A Network Modeling Approach for the Optimization of Internet-Based Advertising Strategies and Pricing with a Quantitative Explanation of Two Paradoxes

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Highlights of this Research

- It is the first attempt to formulate the determination of optimal Internet marketing strategies as a network optimization problem, which allows us to take advantage of network theory in terms of qualitative analysis and computations.

- The results also provide a quantitative perspective to explain such paradoxes as: advertising on more web-sites may cause a decline in the number of clicks and the click rate.

Highlights of this Research

- This research provides a formula for the computation of optimal Internet marketing strategies.

- It provides a pricing index to suggest how firms should shift from one pricing scheme to another.

Network Modeling has been Used in Numerous Applications and Disciplines:

- Transportation Science and Logistics
- Telecommunications and Computer Science
- Regional Science and Economics
- Engineering
- Finance
- Operations Research and Management Science

Some Literature on Internet-based Marketing

Modeling the Clickstream: Implications for web-based advertising Efforts, by P. Chatterjee, D. L. Hofman, and T.P. Novak, 520-541, 2003.

Advertising Competition Under Consumers Inertia, Banerjee, B and S. Bandyopadhyay. (2003)

A Comparison of Online and Offline Consumer Brand Loyalty, by Danaher, P., Wilson, I., and R. Davis (2003)

Assumptions

- A firm is advertising on n Internet market places.
- Response (Amount of click-through) is an increasing, concave function of the amount of advertisement exposures.

Assumptions

- Amount of click-throughs in website i is also impacted by advertisement exposures on other websites.

$$r_i = r_i(e_1, e_2, \ldots, e_n)$$

- A firm's objective is to achieve the maximum amount of click-through.

The Optimization Model

Let c_i $i=1,2,\ldots,n$ be the unit cost of exposure and C be the advertising budget. The firm is to

$$Maximize \sum_{i=1}^{n} r_i(e_1,\ldots,e_n)$$

$$subject\ to: \sum_{i=1}^{n} c_i e_i \le C$$

$$e_i \ge 0, i = 1,2,\ldots n$$

- Let $f_i = c_i e_i$, $f = (f_1, f_{2,} \ldots, f_n)$, then f_i is the amount of money assigned to website i.
- Simple calculus manipulation can show us that:

  the response functions $r_i(f) = r_i(\frac{f_1}{c_1}, \ldots, \frac{f_n}{c_n})$ are increasing and concave functions of f, if they are increasing and concave functions of e.

- The non-linear programming model is equivalent to:

The Network Model

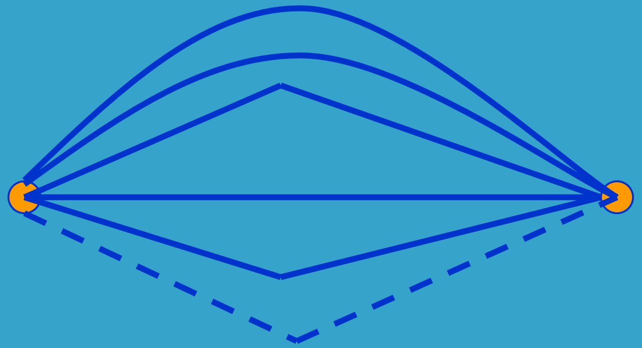
$$\text{Maximize } B(f) = \sum_{i=1}^{n} r_i(f)$$

$$\text{subject to}: \sum_{i=1}^{n} f_i + f_{n+1} = C$$

$$f_i \geq 0, i = 1,2,\ldots n$$

The Equivalent Network Model

The dotted link is the dummy link that absorbs the budget surplus f_{n+1}.



The Variational Inequality Formulation

$$\nabla B(f^*) \bullet (f - f^*) \leq 0$$

$$\forall f \in S = \{f \mid f \geq 0, \sum_{i=1}^{n+1} f_i = C\}$$

This variational inequality can be solved by an iterative scheme where the function in each of the subproblems is separable and quadratic (see Dafermos and Sparrow (1969), Dafermos (1980), Zhao and Dafermos (1991), Nagurney (1999)).

A Special-Purpose Algorithm:

If B(f) is a separable quadratic function in the sense that

$$\frac{\partial B}{\partial f_i} = a_i f_i + b_i$$

Then the optimal strategy can be found by procedure:

1. Sort b_i , without lose of generality, we assume

$$b_1 \leq b_2 \leq ,..., \leq b_n \tag{1}$$

2. Let k=1

- Calculate

$$\lambda_k = \frac{C + \sum_{i=1}^{k} \frac{b_i}{a_i}}{\sum_{i=1}^{k} \frac{1}{a_i}} \qquad (2)$$

- Compare λ_k with b_i in (1)

if $b_p \leq \lambda_k < b_{p+1}$ with p=k then Let $\lambda = \lambda_k$ go to (3)

if $b_p \leq \lambda_k < b_{p+1}$ with p>k , then let k=k+1, go to (2)

if $b_p \leq \lambda_k < b_{p+1}$ with p<k, then let k=k-1, go to (2)

3.

$$f_i^* = \max\{0, \frac{\lambda - b_i}{a_i}\}, i = 1,2,\ldots n;\ f_{n-1}^* = o, if\ \lambda > 0$$

$$f_i^* = \max\{0, \frac{-b_i}{a_i}\}, i = 1,2,\ldots n;\ f_{n-1}^* = C - \sum_{i=1}^{n} f^*_i, if\ \lambda \le 0$$

When the impacts of advertising efforts in websites are interrelated, having more options of choosing websites for ads may be worse off than having fewer options!

An Illustration of the Paradoxes

Example 1: There are two websites to choose for advertising and the budget is $9000.

The response functions on website 1 and 2 are respectively:

$$r_1 = -0.0002\,e_1^2 + 0.4e_1 + 2$$
$$r_2 = -0.0004\,e_2^2 + 0.2e_2 + 1$$

Optimal Strategies

	Expenditures	Exposures	Click-throughs(in 100s)
Site 1	$7764.71	776.471	192
Site 2	$1235.29	82.353	14.758

Example 2: There are three websites to choose for ads and the budget remains at $9000. The response functions on each of the sites are respectively:

$$r_1 = -0.0002e_1^2 + 0.4e_1 + 2 - 0.0002e_3$$
$$r_2 = -0.0004e_2^2 + 0.2e_2 + 1 - 0.01e_3$$
$$r_3 = -0.0002e_3^2 + 0.1e_3 + 2.8 - 0.004e_1 - 0.003e_2$$

Optimal Strategies

	Expenditures	Exposures	Click-throughs(in 100s)
Site1	$7711.60	771.16	191.525
Site 2	$1231.80	82.12	14.670
Site 3	$56.60	5.66	0.029

Total click-throughs are .056 ***fewer*** than in Example 1!

Explanation

Using an additional (new) website causes a change in response, which can be decomposed into three parts:

(1) The change in response in ***old*** websites is impacted by advertisements in the new website. This change may or may not be negative.

Explanation

(2) The change in response in the ***old*** websites caused by the decrease in associated advertising capital is negative.

(3) The response generated in ***new*** website is non-negative.

If item (3) cannot prevail over the sum of (1) and (2), then the use of the new website for advertising makes the firm worse-off!

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http://supernet.som.umass.edu/dart.html

Future Research

- The development of network equilibrium models for the modeling of Internet marketing competition.

- Modeling the impact of asymmetric information on optimal marketing strategies.

For more information see:

http://supernet.som.umass.edu

Thank you!!!