Financial Networks with Intermediation: Risk Management with Variable Weights

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Literature Survey of Financial Networks with Intermediation

- Intermediation
  - Banks, savings institutions, investment and insurance companies
- Quesnay (1758)
  - Original use of networks to represent financial system
  - Depicted the circular flow of funds in an economy
- Thore (1969, 1980)
  - Studied systems of linked portfolios in credit networks
  - Decentralization/decomposition theory
  - Basic intertemporal models

Literature Survey of Financial Networks with Intermediation

- Nagurney, Dong, and Hughes (1992)
  - Multi-sector, multi-instrument, general equilibrium models
- Nagurney and Dong (1995, 1996a, b)
  - Inclusion of transaction costs
- Nagurney and Ke (2001a)
  - First publication that modeled the financial network with intermediation quantifiably.

Literature Survey of Risk Management with Variable Weights

- Behavior of investors
  - Maximization of the expected return
  - Minimization of the risk
- Markowitz (1952, 1959)
  - Mean and variance
- Numerous extensions (Sharpe (1971), Stone (1973), Young (1998))
  - Equal trade-off between two criteria
- Dong and Nagurney (2001) and Nagurney, Dong, and Mokhtarian (2002)
  - Introduction of variable weights into network equilibrium models

Research Motivation

- The financial economy explicitly includes
  - Financial intermediaries
  - Source agents
  - Uses of financial funds
- Construction of a unified, quantifiable framework
- Variable weights to capture the risk attitudes of investors

Research Motivation

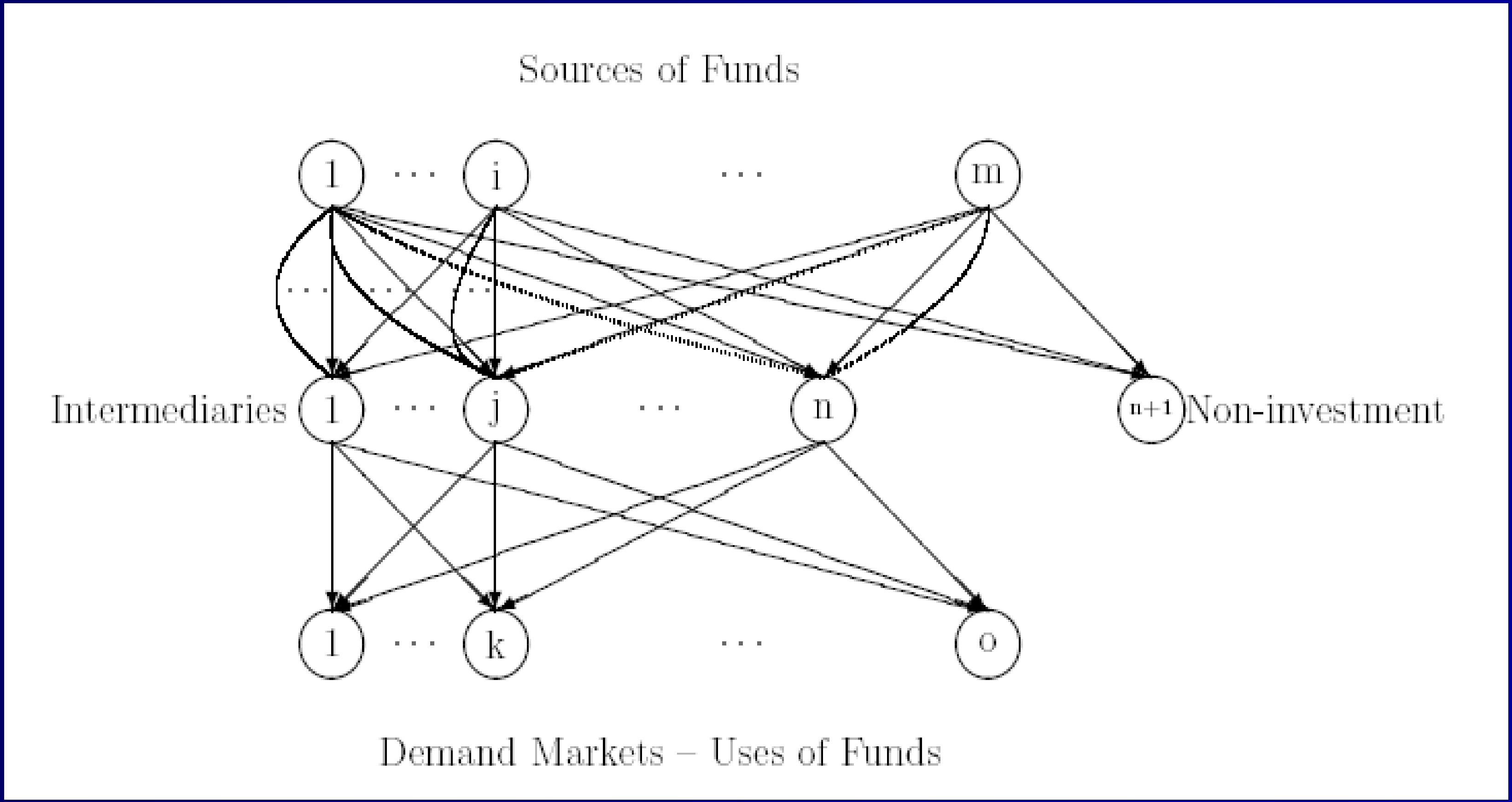
Why variable weights?

In decision-making under uncertainty,

Risk attitude $\longrightarrow$ Decisions $\longrightarrow$ Expected monetary payoffs

To reveal a sector's preference over the return and the risk, we reconstruct the objective functions to include the variable weights.

The Financial Network with Intermediation



Notable Features of Framework

- Models financial systems in disequilibrium or in equilibrium
- Captures interactions
  - Among individual sectors
    - Each facing own objective function
- Emphasizes the advantage of network modeling and computation
- Allows for non-investment
- Incorporates of transaction costs

Notable Features

- Each source agent
  - Maximizes net revenue z_{1i}^{1}
  - Minimizes risk z_{2i}^{1}
- Each intermediary
  - Maximizes net revenue z_{1j}^{2}
  - Minimizes risk z_{2j}^{2}
- Essence: “How much achievement on one objective is the decision-maker willing to give up in order to improve achievement on another objective?”
- Criterion-dependent weights w_{2I}^{t}
  - For source agents: I=i; i=1,…,m, and t=1
  - For intermediaries: I=j; j=1,…,n, and t=2

- Behavior of decision-makers at different tiers
  - Agents with sources of funds

$$\text{Maximize } U^i(q_i^1) = \sum_{j=1}^{n}\sum_{l=1}^{L}(\rho_{ijl}^1 q_{ijl}^1 - c_{ijl}(q_{ijl}^1)) - w_{2i}^1(r^i(q_i^1))r^i(q_i^1)$$

$$\sum_{j=1}^{n}\sum_{l=1}^{L} q_{ijl}^1 \leq S^i$$

  - Intermediaries

$$\text{Maximize } U^j(q_j^2) = \sum_{k=1}^{o}(\rho_{jk}^2 q_{jk}^2 - c_{jk}(q_{jk}^2)) - c_j(Q^1) - \sum_{i=1}^{m}\sum_{l=1}^{L}(\hat{c}_{ijl}(q_{ijl}^1) + \rho_{ijl}^1 q_{ijl}^1)$$
$$-w_{2j}^2(r^j(q_j^2))r^j(q_j^2), \qquad (7.16)$$

Subject to:

$$\sum_{k=1}^{o} q_{jk}^2 \leq \sum_{i=1}^{m}\sum_{l=1}^{L} q_{ijl}^1$$

  - Demand market

$$\rho_{jk}^{2*} + \hat{c}_{jk}(Q^{2*}) \begin{cases} = \rho_k^{3*}, & \text{if} \quad q_{jk}^{2*} > 0 \\ \geq \rho_k^{3*}, & \text{if} \quad q_{jk}^{2*} = 0 \end{cases}$$

$$d_k(\rho^{3*}) \begin{cases} = \sum_{j=1}^{n} q_{jk}^{2*}, & \text{if} \quad \rho_k^{3*} > 0 \\ \leq \sum_{j=1}^{n} q_{jk}^{2*}, & \text{if} \quad \rho_k^{3*} = 0 \end{cases}$$

Variational Inequality Formulation

Theorem: The equilibrium state governing the financial network with intermediation and variable weights is equivalent to the solution of the variational inequality given by:

determine $(Q^{1*}, Q^{2*}, \gamma^*, \rho^{3*}) \in \mathcal{K}$, *satisfying*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L}\left[w_{2i}^1(r^i(q_i^*))\frac{\partial r^i(q_i^*)}{\partial q_{ijl}} + \frac{\partial w_{2i}^1(r^i(q_i^*))}{\partial q_{ijl}}r^i(q_i^*) + w_{2j}^2(r^j(q_j^*))\frac{\partial r^j(q_j^*)}{\partial q_{ijl}} + \frac{\partial w_{2j}^2(r^j(q_j^*))}{\partial q_{ijl}}r^j(q_j^*)\right.$$

$$\left. + \frac{\partial c_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} - \gamma_j^*\right] \times \left[q_{ijl} - q_{ijl}^*\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[w_{2j}^2(r^j(q_j^*))\frac{\partial r^j(q_j^*)}{\partial q_{jk}} + \frac{\partial w_{2j}^2(r^j(q_j^*))}{\partial q_{jk}}r^j(q_j^*) + \frac{\partial c_{jk}(q_{jk}^*)}{\partial q_{jk}} + \hat{c}_{jk}(Q^{2*}) + \gamma_j^* - \rho_k^{3*}\right]\times\left[q_{jk} - q_{jk}^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{L} q_{ijl}^* - \sum_{k=1}^{o} q_{jk}^*\right] \times \left[\gamma_j - \gamma_j^*\right] + \sum_{k=1}^{o}\left[\sum_{j=1}^{n} q_{jk}^* - d_k(\rho^{3*})\right] \times \left[\rho_k^3 - \rho_k^{3*}\right] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, \rho^3) \in \mathcal{K},$$

where $\mathcal{K} \equiv \{\prod_{i=1}^{m} K_i \times R_+^{no+n+o}\}$.

Qualitative Properties

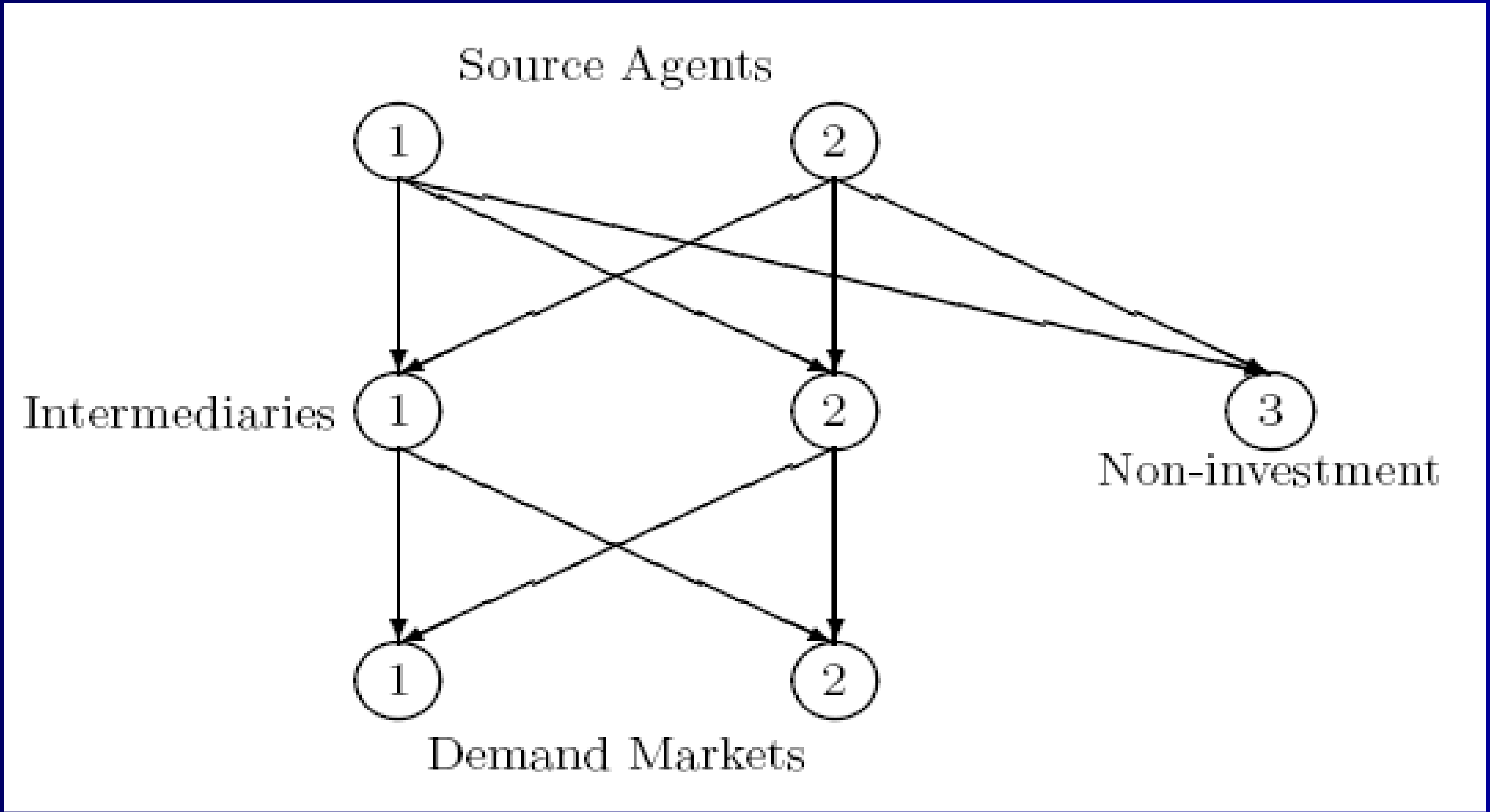
We have established:

- Existence of the solution to the VI
- Uniqueness of the solution to the VI
- Convergence of the modified projection method

Algorithm

- Modified projection method
  - it resolves the VI subproblems into network optimization problems with special structure that can be solved exactly in closed form.
- Computation of financial flow of products and prices

Financial Network Structure for the Numerical Examples



Input Data for Numerical Examples

The transaction cost functions of the source agents	$c_{ijl}(q^1_{ijl}) = 0.5(q^1_{ijl})^2 + 3.5q^1_{ijl}$
The handling costs of the intermediaries	$c_j(Q^1) = 0.5\left(\sum_{i=1}^{2} q^1_{ij1}\right)^2$
The transaction costs of the intermediaries associated with transacting with source agents	$\hat{c}_{ijl}(q^1_{ijl}) = 1.5(q^1_{ijl})^2 + 3q^1_{ijl}$
The transaction costs of the consumers associated with transacting with the intermediaries	$\hat{c}_{jkl}(q^2_{jkl}) = q^2_{jkl} + 5$
The demand functions at the demand markets	$d_1(\rho^3) = -2\rho^3_1 - 1.5\rho^3_2 + 1000$ $d_2(\rho^3) = -2\rho^3_2 - 1.5\rho^3_1 + 1000$

Weights in Examples 2 and 3

- **Example 2:** weights associated with the source agents transacting with the first financial intermediary were doubled

- **Example 3:** $w_{2i}^{1} = z_{2i}^{1}$ for $i =$ 1, 2.

Equilibrium Patterns of the Numerical Examples

			Example 1	Example 2	Example 3
Flows Q^*	Q^{1*}	q_{111}^{1*}	10	9.29	3.10
		q_{121}^{1*}	10	10.71	3.10
		q_{211}^{1*}	10	9.29	3.10
		q_{221}^{1*}	10	10.71	3.10
	Q^{2*}	q_{11}^{2*}	10	9.29	3.10
		q_{12}^{2*}	10	9.29	3.10
		q_{21}^{2*}	10	10.71	3.10
		q_{22}^{2*}	10	10.71	3.10
Lagrange Multipliers	γ_j^*	γ_1^*	245	247.14	269.63
		γ_2^*	245	242.86	269.63
Prices ρ^{3*}	ρ_k^{3*}	ρ_1^{3*}	280	280	283.94
		ρ_2^{3*}	280	280	283.94

Summary and Conclusions

- We developed a framework for the formulation, qualitative analysis, and computation of solutions to financial network equilibrium problems with intermediation and variable weights. The financial network consisted of a multi-tiered network in which non-investment is also permitted.

- Unlike in the earlier literature on financial network equilibrium problems with intermediation, the weights associated with the objectives were no longer assumed to be equal. In particular, we applied risk-penalizing weights, which were variable and dependent on the value of the risk objective in the value function associated with each source agent as well as with each financial intermediary.

Contributions

- Inclusion of variable weights brings the model closer to the “reality” of financial transactions.
- We demonstrated that financial network problems with different tiers of decision-makers in the presence of risk attitudes associated with the source agents and the intermediaries can be formulated and studied in a rigorous fashion.

Future Research

- Empirical studies
- Extension to the international arena
- Inclusion of additional criteria
- Introduction of dynamics

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Thank You!

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