases: high residual fuel oil price (7\$/MMBtu) and low residual fuel oil price (4.4\$/MMBtu)

- We assumed that the natural gas price function (unit: \$/MMBtu) takes the form:

$$\pi_{m_{NG}}(h) = 7 + 6\frac{\sum_{m_{NG}=1}^{M_{NG}} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} q_{m_{NG} r_1 g u} - d_{0_{NG}}}{d_{0_{NG}} + \bar{d}_{0_{NG}}}$$

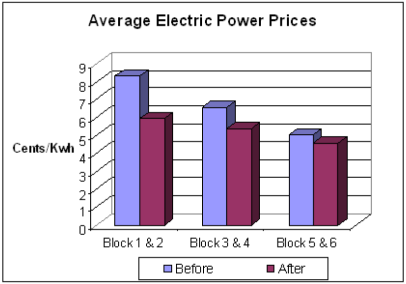
Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

The Price Changes of Natural Gas and Electric Power Under Residual Fuel Oil Price Variation

	Example 3.1		Example 3.2	
	High RFO	Low RFO	High RFO	Low RFO
RFO Price ($/MMBtu)	7.00	4.40	7.00	4.40
NG Demand (Billion MMBtu)	35.95	30.99	41.95	31.80
NG Price ($/MMBtu)	7.00	6.58	7.00	6.27
NG Price Percentage Change	-6.0%		-10.4%	
EP Ave. Price Blocks 1 and 2	8.28	5.94	7.08	5.86
EP Ave. Price Blocks 3 and 4	6.54	5.37	6.25	5.33
EP Ave. Price Blocks 5 and 6	4.99	4.55	4.96	4.44
NG=Natural Gas, RFO=Residual Fuel Oil, EP=Electric Power				

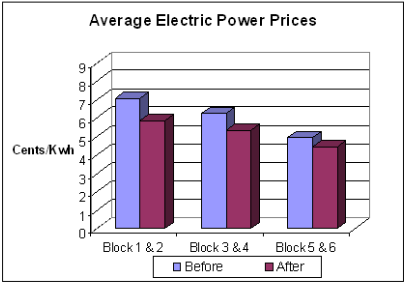
Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

Example 3.1



Natural gas price decreased by 6%

Example 3.2



Natural gas price decreased by 10.4%

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

- When electricity demands increase (or decrease), the electric power prices will increase (or decrease) due to two main reasons:
  - Power plants with higher generating costs (e.g. heat rates) have to operate more (or less) frequently;
  - The demands for various fuels will also rise which may result in higher (or lower) fuel prices/costs.

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

- In August, 2006, the natural gas price soared by 14% because hot weather across the US led to high electricity demand.
- In July 2007, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures.
- We used the first case of Example 3.1 as the base case and assumed that the demand in each block increased by 10%.

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

Prices Before the Demand Increase ($/Mwh)

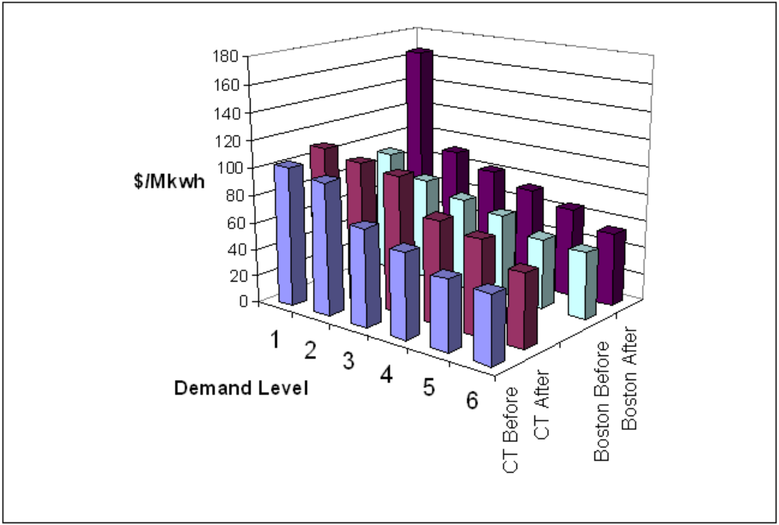
Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
ME	78.73	76.36	67.69	61.56	50.14	49.18
NH	84.82	76.36	67.69	61.56	50.14	49.18
VT	84.82	76.36	67.69	61.56	50.14	49.18
CT	101.81	97.45	71.27	62.22	51.46	49.18
RI	84.82	76.36	67.69	62.22	51.46	49.18
SE MA	84.82	76.36	67.69	62.22	51.46	49.18
WC MA	84.82	76.36	67.69	62.22	51.46	49.18
NE MA	91.30	76.36	67.69	62.22	51.46	49.18
Average	90.23	81.76	68.61	62.08	51.20	49.18
NG Demand	35.95 Billion MMBtu					
NG Price	7.00 $/MMBtu					

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

Prices after the Demand Increase ($/Mwh)

Region	**Block 1**	**Block 2**	**Block 3**	**Block 4**	**Block 5**	**Block 6**
ME	78.73	83.45	81.55	73.33	65.14	53.46
NH	93.68	84.82	81.55	73.33	65.14	53.46
VT	93.68	84.82	81.55	73.33	65.14	53.46
CT	109.09	104.20	100.84	75.74	69.23	53.73
RI	93.68	84.82	81.55	73.33	65.14	53.73
SE MA	93.68	84.82	81.55	73.33	65.14	53.73
WC MA	93.68	84.82	81.55	73.33	65.14	53.73
NE MA	165.16	91.30	81.55	73.33	65.14	53.73
Average	111.48	91.04	86.49	73.95	66.16	53.68
NG Demand	43.62 Billion MMBtu					
NG Price	7.64 $/MMBtu					

Example 4: Electric Power Prices Before and After the Increase of Demands (Connecticut and Boston)



Conclusions

- We developed a new variational inequality model of electric power supply chain networks with fuel markets, which considers both economic transaction networks and physical transmission networks.
- We provided some qualitative properties of the model as well as a computational method.
- We then conducted a case study where our theoretical model was applied to the New England electric power market and fuel supply markets.
- We also conducted sensitivity analysis in order to investigate the electric power prices under fuel price variations.

Conclusions (Cont'd)

- We showed that not only the market responsiveness, but also the electric power market responsiveness, are both crucial to the understanding and determination of the impact of the residual fuel oil price on the natural gas price.
- We applied our model to quantitatively demonstrate how changes in the demand for electricity influence the electric power and fuel markets.
- The model and results presented in this paper are useful in determining and quantifying the interactions between electric power flows and prices and the various fuel supply markets.
- Such information is important to policy-makers who need to ensure system reliability as well as for the energy asset owners and investors who need to manage risk and evaluate their assets.

Thank You!

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