Identification of Vulnerabilities in Fragile Network Systems: Transportation Networks, the Internet, Financial Networks, and Electric Power

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We Are in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- alternative and at times *conflicting criteria* used in decision-making;
- *constraints on resources*: natural, human, financial, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- increasing *risk and uncertainty*, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

Network problems are their own class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.

The study of the efficient operation on transportation networks dates to *ancient Rome* with a classical example being the publicly provided Roman road network and the *time of day chariot policy*, whereby chariots were banned from the ancient city of Rome at particular times of day.

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*;
- the *interactions among networks* themselves such as in transportation versus telecommunications;
- *policies* surrounding networks today may have a *major impact* not only economically but also environmentally, *socially, politically, and security-wise*.

- *alternative behaviors of the users of the network*
  - system-optimized versus
  - user-optimized (network equilibrium),

which may lead to

paradoxical phenomena.

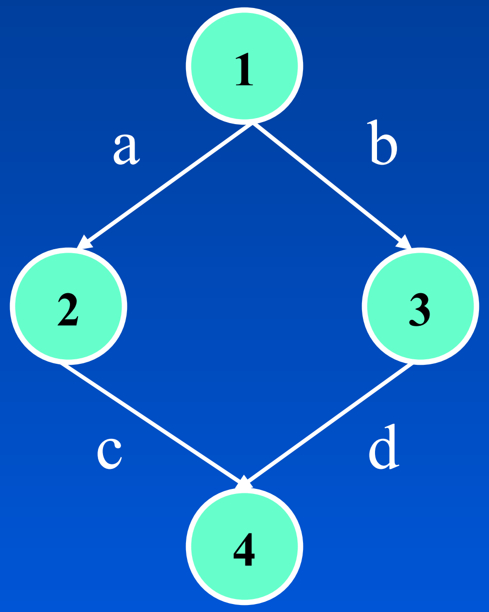
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}^* = x_{p_2}^* = 3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83}$.



$c_a(f_a)=10\ f_a$ $\quad$ $c_b(f_b) = f_b+50$

$c_c(f_c) = f_c+50$ $\quad$ $c_d(f_d) = 10\ f_d$

Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path **p_3=(a,e,d).**

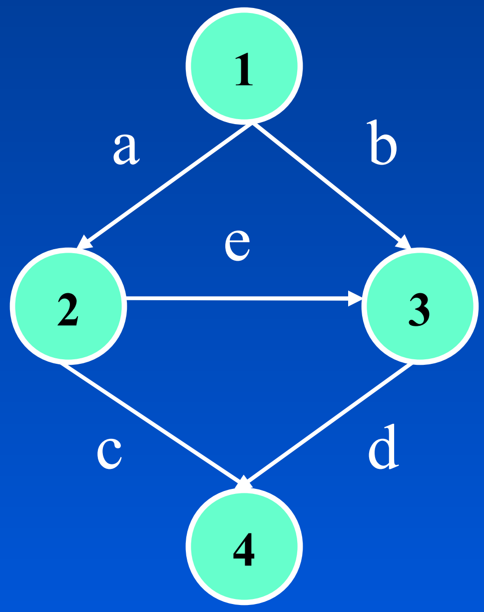
The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^*=2.$

The equilibrium path travel costs:

$C_{p_1} = C_{p_2} = C_{p_3} = 92.$



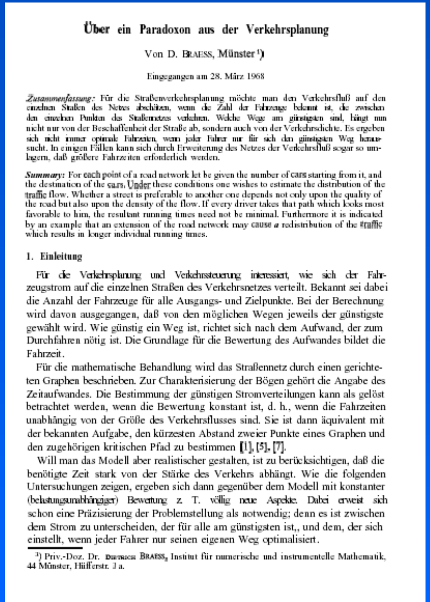
$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation*



Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

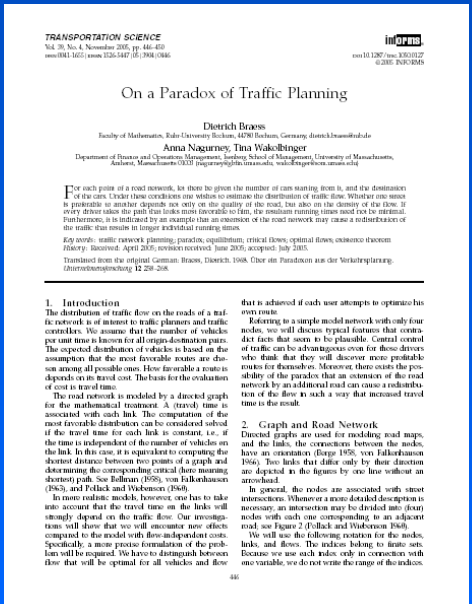
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

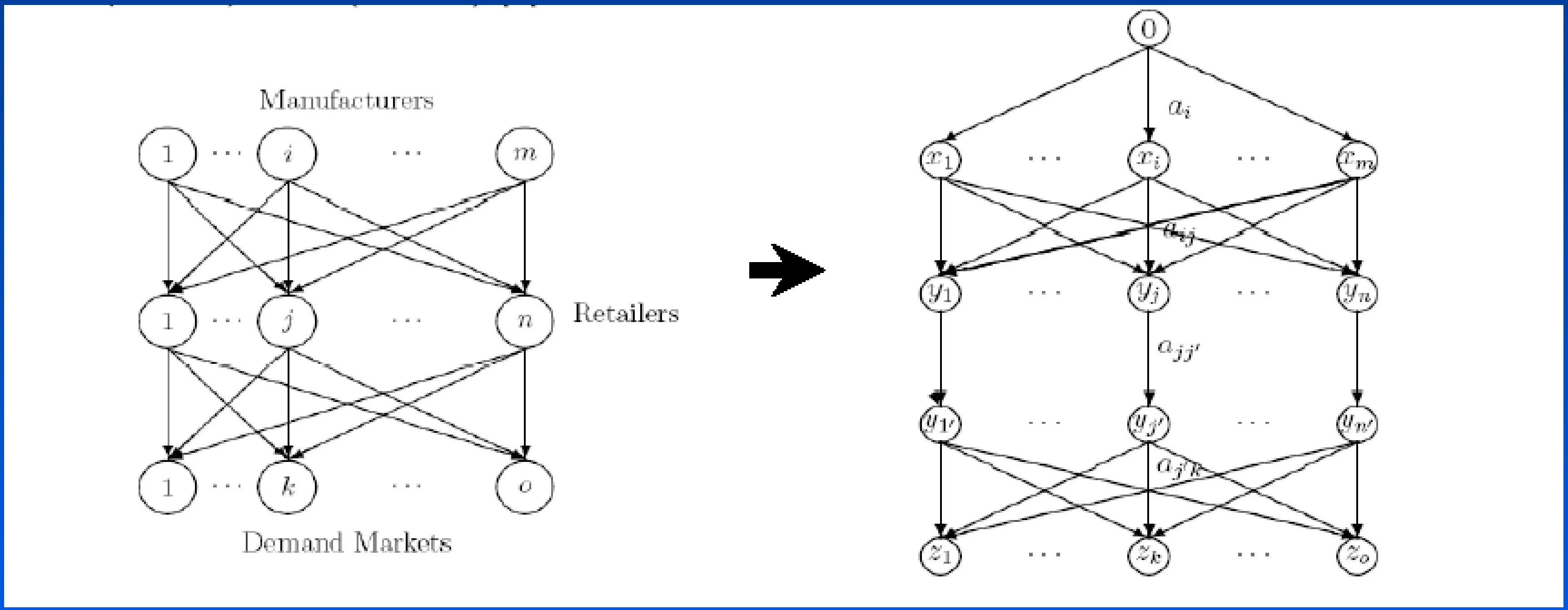
We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

Transportation and Fragile Network Systems

The TNE Paradigm is the Unifying Paradigm for:

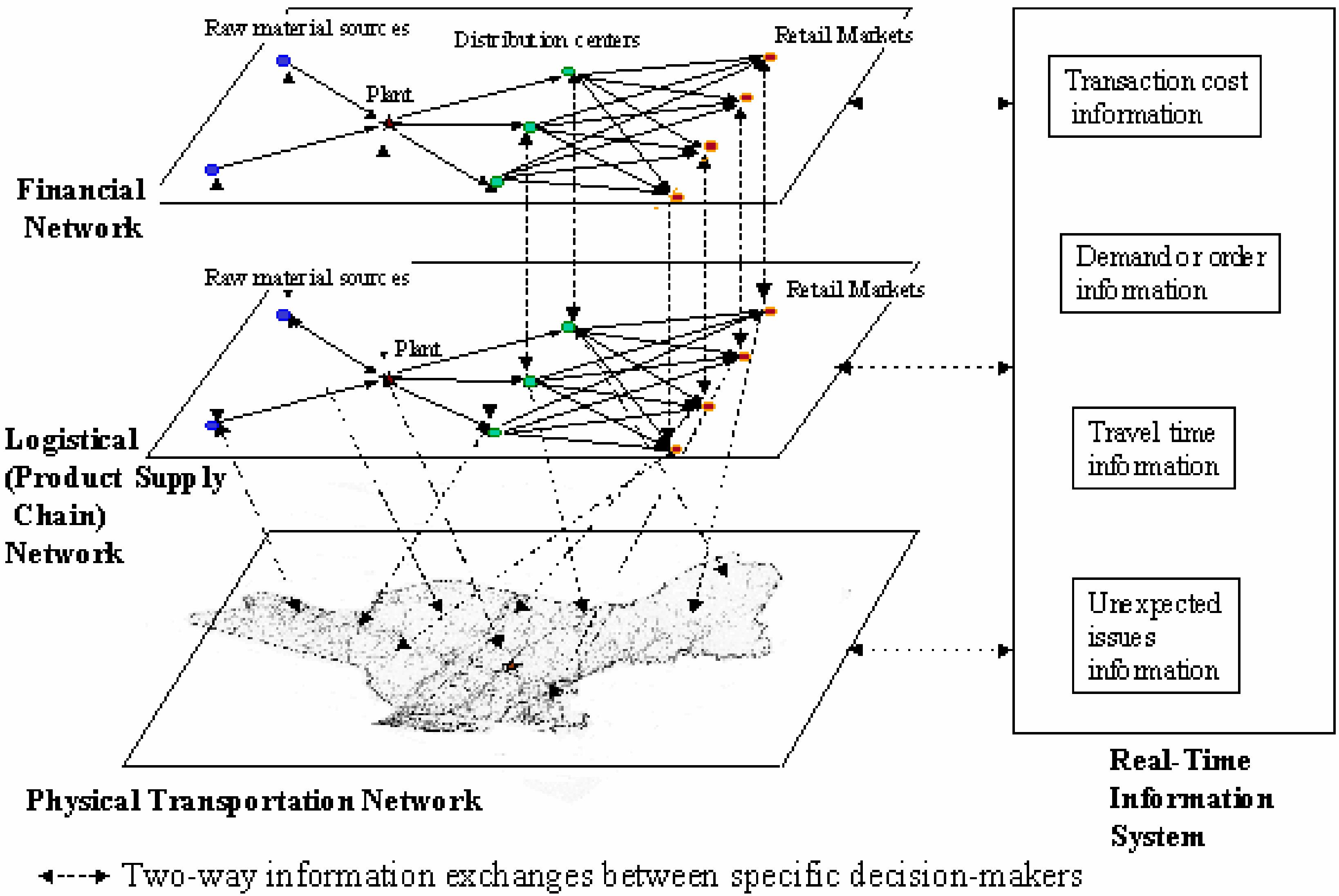
- Transportation Networks
- The Internet
- Financial Networks
- Electric Power Supply Chains.

The Equivalence of Supply Chains and Transportation Networks



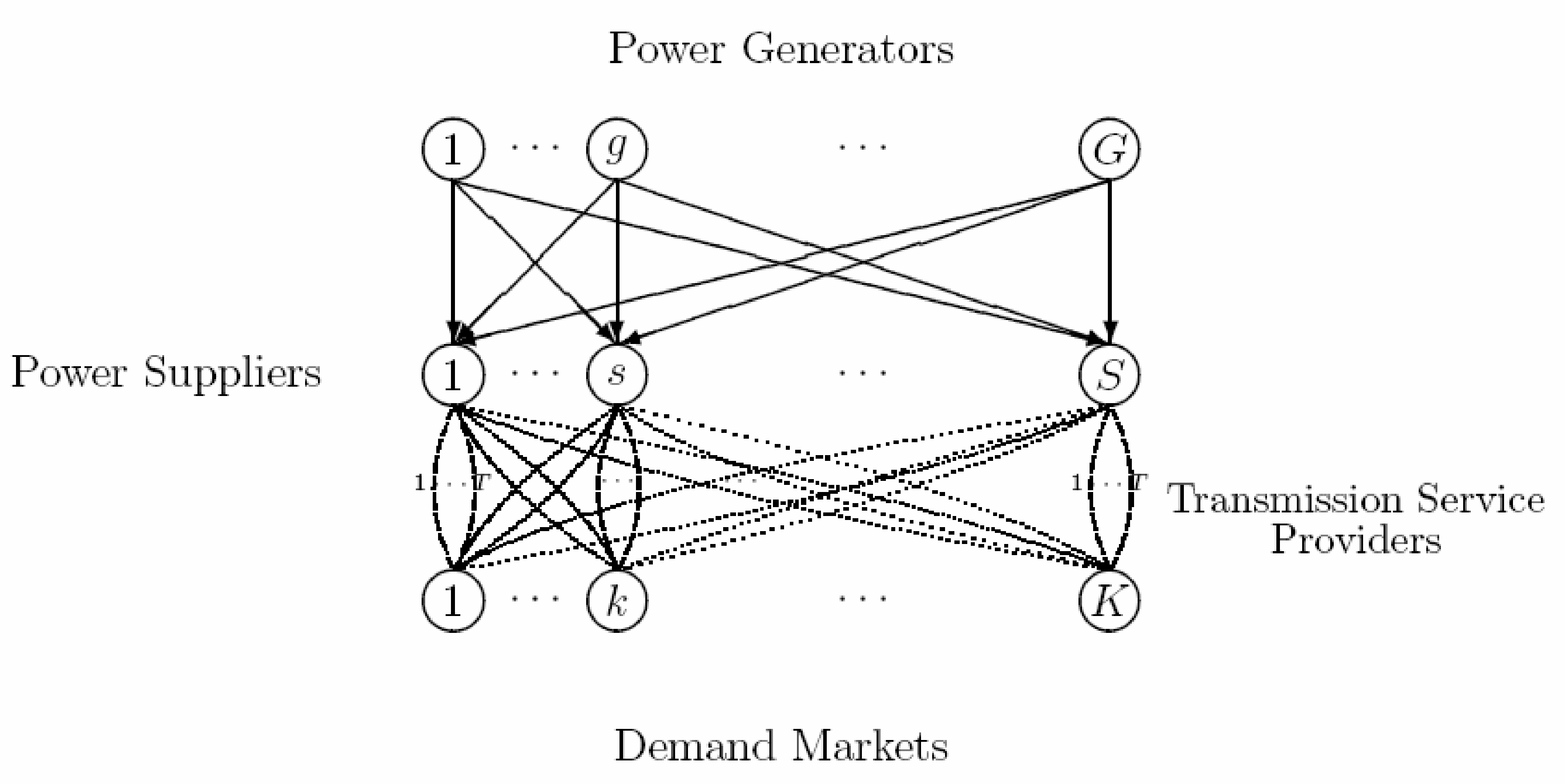
Nagurney, *Transportation Research E* 42: (2006) pp 293-316

Supply Chain -Transportation Supernetwork Representation



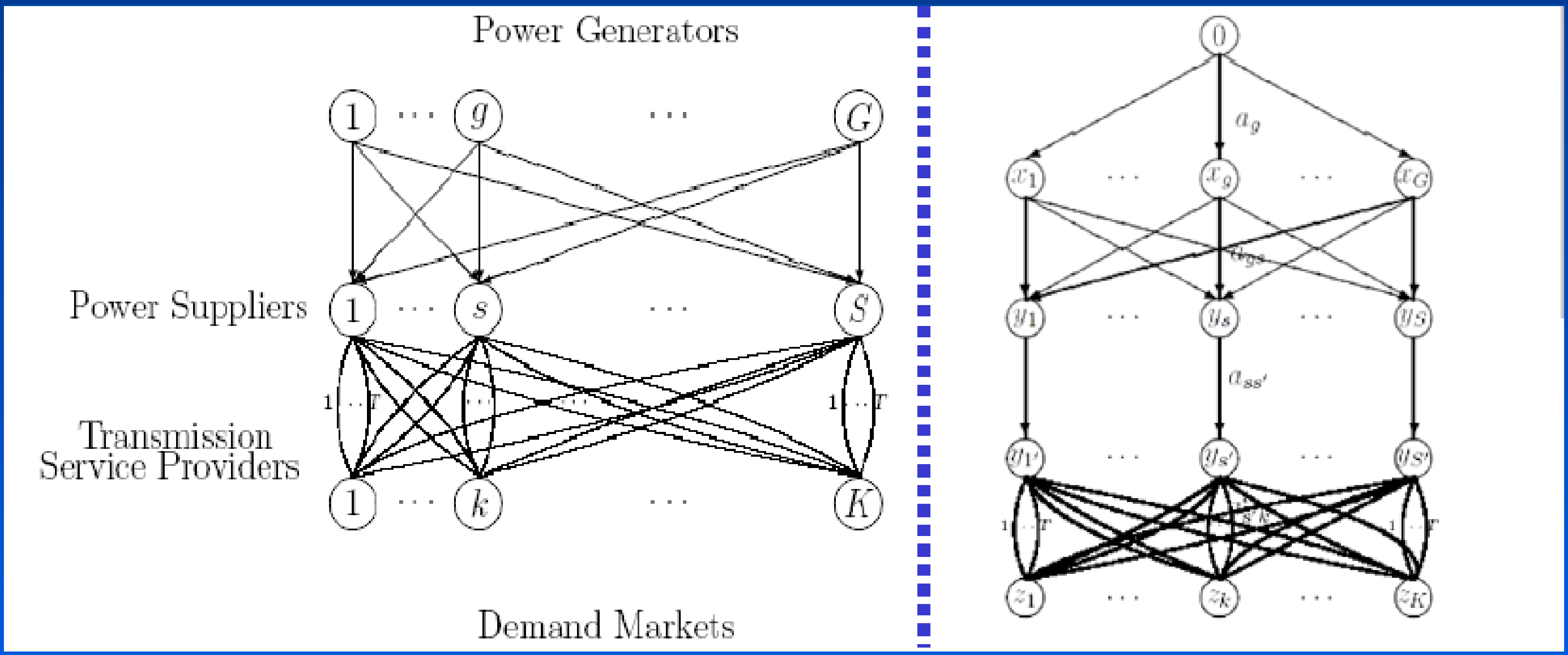
Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* (2002).

The Electric Power Supply Chain Network



Nagurney and Matsypura, *Proceedings of the CCCT* (2004).

The Transportation Network Equilibrium Reformulation of Electric Power Supply Chain Networks



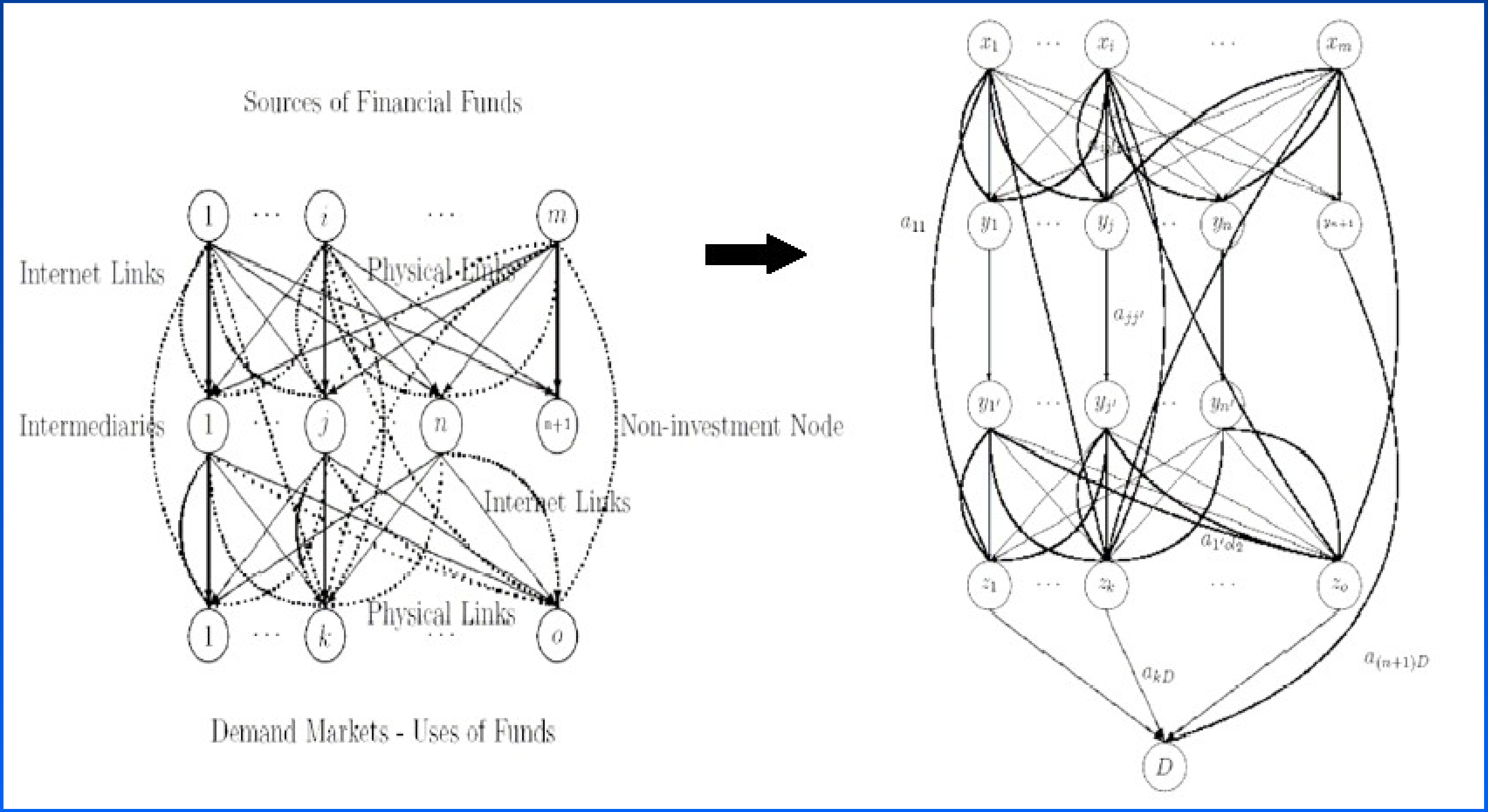
Electric Power Supply Network

Transportation Chain Network

Nagurney et al., *Transportation Research E 43: (2007) pp 624-646*

In 1952, Copeland wondered whether money flows like water or electricity.

The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation



Liu and Nagurney, *Computational Management Science* 4: (2007) pp 243-281

We have shown that *money* as well as *electricity* flow like *transportation* and have answered questions posed fifty years ago by Copeland and by Beckmann, McGuire, and Winsten!

Recent disasters have demonstrated the importance and the vulnerability of network systems.

Examples include:

- 9/11 Terrorist Attacks, September 11, 2001;
- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 -- one in the UK and the other in Italy and Switzerland;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge Collapse, August 1, 2007.

Earthquake Damage

prcs.org.pk

Tsunami

letthesunshinein.wordpress.com

Storm Damage

www.srh.noaa.gov

Infrastructure Collapse

www.10-7.com

Electric Power Network Disasters

media.collegepublisher.com

www.cellar.org

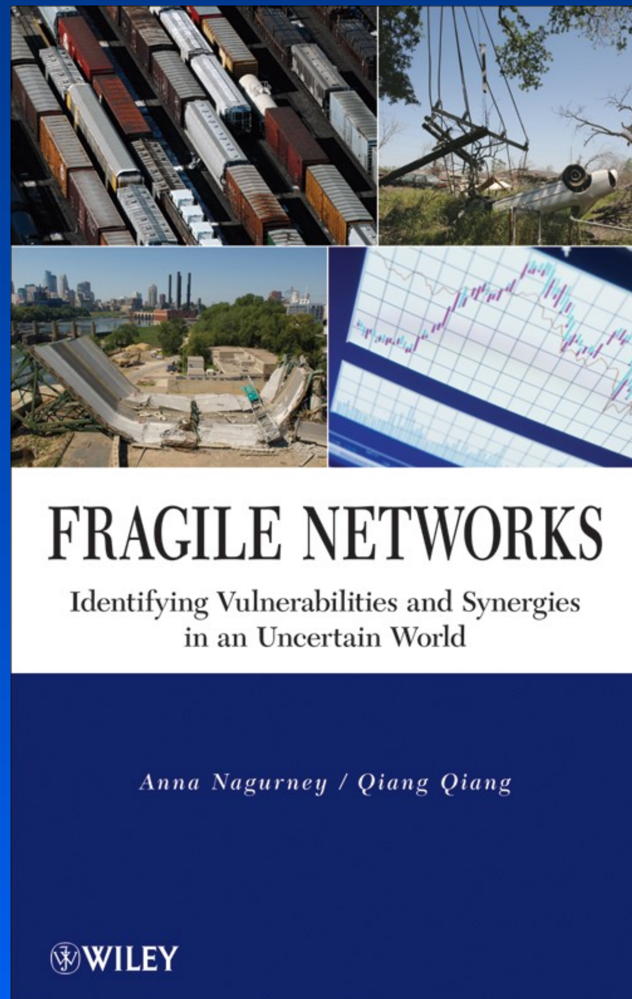
www.crh.noaa.gov

Communication Network Disasters

www.tx.mb21.co.uk

www.w5jgv.com

www.wirelessestimator.com


FRAGILE NETWORKS
Identifying Vulnerabilities and Synergies
in an Uncertain World
Anna Nagurney / Qiang Qiang
WILEY

The Nagurney and Qiang (N-Q) Network Efficiency Measure

The network performance/efficiency measure $\mathcal{E}(G,d)$, for a given network topology G and fixed demand vector d, is defined as

$$\mathcal{E}(G,d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where n_w is the number of O/D pairs in the network and λ_w is the equilibrium disutility for O/D pair w.

Nagurney and Qiang, *Europhysics Letters,* **79** (2007).

Importance of a Network Component

Definition: Importance of a Network Component

The importance, *I(g),* of a network component *gεG* is measured by the relative network efficiency drop after *g* is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)},$$

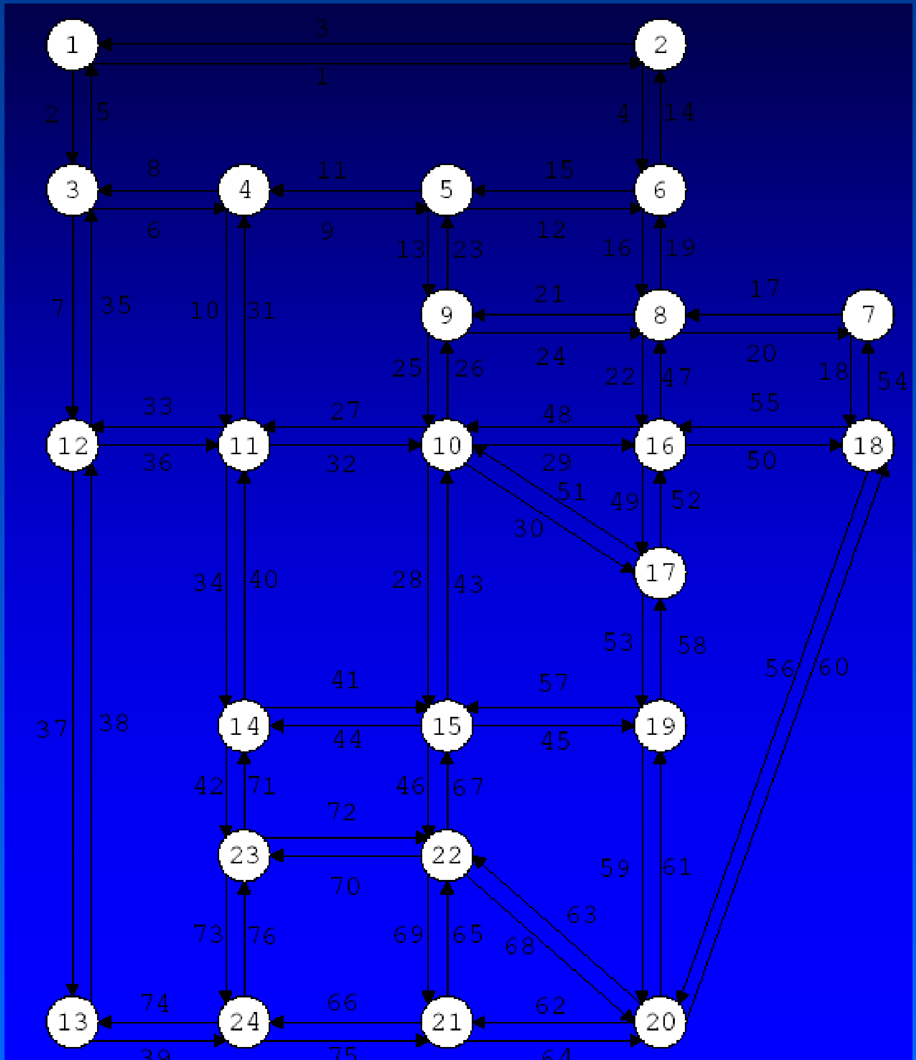
where *G-g* is the resulting network after component g is removed.

Example - Sioux Falls Network

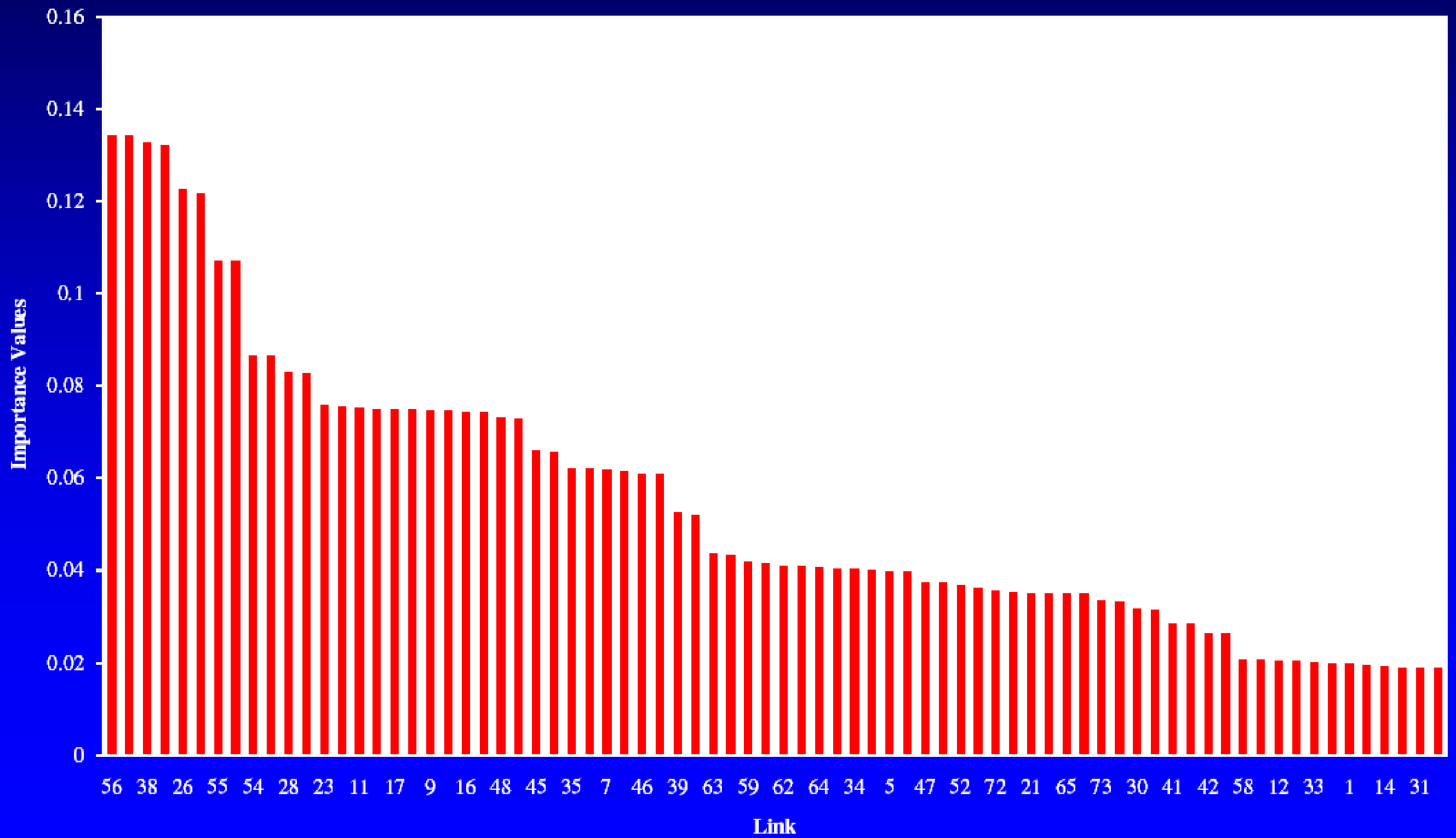
The network data are from LeBlanc, Morlok, and Pierskalla (1975).

The network has 528 O/D pairs, 24 nodes, and 76 links.

The user link cost functions are of Bureau of Public Roads (BPR) form.



Example - Sioux Falls Network Link Importance Rankings



Motivation for Research on Transportation Network Robustness

According to the American Society of Civil Engineering:

Poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of $94 billion in the US in terms of needed repairs for roads alone.

Poor road conditions in the United States cost US motorists $54 billion in repairs and operating costs annually.

The focus of the robustness of networks (and complex networks) has been on the impact of different network measures when facing the removal of nodes on networks.

We focus on the *degradation of links through reductions in their capacities* and the effects on the induced travel costs in the presence of known travel demands and different functional forms for the links.

What About Dynamic Networks?

We are using evolutionary variational inequalities to model dynamic networks with:

- *dynamic (time-dependent)* supplies and demands
- *dynamic (time-dependent)* capacities
- *structural changes* in the networks themselves.

Such issues are important for robustness, resiliency, and reliability of networks (including supply chains and the Internet).

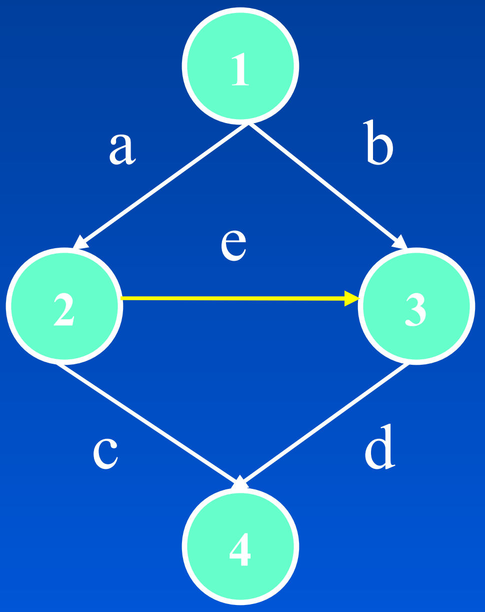
A network like the Internet is volatile. Its traffic patterns can change quickly and dramatically... The assumption of a static model is therefore particularly suspect in such networks. (page 10 of Roughgarden's (2005) book, *Selfish Routing and the Price of Anarchy*).

A Dynamic Model of the Internet

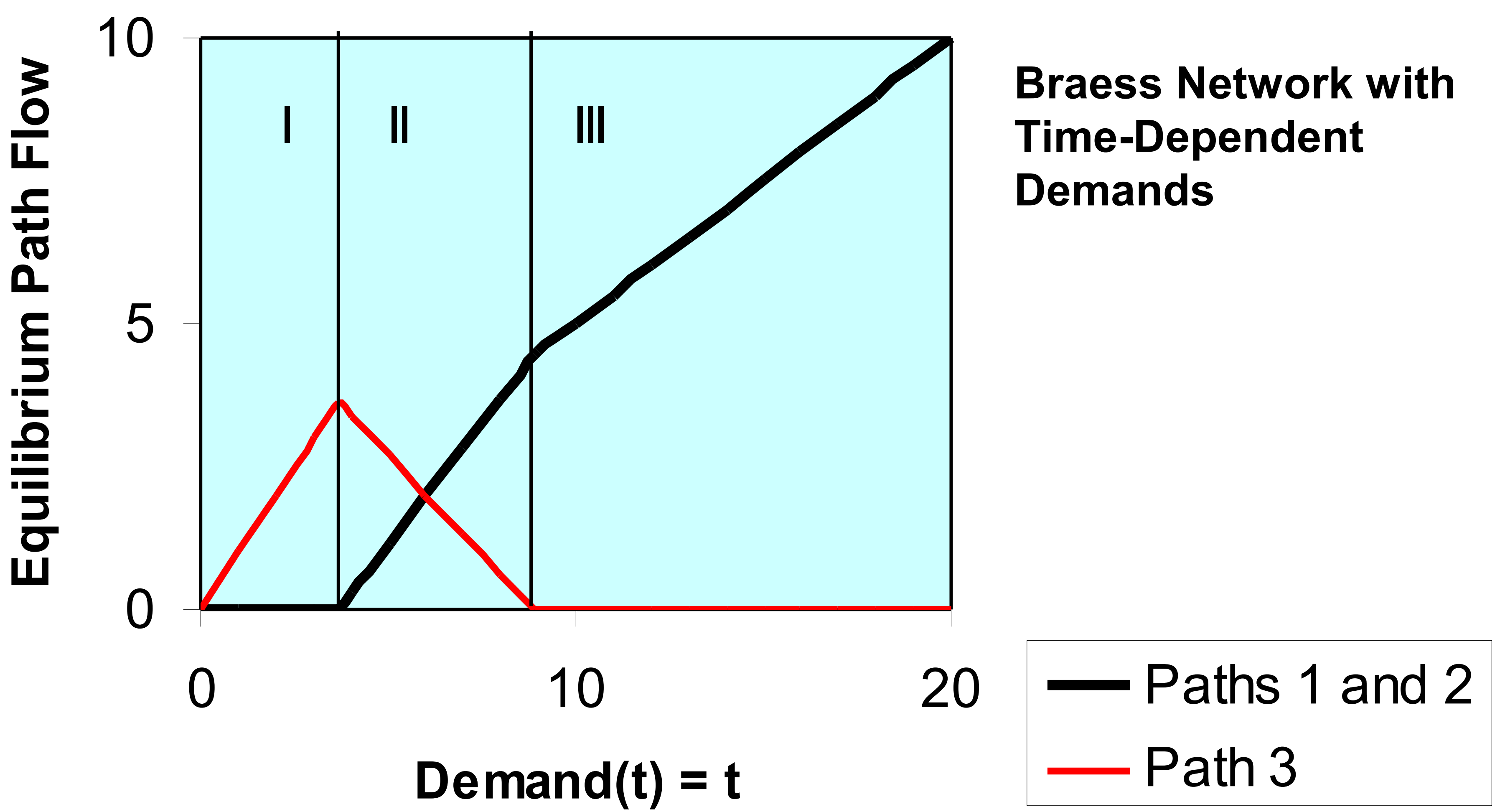
The Internet, Evolutionary Variational Inequalities, and the Time-Dependent Braess Paradox, Nagurney, Parkes, and Daniele, *Computational Management Science* **4** (2007), 355-375.

The Time-Dependent (Demand-Varying) Braess Paradox and Evolutionary Variational Inequalities

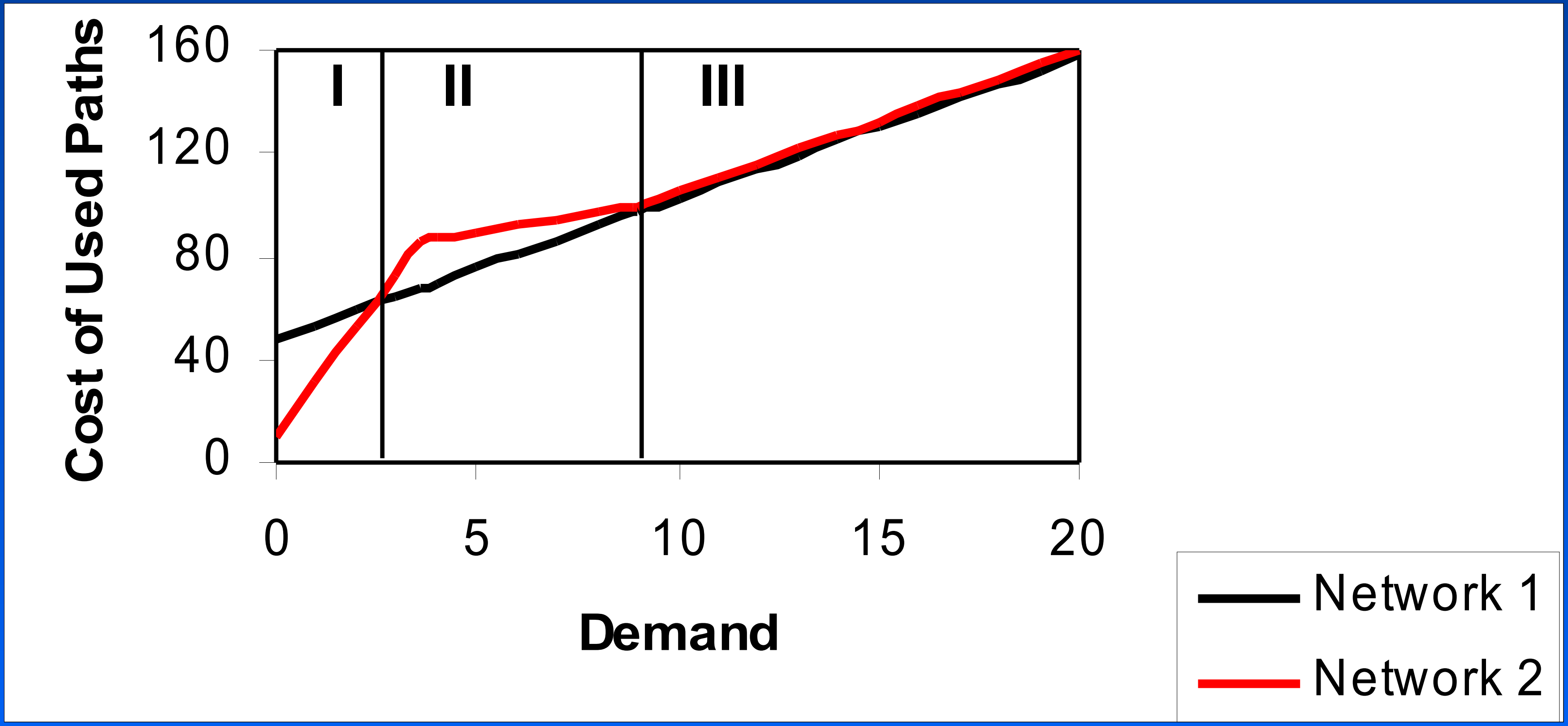
Recall the Braess Network where we add the link e.



The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)



In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

Extension of the Network Efficiency Measure to Dynamic Networks

An Efficiency Measure for Dynamic Networks Modeled as Evolutionary Variational Inequalities with Applications to the Internet and Vulnerability Analysis, Nagurney and Qiang, *Netnomics* (2008).

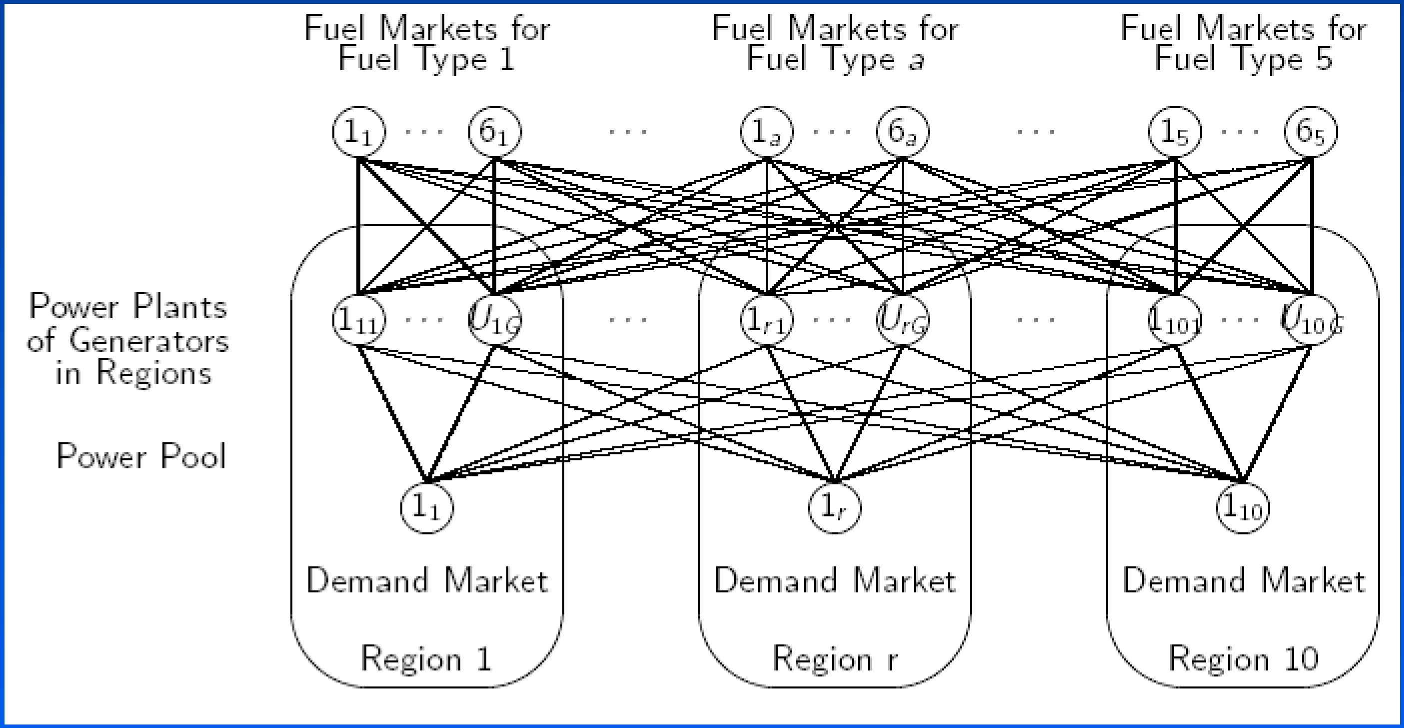
Where Are We Now?

An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England, Liu and Nagurney, *Naval Research Logistics*, 56, pp.600-624 (2009).

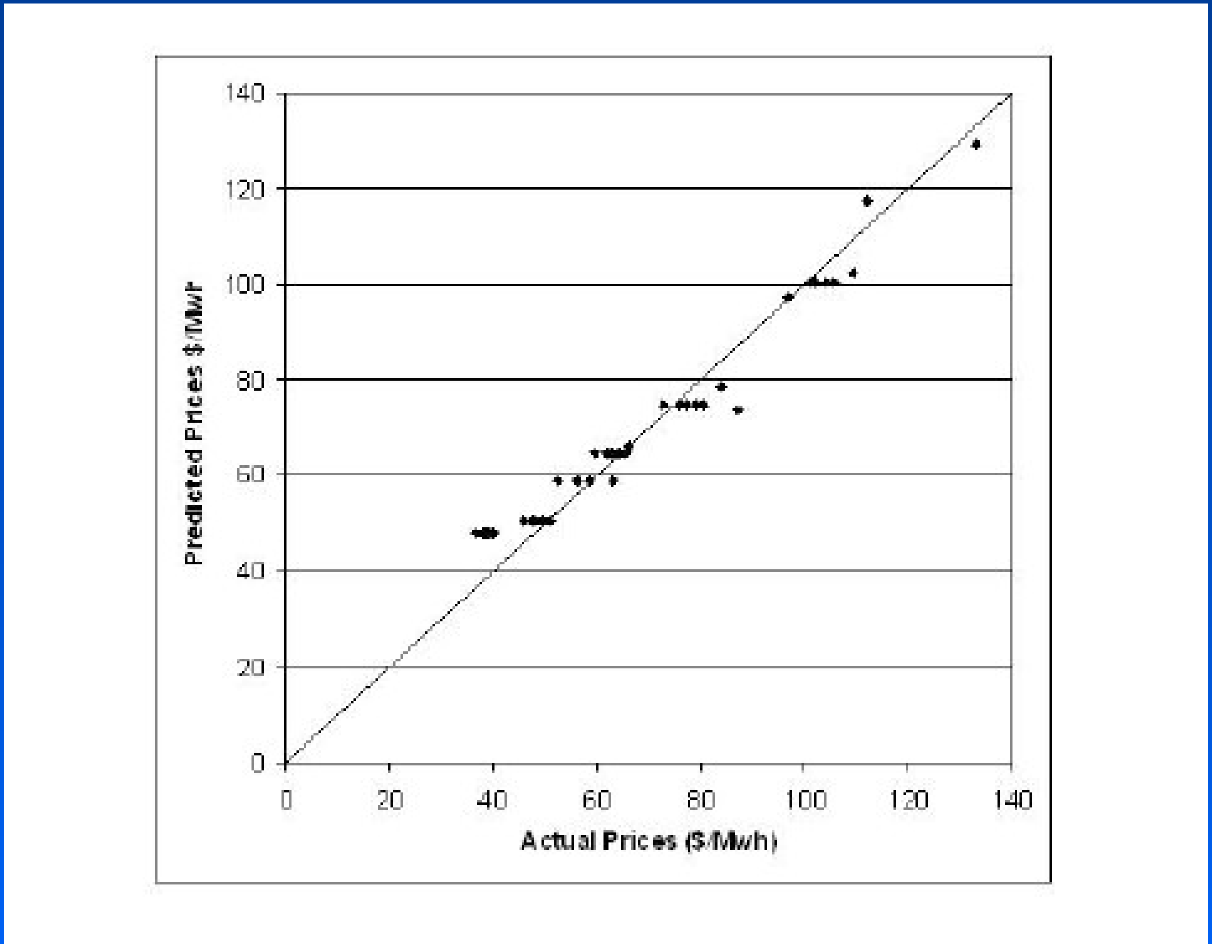
Empirical Case Study

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Ten regions (R=10): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut(excluding Southwest Connecticut), 5. Southwest Connecticut(excluding Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeast Massachusetts, 9. West and Central Massachusetts, 10. Boston/Northeast Massachusetts
- Hourly demand/price data of July 2006 ($24 \times 31 = 744$ scenarios)
- 6 blocks (L1 = 94 hours, and Lw = 130 hours; w = 2, ..., 6)

The New England Electric Power Supply Chain Network with Fuel Suppliers

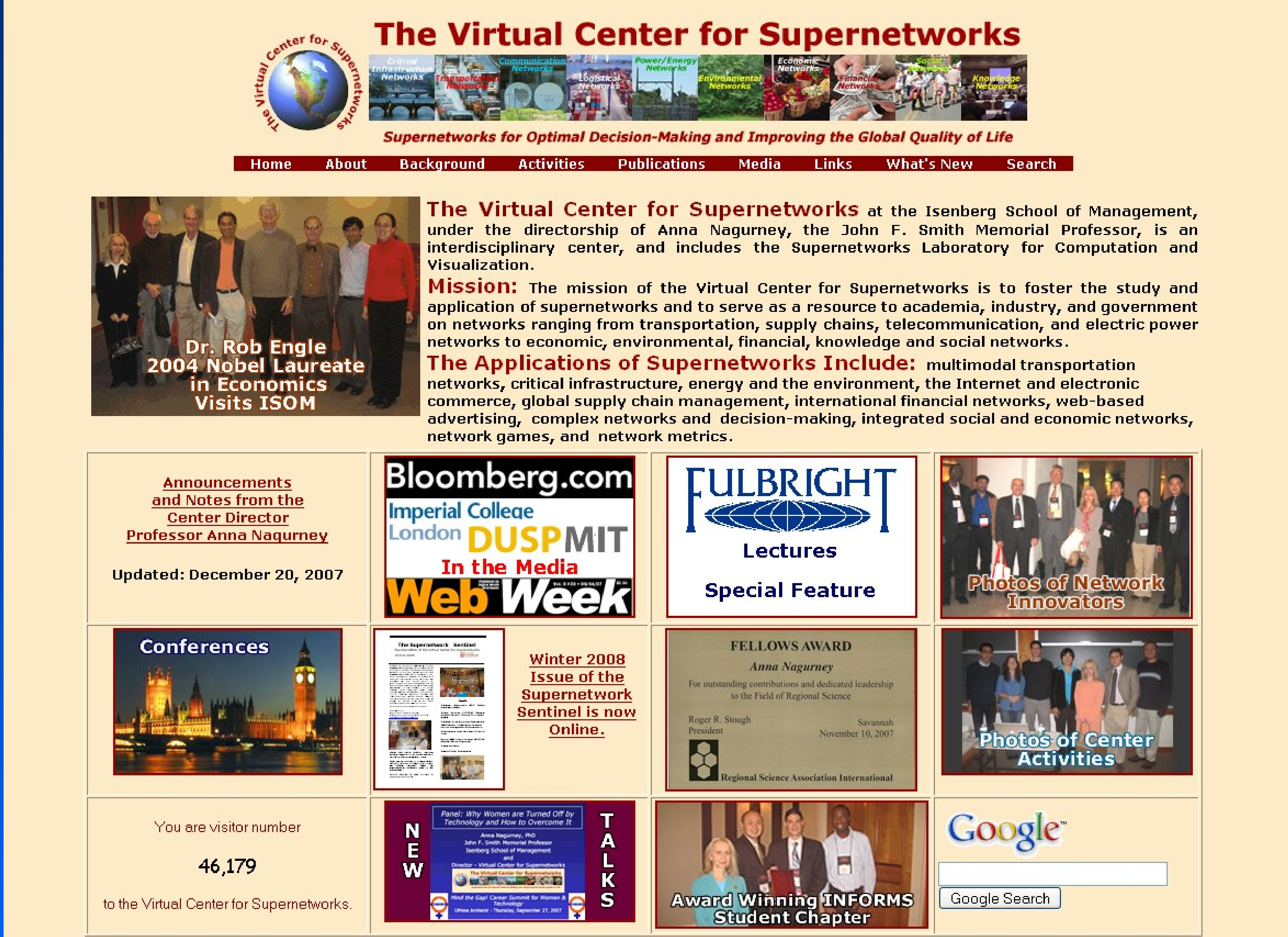


Predicted Prices vs. Actual Prices ($/Mwh)



Ongoing Research and Questions

- Identification of the most important nodes and links in large-scale electric power supply chains as in our empirical case study and in financial networks.
- Design of networks to handle dynamic demands and various uncertainties (cost, demand, etc.) under system-optimizing or user-optimizing behaviors.
- Quantification of network synergies through mergers and acquisitions.
- Further research on network resilience and robustness.
- Extension of the network efficiency measure to handle different kinds of risk; we have completed some work in this dimension.



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The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

Mission: The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, supply chains, telecommunication, and electric power networks to economic, environmental, financial, knowledge and social networks.

The Applications of Supernetworks Include: multimodal transportation networks, critical infrastructure, energy and the environment, the Internet and electronic commerce, global supply chain management, international financial networks, web-based advertising, complex networks and decision-making, integrated social and economic networks, network games, and network metrics.

Announcements and Notes from the Center Director Professor Anna Nagurney

Updated: December 20, 2007

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