Identification of Vulnerabilities in Fragile Network Systems: Transportation Networks, the Internet, Financial Networks, and Electric Power

Anna Nagurney

John F. Smith Memorial Professor
Department of Finance and Operations Management
University of Massachusetts
Amherst, MA 01003

I3P Consortium Meeting
University of Massachusetts
October 29, 2009
Funding for this research has been provided by:

National Science Foundation

AT&T Foundation

John F. Smith Memorial Fund - University of Massachusetts at Amherst

THE ROCKEFELLER FOUNDATION

RADCLIFFE INSTITUTE FOR ADVANCED STUDY
HARVARD UNIVERSITY
We Are in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- alternative and at times *conflicting criteria* used in decision-making;
- *constraints on resources*: natural, human, financial, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- increasing *risk and uncertainty*, and
- *the importance of dynamics* and realizing a fast and sound response to evolving events.
Network problems are their own class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.
The study of the efficient operation on transportation networks dates to ancient Rome with a classical example being the publicly provided Roman road network and the time of day chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.
Characteristics of Networks Today

• large-scale nature and complexity of network topology;

• congestion;

• the interactions among networks themselves such as in transportation versus telecommunications;

• policies surrounding networks today may have a major impact not only economically but also environmentally, socially, politically, and security-wise.
• alternative behaviors of the users of the network

  – system-optimized versus
  
  – user-optimized (network equilibrium),

which may lead to

paradoxical phenomena.
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: \( p_1 = (a, c) \) and \( p_2 = (b, d) \).

For a travel demand of 6, the equilibrium path flows are \( x_{p_1}^* = x_{p_2}^* = 3 \) and

The equilibrium path travel cost is

\[
C_{p_1} = C_{p_2} = 83.
\]
Adding a new link creates a new path $p_3 = (a,e,d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path $p_3$, $C_{p_3} = 70$.

The new equilibrium flow pattern network is

The equilibrium path travel costs:

$C_{p_1} = C_{p_2} = C_{p_3} = 92$. 

$c_e(f_e) = f_e + 10$
The 1968 Braess article has been translated from German to English and appears as

**On a Paradox of Traffic Planning**

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science*.
Transportation and Fragile Network Systems
The TNE Paradigm is the Unifying Paradigm for:

- Transportation Networks
- The Internet
- Financial Networks
- Electric Power Supply Chains.
The Equivalence of Supply Chains and Transportation Networks

Supply Chain - Transportation Supernetwork Representation

- Raw material sources
- Distribution centers
- Retail Markets

Financial Network

Logistical (Product Supply Chain) Network

Physical Transportation Network

Transaction cost information
Demand or order information
Travel time information
Unexpected issues information
Real-Time Information System

Two-way information exchanges between specific decision-makers

The Electric Power Supply Chain Network

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks

Nagurney et al., Transportation Research E 43: (2007) pp 624-646
In 1952, Copeland wondered whether money flows like water or electricity.
The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation

We have shown that money as well as electricity flow like transportation and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!
Recent disasters have demonstrated the importance and the vulnerability of network systems.

Examples include:

- 9/11 Terrorist Attacks, September 11, 2001;
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge Collapse, August 1, 2007.
Electric Power Network Disasters

www.cellar.org

media.collegepublisher.com

www.crh.noaa.gov
Communication Network Disasters

www.tx.mb21.co.uk

www.w5jgv.com

www.wirelessestimator.com
FRAGILE NETWORKS
Identifying Vulnerabilities and Synergies in an Uncertain World

Anna Nagurney / Qiang Qiang

WILEY
The network performance/efficiency measure $\varepsilon(G,d)$, for a given network topology $G$ and fixed demand vector $d$, is defined as

$$\varepsilon(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where $n_w$ is the number of O/D pairs in the network and $\lambda_w$ is the equilibrium disutility for O/D pair $w$.

Definition: Importance of a Network Component

The importance, $I(g)$, of a network component $g \in G$ is measured by the relative network efficiency drop after $g$ is removed from the network:

$$I(g) = \frac{\Delta \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G, d) - \mathcal{E}(G - g, d)}{\mathcal{E}(G, d)},$$

where $G-g$ is the resulting network after component $g$ is removed.
Example - Sioux Falls Network

The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of Bureau of Public Roads (BPR) form.
Example - Sioux Falls Network
Link Importance Rankings
Motivation for Research on Transportation Network Robustness

According to the American Society of Civil Engineering:

Poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of $94 billion in the US in terms of needed repairs for roads alone.

Poor road conditions in the United States cost US motorists $54 billion in repairs and operating costs annually.
The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.

We focus on the *degradation of links through reductions in their capacities* and the effects on the induced travel costs in the presence of known travel demands and different functional forms for the links.
What About Dynamic Networks?
We are using evolutionary variational inequalities to model dynamic networks with:

- **dynamic (time-dependent)** supplies and demands
- **dynamic (time-dependent)** capacities
- **structural changes** in the networks themselves.

Such issues are important for robustness, resiliency, and reliability of networks (including supply chains and the Internet).
A network like the Internet is volatile. Its traffic patterns can change quickly and dramatically... The assumption of a static model is therefore particularly suspect in such networks. (page 10 of Roughgarden’s (2005) book, *Selfish Routing and the Price of Anarchy*).

A Dynamic Model of the Internet

The Time-Dependent (Demand-Varying) Braess Paradox and Evolutionary Variational Inequalities
Recall the Braess Network where we add the link e.
The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)

Braess Network with Time-Dependent Demands

Paths 1 and 2

Path 3
In Demand Regime I, only the new path is used.  
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!  
In Demand Regime III, only the original paths are used.

Network 1 is the Original Braess Network - Network 2 has the added link.
The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!
Extension of the Network Efficiency Measure to Dynamic Networks

An Efficiency Measure for Dynamic Networks Modeled as Evolutionary Variational Inequalities with Applications to the Internet and Vulnerability Analysis, Nagurney and Qiang, Netnomics (2008).
Where Are We Now?

Empirical Case Study

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Hourly demand/price data of July 2006 (24 × 31 = 744 scenarios)
- 6 blocks (L1 = 94 hours, and Lw = 130 hours; w = 2, ..., 6)
The New England Electric Power Supply Chain Network with Fuel Suppliers
Predicted Prices vs. Actual Prices ($/Mwh)
Ongoing Research and Questions

- Identification of the most important nodes and links in large-scale electric power supply chains as in our empirical case study and in financial networks.
- Design of networks to handle dynamic demands and various uncertainties (cost, demand, etc.) under system-optimizing or user-optimizing behaviors.
- Quantification of network synergies through mergers and acquisitions.
- Further research on network resilience and robustness.
- Extension of the network efficiency measure to handle different kinds of risk; we have completed some work in this dimension.
The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

**Mission:** The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, supply chains, telecommunication, and electric power networks to economic, environmental, financial, knowledge and social networks.

**The Applications of Supernetworks Include:** multimodal transportation networks, critical infrastructure, energy and the environment, the Internet and electronic commerce, global supply chain management, international financial networks, web-based advertising, complex networks and decision-making, integrated social and economic networks, network games, and network metrics.