

Supernetworks for the Management of Knowledge Intensive Dynamic Systems Workshop

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Agenda

- **Introduction**
- **The importance of networks**
- **The concept of supernetworks and their applications**
- **Knowledge supernetworks**
- **Discussion**

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The Supernetwork Team



New Era of Decision-Making

- Increasing ***risk*** and uncertainty
- Importance of ***dynamics*** and realizing a fast and sound response to evolving events
- ***Complex interactions*** among decision-makers in organizations
- Alternative and at times conflicting ***criteria*** used in decision-making
- ***Global reach*** of many decisions
- ***High impact*** of many decisions

Network-Based New Era

- **Internet**
- **Transportation/logistical networks**
- **Other telecommunication networks**
- **Energy/power networks**

No longer are *networks* independent of one another but *critically linked* with major questions arising regarding decision-making and appropriate management tools.

Classical Networks

| Network System | Nodes | Links | Flows |
|--------------------------------|--|--|---|
| Transportation | | | |
| Urban | Intersections, Homes, Places of Work | Roads | Autos |
| Air | Airports | Airline Routes | Planes |
| Rail | Railyards | Railroad Track | Trains |
| Manufacturing and Logistics | Distribution Points, Processing Points | Routes Assembly Line | Parts, Products |
| Communication | Computers Satellites Phone Exchanges | Cables Radio Cables, Microwaves | Messages Messages Voice, Video |
| Energy | Pumping Stations Plants | Pipelines Pipelines | Water Gas, Oil |

Importance of Networks

- ***Throughout history, networks have served as the foundation for connecting humans to one another and their activities.***
- **Roads were laid, bridges built, and waterways crossed so that humans, be they on foot, on animal, or vehicle could traverse physical distance through *transportation*. The airways were conquered through flight.**
- ***Communications* were conducted using the available means of the period, from smoke signals, drum beats, and pigeons, to the telegraph, telephone, and computer networks of today.**

Importance of Networks to the Economy

- US consumers, businesses, and governments spent ***\$950 billion on transportation*** in 1998 (US DOT).
- Corporate buyers spent ***\$517.6 billion on telecommunications*** in 1999 (Purchasing).
- ***Energy expenditures*** in the United States were ***\$515.8 billion*** in 1995 (US Dept. of Commerce).

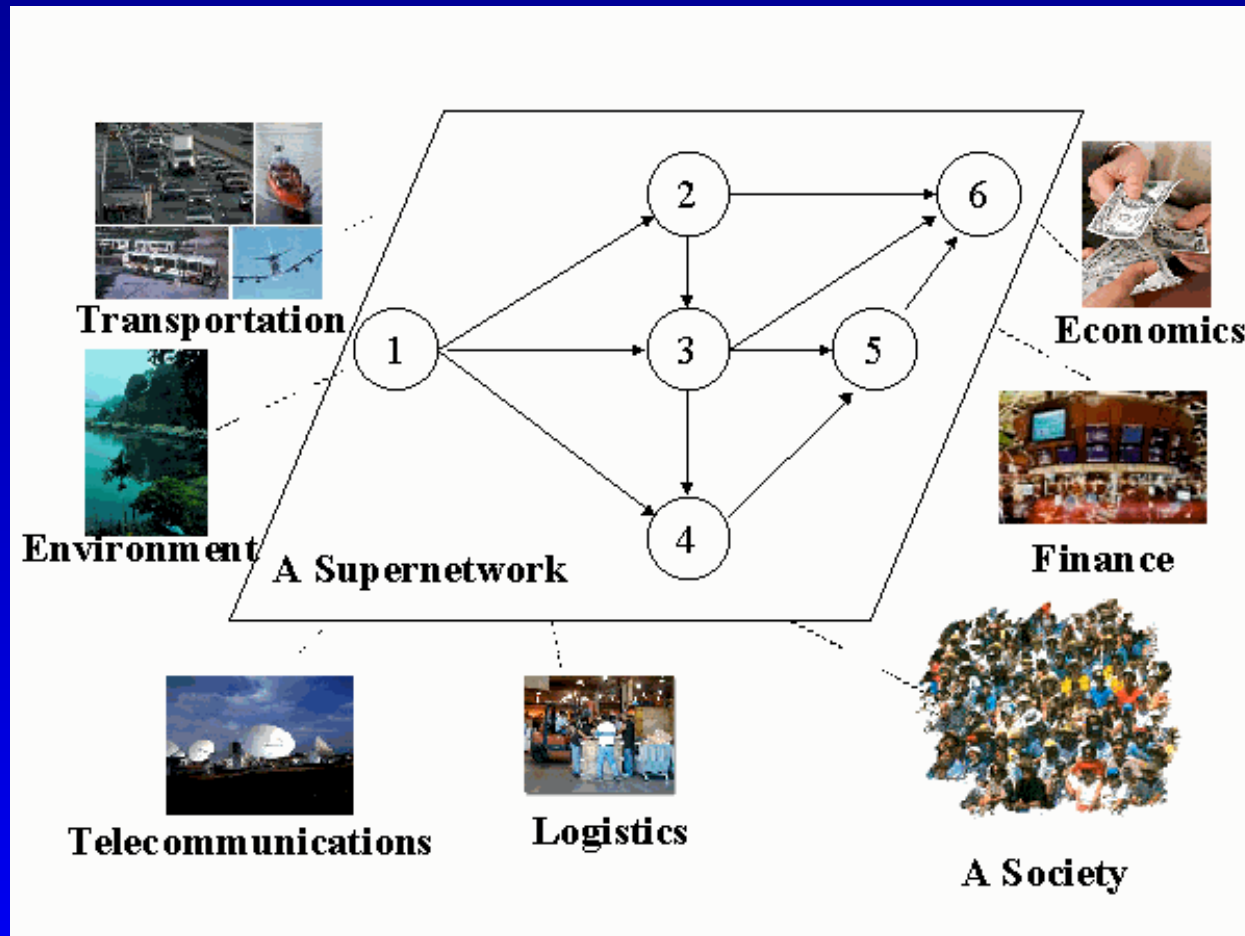
Reality of Today's Networks:

- ***Large-scale nature*** and complexity of network topology
- ***Congestion***
- **Alternative behavior of users of the network, which may lead to *paradoxical phenomena***
- **The *interactions among networks* themselves such as in transportation versus telecommunications networks**
- ***Policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise.***

Supernetworks: A New Paradigm

- **Supernetworks may be comprised of such networks as *transportation, telecommunication, logistical, and/or financial networks.***
- **They may be *multilevel* as when they formalize the study of supply chain networks or *multitiered* as in the case of financial networks with intermediation.**
- **Decision-makers may be faced with multiple criteria; thus, the study of supernetworks also includes the study of *multicriteria decision-making.***

A Supernetwork



Supernetworks

- ***Provide tools*** to study interrelated networks
- Allow us to apply ***efficient algorithms*** for computation
- Provide ***visual aids*** to see the dynamic changes



Supernetworks

Decision-Making for the Information Age

Anna Nagurney

June Dong

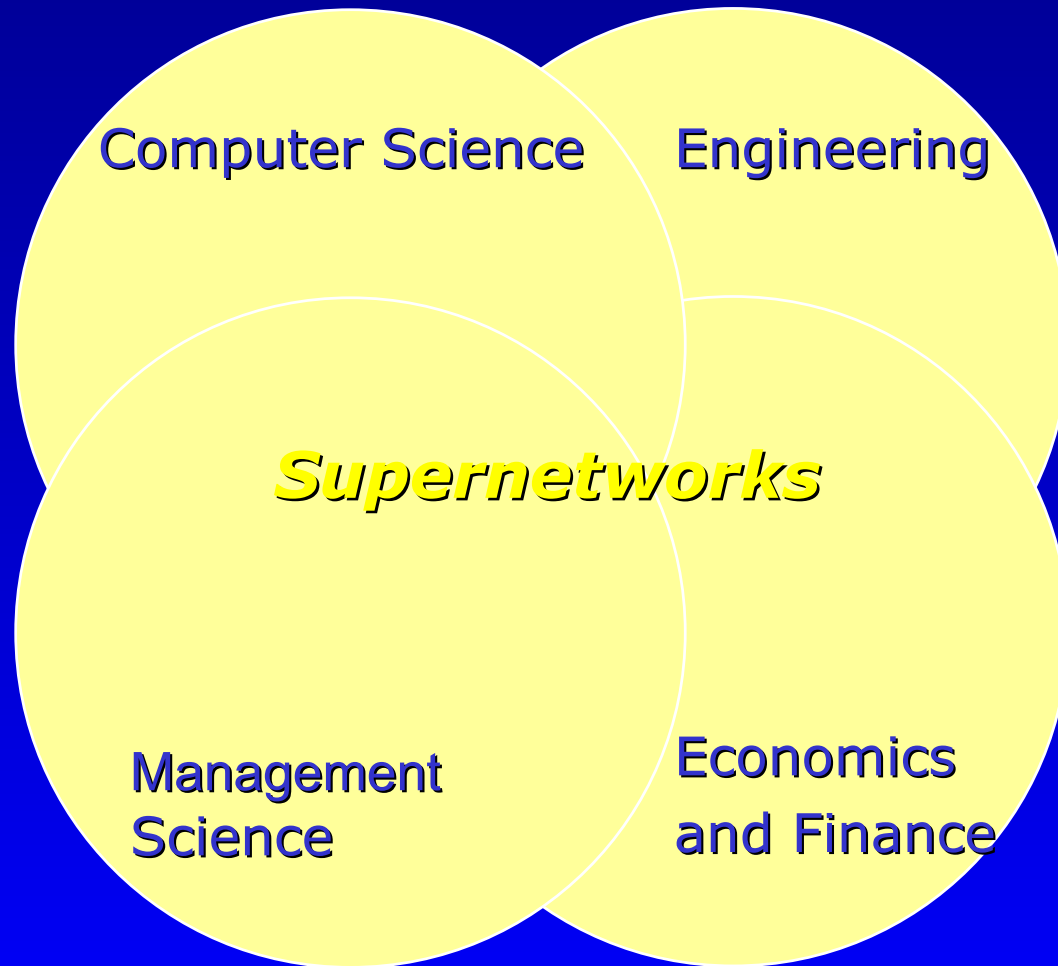


New Dimensions in Networks

Tools That We Have Been Using

- **Network theory**
- **Optimization theory**
- **Game theory**
- **Variational inequality theory**
- **Projected dynamical systems theory
(which we have been instrumental in
developing)**
- **Network visualization tools**

A Multidisciplinary Paradigm



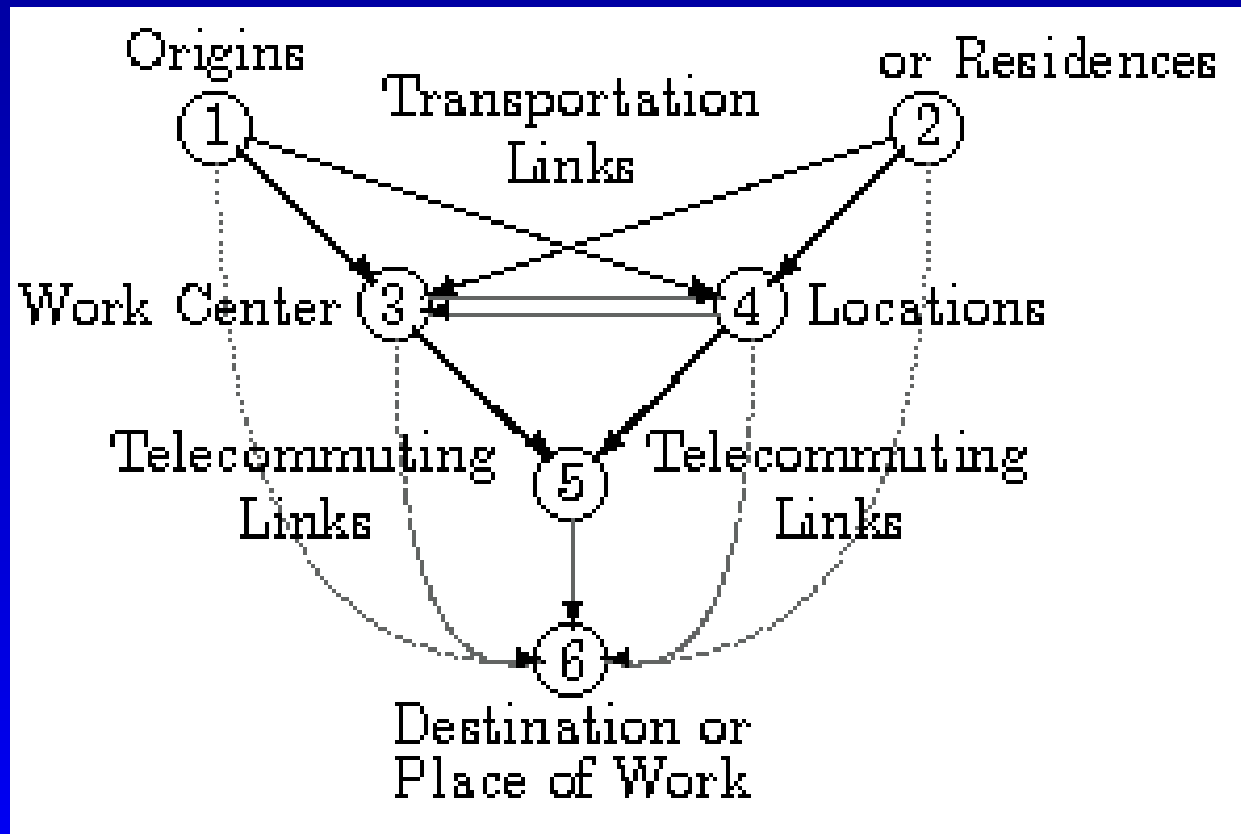
Applications of Supernetworks

- **Telecommuting/Commuting Decision-Making**
- **Teleshopping/Shopping Decision-Making**
- **Supply Chain Networks with Electronic Commerce**
- **Financial Networks with Electronic Transactions**
- **Reverse Supply Chains with E-Cycling**
- **Energy Networks/Power Grids**
- **Knowledge Networks**

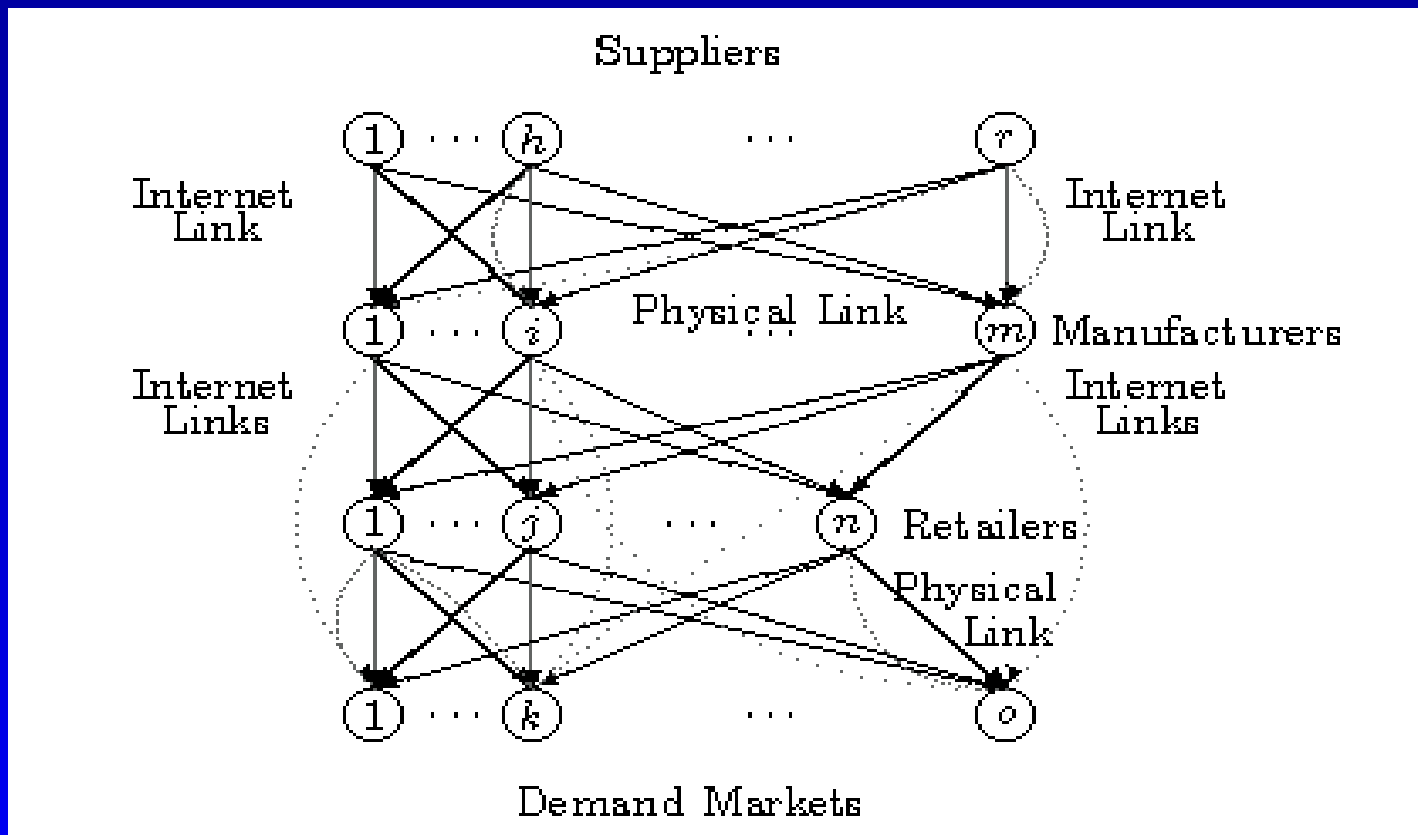
Examples from the Supernetwork Literature

- **A Space-Time Network for Telecommuting versus Commuting Decision Making**
Anna Nagurney, June Dong, and Patricia L. Mokhtarian, *Papers in Regional Science* 82: (2003) pp 451-473.
- **Dynamics of Supply Chains: A Multilevel (Logistical/Informational/Financial) Network Perspective**
Anna Nagurney, Ke Ke, Jose Cruz, Kitty Hancock, and Frank Southworth, *Environment and Planning B* 29: (2002) pp 795-818.

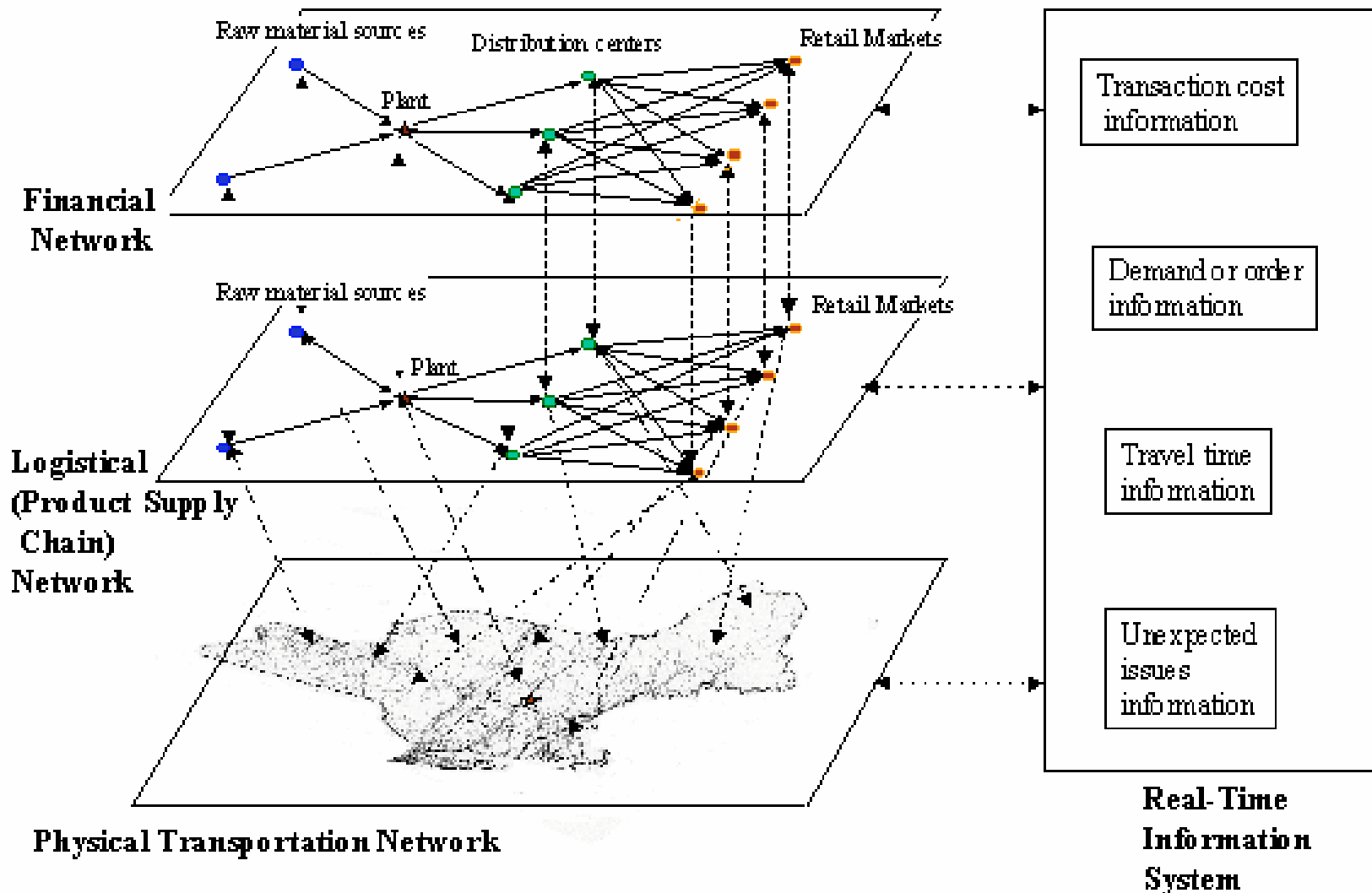
A Supernetwork Conceptualization of Commuting vs. Telecommuting



The Supernetwork Structure of a Supply Chain Network



Supply Chain -Transportation Supernetwork Representation

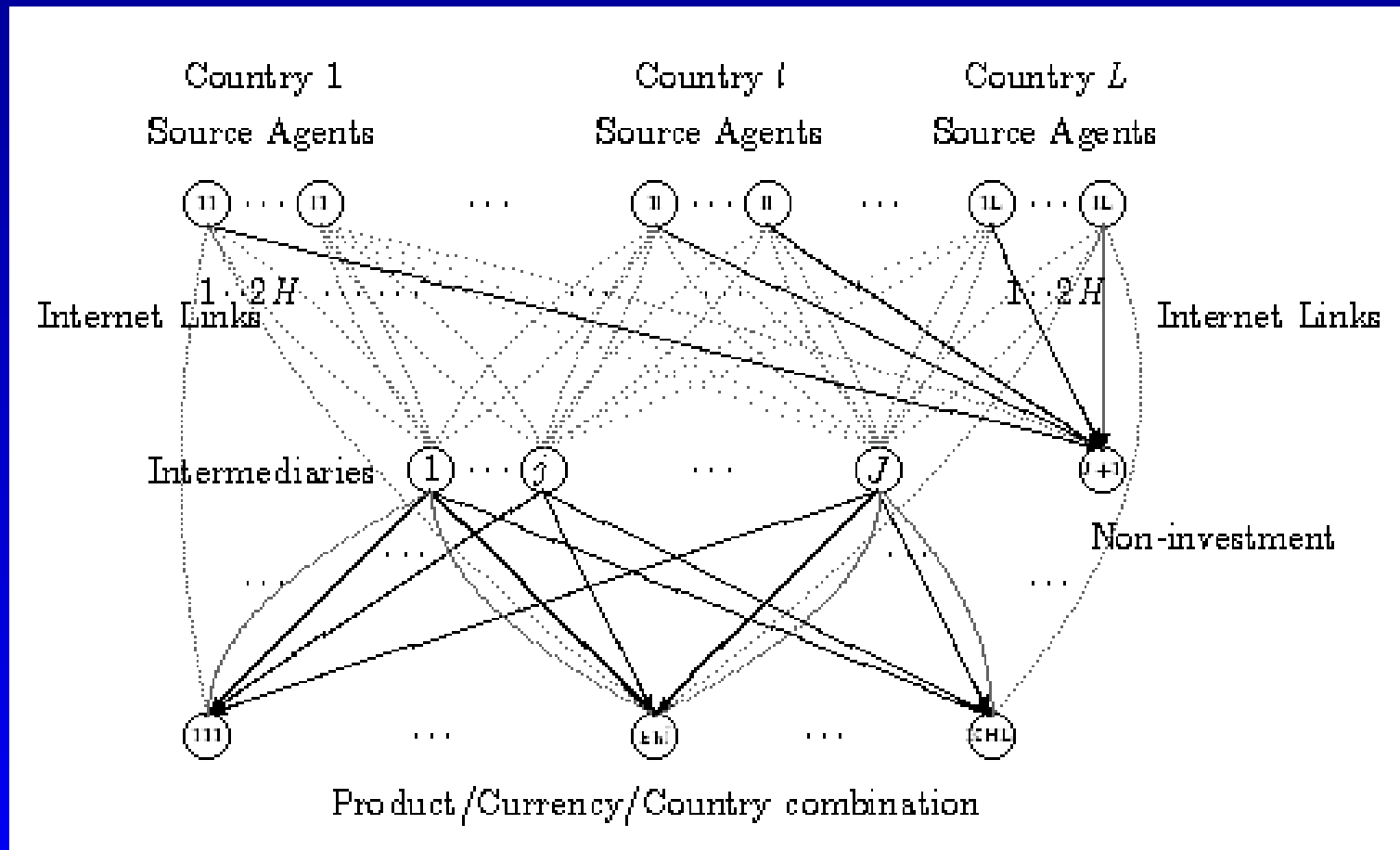


←---→ Two-way information exchanges between specific decision-makers

Examples from the Financial Networks Literature

- **Financial Networks with Electronic Transactions: Modeling, Analysis and Computations**
Anna Nagurney and Ke Ke, *Quantitative Finance* 3 (2003): pp 71-87.
- **International Financial Networks with Electronic Transactions**
Anna Nagurney and Jose Cruz in *Innovations in Financial and Economic Networks*, Anna Nagurney, editor, Edward Elgar Publishing (2003): pp 136-168.

International Financial Network with Electronic Transactions



Supernetworks Integrating Social Networks with Other Networks

- **Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations**
Tina Wakolbinger and Anna Nagurney, 2004
- **The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels**
Anna Nagurney, Tina Wakolbinger and Li Zhao, 2004

Supernetworks Integrating Social Networks with Other Networks

Models explicitly consider the role that relationship levels play in supply chain networks/financial networks.

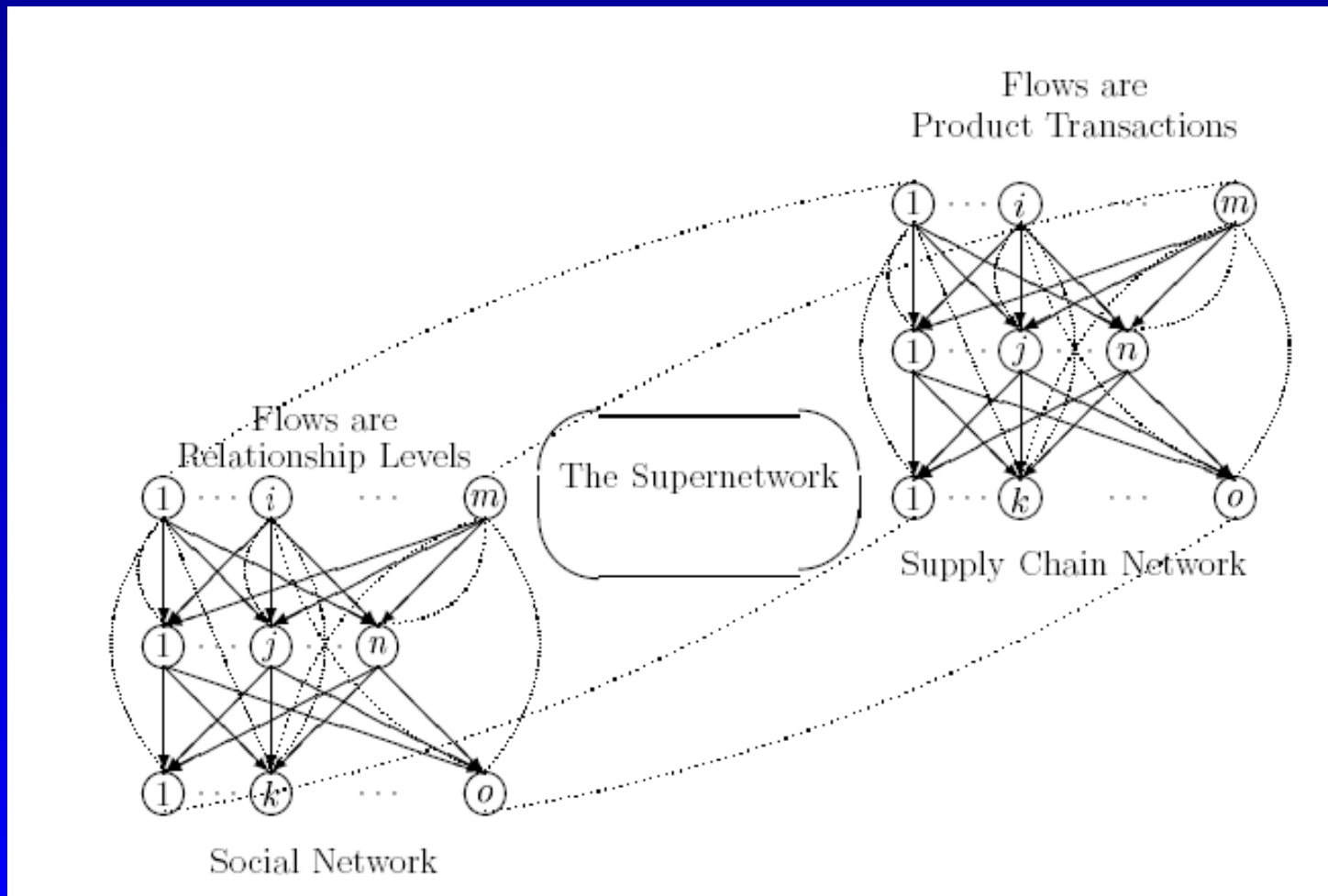
Supernetworks Integrating Social Networks with Other Networks

- Decision-makers in the network can decide about the the ***relationship levels*** [0,1] that they want to establish.
- Establishing relationship levels incurs some ***costs***.
- Higher relationship levels
 - ***Reduce transaction costs***
 - ***Reduce risk***
 - Have some additional value ("***relationship value***")

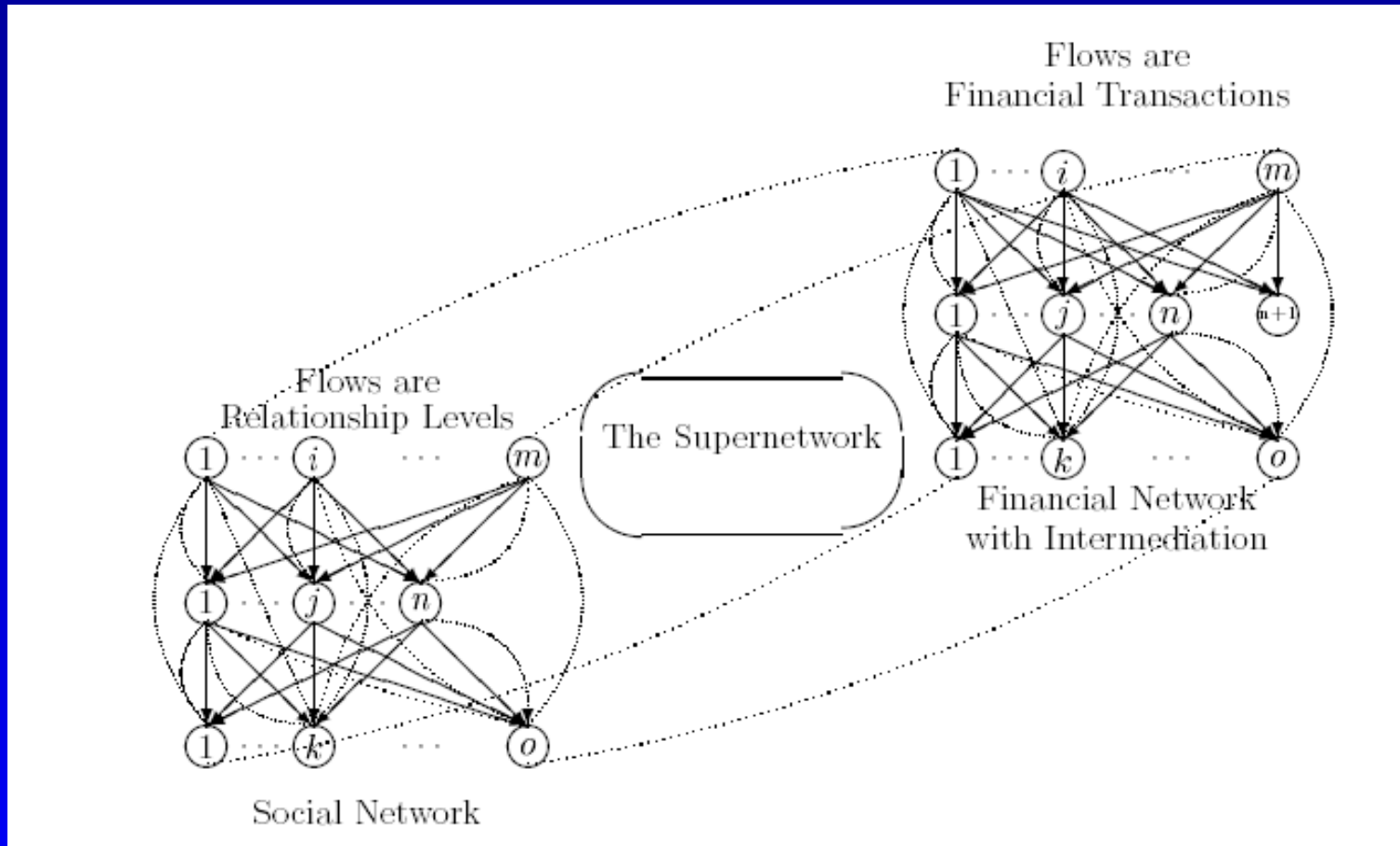
Dynamic evolution of:

- **Product transactions/financial flows and associated prices on the supply chain network/financial network with intermediation**
- **Relationship levels on the social network**

Supernetwork Structure: Integrated Supply Chain/Social Network System



Supernetwork Structure: Integrated Financial/Social Network System



Management of Knowledge Intensive Dynamic Systems (MKIDS) Project

- ***Research objective***
 - **How information technologies can help streamline processes for organizations that must respond rapidly to incoming knowledge situations and uncertainty**
- ***Research paper***
 - **Management of Knowledge Intensive Systems as Supernetworks: Modeling, Analysis, Computations, and Applications; Anna Nagurney and June Dong
(To appear in *Mathematical and Computer Modelling*.)**

Challenges for Decision-Makers

- How to respond in a ***timely manner*** to new events
- How to best manage the ***scale and scope*** of their coverage and ultimately reach globally
- How to best manage various production allocation processes ***efficiently***
- Respond to their customers' needs in an environment of increasing ***uncertainty*** and ***risk***

Challenges for Decision-Makers

- **Large-scale**
- **Complexity**
- **Multicriteria**
- **Time**

Information management tools are needed

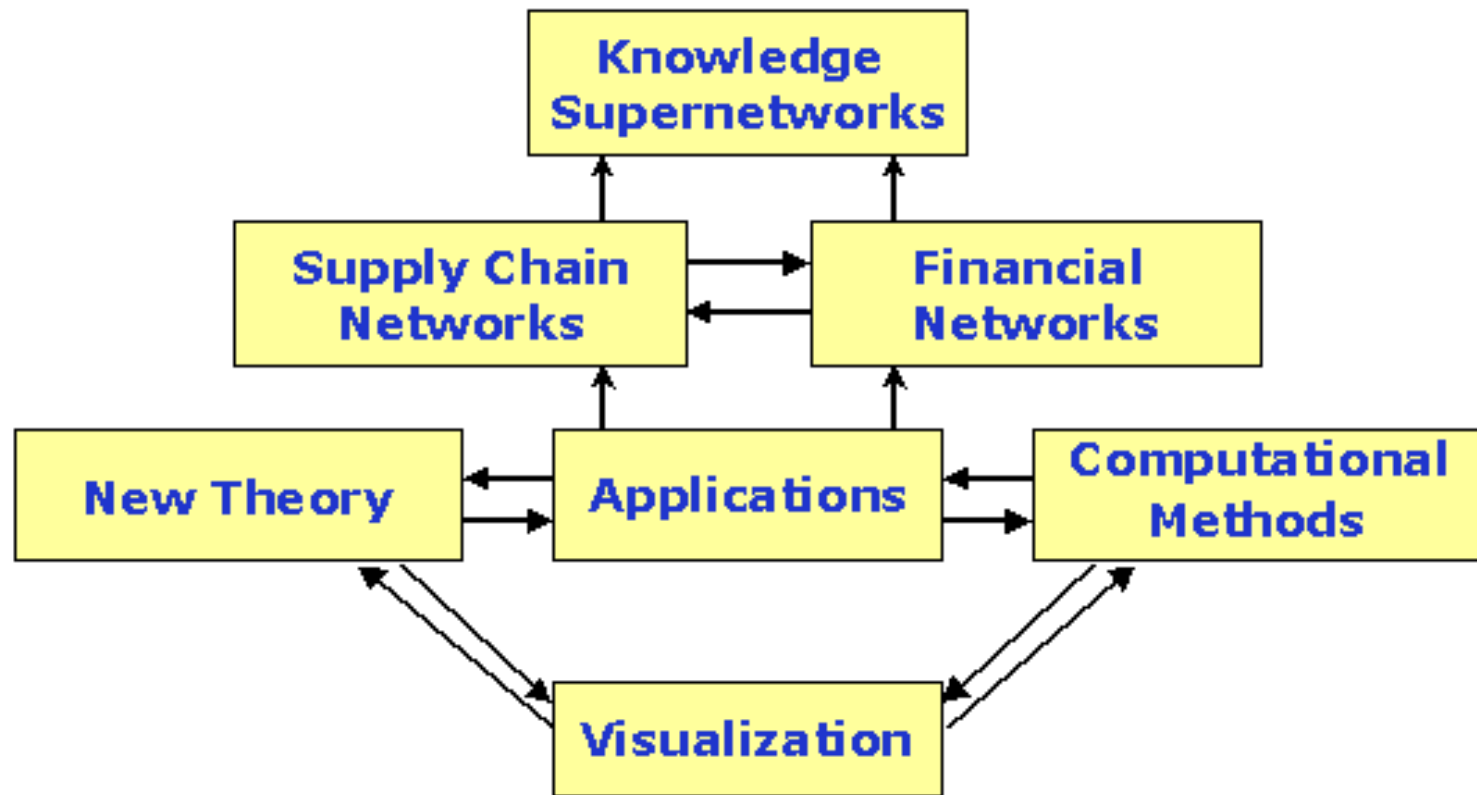
Research Discussion Questions

- **How would you define a knowledge product?**
- **What would be good examples of knowledge products?**
- **What are the differences between knowledge products and “regular” products?**
- **How does the production process of knowledge products differ from a “regular” production process?**
- **Who are the participants in the knowledge production process?**
- **What are their decision criteria?**

Goal of Knowledge Supernetworks

- **To capture graphical format**
- **To provide alternatives**
- **To determine the optimal allocation of resources**
- **To schedule the activities**

Schematic of Project Goals and Objectives



Examples of Knowledge Definitions

- **“Justified true belief” (Plato)**
- **“The whole set of insights, experiences and procedures which are considered correct and true and which, therefore, guide the thoughts, behaviors and communication of people”
(Van der Spek, 1997)**
- **“A fluid mix of framed experience, values, contextual information, and expert insight” (Davenport and Prusak, 1998)**

Knowledge Production

Knowledge production is “a series of transformations, by which standard resources, which are available in open markets, are used and combined within the organizational context in order to produce capabilities” (Ciborra and Andreu, 2001)

Measures of Knowledge Quality

- **Correctness**
- **Uniqueness**
- **Timeliness**

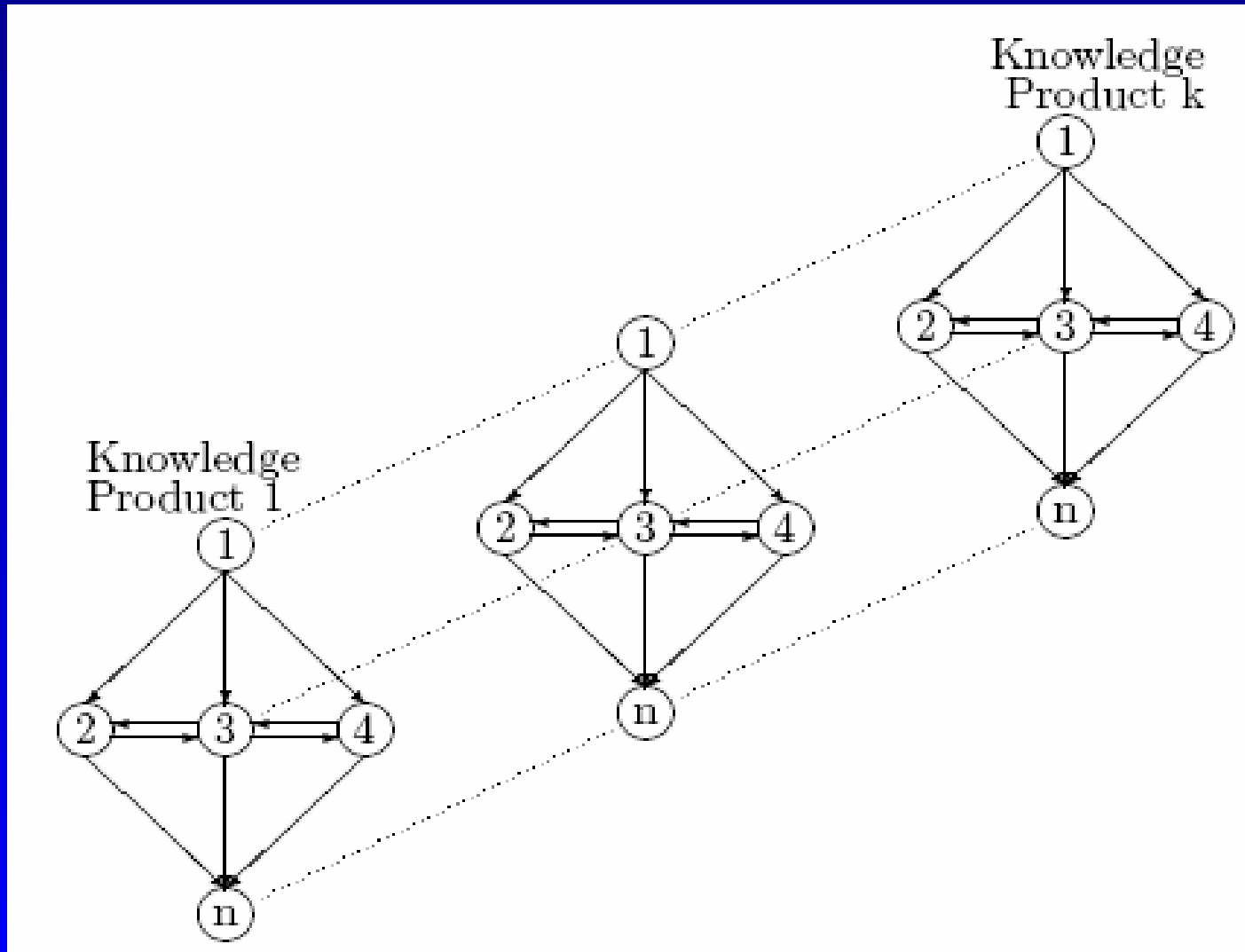
Special Characteristics of Knowledge-Based Products

- **Nonrival consumption**
 - **Special demand situation**
 - **Problem of copies**
- **Quality depends on organizational and individual knowledge**
- **Standardization of production is more difficult or even impossible**

The Knowledge Supernetwork Model

- **Network $G = [N, L]$**
 - **N nodes**
 - **L directed links**
 - **Physical links**
 - **Abstract links**
 - **Associated with a factor of production or activity required for knowledge production**
 - **A path corresponds to a production process.**
- **O/D pairs**
 - **An O/D pair corresponds to the beginning and the end of knowledge production.**

The Knowledge Supernetwork



Conservation of Flow Equations

- Consider **k** knowledge products, a typical one as **i**
- f_a^i : the flow of knowledge product **i** on link **a**
- x_p^i : Path flow of knowledge product **i** on path **p**

$$f_a^i = \sum_{p \in P} x_p^i \delta_{ap}, \quad \forall i, a,$$

$$d_\omega^i = \sum_{p \in P_\omega} x_p^i, \quad \forall i, \omega,$$

The Multicriteria Decision-Making Problem

Assume there are H criteria on each link a for each product i . A unit cost function associated with criterion h , link a , and product i is

$$c_{ha}^i = c_{ha}^i(f), \quad \forall i, h, a,$$

The total function is

$$\hat{c}_{ha}^i = c_{ha}^i(f) \times f_a^i, \quad \forall i, h, a.$$

The Multicriteria Decision-Making Problem

Assume there are **H** objective functions for the decision-maker:

$$\phi_h(f) \quad h=1,\dots,H$$

where

$$\phi_H(f) = \sum_{i,a} \hat{c}_{Ha}^i(f)$$

Examples of Criteria

- **Total (production) cost:**

$$\sum_{i,a} \pi_a^i(f) \times f_a^i.$$

- **Total (production) time:**

$$\sum_{i,a} \tau_a^i(f) \times f_a^i.$$

- **Total risk:**

$$\sum_{i,a} \rho_a^i(f) \times f_a^i.$$

The Multicriteria Decision-Making Problem

The decision-maker seeks to

$$\text{Minimize } Z(\phi_1(f), \phi_2(f), \dots, \phi_H(f))$$

subject to:

$$f \in \mathcal{K}.$$

The Multicriteria Decision-Making Problem

- **Assume the decision-maker weights the various objectives with weight w_h associated with objective function h .**
- **Hence we have the following optimization problem**

$$\text{Minimize}_{f \in \mathcal{K}} \quad Z(f) = \sum_{h=1}^H w_h \phi_h(f).$$

The Knowledge Supernetwork Optimality Conditions

- **At optimality, all the utilized paths for a product connecting an O/D pair have *equal and minimal generalized marginal total costs*:**

$$\frac{\partial Z(f^*)}{\partial x_p^i} \equiv \hat{C}_p^{i*}(f^*) \begin{cases} = \lambda_{\omega}^i, & \text{if } x_p^{i*} > 0 \\ \geq \lambda_{\omega}^i, & \text{if } x_p^{i*} = 0, \end{cases}$$

where

$$\hat{C}_p^{i*}(f) = \sum_{j=1}^k \sum_{h=1}^H \sum_{a,b \in \mathcal{L}} w_h \frac{\partial \hat{c}_{hb}^j(f)}{\partial f_a^i} \delta_{ap}.$$

A Special Case

- With three criteria **cost**, **time**, and **risk**, the optimization problem can be expressed as:

$$\text{Minimize}_{f \in \mathcal{K}} \quad w_1 \sum_{i,a} \pi_a^i(f) \times f_a^i + w_2 \sum_{i,a} \tau_a^i(f) \times f_a^i + w_3 \sum_{i,a} \rho_a^i(f) \times f_a^i.$$

- At optimality, all the utilized paths for a product connecting an O/D pair have **equal and minimal generalized marginal total** costs.

$$\frac{\partial Z(f^*)}{\partial x_p^i} \equiv \hat{C}_p^{i*}(f^*) \begin{cases} = \lambda_\omega^i, & \text{if } x_p^{i*} > 0 \\ \geq \lambda_\omega^i, & \text{if } x_p^{i*} = 0, \end{cases}$$

Variational Inequality Formulation (Fixed Demand)

- The optimal solution f^* to the optimization problem is equivalent to the solution of the following variational inequality problem:

$$\langle \nabla Z(f^*), f - f^* \rangle \geq 0, \quad \forall f \in \mathcal{K},$$

where ∇Z is the gradient of the function Z .

- In expanded notation:

$$\sum_{i,a} \sum_{h=1}^H \sum_{j=1}^k \sum_{b \in \mathcal{L}} w_h \frac{\partial c_{hb}^j(f^*)}{\partial f_a^i} \times (f_a^i - f_a^{i*}) \geq 0, \quad \forall f \in \mathcal{K}.$$

Modeling Extensions

- **Elastic Demand Model with Known Price Functions**
- **Elastic Demand Model with Known Demand Functions**

Elastic Demand Model with Known Price Functions

Equilibrium Conditions in the Case of Known Price Functions

We can immediately extend optimality conditions (17), in the fixed demand case, as follows: for all products i ; $i = 1, \dots, k$; all O/D pairs $\omega \in W$, and all paths $p \in P_\omega$, a link flow and demand pattern (f^*, d^*) is said to be in equilibrium if the following holds:

$$\hat{C}_p^{ii}(f^*) \begin{cases} = \lambda_\omega^i(d^*), & \text{if } x_p^{i*} > 0 \\ \geq \lambda_\omega^i(d^*), & \text{if } x_p^{i*} = 0. \end{cases} \quad (20)$$

Theorem 2: Variational Inequality Formulation (Price Functions Known)

The link flow and demand pattern $(f^*, d^*) \in \mathcal{K}$ satisfying equilibrium conditions (20) also satisfies the variational inequality problem:

$$\sum_{i,a} \sum_{j=1}^k \sum_{h=1}^H \sum_{b \in \mathcal{L}} w_h \frac{\partial \hat{C}_{hb}^j(f)}{\partial f_{ha}^i} \times (f_a^i - f_a^{i*}) - \sum_{i,\omega} \lambda_\omega^i(d^*) \times (d_\omega^i - d_\omega^{i*}) \geq 0, \quad \forall (f, d) \in \mathcal{K}$$

Elastic Demand Model with Known Demand Functions

Equilibrium Conditions in the Case of Known Demand Functions

We now give the equilibrium conditions governing the knowledge supernetwork model in which the demand functions are known. In particular, we have that (cf. (17) and (20)): for all products i ; $i = 1, \dots, m$; all O/D pairs $\omega \in W$, and all paths $p \in P_\omega$, a pattern of link flows, prices, and demands is in equilibrium if it satisfies the conditions:

$$\hat{C}_p^{i'}(f^*) \begin{cases} = \lambda_\omega^{i*}, & \text{if } x_p^{i*} > 0 \\ \geq \lambda_\omega^{i*}, & \text{if } x_p^{i*} = 0 \end{cases} \quad (23)$$

and

$$d_\omega^i(\lambda^*) \begin{cases} = \sum_{p \in P_\omega} x_p^{i*}, & \text{if } \lambda_\omega^{i*} > 0 \\ \leq \sum_{p \in P_\omega} x_p^{i*}, & \text{if } \lambda_\omega^{i*} = 0. \end{cases} \quad (24)$$

Elastic Demand Model with Known Demand Functions

Theorem 3: Variational Inequality Formulation (Demand Functions Known)

Let \mathcal{K} now denote the feasible set defined by $\mathcal{K} \equiv \{(f, d, \lambda) | \lambda \geq 0, \exists x \geq 0 \text{ such that (1), (2) hold}\}$. The vector $(f^*, d^*, \lambda^*) \in \mathcal{K}$ is an equilibrium according to (23) and (24) if and only if it satisfies the variational inequality problem:

$$\begin{aligned} & \sum_{i, \alpha} \sum_{j=1}^k \sum_{h=1}^H \sum_{b \in \mathcal{L}} w_h \frac{\partial \tilde{c}_{hb}^j(f^*)}{\partial f_a^i} \times (f_a^i - f_a^{i*}) - \sum_{i, \omega} \lambda_\omega^{i*} \times (d_\omega^i - d_\omega^{i*}) \\ & + \sum_{i, \omega} (d_\omega^{i*} - d_\omega^i(\lambda^*)) \times (\lambda_\omega^i - \lambda_\omega^{i*}) \geq 0, \quad \forall (f, d, \lambda) \in \mathcal{K}. \end{aligned} \quad (25)$$

Computational Procedure

We use the ***Euler method*** to solve the Variational Inequality problems. The VI is in standard form:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

Solution Properties

- **A solution f^* to the VI exists, given that the feasible set K is compact and the marginal total costs on the links are continuous for all the products.**
- **The solution to the VI is unique under a strict monotonicity assumption.**

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \rightarrow 0$, as $T \rightarrow \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set \mathcal{K} defined as:

$$y = P_{\mathcal{K}}X = \arg \min_{z \in \mathcal{K}} \|X - z\|.$$

Step 2: Convergence Verification

If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1,

Why the Euler Method?

- In the context of the knowledge supernetwork models the induced ***subproblems*** can be ***solved exactly and in closed form.***
- The Euler method suggests natural underlying ***dynamics*** to the problems.
- The proposed scheme further lays the ***foundations for*** the ultimate development of ***dynamic versions*** of the models.

Applications of the Knowledge Supernetwork Model

- **News organizations**
- **Multinational research corporations**
- **Global financial institutions**
- **Intelligence agencies**

Application to a News Organization (Eg. CNN)

- **A knowledge organization**
- **Produces knowledge**
- **Disseminates knowledge**
- **Its product is in the form of processed information or knowledge**

Application to a News Organization (Eg. CNN)

- **An O/D pair**
 - News programs
- **Products**
 - News segments
- **Demand for each product**
 - Minutes
- **Links**
 - Activities that are needed to produce the news segment

Application to an Intelligence Agency

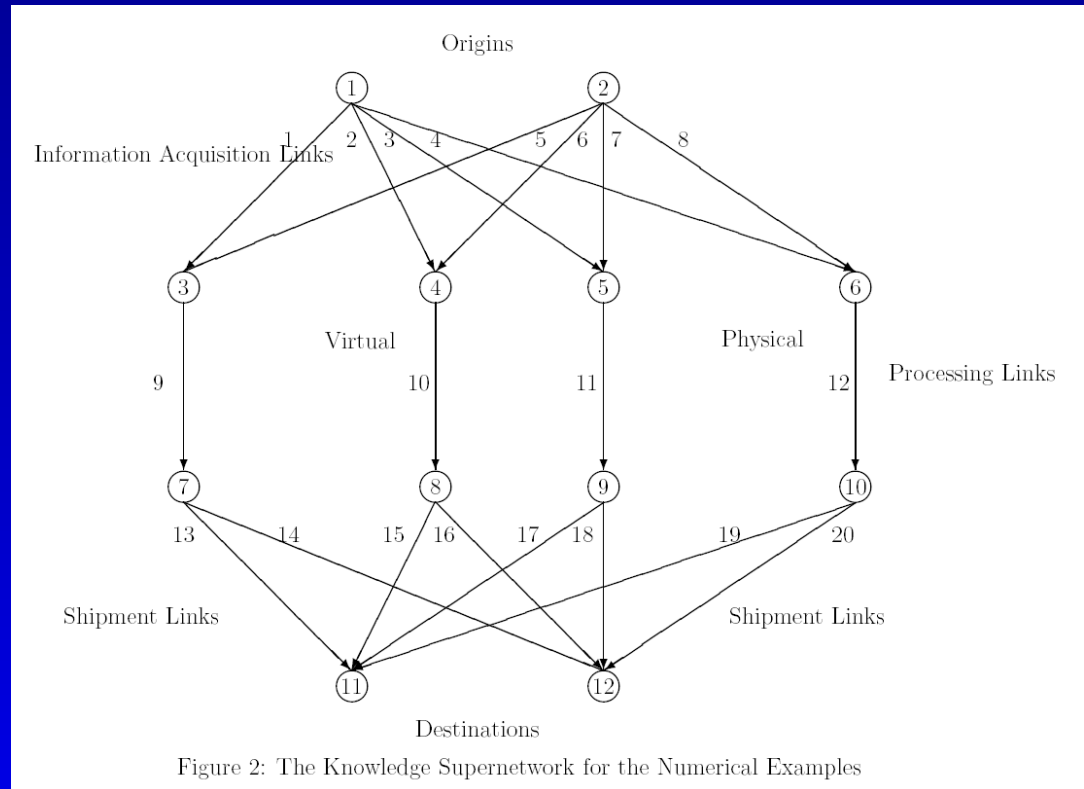
- **O/D pairs**
 - **Reports/studies**
- **Demand**
 - **Pages**
- **Links**
 - **Information processing**
 - **Transformation**
 - **Acquisition**
 - **Synthesizing**
 - **Transportation and communication links may be involved**

Application to a Global Financial Institution

- **O/D pairs**
 - **Clients**
- **Demands**
 - **Financial products**
- **Links**
 - **Transaction links (physical or electronic)**

Numerical Examples

- **Two O/D pairs**
 - $w_1 = (1, 11)$
 - $w_2 = (2, 12)$
- **Demands**
 - $d_{w_1} = 80$
 - $d_{w_2} = 160$
- **Three Criteria**
 - Cost, time, risk



Equilibrium Link and Path Flows and the Generalized Path Marginal Costs - Example 1

| Link a | f_a^* |
|----------|---------|
| 1 | 23.31 |
| 2 | 16.21 |
| 3 | 21.05 |
| 4 | 19.43 |
| 5 | 65.03 |
| 6 | 30.11 |
| 7 | 31.16 |
| 8 | 33.70 |
| 9 | 88.34 |
| 10 | 46.32 |
| 11 | 52.21 |
| 12 | 53.13 |
| 13 | 23.31 |
| 14 | 65.03 |
| 15 | 16.21 |
| 16 | 30.11 |
| 17 | 21.05 |
| 18 | 31.16 |
| 19 | 19.43 |
| 20 | 33.70 |

**Fixed demand and criteria weighted
equally: $w_1=1, w_2=1, w_3=1$**

| Path p | x_p^* |
|----------|---------|
| p_1 | 23.31 |
| p_2 | 16.21 |
| p_3 | 21.05 |
| p_4 | 19.43 |
| p_5 | 65.03 |
| p_6 | 30.11 |
| p_7 | 31.16 |
| p_8 | 33.70 |

The incurred generalized path marginal total costs (cf. (18)) were:

O/D pair ω_1 :

$$\hat{C}'_{p_1} = 1595.36, \quad \hat{C}'_{p_2} = 1595.41, \quad \hat{C}'_{p_3} = 1595.42, \quad \hat{C}'_{p_4} = 1595.42,$$

O/D pair ω_2 :

$$\hat{C}'_{p_5} = 2078.50, \quad \hat{C}'_{p_6} = 2078.42, \quad \hat{C}'_{p_7} = 2078.42, \quad \hat{C}'_{p_8} = 2078.43.$$

Equilibrium Link and Path Flows and the Generalized Path Marginal Costs - Example 2

Fixed demand and criteria weighted unequally: $w_1=1, w_2=0.5, w_3=1$

| link a | f_a^* |
|----------|---------|
| 1 | 21.53 |
| 2 | 15.21 |
| 3 | 21.88 |
| 4 | 21.38 |
| 5 | 62.54 |
| 6 | 30.08 |
| 7 | 32.86 |
| 8 | 34.52 |
| 9 | 84.07 |
| 10 | 45.29 |
| 11 | 54.74 |
| 12 | 55.91 |
| 13 | 21.52 |
| 14 | 62.54 |
| 15 | 51.21 |
| 16 | 30.08 |
| 17 | 21.88 |
| 18 | 32.86 |
| 19 | 21.38 |
| 20 | 34.52 |

| Path p | x_p^* |
|----------|---------|
| p_1 | 21.52 |
| p_2 | 15.21 |
| p_3 | 21.88 |
| p_4 | 21.38 |
| p_5 | 62.54 |
| p_6 | 30.08 |
| p_7 | 32.86 |
| p_8 | 34.52 |

The incurred generalized marginal total costs on the paths were now:

O/D pair ω_1 :

$$\hat{C}'_{p_1} = 1312.15, \quad \hat{C}'_{p_2} = 1312.09, \quad \hat{C}'_{p_3} = 1312.09, \quad \hat{C}'_{p_4} = 1312.10,$$

O/D pair ω_2 :

$$\hat{C}'_{p_5} = 1749.99, \quad \hat{C}'_{p_6} = 1749.93, \quad \hat{C}'_{p_7} = 1749.98, \quad \hat{C}'_{p_8} = 1749.92.$$

New Criterion and Demand Price Functions - Examples 3 and 4

New criterion security cost:

| link a | $\sum_{b \in \mathcal{L}} \frac{\partial \mathcal{E}_a(\mathbf{f})}{\partial f_a}$ |
|----------|--|
| 1 | $f_1 + 1$ |
| 2 | $f_2 + 2$ |
| 3 | $f_3 + 1$ |
| 4 | $f_4 + 1$ |
| 5 | $f_5 + 1$ |
| 6 | $2f_6 + 1$ |
| 7 | $f_7 + 1$ |
| 8 | $f_8 + 1$ |
| 9 | $f_9 + 1$ |
| 10 | $f_{10} + 11$ |
| 11 | $f_{11} + 1$ |
| 12 | $f_{12} + 1$ |
| 13 | $f_{13} + .5$ |
| 14 | $2f_{14} + 1$ |
| 15 | $f_{15} + .5$ |
| 16 | $f_{16} + 1$ |
| 17 | $f_{17} + 1$ |
| 18 | $f_{18} + 1$ |
| 19 | $f_{19} + 1$ |
| 20 | $f_{20} + 1$ |

Demand price functions:

$$\lambda_{\omega_1}(d) = -.5d_{\omega_1} + 1200, \quad \lambda_{\omega_2}(d) = -.1d_{\omega_2} + 900.$$

Equilibrium Link and Path Flows and the Generalized Path Marginal Costs - Example 3

Elastic demand and criteria weighted equally: $w_1=1, w_2=1, w_3=1, w_4=1$

| link a | f_a^* |
|----------|---------|
| 1 | 8.28 |
| 2 | 5.66 |
| 3 | 8.78 |
| 4 | 5.43 |
| 5 | 0.00 |
| 6 | 0.00 |
| 7 | 0.00 |
| 8 | 0.00 |
| 9 | 8.28 |
| 10 | 5.66 |
| 11 | 8.78 |
| 12 | 5.43 |
| 13 | 8.28 |
| 14 | 0.00 |
| 15 | 5.66 |
| 16 | 0.00 |
| 17 | 8.78 |
| 18 | 0.00 |
| 19 | 5.43 |
| 20 | 0.00 |

| Path p | x_p^* |
|----------|---------|
| p_1 | 8.28 |
| p_2 | 5.66 |
| p_3 | 8.78 |
| p_4 | 5.43 |
| p_5 | 0.00 |
| p_6 | 0.00 |
| p_7 | 0.00 |
| p_8 | 0.00 |

The incurred generalized marginal total costs on the paths were:

O/D pair ω_1 :

$$\hat{C}'_{p_1} = 11985.93, \quad \hat{C}'_{p_2} = 11985.75, \quad \hat{C}'_{p_3} = 11985.97, \quad \hat{C}'_{p_4} = 11986.03,$$

O/D pair ω_2 :

$$\hat{C}'_{p_5} = 3951.88, \quad \hat{C}'_{p_6} = 6553.36, \quad \hat{C}'_{p_7} = 4059.84, \quad \hat{C}'_{p_8} = 2499.29.$$

$$\lambda_{\omega_1}(d^*) = 11985.93 \text{ and } \lambda_{\omega_2}(d^*) = 11985.93.$$

Equilibrium Link and Path Flows and the Generalized Path Marginal Costs - Example 4

Elastic demand and criteria weighted
unequally: $w_1=1, w_2=1, w_3=2, w_4=1$

| link a | f_a^* |
|----------|---------|
| 1 | 7.09 |
| 2 | 4.02 |
| 3 | 8.68 |
| 3 | 5.41 |
| 5 | 0.00 |
| 6 | 0.00 |
| 7 | 0.00 |
| 8 | 0.00 |
| 9 | 7.09 |
| 10 | 4.02 |
| 11 | 8.68 |
| 12 | 5.41 |
| 13 | 7.09 |
| 14 | 0.00 |
| 15 | 4.02 |
| 16 | 0.00 |
| 17 | 8.68 |
| 18 | 0.00 |
| 19 | 5.41 |
| 20 | 0.00 |

| Path p | x_p^* |
|----------|---------|
| p_1 | 7.09 |
| p_2 | 4.02 |
| p_3 | 8.68 |
| p_4 | 5.41 |
| p_5 | 0.00 |
| p_6 | 0.00 |
| p_7 | 0.00 |
| p_8 | 0.00 |

The generalized marginal total costs on the paths were now:

O/D pair ω_1 :

$$\hat{C}'_{p_1} = 11987.41, \quad \hat{C}'_{p_2} = 11987.80, \quad \hat{C}'_{p_3} = 11987.41, \quad \hat{C}'_{p_4} = 11987.39,$$

O/D pair ω_2 :

$$\hat{C}'_{p_5} = 5918.69, \quad \hat{C}'_{p_6} = 7082.38, \quad \hat{C}'_{p_7} = 4014.71, \quad \hat{C}'_{p_8} = 2513.26.$$

$$\lambda_{\omega_1}(d^*) = 11987.40 \text{ and } \lambda_{\omega_2}(d^*) = 900$$

What Do We Gain from the Model?

- ***Incorporate various related elements*** (networks) into the supernetwork structure
- **View problems in a *systematic way*** to see the whole picture
- **Incorporate *multicriteria*** into the ***decision-making*** process to capture the sometimes conflicting issues

What Do We Gain from the Model?

- The model provides us with the ***optimal allocation*** of activities and the allocation of resources
- It also will allow us to see ***the dynamic*** of the changes in reaching the optimal solution

Future Directions

- **Incorporate *competition***
 - **Several knowledge organizations may share a subset of links (information resources, transformation processor, etc.)**
- **Introduce *uncertainty* into the framework**
- **Conduct *empirical tests* to validate the models**

**The full text of the papers can be found
under Downloadable Articles at:**

<http://supernetworks.som.umass.edu>



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Additional Discussion Questions

- **Which other applications of the knowledge supernetwork model, besides news organizations, multinational research corporations, global financial institutions and intelligence agencies, could you think of?**
- **What could be future directions for research?**

Thank you!



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