Multiperiod Competitive Supply Chain Networks with Inventorying and A Transportation Network Equilibrium Reformulation

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Outline

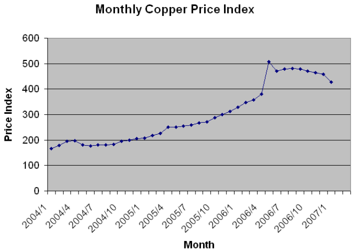
- Introduction
- The related literature
- Multiperiod competitive supply chain networks
- Transportation network reformulation of the multiperiod competitive supply chain network
- Numerical examples

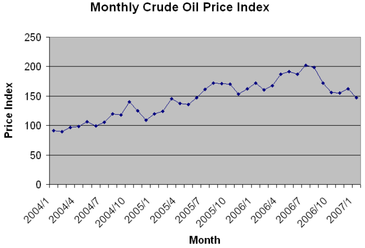
Dynamic Supply Chains

- A supply chain is a network of manufacturers, storage facility managers, transporters and retailers that perform the functions of production, storage, transportation, and sale of a particular product (cf. Nagurney (2006b), Ganeshan and Harrison (1995), Lee and Billington (1995), and Swaminathan, Sadeh, and Smith (1995))
- The fundamental function of a supply chain is to match supply with demand at the lowest cost (cf. Cachon and Terwiesch (2005))
- Highly dynamic and competitive business environments
    - Fluctuating costs and demands

Varying Costs

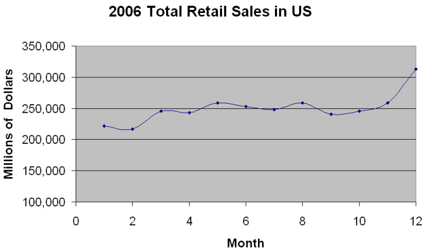
- In the past three years, iron and steel prices increased by 47%
- The price of industrial chemicals increased by 51%
- Prices of natural gas and crude petroleum increased by 68%, and 112%, respectively
- Truck freight transportation cost increased by 6% annually
- Water freight transportation cost increased by only 2% annually (source: Bureau of Labor Statistics (2007))

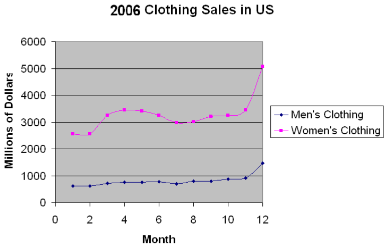




Fluctuating Demands

- Seasonal fluctuations
  - E.g. The US automobile market: sales in summer 25 – 35% higher than that in winter
  - E.g. Many products such as TV, sporting goods and electronics soar by more than 100% during the holiday season (sources: U.S. Census Bureau (2007))





Some of the Related Literature

- Decentralized decision-making and competition
  - Lederer and Li (1997), Cachon and Zipkin (1999), Cachon and Netessine (2003), Geunes and Pardalos (2003), etc.
- Dynamic production, inventory, and pricing models
  - Hafsi and Bai (1998), Federgruen and Tzur (1991), Stadtler (2000), Goyal and Giri (2001), Teng et al. (2002), Perakis and Sood (2003, 2006), and Bernstein and Federgruen (2003), etc.
- Multiperiod spatial pricing models
  - Nagurney and Aronson (1988, 1989)

Some of the Related Literature

- Nagurney, A., Dong, J., Zhang, D., 2002. A Supply Chain Network Equilibrium Model. *Transportation Research E 38*, 281-303

- Nagurney, A., 2006. On the Relationship Between Supply Chain and Transportation Network Equilibria: A Supernetwork Equivalence with Computations. *Transportation Research E 42*, 293-316

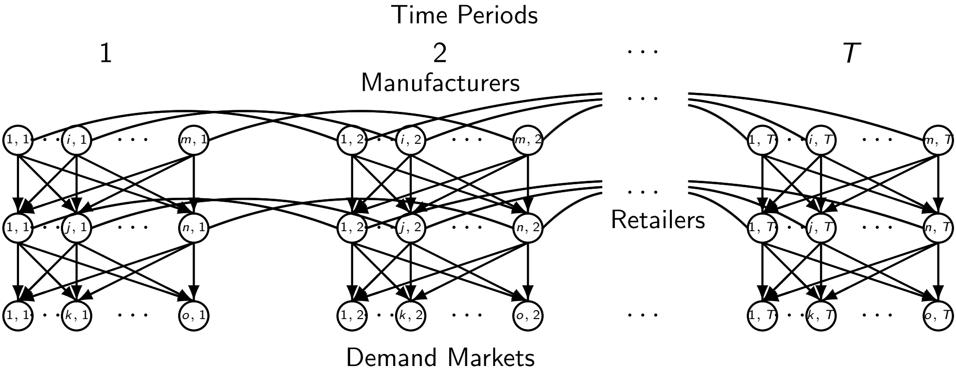
Some of the Related Literature

- Nagurney, A., Liu, Z., Cojocaru, M., Daniele, P., 2006. Dynamic Electric Power Supply Chains and Transportation Networks: An Evolutionary Variational Inequality Formulation. *Transportation Research E*, in press
- Wu, K., Nagurney, A., Liu, Z., Stranlund, J., 2006. Modeling Generator Power Plant Portfolios and Pollution Taxes in Electric Power Supply Chain Networks: A Transportation Network Equilibrium Transformation. *Transportation Research D 11*, 171-190
- Liu, Z., Nagurney, A., 2006. Financial Networks with Intermediation and Transportation Network Equilibria: A Supernetwork Equivalence and Reinterpretation of the Equilibrium Conditions with Computations. *Computational Management Science*, in press

Multiperiod Supply Chain Networks with Inventorying

- Focuses on the supply chain planning process
- Captures the changing costs and demands
- Provides a flexible platform so that perishable products and transportation delays can be easily incorporated
- The transportation network reformulation provides novel theoretical insights and efficient computational methods for the multiperiod supply chain network equilibrium model

The Network Structure of the Multiperiod Supply Chain



Manufacturer's Maximization Problem

$$\text{Max} \quad \sum_{t=1}^{T}\sum_{j=1}^{n}\rho^*_{1ijt}q_{ijt} - \sum_{t=1}^{T} f_{it}(q_t) - \sum_{t=1}^{T} cv_{it}(u_{it}) - \sum_{t=1}^{T}\sum_{j=1}^{n} c_{ijt}(q_{ijt})$$

subject to

$$\sum_{j=1}^{n} q_{ij1} + u_{i1} = q_{i1},$$

$$u_{it} + \sum_{j=1}^{n} q_{ijt} = q_{it} + u_{i(t-1)}, \quad t = 2, \ldots, T-1,$$

$$\sum_{j=1}^{n} q_{ijT} = q_{iT} + u_{i(T-1)},$$

$$q_{ijt} \geq 0, \quad j = 1, \ldots, n; \quad t = 1, \ldots, T,$$

$$q_{it} \geq 0, \quad t = 1, \ldots, T,$$

$$u_{it} \geq 0, \quad t = 1, \ldots, T-1$$

- We assume that the cost functions of the manufacturers are convex and continuously differentiable and that the manufacturers compete with one another in a noncooperative manner

The Optimal Conditions for All Manufacturers

Determine:$(q^*, u^{1*}, Q^{1*}) \in \mathcal{K}^1$ satisfying:

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\frac{\partial f_{it}(q_t^*)}{\partial q_{it}} \times [q_{it} - q_{it}^*] + \sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_{ijt}(q_{ijt}^*)}{\partial q_{ijt}} - \rho_{1ijt}^*\right] \times [q_{ijt} - q_{ijt}^*]$$

$$+\sum_{t=1}^{T}\sum_{i=1}^{m}\frac{\partial cv_{it}(u_{it}^*)}{\partial u_{it}} \times [u_{it} - u_{it}^*] \geq 0, \quad \forall (q, u^1, Q^1) \in \mathcal{K}^1,$$

where $\mathcal{K}^1 \equiv \{(q, u^1, Q^1) | (q, u^1, Q^1) \in$
$R_+^{Tm(2+n)}$ and the conservation of flow equations hold$\}$

Retailer's Maximization Problem

$$\text{Max} \quad \sum_{t=1}^{T}\sum_{k=1}^{o}\rho_{2jt}^{*}q_{jkt}-\sum_{t=1}^{T}c_{jt}(h_t)-\sum_{t=1}^{T}cv_{jt}(u_{jt})-\sum_{t=1}^{T}\sum_{i=1}^{m}\rho_{1ijt}^{*}q_{ijt}$$

subject to

$$h_{jt}=\sum_{i=1}^{m}q_{ijt}, \quad t=1,...,T,$$

$$\sum_{k=1}^{o}q_{jk1}+u_{j1}=\sum_{i=1}^{m}q_{ij1},$$

$$\sum_{k=1}^{o}q_{jkt}+u_{jt}=\sum_{i=1}^{m}q_{ijt}+u_{j(t-1)}, \quad t=2,...,T-1,$$

$$\sum_{k=1}^{o}q_{jkT}=\sum_{i=1}^{m}q_{ijT}+u_{j(T-1)},$$

$$q_{ijt}\geq 0, \quad i=1,\ldots,m; t=1,\ldots,T,$$

$$q_{jkt}\geq 0, \quad k=1,\ldots,m; t=1,\ldots,T,$$

$$u_{jt}\geq 0, \quad t=1,\ldots,T$$

- We assume that the cost functions of the retailers are convex and continuously differentiable and that the retailers compete with one another in a noncooperative manner

The Optimal Conditions for All Retailers

determine $(Q^{1*}, h^*, u^{2*}, Q^{2*}) \in \mathcal{K}^2$ satisfying:

$$\sum_{t=1}^{T}\sum_{j=1}^{n}\frac{\partial c_{jt}(h_t^*)}{\partial h_{jt}} \times \left[h_{jt} - h_{jt}^*\right]$$

$$+\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\rho_{1ijt}^* \times \left[q_{ijt} - q_{ijt}^*\right] - \sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{o}\rho_{2jt}^* \times \left[q_{jkt} - q_{jkt}^*\right]$$

$$+\sum_{t=1}^{T}\sum_{j=1}^{n}\frac{\partial cv_{jt}(u_{jt}^*)}{\partial u_{jt}} \times \left[u_{jt} - u_{jt}^*\right] \geq 0, \quad \forall (Q^1, h, u^2, Q^2) \in \mathcal{K}^2,$$

where $\mathcal{K}^2 \equiv \{(Q^1, h, u^2, Q^1) | (Q^1, h, u^2, Q^1) \in R_+^{Tn(m+o+2)}$ and the conservation of flow equations hold$\}$

The Equilibrium Conditions for Consumers at the Demand Markets

The equilibrium conditions for consumers at demand market k, take the form: for all retailers j; $j = 1, \ldots, n$ and time periods t; $t = 1, \ldots, T$:

$$\rho^*_{2jt} + \hat{c}_{jkt}(Q^{2*}_t) \begin{cases} = \rho^*_{3kt}, & \text{if} \quad q^*_{jkt} > 0, \\ \geq \rho^*_{3kt}, & \text{if} \quad q^*_{jkt} = 0, \end{cases}$$

and

$$d_{kt}(\rho^*_{3t}) \begin{cases} = \sum_{j=1}^{n} q^*_{jkt}, & \text{if} \quad \rho^*_{3kt} > 0, \\ \leq \sum_{j=1}^{n} q^*_{jkt}, & \text{if} \quad \rho^*_{3kt} = 0 \end{cases}$$

Equilibrium Conditions of the Multiperiod Supply Chain Network

Definition: Multiperiod Supply Chain Network Equilibrium

The equilibrium state of the multiperiod supply chain network is one where the flows of the product between the tiers of the decision-makers coincide and the flows and prices satisfy the sum of the optimality conditions and equilibrium conditions so that no decision-maker has any incentive to alter his transactions.

Theorem: Variational Inequality Formulation of the Multiperiod Supply Chain Network Equilibrium

Determine: $(q^*, h^*, u^{1*}, Q^{1*}, u^{2*}, Q^{2*}, d^*, \rho_3^*) \in \mathcal{K}^3$ *satisfying:*

$$\sum_{t=1}^{T}\sum_{i=1}^{m}\frac{\partial f_{it}(q_t^*)}{\partial q_{it}}\times[q_{it}-q_{it}^*]+\sum_{t=1}^{T}\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial c_{ijt}(q_{ijt}^*)}{\partial q_{ijt}}\times[q_{ijt}-q_{ijt}^*]$$

$$+\sum_{t=1}^{T}\sum_{j=1}^{n}\frac{\partial c_{jt}(h_t^*)}{\partial h_{jt}}\times[h_{jt}-h_{jt}^*]+\sum_{t=1}^{T}\sum_{i=1}^{m}\frac{\partial cv_{it}(u_{it}^*)}{\partial u_{it}}\times[u_{it}-u_{it}^*]$$

$$+\sum_{t=1}^{T}\sum_{j=1}^{n}\frac{\partial cv_{jt}(u_{jt}^*)}{\partial u_{jt}}\times[u_{jt}-u_{jt}^*]+\sum_{t=1}^{T}\sum_{j=1}^{n}\sum_{k=1}^{o}\hat{c}_{jkt}(Q_t^{2*})\times[q_{jkt}-q_{jkt}^*]$$

$$+\sum_{t=1}^{T}\sum_{k=1}^{o}\rho_{3kt}^*\times[d_{kt}-d_{kt}^*]+\sum_{t=1}^{T}\sum_{k=1}^{o}[d_{kt}^*-d_{kt}(\rho_{3t}^*)]\times[\rho_{3kt}-\rho_{3kt}^*]\geq 0,$$

$$\forall (q, h, u^1, Q^1, u^2, Q^2, d, \rho_3) \in \mathcal{K}^3,$$

where $\mathcal{K}^3 \equiv \{(q, h, u^1, Q^1, u^2, Q^2, d, \rho_3) | (q, h, u^1, Q^1, u^2, Q^2, d, \rho_3) \in R_+^{T(2m+mn+2n+no+2o)}$ and the conservation of flow equations hold$\}$

Transportation Network Equilibrium Model with Known Demand Functions (Dafermos and Nagurney (1984))

- Define the set of origin/destination (O/D) pairs W with Z elements; the set of paths P with Q elements; the set of links L with K elements
- The conservation of flow equations must hold:

$$d_w = \sum_{p \in P_w} x_p, \quad \forall w \in W$$

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L$$

- The user cost on a path is equal to the sum of user costs on links the path consists of

$$C_p = \sum_{a \in L} c_a \delta_{ap}, \quad \forall p \in P$$

Transportation Network Equilibrium Model with Known Demand Functions

Theorem

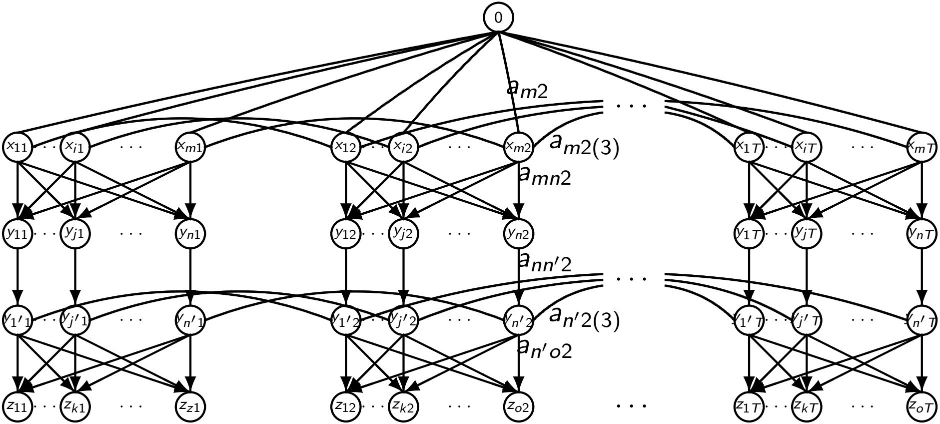
A travel link flow pattern and associated travel demand and disutility pattern is a transportation network equilibrium if and only if it satisfies the variational inequality problem: determine $(f^*, d^*, \lambda^*) \in \mathcal{K}^4$ *satisfying*

$$\sum_{a \in L} c_a(f^*) \times (f_a - f_a^*) - \sum_{w \in W} \lambda_w^* \times (d_w - d_w^*)$$

$$+ \sum_{w \in W} [d_w^* - d_w(\lambda^*)] \times [\lambda_w - \lambda_w^*] \geq 0, \quad \forall (f, d, \lambda) \in \mathcal{K}^4,$$

where $\mathcal{K}^4 \equiv \{(f, d, \lambda) \in R_+^{K+2Z}|$ *there exists an* x *satisfying the conservation of flow equations*$\}$

The $\mathcal{G}_S$ Supernetwork Representation of the Multiperiod Supply Chain Network



Transportation Network Equilibrium Reformulation of the Multiperiod Supply Chain Network

- New interpretation of the supply chain network equilibrium in terms of paths and path flows
- Competition among end-to-end supply chains
- Efficient computational method

The Euler Method

Let $\mathcal{T}$ denote an iteration counter

Step 0: Initialization
Set $X^0 \in \mathcal{K}$
Let $\mathcal{T} = 1$ and set the sequence $\{\alpha_{\mathcal{T}}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} \alpha_{\mathcal{T}} = \infty$, $\alpha_{\mathcal{T}} > 0$ for all $\mathcal{T}$, and $\alpha_{\mathcal{T}} \to 0$ as $\mathcal{T} \to \infty$

Step 1: Computation
Compute $X^{\mathcal{T}} \in \mathcal{K}$ by solving the variational inequality subproblem:

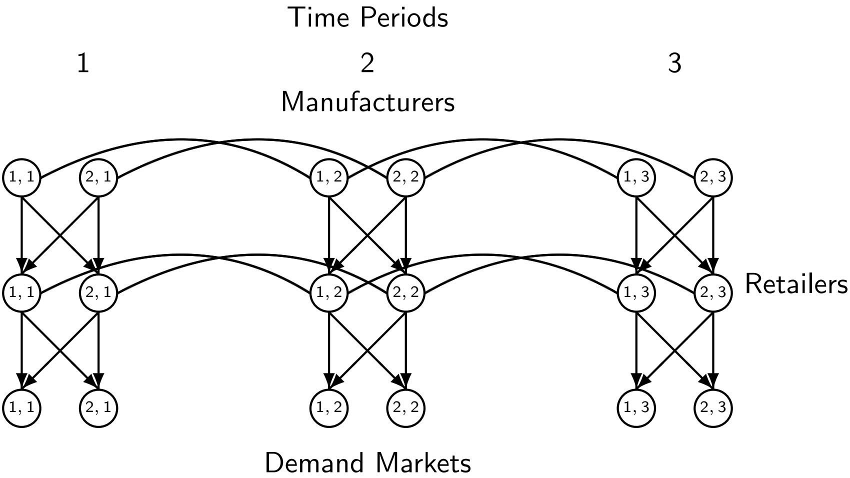
$$\langle X^{\mathcal{T}} + \alpha_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}$$

Step 2: Convergence Verification
If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1

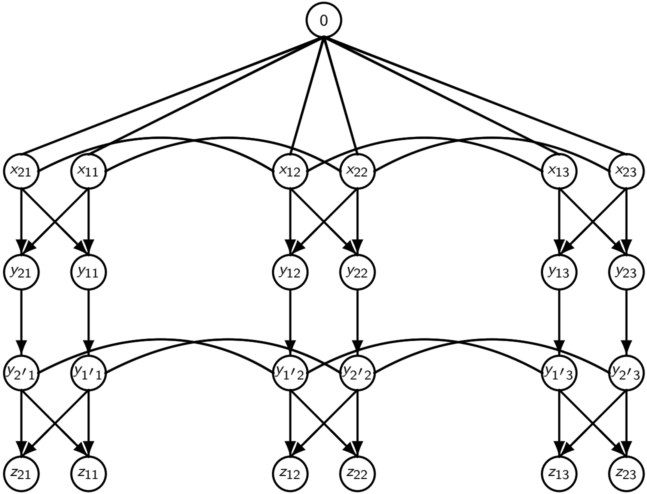
Numerical Examples 1 and 2

- Example 1 has 2 manufacturers, 2 retailers, 2 demand markets and 3 periods
- Example 2 has the same data as Example 1 except that the product in Example 2 is perishable ($L = 2$)

The Network Structure of Numerical Examples 1 and 2



Supernetwork Structure of the Transportation Network Equilibrium Reformulation of Examples 1 and 2



Cost Functions of Examples 1 and 2

- The production cost functions:

$$f_{1t}(q_{1t}) = 0.1q_{1t}^2 + 5q_{1t} + 20, \quad f_{2t}(q_{2t}) = 0.15q_{2t}^2 + 4q_{2t} + 10, \quad t = 1, 2, 3$$

- The transaction/transportation cost functions:

$$c_{11t}(q_{11t}) = 0.005q_{11t}^2 + q_{11t}, \quad c_{12t}(q_{12t}) = 0.005q_{12t}^2 + q_{12t},$$
$$c_{21t}(q_{21t}) = 0.005q_{21t}^2 + q_{21t}, \quad c_{22t}(q_{22t}) = 0.005q_{22t}^2 + q_{22t}, \quad t = 1, 2, 3$$

- The handling costs of the retailers:

$$c_{1t}(h_{1t}) = 0.05h_{1t}^2 + 1.5h_{1t} + 20, \quad c_{2t}(h_{2t}) = 0.1h_{2t}^2 + h_{2t} + 30, \quad t = 1, 2, 3$$

Cost Functions of Examples 1 and 2 (Cont')

- The inventory cost functions of the manufacturers:

$$cv_{1t}(u_{1t}) = 0.025u_{1t}^2 + 0.5u_{1t} + 10, \quad cv_{2t}(u_{2t}) = 0.025u_{2t}^2 + 0.5u_{2t} + 20, \quad t = 1, 2$$

- The inventory costs functions of the retailers:

$$cv_{1t}(u_{1t}) = 0.025u_{1t}^2 + 0.5u_{1t}, \quad cv_{2t}(u_{2t}) = 0.025u_{2t}^2 + 0.5u_{2t}, \quad t = 1, 2$$

- The demand functions:

$$d_{11}(\rho_{311}) = 15 - \frac{1}{2}\rho_{311}, \quad d_{12}(\rho_{312}) = 20 - \frac{1}{3}\rho_{312}, \quad d_{13}(\rho_{313}) = 80 - \frac{1}{10}\rho_{313},$$
$$d_{21}(\rho_{321}) = 10 - \frac{1}{2}\rho_{321}, \quad d_{22}(\rho_{322}) = 15 - \frac{1}{3}\rho_{322}, \quad d_{23}(\rho_{323}) = 90 - \frac{1}{10}\rho_{323}$$

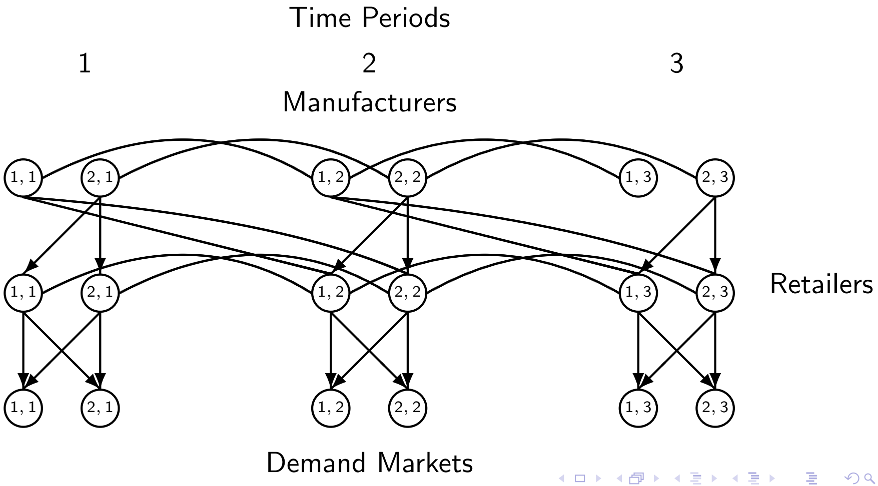
Equilibrium Solutions of Examples 1 and 2

	Example 1			Example 2		
Variable	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$
$f^*_{a_{1t}} = q^*_{1t}$	32.6997	36.3997	40.5709	18.5877	45.5601	49.1646
$f^*_{a_{2t}} = q^*_{2t}$	25.2589	27.7443	30.5455	15.7721	33.9027	36.3221
$f^*_{a_{11t}} = q^*_{11t}$	16.6645	21.3144	29.6492	11.2588	24.2616	33.5997
$f^*_{a_{12t}} = q^*_{12t}$	11.2351	13.2002	17.6067	7.3289	16.8807	19.9826
$f^*_{a_{21t}} = q^*_{21t}$	12.8955	16.9915	24.6881	9.8501	18.3844	27.2268
$f^*_{a_{22t}} = q^*_{22t}$	7.4509	8.8588	12.6637	5.9219	11.0021	13.6114
$f^*_{a_{11't}} = h^*_{1t}$	29.5601	38.3060	54.3373	21.1089	42.6460	60.8265
$f^*_{a_{22't}} = h^*_{2t}$	18.6860	22.0591	30.2705	13.2509	27.8829	33.5940
$f^*_{a_{1'1t}} = q^*_{11t}$	5.4186	12.8893	38.7451	6.4787	5.2527	46.0004
$f^*_{a_{1'2t}} = q^*_{12t}$	0.9186	8.2277	56.0039	2.4797	6.8976	57.4722
$f^*_{a_{2'1t}} = q^*_{21t}$	0.0000	0.1694	38.9204	0.8006	9.2310	31.4244
$f^*_{a_{2'2t}} = q^*_{22t}$	0.0000	0.1643	31.7615	0.2996	2.9194	30.0525
$f^*_{a_{1t(t+1)}} = u^*_{1t}$	4.8001	6.6850	0.0000	0.0000	4.4177	0.0000
$f^*_{a_{2t(t+1)}} = u_{2t}$	4.9124	6.8063	0.0000	0.0000	4.5162	0.0000
$f^*_{a_{1't(t+1)}} = u^*_{1t}$	23.2227	40.4117	0.0000	12.1504	42.6460	0.0000
$f^*_{a_{2't(t+1)}} = u^*_{2t}$	18.6860	40.4114	0.0000	12.1505	27.8829	0.0000
$d^*_{w_1t} = d^*_{1t}$	5.4186	13.0587	77.6655	7.2794	14.4838	77.4248
$d^*_{w_2t} = d^*_{2t}$	0.9186	8.3920	87.7655	2.7794	9.8171	87.5248
$\lambda^*_{w_1t} = \rho^*_{31t}$	19.1626	20.8237	23.3443	15.4410	16.5485	25.7515
$\lambda^*_{w_2t} = \rho^*_{32t}$	18.1626	19.8237	22.3443	14.4410	15.5485	24.7515

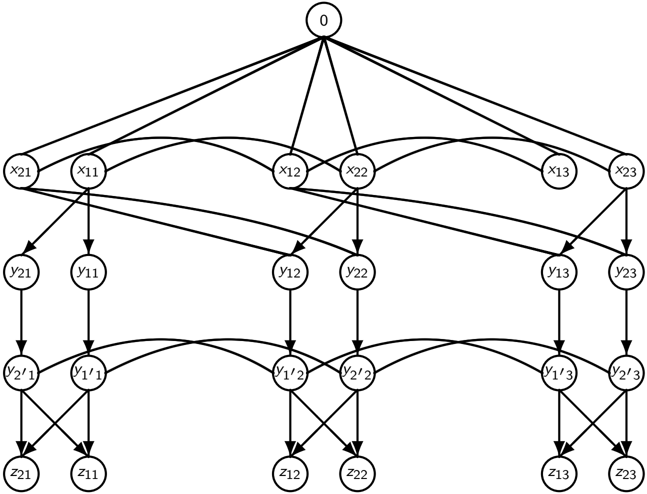
Numerical Examples 3 and 4

- Examples 3 and 4 have inseparable cost functions
- Example 3 has the same network structure as Examples 1 and 2
- In Example 4, manufacturer 1 has one-period transportation delay

The Network Structure of Example 4 (Transportation Delay)



Supernetwork Structure of the Transportation Network Equilibrium Reformulation of Example 4



Cost Functions of Examples 3 and 4

- The production cost functions:

$$f_{1t}(q_{1t}, q_{2t}) = 5 + q_{1t} + 0.4q_{1t}^2 + 0.2q_{1t}q_{2t}, \quad t = 1, 2, 3,$$
$$f_{2t}(q_{1t}, q_{2t}) = 10 + 2q_{2t} + 1.5q_{2t}^2 + 0.4q_{1t}q_{2t}, \quad t = 1, 2, 3$$

- The transaction/transportation cost functions:

$$c_{11t}(q_{11t}) = 3q_{11t} + 0.1q_{11t}^2, \quad c_{12t}(q_{12t}) = 3q_{12t} + 0.1q_{12t}^2,$$
$$c_{21t}(q_{21t}) = q_{21t} + 0.1q_{21t}^2, \quad c_{22t}(q_{22t}) = q_{22t} + 0.1q_{22t}^2, \quad t = 1, 2, 3$$

- The handling costs of the retailers:

$$c_{1t}(h_{1t}, h_{2t}) = 0.1h_{1t}^2 + 0.1h_{1t}h_{2t}, \quad t = 1, 2, 3, \quad c_{2t}(h_{1t}, h_{2t}) = 0.1h_{2t}^2 + 0.1h_{1t}h_{2t},$$

Cost Functions of Examples 3 and 4 (Cont')

- The inventory cost functions of the manufacturers:

$$cv_{1t}(u_{1t}) = 0.2u_{1t}^2 + u_{1t}, \quad t = 1,2, \quad cv_{2t}(u_{2t}) = 0.5u_{2t}^2 + 2u_{2t}, \quad t = 1,2$$

- The inventory costs functions of the retailers:

$$cv_{1t}(u_{1t}) = 0.05u_{1t}^2 + u_{1t}, \quad t = 1,2, \quad cv_{2t}(u_{2t}) = 0.02u_{2t}^2 + u_{2t}, \quad t = 1,2.$$

- The demand functions:

$$d_{11}(\rho_{311}) = 70 - \rho_{311}, \quad d_{12}(\rho_{312}) = 80 - \rho_{312}, \quad d_{13}(\rho_{313}) = 90 - \rho_{313},$$

$$d_{21}(\rho_{321}) = 55 - 0.2\rho_{321}, \quad d_{22}(\rho_{322}) = 55 - 0.2\rho_{322}, \quad d_{23}(\rho_{323}) = 60 - 0.2\rho_{323}$$

Equilibrium Solutions of Examples 3 and 4

		Example 3			Example 4	
Variable	$t=1$	$t=2$	$t=3$	$t=1$	$t=2$	$t=3$
$f^*_{a_{1t}} = q^*_{1t}$	50.8291	51.9564	53.1257	49.4331	53.4121	0.0000
$f^*_{a_{2t}} = q^*_{2t}$	9.1084	9.3028	9.5044	30.4517	9.7468	16.4581
$f^*_{a_{11t}} = q^*_{11t}$	25.4145	25.9782	26.5628	24.7165	26.7060	0.0000
$f^*_{a_{12t}} = q^*_{12t}$	25.4145	25.9782	26.5628	24.7165	26.7060	0.0000
$f^*_{a_{21t}} = q^*_{21t}$	4.5542	4.6514	4.7522	15.2258	4.8734	8.2290
$f^*_{a_{22t}} = q^*_{22t}$	4.5542	4.6514	4.7522	15.2258	4.8734	8.2290
$f^*_{a_{11't}} = h^*_{1t}$	29.9688	30.6296	31.3150	15.2258	29.5900	34.9351
$f^*_{a_{22't}} = h^*_{2t}$	29.9688	30.6296	31.3150	15.2258	29.5900	34.9351
$f^*_{a_{1'1t}} = q^*_{11t}$	2.3862	6.3799	14.7593	0.0000	7.6204	12.3561
$f^*_{a_{1'2t}} = q^*_{12t}$	21.2911	23.0842	24.0125	15.2258	20.8777	23.6708
$f^*_{a_{2'1t}} = q^*_{21t}$	6.0551	10.8097	11.1321	0.0000	7.9223	12.1430
$f^*_{a_{2'2t}} = q^*_{22t}$	21.3970	19.3536	23.1657	15.2258	21.2307	23.2289
$f^*_{a_{1t(t+1)}} = u^*_{1t}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$f^*_{a_{2t(t+1)}} = u^*_{2t}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$f^*_{a_{1't(t+1)}} = u^*_{1t}$	6.2914	7.4568	0.0000	0.0000	1.0917	0.0000
$f^*_{a_{2't(t+1)}} = u^*_{2t}$	2.5165	2.9827	0.0000	0.0000	0.4368	0.0000
$d^*_{w_1 t} = d^*_{1t}$	8.4413	17.1897	25.8914	0.0000	15.5428	24.4991
$d^*_{w_2 t} = d^*_{2t}$	42.6882	42.4379	46.3314	30.4517	42.1085	46.8998
$\lambda^*_{w_1 t} = \rho^*_{31t}$	61.5586	62.8102	64.1085	70.0000	64.4571	65.5008
$\lambda^*_{w_2 t} = \rho^*_{32t}$	61.5586	62.8102	64.1085	122.7413	64.4571	65.5008

Conclusions

- Developed the multiperiod competitive supply chain network model with varying costs and demands
- Established the supernetwork equivalence between transportation networks and multiperiod competitive supply chain networks
- The model provides a flexible platform so that perishable products and transportation delays can be easily incorporated
- The transportation network reformulation provides novel theoretical insights and efficient computational methods for the multiperiod supply chain network equilibrium model

Thank You!

For more information, please see:
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