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Source: http://www.nasa.gov
Outline

- Introduction
- Literature review
- An integrated electric power supply chain and fuel market network framework
- Empirical case study and examples
- Conclusions
Electric Power Supply Chains
Electric Power Supply Chains

- The US electric power industry: Half a trillion dollars of net assets, $220 billion annual sales, 40% of domestic primary energy (Energy Information Administration (2000, 2005))

- Deregulation

- Electric power supply chain networks
  - Various generation technologies
  - Insensitive demands
  - Transmission congestion
  - High storage costs
Sources of Electricity in the U.S. in 2007

Source: http://www.eia.doe.gov
Electric Power Industry Fuel Costs, July 2006 through June 2007

Source: http://www.eia.doe.gov
Electric Power Supply Chains and Fuel Markets

- In the U.S., electric power generation accounts for 30% of the natural gas demand (over 50% in the summer), 90% of the coal demand, and over 45% of the residual fuel oil demand.

- The demand from electric power generation has been significantly and steadily increasing while demands from all other sectors (industrial, commercial, and residential) have been slightly decreasing.
Electric Power Supply Chains and Fuel Markets (Cont’d)

The interactions between electric power supply chains and fuel markets affect demands and prices of electric power and fuels.

- From December 1, 2005 to April 1, 2006, the wholesale electricity price in New England decreased by 38% mainly because the delivered natural gas price declined by 45%.

- In August, 2006, the natural gas price jumped 14% because hot weather across the US led to elevated demand for electricity. This high electricity demand also caused the crude oil price to rise by 1.6%.
The availability and the reliability of diversified fuel supplies also affect national security.

- In January 2004, over 7000MW of electric power generation, which accounts for almost one fourth of the total capacity of New England, was unavailable during the electric system peak due to the limited natural gas supply.

- The American Association of Railroads has requested that the Federal Energy Regulatory Commission (FERC) investigate the reliability of the energy supply chain with a focus on electric power and coal transportation.
Beckmann, McGuire, and Winsten (1956): How are electric power flows related to transportation flows?

Deregulation


Electric power wholesale and retail markets

- Visudhiphan and Ilic (1999), Raineri, Rios, and Vasquez (2005), Ruff (2002), Borenstein and Holland (2003), Borenstein (2005a), Joskow and Tirole (2004), and Garcia, Campos, and Reitzes (2005), etc.
Emery and Liu (2001)


Deng, Johnson, and Sogomonian (2001)


An Integrated Electric Power Supply Chain and Fuel Market Network Framework

The Electric Power Supply Chain Network with Fuel Supply Markets

- Fuel Markets for Fuel Type 1
  - Power Plants of Generators in Regions
  - Power Pool
- Fuel Markets for Fuel Type \(a\)
- Fuel Markets for Fuel Type \(A\)

- Demand Markets
  - Region 1
  - Region \(r\)
  - Region \(R\)
The Equilibrium Conditions for the Fuel Supply Markets

We assume that the following conservation of flow equations must hold for all fuel supply markets $m_a = 1, \ldots, M_A; \ a = 1, \ldots, A$:

$$
\sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U} q_{m_ar_1gu} + \bar{q}_{ma} = h_m.
$$

The equilibrium conditions for suppliers at fuel supply market $m_a; m_a = 1, \ldots, M_a; \ a = 1, \ldots, A$, take the form: for each power plant $u; u = 1, \ldots, U_{r_1g}; g = 1, \ldots, G; r_1 = 1, \ldots, R$:

$$
\pi_{m_a}(h^*) + c_{m_ar_1gu} \begin{cases} 
\geq \rho_{m_ar_1gu}^*, & \text{if } q_{m_ar_1gu}^* > 0, \\
= \rho_{m_ar_1gu}^*, & \text{if } q_{m_ar_1gu}^* = 0.
\end{cases}
$$
The Equilibrium Conditions for the Fuel Supply Markets

We can express these equilibrium conditions as the following variational inequality: determine $Q^{1*} \in \mathcal{K}^1$, such that

$$
\sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left[ \pi_{m_a} (Q^{1*}) + c_{m_a r_1 g u} - \rho_{m_a r_1 g u}^{*} \right] \times [q_{m_a r_1 g u} - q_{m_a r_1 g u}^{*}] \geq 0,
$$

$$
\forall Q^1 \in \mathcal{K}^1,
$$

where $\mathcal{K}^1 \equiv \{ Q^1 | Q^1 \in \mathbb{R}^{MU}_{+} \}$. 
Power Generator’s Maximization Problem

Maximize \( \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} \sum_{r_2=1}^{K_{r_2}} \rho_{wr_2 k}^{r_{1 g u}} q_{wr_2 k}^{r_{1 g u}} \)

\[+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} \rho_{wr_2}^{*} y_{wr_2}^{r_{1 g u}} + \sum_{w=1}^{W} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} L_w \varphi_{wr_1}^{*} z_{wr_1}^{r_{1 g u}} \]

\[- \sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} \rho_{m_a r_1 gu}^{*} q_{m_a r_1 gu} \]

\[- \sum_{w=1}^{W} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} f_{wr_1 gu}^{(q_{wr_1 gu}^{r_{1 g u}})} - \sum_{w=1}^{W} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} \sum_{r_2=1}^{K_{r_2}} \sum_{k=1}^{c_{wr_1 gu r_2 k}} c_{wr_1 gu r_2 k}^{(q_{wr_2 k}^{r_{1 g u}})} \]

\[- \sum_{w=1}^{W} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} \sum_{r_2=1}^{K_{r_2}} c_{wr_1 gu r_2}^{(y_{wr_2}^{r_{1 g u}})} - \sum_{w=1}^{W} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} c_{wr_1 gu}^{(z_{wr_1}^{r_{1 g u}})} \]

\[- \sum_{w=1}^{W} \sum_{r_1=1}^{R} U_{r_1 g} \sum_{u=1}^{R} \sum_{b=1}^{B} \sum_{r_2=1}^{R} \mu_{wb}^{*} \alpha_{r_1 r_2 b}^{[K_{r_2}} q_{wr_2 k}^{r_{1 g u}} + y_{wr_2}^{r_{1 g u}}] \]
Power Generator’s Maximization Problem (Cont’d)

subject to:

\[
\begin{align*}
\sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} q_{w_{r_2} k}^{r_1 u} + \sum_{r_2=1}^{R} y_{w_{r_2}}^{r_1 u} &= q_{w}^{r_1 u}, \quad \forall r_1, u, w, \\
\sum_{a=1}^{A} \sum_{m_a=1}^{M_a} q_{m_a r_1 u} + \sum_{w=1}^{W} L_w \beta_{r_1 u} q_{w}^{r_1 u} &= \sum_{w=1}^{W} L_w q_{w}^{r_1 u}, \quad \forall r_1, u,
\end{align*}
\]  

(5)

(6)

\[q_{w}^{r_1 u} + z_{w}^{r_1 u} \leq Cap_{r_1 u}, \quad \forall r_1, u, w,\]

\[z_{w}^{r_1 u} \leq OP_{r_1 u}, \quad \forall r_1, u, w,\]

\[q_{w_{r_2} k}^{r_1 u} \geq 0, \quad \forall r_1, u, w, r_2, k,\]

\[q_{m_a r_1 u} \geq 0, \quad \forall m_a, r_1, u,\]

\[y_{w_{r_2}}^{r_1 u} \geq 0, \quad \forall r_1, u, w, r_2,\]

\[z_{w}^{r_1 u} \geq 0, \quad \forall r_1, u, w.\]
Power Generators’ Optimization Conditions

Determine \((Q^1*, q^*, Q^2*, Y^1*, Z^*, \eta^*, \lambda^*) \in K^2\) satisfying

\[
\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left[ \frac{\partial f_{wr_1 gu}(q_{w r_1 gu}^*)}{\partial q_{w r_1 gu}^*} \right] \times \left[ q_{w r_1 gu}^* - q_{w r_1 gu} \right] + \sum_{b=1}^{B} \mu^*_w \alpha_{r_1 r_2 b} - \rho_{r_2 k}^* \times \left[ q_{w r_2 k}^* - q_{w r_2 k} \right] \]

\[
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} K_{r_2} \left[ \frac{\partial c_{wr_1 gur_2 k}(q_{w r_2 k}^*)}{\partial q_{w r_2 k}^*} \right] + \sum_{b=1}^{B} \mu^*_w \alpha_{r_1 r_2 b} - \rho_{r_2 k}^* \times \left[ q_{w r_2 k}^* - q_{w r_2 k} \right] \]

\[
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \left[ \frac{\partial c_{wr_1 gur_2 k}(q_{w r_2 k}^*)}{\partial q_{w r_2 k}^*} \right] + \sum_{b=1}^{B} \mu^*_w \alpha_{r_1 r_2 b} - \rho_{w r_2}^* \times \left[ y_{w r_2}^* - y_{w r_2} \right] \]

\[
+ \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \rho^*_{m_a r_1 gu} \times \left[ q_{m_a r_1 gu} - q_{m_a r_1 gu}^* \right] \]
Power Generators’ Optimization Conditions (Cont’d)

\[ + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} \left[ \frac{\partial c_{wr_1gu}(z_{r_1gu}^*)}{\partial z_{r_1gu}} + \lambda_{r_1gu}^* + \eta_{r_1gu}^* - \varphi_{r_1gu}^* \right] \times [z_{r_1gu}^* - z_{r_1gu}] \]

\[ + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} \left[ Cap_{r_1gu} - q_{r_1gu}^* - z_{r_1gu}^* \right] \times [\eta_{r_1gu}^* - \eta_{r_1gu}] \]

\[ + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} \left[ OP_{r_1gu} - z_{r_1gu}^* \right] \times [\lambda_{r_1gu}^* - \lambda_{r_1gu}] \geq 0, \]

\[ \forall (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in \mathcal{K}^2, \quad (14) \]

where \( \mathcal{K}^2 \equiv \{ (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) | (Q^1, q, Q^2, Y^1, Z, \eta, \lambda) \in R_+^{WMU+UKW+WUR+4WU} \}, \) and (5) and (6) hold.
The ISO’s Role

The following conditions must hold for each interface $b$ and at each demand level $w$, where $b = 1, \ldots, B$; $w = 1, \ldots, W$:

$$
\sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} \sum_{k=1}^{K_{r_2}} q_{wr_2k}^* + \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} y_{wr_2}^* + \sum_{k=1}^{K_{r_2}} y_{wr_1r_2k}^* \left\{ \begin{array}{ll}
= \text{Cap}_b, & \text{if } \mu_{wb}^* > 0, \\
\leq \text{Cap}_b, & \text{if } \mu_{wb}^* = 0.
\end{array} \right.
$$

The ISO ensures that the regional electricity markets $r = 1, \ldots, R$ clear at each demand level $w = 1, \ldots, W$, that is,

$$
\sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} y_{wr}^* \left\{ \begin{array}{ll}
= \sum_{r_2=1}^{R} \sum_{k=1}^{K_r} y_{wrr_2k}^*, & \text{if } \rho_{wr}^* > 0, \\
\geq \sum_{r_2=1}^{R} \sum_{k=1}^{K_r} y_{wrr_2k}^*, & \text{if } \rho_{wr}^* = 0.
\end{array} \right.
$$

The ISO also ensures that the regional operating reserve markets; hence, $r_1 = 1, \ldots, R$ clear at each demand level $w = 1, \ldots, W$, that is,

$$
\sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} z_{wr}^* \left\{ \begin{array}{ll}
= \text{OPR}_{wr_1}, & \text{if } \varphi_{wr_1}^* > 0, \\
\geq \text{OPR}_{wr_1}, & \text{if } \varphi_{wr_1}^* = 0.
\end{array} \right.
$$
The ISO’s Role (Cont’d)

We can express these equilibrium conditions using the following variational inequality: determine \((\mu^*, \rho_3^*, \varphi^*) \in R_+^{WB+2WR}\), such that

\[
\sum_{w=1}^{W} L_w \sum_{b=1}^{B} \left[ \text{Cap}_b - \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} \sum_{k=1}^{K_{r_2}} q_{wr_2k}^{r_1gu^*} + \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} y_{wr_2}^{r_1gu^*} + \sum_{k=1}^{K_{r_2}} y_{wr_1r_2k}^{*} \right] \alpha_{r_1r_2b} \times \left[ \mu_{wb} - \mu_{wb}^* \right]
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{r=1}^{R} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} y_{wr}^{r_1gu^*} - \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} y_{wr_2k}^{*} \right] \times \left[ \rho_{wr} - \rho_{wr}^* \right]
\]

\[
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} \left[ z_{w}^{r_1gu^*} - OPR_{wr_1} \right] \times \left[ \varphi_{wr_1} - \varphi_{wr_1}^* \right] \geq 0, \quad \forall (\mu, \rho_3, \varphi) \in R_+^{WB+2WR}.
\]
The Equilibrium Conditions for the Demand Markets

We assume that all demand markets have fixed and known demands and the following conservation of flow equations, hence, must hold for all demand markets \( k = 1, \ldots, K_{r_2} \), all regions \( r_2 = 1, \ldots, R \), and at all demand levels \( w = 1, \ldots, W \):

\[
\sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1g}} q_{w_{r_2k}}^{r_1gu*} + \sum_{r_1=1}^{R} y_{w_1r_2k}^* = (1 + \kappa_{w_{r_2k}})d_{w_{r_2k}}. \tag{19}
\]

The equilibrium conditions for consumers at demand market \( k \) in region \( r_2 \) take the form: for each power plant \( u; u = 1, \ldots, U_{r_1g} \); each generator \( g = 1, \ldots, G \); each region \( r_1 = 1, \ldots, R \), and each demand level \( w; w = 1, \ldots, W \):

\[
\begin{align*}
\rho_{w_{r_2k}}^{r_1gu*} + \hat{c}_{r_2k}^{wr_1gu}(Q_w^{2*}) & \begin{cases} = \rho_{w_{r_2k}}^*, & \text{if } q_{w_{r_2k}}^{r_1gu*} > 0, \\
\geq \rho_{w_{r_2k}}^*, & \text{if } q_{w_{r_2k}}^{r_1gu*} = 0; \end{cases}
\end{align*}
\]

and

\[
\begin{align*}
\rho_{w_1}^* + \sum_{b=1}^{B} \mu_{wb}^{r_1r_2b} + \hat{c}_{r_1r_2k}(Y_w^{2*}) & \begin{cases} = \rho_{w_{r_2k}}^*, & \text{if } y_{w_1r_2k}^* > 0, \\
\geq \rho_{w_{r_2k}}^*, & \text{if } y_{w_1r_2k}^* = 0. \end{cases}
\end{align*}
\]
The Equilibrium Conditions for the Demand Markets
(Cont’d)

We can express these equilibrium conditions using the following variational
inequality: determine \((Q_2^*, Y_2^*) \in \mathcal{K}^3\), such that

\[
\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} U_{r_1g} \sum_{u=1}^{R} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \left[ \rho_{wr_2k}^{r_1gu} + \hat{c}_{r_2k}^{wr_1gu}(Q_2^*) \right] \times [q_{wr_2k}^{r_1gu} - q_{wr_2k}^{r_1gu*}] \\
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{k=1}^{K_{r_2}} \left[ \rho_{wr_2}^{r_1gu} + \sum_{b=1}^{B} \mu_{wb}^{r_1r_2b} \alpha_{r_1r_2b} + \hat{c}_{wr_2k}(Y_2^*) \right] \times [y_{wr_2k} - y_{wr_1r_2k}^{*}] \geq 0,
\]

\forall (Q^2, Y^2) \in \mathcal{K}^3,

(22)

where \(\mathcal{K}^3 \equiv \{(Q^2, Y^2)|(Q^2, Y^2) \in R_{+}^{WUK+WRK} \text{ and (19) holds}\}.\)
Definition 1: Electric Power Supply Chain Network Equilibrium

The equilibrium state of the electric power supply chain network with fuel supply market is one where the fuel flows and electric power flows satisfy the sum of conditions (4), (14), (18), and (22).
Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium

The equilibrium conditions governing the electric power supply chain network according to Definition 1 coincide with the solution of the variational inequality given by: determine \((Q_1^*, q^*, Q_2^*, Y_1^*, Y_2^*, Z^*, \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*) \in K^4\) satisfying

\[
\begin{align*}
\sum_{a=1}^{A} \sum_{m_a=1}^{M_a} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_r_1 g} \left[ \pi_{m_a}(Q_1^*) + c_{ma r_1 gu} \right] \times [q_{ma r_1 gu} - q_{ma r_1 gu}^*] \\
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_r_1 g} \sum_{r_2=1}^{K_{r_2}} \left[ \frac{\partial c_{wr_1 gu r_2 k}(q_{wr_2 k}^*)}{\partial q_{wr_2 k}} + \sum_{b=1}^{B} \mu_{wb} \alpha_{r_1 r_2 b} + \hat{c}_{r_2 k}^* (Q_2^*) \right] \times [q_{wr_2 k}^* - q_{wr_2 k}] \\
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_r_1 g} \sum_{r_2=1}^{K_{r_2}} \left[ \frac{\partial c_{wr_1 gu r_2 k}(y_{wr_2 k}^*)}{\partial y_{wr_2 k}} + \sum_{b=1}^{B} \mu_{wb} \alpha_{r_1 r_2 b} - \rho_{wr_2}^* \right] \times [y_{wr_2 k}^* - y_{wr_2 k}] \\
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_r_1 g} \sum_{r_2=1}^{K_{r_2}} \left[ \frac{\partial c_{wr_1 gu}(z_{w}^*)}{\partial z_w} + \lambda_{w}^* + \eta_{w}^* - \varphi_{w}^* \right] \times [z_{w}^* - z_{w}] \\
+ \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{K_{r_2}} \left[ \rho_{wr_1}^* + \hat{c}_{wr_1 r_2 k}(Y_2^*) + \sum_{b=1}^{B} \mu_{wb} \alpha_{r_1 r_2 b} \right] \times [y_{wr_1 r_2 k}^* - y_{wr_1 r_2 k}] 
\end{align*}
\]
Theorem 1: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium (Cont’d)

\[ + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left( Cap_{r_1 gu} - q_{w}^{r_1 gu} - z_{w}^{r_1 gu} \right) \times \left[ \eta_{w}^{r_1 gu} - \eta_{w}^{r_1 gu} \right] + \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \left( OP_{r_1 gu} - z_{w}^{r_1 gu} \right) \times \left[ \lambda_{w}^{r_1 gu} - \lambda_{w}^{r_1 gu} \right] + \sum_{w=1}^{W} \sum_{b=1}^{B} [Cap_{b} - \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{k=1}^{K_{r_2}} q_{w r_2 k}^{r_1 gu} - \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} y_{w r_2}^{r_1 gu} + \sum_{k=1}^{K_{r_2}} y_{w r_2}^{r_1 gu} \times \alpha_{r_1 r_2 b}] \times [\mu_{wb} - \mu_{wb}^*] + \sum_{w=1}^{W} \sum_{r_2=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} \sum_{k=1}^{K_{r_2}} y_{w r_2}^{r_1 gu} \times [\rho_{wr} - \rho_{wr}^*] + \sum_{w=1}^{W} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} [\sum_{g=1}^{G} \sum_{u=1}^{U_{r_1 g}} z_{w}^{r_1 gu} - OP_{r_1}] \times [\phi_{wr_1} - \phi_{wr_1}^*] \geq 0, \quad \forall (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in K^4, \]  

where \( K^4 \equiv \{(Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) | (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in R^{MU+WUK+WUR+4WU+WRK+WB+2WR} + (5), (6), \) and (19) hold.}
Some Qualitative Properties

**Theorem 2: Existence**
Assume that the feasible set $\mathcal{K}$ is nonempty. Then variational inequality (23) admits a solution.

**Theorem 3: Uniqueness**
Assume the conditions in Theorem 2 and that the function $F(X)$ that enters variational inequality (23) is strictly monotone on $\mathcal{K}$, that is,

$$\langle (F(X') - F(X''))^T, X' - X'' \rangle > 0, \hspace{1cm} \forall X', X'' \in \mathcal{K}, X' \neq X''$$

Then the solution $X^*$ to variational inequality (23) is unique.
Computational Method

- We let $VIP$ denote the original variational inequality problem, and we decompose $VIP$ into $W$ subproblems based on $W$ demand levels.

- We let $\bar{\pi}_{m_a}$ denote the price of fuel $a$ at market $m_a$, and group all the $\bar{\pi}_{m_a}$s into vector $\bar{\pi}$.

- Each subproblem is denoted by $SVIP_w(\bar{\pi})$.

- Note that in each $SVIP_w(\bar{\pi})$ the fuel prices $\bar{\pi}$ are fixed and given.
Computational Method (Cont’d)

We first define $X^T \equiv (Q^{3T}, q^T, Q^{2T}, Y^{1T}, Y^{2T}, Z^T, \eta^T, \chi^T, \mu^T, \rho_3^T, \varphi^T)^T$.

**Step 0: Initialization**
Initialize $X^0$. Let $T = 1$.

**Step 1: Compute Fuel Prices**
Use $Q^{3(T-1)}$ to compute $\pi_{ma}^T = \pi_{ma}(Q^{3(T-1)})$, $\forall a, m_a$.

**Step 2: Solve Subproblems**
For each $w$, compute $X^T$ by solving each $SVIP_w(\pi^T)$ individually.

**Step 3: Convergence Verification**
If $||X^T - X^{T-1}||_\infty \leq \epsilon$ with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $T := T + 1$ and go to Step 1. We set the tolerance $\epsilon = 10^{-3}$ for all computations in this paper.

Each subproblem $SVIP(\pi)$ can be reformulated and solved as the user-optimal transportation network model with Lagrangian multipliers.
Empirical Case Study and Examples

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Hourly demand/price data of July 2006 ($24 \times 31 = 744$ scenarios)
- 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$)
Empirical Case Study and Examples

- Example 1: Predictions to the actual regional electric power prices

- Example 2: Sensitivity analysis for peak-hour electricity prices under natural gas and oil price variations

- Example 3: The impact of the oil price on the natural gas price through electric power markets

- Example 4: The impact of changes in the electricity demands for electricity on the electric power and fuel supply markets
The New England Electric Power Supply Chain Network with Fuel Supply Markets
Example 1: Predictions to the actual regional electric power prices

Average Regional Demands for Each Demand Level (Mwh)

<table>
<thead>
<tr>
<th>Region</th>
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<th>Block 3</th>
<th>Block 4</th>
<th>Block 5</th>
<th>Block 6</th>
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<tbody>
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<td>1</td>
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<td>2</td>
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<tr>
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<td>760</td>
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<td>560</td>
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<td>1798</td>
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<td>16350</td>
<td>14252</td>
<td>12168</td>
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Example 1: Predictions to the actual regional electric power prices

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<th>Block 3</th>
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<td>56.31</td>
<td>48.20</td>
<td>38.35</td>
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<td>80.69</td>
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<td>58.39</td>
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<td>65.97</td>
<td>50.92</td>
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Example 1: Predictions to the actual regional electric power prices

<table>
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<td>74.62</td>
<td>64.77</td>
<td>58.71</td>
<td>50.31</td>
<td>48.00</td>
</tr>
<tr>
<td>VT</td>
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<td>74.62</td>
<td>64.77</td>
<td>58.71</td>
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<td>48.00</td>
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<td>117.21</td>
<td>73.50</td>
<td>65.91</td>
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<td>100.28</td>
<td>74.62</td>
<td>64.77</td>
<td>58.71</td>
<td>50.31</td>
<td>48.00</td>
</tr>
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<td>SE MA</td>
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<td>74.62</td>
<td>64.77</td>
<td>58.71</td>
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<td>48.00</td>
</tr>
<tr>
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<td>74.62</td>
<td>64.77</td>
<td>58.71</td>
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<td>48.00</td>
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<td>78.43</td>
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<td>48.00</td>
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<td>67.01</td>
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<td>48.00</td>
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<tr>
<td>Average (*)</td>
<td>100.28</td>
<td>76.19</td>
<td>65.07</td>
<td>58.71</td>
<td>50.31</td>
<td>48.00</td>
</tr>
</tbody>
</table>

(*) is the predicted weighted average electricity price without the consideration of physical transmission constraints
Actual Prices vs. Predicted Prices ($/Mwh)
Example 2: Peak Electric Power Prices under Fuel Price Variations

- Natural gas units and oil units generate 38% and 24% of electric power in New England, respectively.

- Generating units that burn gas or oil set electric power market price 85% of the time.
Example 2: Peak Electric Power Prices under Fuel Price Variations

<table>
<thead>
<tr>
<th>Electricity Price (cents/kwh)</th>
<th>5.00</th>
<th>7.00</th>
<th>9.00</th>
<th>11.00</th>
<th>13.00</th>
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</thead>
<tbody>
<tr>
<td>Natural Gas ($/MMBtu)</td>
<td>6.00</td>
<td>7.00</td>
<td>7.00</td>
<td>8.00</td>
<td>11.00</td>
</tr>
<tr>
<td>4.00</td>
<td>5.76</td>
<td>6.74</td>
<td>7.73</td>
<td>8.70</td>
<td>9.45</td>
</tr>
<tr>
<td>5.00</td>
<td>6.06</td>
<td>7.24</td>
<td>8.22</td>
<td>9.20</td>
<td>9.95</td>
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<tr>
<td>6.00</td>
<td>6.45</td>
<td>7.82</td>
<td>8.81</td>
<td>9.79</td>
<td>10.54</td>
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<tr>
<td>7.00</td>
<td>6.67</td>
<td>8.19</td>
<td>9.39</td>
<td>10.36</td>
<td>11.12</td>
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<tr>
<td>8.00</td>
<td>7.08</td>
<td>8.44</td>
<td>9.97</td>
<td>10.96</td>
<td>11.71</td>
</tr>
</tbody>
</table>
Example 2: Peak Electric Power Prices under Fuel Price Variations
Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

- Two cases: high residual fuel oil price (7$/MMBtu) and low residual fuel oil price (4.4$/MMBtu)

- We assumed that the natural gas price function (unit: $/MMBtu) takes the form:

  \[ \pi_{m_{NG}}(h) = 7 + 6 \sum_{m_{NG}=1}^{M_{NG}} \sum_{r_1=1}^{R} \sum_{g=1}^{G} \sum_{u=1}^{U_{rg}} q_{m_{NG}r_1gu} - d_{0_{NG}} \]
  \[ \frac{1}{d_{0_{NG}} + d_{0_{NG}}} \]
Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

The Price Changes of Natural Gas and Electric Power Under Residual Fuel Oil Price Variation

<table>
<thead>
<tr>
<th></th>
<th>Example 3.1</th>
<th>Example 3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High RFO</td>
<td>Low RFO</td>
</tr>
<tr>
<td>RFO Price ($/MMBtu)</td>
<td>7.00</td>
<td>4.40</td>
</tr>
<tr>
<td>NG Demand (Billion MMBtu)</td>
<td>35.95</td>
<td>30.99</td>
</tr>
<tr>
<td>NG Price ($/MMBtu)</td>
<td>7.00</td>
<td>6.58</td>
</tr>
<tr>
<td>NG Price Percentage Change</td>
<td>-6.0%</td>
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<tr>
<td>EP Ave. Price Blocks 1 and 2</td>
<td>8.28</td>
<td>5.94</td>
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<tr>
<td>EP Ave. Price Blocks 3 and 4</td>
<td>6.54</td>
<td>5.37</td>
</tr>
</tbody>
</table>

NG=Natural Gas, RFO=Residual Fuel Oil, EP=Electric Power
Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

Example 3.1

Natural gas price decreased by 6%

Example 3.2

Natural gas price decreased by 10.4%
Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

- When electricity demands increase (or decrease), the electric power prices will increase (or decrease) due to two main reasons:
  - Power plants with higher generating costs (e.g. heat rates) have to operate more (or less) frequently;
  - The demands for various fuels will also rise which may result in higher (or lower) fuel prices/costs.
Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

- In August, 2006, the natural gas price soared by 14% because hot weather across the US led to high electricity demand.

- In July 2007, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures.

- We used the first case of Example 3.1 as the base case and assumed that the demand in each block increased by 10%.
Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

<table>
<thead>
<tr>
<th>Region</th>
<th>Block 1</th>
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<tr>
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<td>76.36</td>
<td>67.69</td>
<td>61.56</td>
<td>50.14</td>
<td>49.18</td>
</tr>
<tr>
<td>NH</td>
<td>84.82</td>
<td>76.36</td>
<td>67.69</td>
<td>61.56</td>
<td>50.14</td>
<td>49.18</td>
</tr>
<tr>
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<td>76.36</td>
<td>67.69</td>
<td>61.56</td>
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<td>49.18</td>
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<td>62.22</td>
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<td>67.69</td>
<td>62.22</td>
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<td>49.18</td>
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<td>67.69</td>
<td>62.22</td>
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<td>76.36</td>
<td>67.69</td>
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<tr>
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<td>67.69</td>
<td>62.22</td>
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<td>49.18</td>
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<td>90.23</td>
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<td>68.61</td>
<td>62.08</td>
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<td>49.18</td>
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</table>

NG Demand: 35.95 Billion MMBtu
NG Price: 7.00 $/MMBtu
Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

<table>
<thead>
<tr>
<th>Region</th>
<th>Block 1</th>
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<th>Block 4</th>
<th>Block 5</th>
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<td>84.82</td>
<td>81.55</td>
<td>73.33</td>
<td>65.14</td>
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<tr>
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<td>104.20</td>
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<td>75.74</td>
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<td>53.73</td>
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<tr>
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<td>81.55</td>
<td>73.33</td>
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<td>NG Price</td>
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<td></td>
<td></td>
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</table>
Example 4: Electric Power Prices Before and After the Increase of Demands (Connecticut and Boston)
Conclusions

- We developed a new variational inequality model of electric power supply chain networks with fuel markets, which considers both economic transaction networks and physical transmission networks.

- We provided some qualitative properties of the model as well as a computational method.

- We then conducted a case study where our theoretical model was applied to the New England electric power market and fuel supply markets.

- We also conducted sensitivity analysis in order to investigate the electric power prices under fuel price variations.
We showed that not only the market responsiveness, but also the electric power market responsiveness, are both crucial to the understanding and determination of the impact of the residual fuel oil price on the natural gas price.

We applied our model to quantitatively demonstrate how changes in the demand for electricity influence the electric power and fuel markets.

The model and results presented in this paper are useful in determining and quantifying the interactions between electric power flows and prices and the various fuel supply markets.

Such information is important to policy-makers who need to ensure system reliability as well as for the energy asset owners and investors who need to manage risk and evaluate their assets.
Thank You!

For more information, please see:
The Virtual Center for Supernetworks
http://supernet.som.umass.edu