Statics and Dynamics of Complex Network Systems: Supply Chain Analysis and Financial Networks with Intermediation

Dissertation Defense
by
Ke Ke
June 21, 2004
Complex network systems
  - Financial networks with intermediation
  - Supply chains

In
  - Static setting
  - Dynamic setting
Dissertation Structure

- Introduction
  - Literature survey
  - Research motivation
- Methodologies
  - Variational inequality theory
  - Projected dynamical systems
- Multilevel network perspective for supply chain
- Financial networks with intermediation
  - Statics
  - Dynamics
  - Electronic transaction
  - Risk management with variable weights
Literature Survey for Supply Chain Analysis

- Interdisciplinary topic that received great attention
  - Theoretical challenges
  - Practical importance
- Conceptual in nature
- Analytical study
Development of supply chain network models

- Study of interactions among decision-makers
  - Compete within a tier
  - Cooperate between tiers
- Nagurney, Dong, and Zhang (2002)
  - First multitiered model studying the supply chains in a unified manner in network equilibrium context
- Nagurney and Dong (2002b)
  - For further background
- Nagurney, Loo, Dong, and Zhang (2002)
  - Static models over a single network
  - Unified treatment of complex network systems
    - Teleshopping vs shopping
    - Telecommuting vs commuting
Research Motivation for Supply Chain Analysis

- To capture distinct flows
  - Logistical
  - Informational
  - Financial
  within the *same* supply chain network system

- Observe *evolution* of
  - Prices
  - Commodity shipments
  through a discrete-time process
Novelty of My Research

- Nagurney, Ke, Cruz, Hancock and Southworth (2002)
  - *Environment and Planning B*
  - First proposed the multilevel (Logistical/Informational/Financial) network perspective to study the supply chain problem in network equilibrium context.
  - In the supply chain model, the financial and informational networks are made explicit.
  - The multilevel network was widely used thereafter and inspired the idea of supernetworks (http://supernet.som.umass.edu/).
Literature Survey for Financial Networks with Intermediation

- Intermediation
  - Banks, savings institutions, investment and insurance companies
- Quesnay (1758)
  - Origin use of networks to represent financial system
  - Depict the circular flow of funds
  - Study systems of linked portfolios in credit networks
  - Decentralization/decomposition theory
  - Basic intertemporal models
Literature Survey for Financial Networks with Intermediation

- Nagurney, Dong, and Hughes (1992)
  - Multi-sector, multi-instrument
- Dong, Zhang, and Nagurney (1996)
  - Dynamic model
- Nagurney and Dong (1995, 1996a, b)
  - Inclusion of transaction costs
Literature Survey for Electronic Finance

- **A growing area of study**
  - Increasing impact on
    - Financial markets
    - Financial intermediation
    - Related regulatory issues and governance

- **Particular emphasis**
  - Conceptualization
  - Role of networks in the transformations
Literature Survey for Risk Management with Variable Weights

- Behavior of investors
  - Maximize the expected return
  - Minimize the risk
- Markowitz (1952, 1959)
  - Mean and variance
- Numerous extension (Sharpe (1971), Stone (1973), Young (1998))
  - Equal trade-off between two criteria
- Dong and Nagurney (2001) and Nagurney, Dong, and Mokhtarian (2002)
  - Variable weights
Research Motivation for Financial Networks with Intermediation

- The financial economy explicitly includes
  - Financial intermediaries
  - Source agents
  - Uses of financial funds
- Advances of telecommunications have enormous effect
  - Financial services
  - Options available for financial transactions
  - Distribution channels
  - New types of products
  - Role of financial intermediaries
- Construction of a unified, quantifiable framework
- Variable weights to catch the risk attitudes of investors
Novelty of My Research

- Nagurney and Ke (2001a)
  - Published in *Quantitative Finance*
  - First publication that modeled the financial network with intermediation quantifiably.

- Nagurney and Ke (2001b)
  - Disequilibrium dynamics
  - Foundation for further study

  - Published in *Quantitative Finance*
  - First considered the impact of electronic transaction
  - Capture competition within a tier of nodes and cooperation between tiers.

  - First studied the bicriteria decision-making problem in multitiered financial networks.
Chapter 2: Overview of Methodologies

- **Introduction**
- **Methodologies**
  - Variational inequality theory
  - Projected dynamical systems
- Multilevel network perspective for supply chain
- Financial networks with intermediation
- Conclusion

- **Definition of variational inequality problem:**

  To determine a vector \( X^* \in \mathcal{K} \subset \mathbb{R}^n \) such that

  \[
  F(X^*)^T \cdot (X - X^*) \geq 0, \quad \forall X \in \mathcal{K}
  \]

- **Qualitative properties**
  - Existence
  - Monotonicity
  - Lipschitz continuity
  - Uniqueness
Chapter 2: Overview of Methodologies

- Introduction
- **Methodologies**
  - Variational inequality theory
  - Projected dynamical systems
- Multilevel network perspective for supply chain
- Financial networks with intermediation
- Conclusion

Algorithms

- Projection method
  - Initialization
  - Computation
  
\[
[X^\tau + \alpha G^{-1} F(X^{\tau-1}) - X^{\tau-1}]^T \cdot [X - X^\tau] \geq 0, \quad \forall X \in \mathcal{K}
\]

- Convergence verification
Chapter 2: Overview of Methodologies

- Introduction
- Methodologies
  - Variational inequality theory
  - Projected dynamical systems
- Multilevel network perspective for supply chain
- Financial networks with intermediation
- Conclusion

- Modified projection method
  - Initialization
  - Computation
    \[
    \left[ \bar{X}^\tau + \alpha F(X^{\tau-1}) - X^{\tau-1} \right]^T
    \cdot [X - \bar{X}^\tau] \geq 0, \quad \forall X \in \mathcal{K}
    \]
  - Adaptation
    \[
    \left[ X^\tau + \alpha F(\bar{X}^\tau) - X^{\tau-1} \right]^T
    \cdot [X - X^\tau] \geq 0, \quad \forall X \in \mathcal{K}
    \]
  - Convergence verification
Chapter 2: Overview of Methodologies

- Introduction
- **Methodologies**
  - Variational inequality theory
  - **Projected dynamical systems**
- Multilevel network perspective for supply chain
- Financial networks with intermediation
- Conclusion

- **Definition of Projected Dynamical Systems**

  Given \( X \in \mathcal{K} \) and \( v \in F^n \), define the projection of \( v \) at \( X \) (with respect to \( K \)) by

  \[ \pi_K(X, v) = \lim_{\delta \to 0} \frac{P_K(X + \delta v) - X}{\delta} \]

  And

  \[ P_K(X) = \arg\min_{X' \in \mathcal{K}} \|X' - X\| \]

  Definition: The projected dynamical system (PDS) is defined as map: \( \Phi: \mathcal{K} \times \mathcal{R} \mapsto \mathcal{K} \) where:

  \[ \Phi(X, t) = \phi_X(t) \]

  solves the initial value problem:

  \[ \dot{\phi}_X(t) = \pi_K(\phi_X(t), -F(\phi_X(t))), \]

  \[ \phi_X(0) = X \]
Chapter 2: Overview of Methodologies

- Introduction
- Methodologies
  - Variational inequality theory
  - Projected dynamical systems
- Multilevel network perspective for supply chain
- Financial networks with intermediation
- Conclusion

- Qualitative properties
  - Existence, uniqueness, and continuous dependence (Theorem 2.8)
  - Lipschitz continuity (Theorem 2.9)
  - Stability analysis
Chapter 2: Overview of Methodologies

- Introduction
- Methodologies
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- Financial networks with intermediation
- Conclusion

- Algorithm (the Euler method)
  - Initialization
    - Set the sequence $\{\alpha_\tau\}$
  - Computation
    \[ X^\tau = P_K(X^{\tau-1} - \alpha_{\tau-1} F(X^{\tau-1})) \]
  - Convergence verification
Chapter 3: Dynamics of Supply Chains: A Multilevel Network Perspective
Notable Features

- Within the same network:
  - Logistical network
    - Depict the physical flows of products
    - Examine the relationships of various individual entities
  - Informational network
    - Processing information
    - Demand forecasting information
    - Management information
    - Price information
  - Financial network
    - Prices
The Dynamics

- The quantity flows over logistical network
  - The firms adjust commodity shipment
  - The consumers adjust consumption based on prices
  - The retailers operating between them
- The prices over financial network
The Projected Dynamical System

- The equilibrium point (the variational inequality formulation)
- Qualitative properties
  - Monotonicity
  - Lipschitz continuity
  - Existence and uniqueness
  - Stability of the system
- A discrete-time adjustment process
  - The Euler method
  - Computation of the commodity shipments at each iteration
  - Computation of the prices at each iteration
Numerical Examples

Figure 3.2: Multilevel Network for Examples 3.1 and 3.2

Figure 3.3: Multilevel Network for Example 3.3
## Input Data for Examples

<table>
<thead>
<tr>
<th></th>
<th>Example 3.1: ( f_i(q) = 2.5q_i^2 + q_1q_2 + 2q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Example 3.2: ( f_1(q) = 2.5q_1^2 + q_1q_2 + 12q_1 )</td>
</tr>
<tr>
<td></td>
<td>Example 3.3: ( f_2(q) = 2.5q_2^2 + q_1q_2 + 12q_2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The production cost functions of the manufacturers</th>
<th>Example 3.1: ( c_{ij}(Q^1) = .5q_{ij}^2 + 3.5q_{ij}^1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>The transaction cost functions faced by the manufacturers and associated with retailers</td>
<td>Example 3.2: ( c_{1j}(Q^1) = q_{1j}^2 + 3.5q_{1j}^1 )</td>
</tr>
<tr>
<td>The handling costs of the retailers</td>
<td>Example 3.3: ( c_{i3}(Q^1) = .5q_{i3}^1 + 5q_{i3}^1 )</td>
</tr>
<tr>
<td>The demand functions</td>
<td>( c_j(Q^1) = .5(\sum_{i=1}^{2} q_{ij}^1)^2 )</td>
</tr>
<tr>
<td>The transaction costs between the retailers and the consumers</td>
<td>( d_1(\rho^3) = -2\rho_1^3 - 1.5\rho_2^3 + 1000 )</td>
</tr>
<tr>
<td></td>
<td>( d_2(\rho^3) = -2\rho_2^3 - 1.5\rho_1^3 + 1000 )</td>
</tr>
<tr>
<td></td>
<td>( c_{jk}(Q^2) = q_{jk}^2 + 5 )</td>
</tr>
</tbody>
</table>
Equilibrium Patterns of the Numerical Examples

<table>
<thead>
<tr>
<th>Flows $Q^*$</th>
<th>$Q_{1*}^{1*}$ $(q_{ij}^{1*})$</th>
<th>Example 3.1</th>
<th>Example 3.2</th>
<th>Example 3.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{11}$</td>
<td>16.608</td>
<td>14.507</td>
<td>9.243</td>
<td></td>
</tr>
<tr>
<td>$q_{12}$</td>
<td>16.608</td>
<td>14.507</td>
<td>9.243</td>
<td></td>
</tr>
<tr>
<td>$q_{21}$</td>
<td>16.608</td>
<td>17.230</td>
<td>13.567</td>
<td></td>
</tr>
<tr>
<td>$q_{22}$</td>
<td>16.608</td>
<td>17.230</td>
<td>13.567</td>
<td></td>
</tr>
<tr>
<td>$q_{13}$</td>
<td>16.608</td>
<td></td>
<td>14.645</td>
<td></td>
</tr>
<tr>
<td>$q_{23}$</td>
<td></td>
<td></td>
<td>9.726</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Q_{2*}^{2*}$ $(q_{jk}^{2*})$</th>
<th>Example 3.1</th>
<th>Example 3.2</th>
<th>Example 3.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{11}$</td>
<td>16.608</td>
<td>15.869</td>
<td>11.404</td>
</tr>
<tr>
<td>$q_{12}$</td>
<td>16.608</td>
<td>15.869</td>
<td>11.404</td>
</tr>
<tr>
<td>$q_{21}$</td>
<td>16.608</td>
<td>15.869</td>
<td>11.404</td>
</tr>
<tr>
<td>$q_{22}$</td>
<td>16.608</td>
<td>15.869</td>
<td>11.404</td>
</tr>
<tr>
<td>$q_{13}$</td>
<td></td>
<td></td>
<td>12.184</td>
</tr>
<tr>
<td>$q_{23}$</td>
<td></td>
<td></td>
<td>12.184</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>prices $\rho^{3*}$</th>
<th>$\rho_{k}^{3*}$</th>
<th>$\rho_{1}^{3*}$</th>
<th>$\rho_{2}^{3*}$</th>
<th>$\rho_{1}^{3*}$</th>
<th>$\rho_{2}^{3*}$</th>
<th>$\rho_{1}^{3*}$</th>
<th>$\rho_{2}^{3*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>276.224</td>
<td>276.646</td>
<td>275.717</td>
<td>276.224</td>
<td>276.646</td>
<td>275.717</td>
<td>276.224</td>
</tr>
</tbody>
</table>
Chapter 4: Financial Networks with Intermediation
Notable Features

- Financial systems in disequilibrium or in equilibrium
- Interactions
  - Among individual sectors
    - Each facing own objective function
- Emphasize the advantage of network modeling and computation
- Allowing non-investment
- Consideration of transaction costs
The behavior and optimality conditions

- **Agents with sources of funds**

Maximize \( U^i(q^1_i) = \sum_{j=1}^{n} \sum_{l=1}^{L} \rho_{ijl}^1 q_{ijl}^1 - \sum_{j=1}^{n} \sum_{l=1}^{L} c_{ijl}(q_{ijl}^1) - q_i^1 V^i q_i^1 \)

\[ \sum_{j=1}^{n} \sum_{l=1}^{L} q_{ijl}^1 \leq S^i \]

- **Intermediaries**

Maximize \( U^j(q^2_j) = \sum_{k=1}^{o} (\rho^2_{jk} q^2_{jk} - c_{jk}(q^2_{jk})) - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{L} (\hat{c}_{ijl}(q_{ijl}^1) + \rho_{ijl}^1 q_{ijl}^1) - q_j^2 V^i q_j^2 \)

Subject to:
\[ \sum_{k=1}^{o} q^2_{jk} \leq \sum_{i=1}^{m} \sum_{l=1}^{L} q_{ijl}^1 \]

- **Demand markets**

\[ \rho^2_{jk} + \hat{c}_{jk}(Q^{2*}) \begin{cases} = \rho^3_{k}, & \text{if } q^2_{jk} > 0 \\ \geq \rho^3_{k}, & \text{if } q^2_{jk} = 0 \end{cases} \]

\[ d_k(\rho^{3*}) \begin{cases} = \sum_{j=1}^{n} q^2_{jk}, & \text{if } \rho^3_{k} > 0 \\ \leq \sum_{j=1}^{n} q^2_{jk}, & \text{if } \rho^3_{k} = 0 \end{cases} \]
Variational inequality formulation

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{L} \left[ 2V_{zi}^i \cdot q_{ij}^{1*} + 2V_{zi}^j \cdot q_{ij}^{2*} + \frac{\partial c_{ijkl}(q_{ij}^{1*})}{\partial q_{ij}^{1*}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ij}^{1*}} 
\right]
+ \frac{\partial \hat{c}_{iijl}(q_{ij}^{1*})}{\partial q_{ij}^{1*}} - \gamma_j \right] \times [q_{ij}^{1*} - q_{ij}^{1*}]

+ \sum_{j=1}^{n} \sum_{k=1}^{o} \left[ 2V_{z_{mL+k}}^j \cdot q_{jk}^{2*} + \frac{\partial c_{jkl}(q_{jk}^{2*})}{\partial q_{jk}^{2*}} + \hat{c}_{jk}(Q^{2*}) + \gamma_j - \rho_k^{3*} \right] \times [q_{jk}^{2*} - q_{jk}^{2*}]

+ \sum_{j=1}^{n} \left[ \sum_{i=1}^{m} \sum_{l=1}^{L} q_{ijl}^{1*} - \sum_{k=1}^{o} q_{jk}^{2*} \right] \times [\gamma_j - \gamma_j]

+ \sum_{k=1}^{o} \left[ \sum_{j=1}^{n} q_{jk}^{2*} - d_k(\rho^{3*}) \right] \times [\rho_k^{3*} - \rho_k^{3*}] \geq 0, \ \forall (Q^1, Q^2, \gamma, \rho^3) \in \mathcal{K},
\]

Qualitative properties

- Uniqueness
- Existence

The algorithm

- Modified projection method
- Computation of financial flow of products and prices

Numerical examples
Chapter 5: Dynamics of Financial Networks with Intermediation

- Network structure
- The dynamics
  - Financial flows
    - Source agents adjust investment in various instruments
    - Intermediaries adjust transactions
    - Consumers obtain the financial products
  - Prices over time
The projected dynamical system
The equilibrium point (the variational inequality formulation)
Qualitative properties
- Existence and uniqueness
- Stability of the system
A discrete-time adjustment process
- The Euler method
- Computation of financial flows and prices at each iteration
Numerical examples
- Verify that the set of stationary points of PDS coincides with the set of solutions to the VI problem
Chapter 6: Dynamics of Financial Networks with Electronic Transactions

Sources of Financial Funds

Internet Links

Intermediaries

Physical Links

Internet Links

Physical Links

Demand Markets – Uses of Funds

1 \quad \cdots \quad i \quad \cdots \quad m

1 \quad \cdots \quad j \quad \cdots \quad n \quad \cdots \quad n+1

1 \quad \cdots \quad k \quad \cdots \quad o
Notable Features

- Electronic transactions
- Source agents can transact:
  - With the intermediaries either physically or electronically
  - With the consumers in an electronic manner
- Intermediaries transact with the consumers
  - Physically
  - Electronically
The behavior and optimality conditions

- Agents with sources of funds
  - Maximize the net revenue
  - Minimize the risk

- Intermediaries
  - Maximize the net revenue
  - Minimize the risk

- Demand markets

Variational inequality formulation

The dynamics

- The projected dynamical system
Qualitative properties
- Uniqueness
- Existence
- Lipschitz continuity
- Stability of the system

The algorithm
- The Euler method
- Computation of financial flows and prices

Numerical examples
Numerical Examples

Figure: The Financial Network Structure of Example 6.1
Numerical Examples (Cont.)

Figure: The Financial Network Structure of Example 6.2
Numerical Examples (Cont.)

Figure: The Financial Network Structure of Example 6.3
## Input Data for Examples 6.1-6.3

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The transaction cost functions of the source agents</td>
<td>( c_{ijl}(q_{ijl}) = 0.5(q_{ijl}^1)^2 + 3.5q_{ijl}^1 )</td>
</tr>
<tr>
<td>The handling costs of the intermediaries</td>
<td>( c_j(Q^1) = 0.5 \left( \sum_{i=1}^{2} q_{ij1}^1 \right)^2 )</td>
</tr>
<tr>
<td>The transaction costs of the intermediaries associated with transacting with source agents</td>
<td>( \hat{c}<em>{ijl}(q</em>{ijl}) = 1.5(q_{ijl}^1)^2 + 3q_{ijl}^1 )</td>
</tr>
<tr>
<td>The transaction costs of the consumers transacting with the intermediaries</td>
<td>( \hat{c}<em>{jkl}(q</em>{jkl}^2) = q_{jkl}^2 + 5 )</td>
</tr>
<tr>
<td>The demand functions at the demand markets</td>
<td>( d_1(\rho^3) = -2\rho_1^3 - 1.5\rho_2^3 + 1000 )</td>
</tr>
<tr>
<td></td>
<td>( d_2(\rho^3) = -2\rho_2^3 - 1.5\rho_1^3 + 1000 )</td>
</tr>
</tbody>
</table>
### Additional transaction costs for Example 6.2 and 6.3:

<table>
<thead>
<tr>
<th>The transaction cost functions of the source agents for dealing with demand markets:</th>
<th>$c_{ik}(q_{ik}^3) = 0.5(q_{ik}^3)^2 + q_{ik}^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction cost functions from the demand markets:</td>
<td>$\hat{c}<em>{ik}(Q_i^2, Q_j^3) = q</em>{ik}^3 + 1$</td>
</tr>
</tbody>
</table>

Based on Example 6.2, the additional transaction costs for Example 6.3 are:

<table>
<thead>
<tr>
<th>The transaction costs of the source agents transacting with the financial intermediaries electronically:</th>
<th>$c_{ij2}(q_{ij2}^1) = 0.5(q_{ij2}^1)^2 + 0.5q_{ij2}^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transaction costs of the intermediaries transacting with sources electronically:</td>
<td>$\hat{c}<em>{ij2}(q</em>{ij2}^1) = 0.5(q_{ij2}^1)^2 + 0.5q_{ij2}^1$</td>
</tr>
</tbody>
</table>
Table 6.1: Equilibrium Patterns of the Numerical Examples

<table>
<thead>
<tr>
<th>Flows $Q^*$</th>
<th>Example 6.1</th>
<th>Example 6.2</th>
<th>Example 6.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^{1+}$</td>
<td>$q_{i1}^{1+}$</td>
<td>10</td>
<td>4.8</td>
</tr>
<tr>
<td>($q_{ij}^{1+}$)</td>
<td>$q_{i2}^{1+}$</td>
<td>10</td>
<td>4.8</td>
</tr>
<tr>
<td>$q_{11}^{1+}$</td>
<td>10</td>
<td>4.8</td>
<td>3.09</td>
</tr>
<tr>
<td>$q_{21}^{1+}$</td>
<td>10</td>
<td>4.8</td>
<td>3.09</td>
</tr>
<tr>
<td>$q_{12}^{1+}$</td>
<td>10</td>
<td>4.8</td>
<td>3.09</td>
</tr>
<tr>
<td>$q_{112}$</td>
<td></td>
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<td>3.33</td>
</tr>
<tr>
<td>$q_{122}$</td>
<td></td>
<td></td>
<td>3.33</td>
</tr>
<tr>
<td>$q_{212}$</td>
<td></td>
<td></td>
<td>3.33</td>
</tr>
<tr>
<td>$q_{222}$</td>
<td></td>
<td></td>
<td>3.33</td>
</tr>
<tr>
<td>$Q^{2+}$</td>
<td>$q_{i1}^{2+}$</td>
<td>10</td>
<td>4.8</td>
</tr>
<tr>
<td>($q_{ijkl}^{2+}$)</td>
<td>$q_{i2}^{2+}$</td>
<td>10</td>
<td>4.8</td>
</tr>
<tr>
<td>$q_{211}^{2+}$</td>
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<td>4.8</td>
<td>6.42</td>
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<tr>
<td>$q_{211}^{2+}$</td>
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<td>4.8</td>
<td>6.42</td>
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<tr>
<td>$Q^{3+}$</td>
<td>$q_{i1}^{3+}$</td>
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<td>5.2</td>
</tr>
<tr>
<td>($q_{ijk}^{3+}$)</td>
<td>$q_{i2}^{3+}$</td>
<td></td>
<td>5.2</td>
</tr>
<tr>
<td>$q_{211}^{3+}$</td>
<td></td>
<td></td>
<td>5.2</td>
</tr>
<tr>
<td>$q_{221}^{3+}$</td>
<td></td>
<td></td>
<td>5.2</td>
</tr>
<tr>
<td>Lagrange Multipliers</td>
<td>$\gamma_i$</td>
<td>245.02</td>
<td>260.59</td>
</tr>
<tr>
<td>$\gamma^*$</td>
<td>$\gamma_1$</td>
<td>245.02</td>
<td>260.59</td>
</tr>
<tr>
<td>Prices $\rho^{3*}$</td>
<td>$\rho_k^{3*}$</td>
<td>280</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>$\rho_1^{3*}$</td>
<td>280</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>$\rho_2^{3*}$</td>
<td>280</td>
<td>280</td>
</tr>
</tbody>
</table>
Chapter 7: Financial Networks with Intermediation: Risk Management with Variable Weights

Why variable weights?
In decision-making under uncertainty,

Risk attitude → Decisions → Expected monetary payoffs

To reveal a sector’s preference over the return and the risk, we reconstruct the objective functions to include the variable weights.
Notable Features

- Each source agent
  - Maximize net revenue \( z_{1i} \)
  - Minimize risk \( z_{2i} \)

- Each intermediary
  - Maximize net revenue \( z_{1j} \)
  - Minimize risk \( z_{2j} \)

- Essence: “How much achievement on one objective is the decision-maker willing to give up in order to improve achievement on another objective?”

- Criterion-dependent weight \( w_{2I} \)
  - For source agents: \( I=i; \ i=1,\ldots,m, \ \text{and} \ t=1 \)
  - For intermediaries: \( I=j; \ j=1,\ldots,n, \ \text{and} \ t=2 \)
**Risk-penalizing value function**

- **Source Agents**

Maximize $U^i(q^1_i) = \sum_{j=1}^{n} \sum_{l=1}^{L} (\rho_{ijl}^1 q^1_{ijl} - c_{ijl}(q^1_{ijl})) - w^1_{2i}(r^i(q^1_i))r^i(q^1_i)$

- **Intermediaries**

Maximize $U^j(q^2_j) = \sum_{k=1}^{o} (\rho_{jk}^2 q^2_{jk} - c_{jk}(q^2_{jk})) - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{L} (\hat{c}_{ijl}(q^1_{ijl}) + \rho_{ijl}^1 q^1_{ijl})$

$- w^2_{2j}(r^j(q^2_j))r^j(q^2_j)$, \hspace{1cm} (7.16)

Subject to: $\sum_{k=1}^{o} q^2_{jk} \leq \sum_{i=1}^{m} \sum_{l=1}^{L} q^1_{ijl}$

- **Demand Markets**
Variational inequality formulation

Qualitative properties
  - Uniqueness
  - Existence

The algorithm
  - Modified projection method
  - Computation of financial flow of products and prices

Numerical examples
## Input Data for Examples 7.1-7.3

<table>
<thead>
<tr>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>The transaction cost functions of the source agents</td>
<td>( c_{ijl}(q_{ijl}^1) = 0.5(q_{ijl}^1)^2 + 3.5q_{ijl}^1 )</td>
</tr>
<tr>
<td>The handling costs of the intermediaries</td>
<td>( c_j(Q^1) = 0.5\left(\sum_{i=1}^{2} q_{ijl}^1\right)^2 )</td>
</tr>
<tr>
<td>The transaction costs of the intermediaries associated with transacting with source agents</td>
<td>( \hat{c}<em>{ijl}(q</em>{ijl}^1) = 1.5(q_{ijl}^1)^2 + 3q_{ijl}^1 )</td>
</tr>
<tr>
<td>The transaction costs of the consumers associated with transacting with the intermediaries</td>
<td>( \hat{c}<em>{jkl}(q</em>{jkl}^2) = q_{jkl}^2 + 5 )</td>
</tr>
<tr>
<td>The demand functions at the demand markets</td>
<td>( d_1(\rho^3) = -2\rho_1^3 - 1.5\rho_2^3 + 1000 ) ( d_2(\rho^3) = -2\rho_2^3 - 1.5\rho_1^3 + 1000 )</td>
</tr>
</tbody>
</table>
Weights in Examples 7.2 and 7.3

- Example 7.2: weights associated with the source agents transacting with the first financial intermediary were doubled.
- Example 7.3: $w_{2i}^1 = z_{2i}^1$ for $i = 1, 2$. 
Table 7.1: Equilibrium Patterns of the Numerical Examples

<table>
<thead>
<tr>
<th></th>
<th>Example 7.1</th>
<th>Example 7.2</th>
<th>Example 7.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flows $Q^*$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q^1*$</td>
<td>$q_{111}^1$</td>
<td>10</td>
<td>9.29</td>
</tr>
<tr>
<td>$(q_{ijl}^1)$</td>
<td>$q_{121}^1$</td>
<td>10</td>
<td>10.71</td>
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<tr>
<td></td>
<td>$q_{211}^1$</td>
<td>10</td>
<td>9.29</td>
</tr>
<tr>
<td></td>
<td>$q_{221}^1$</td>
<td>10</td>
<td>10.71</td>
</tr>
<tr>
<td>$Q^2*$</td>
<td>$q_{11}^2$</td>
<td>10</td>
<td>9.29</td>
</tr>
<tr>
<td>$(q_{ijkl}^2)$</td>
<td>$q_{12}^2$</td>
<td>10</td>
<td>9.29</td>
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<tr>
<td></td>
<td>$q_{21}^2$</td>
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<td>10.71</td>
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<tr>
<td></td>
<td>$q_{22}^2$</td>
<td>10</td>
<td>10.71</td>
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<tr>
<td>Lagrange Multipliers</td>
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</tr>
<tr>
<td>$\gamma^*$</td>
<td>$\gamma_j^*$</td>
<td>245.00</td>
<td>247.14</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma_2^*$</td>
<td>245.00</td>
<td>242.86</td>
</tr>
<tr>
<td>Prices $\rho^{3*}$</td>
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</tr>
<tr>
<td>$\rho_{k}^{3*}$</td>
<td>$\rho_1^{3*}$</td>
<td>280</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>$\rho_2^{3*}$</td>
<td>280</td>
<td>280</td>
</tr>
</tbody>
</table>
Comparison of the Two Areas

- **Similarities**
  - Network structure
  - Variables
  - Cost structure
  - Methodology

- **Differences**
  - Construction
  - Flows on the links
  - Direction of flows
  - Behavior of top two tiers
  - Role of middle tier
  - Constraints
Contributions

- First dynamic multilevel supply chain model within context of network equilibrium (Nagurney, Ke, Cruz, Hancock, and Southworth (2002))
- First financial network study to explicitly include the financial intermediaries quantifiably (Nagurney and Ke (2001a))
- First financial network study to consider the impact of electronic transactions quantifiably (Nagurney and Ke (2003))
- Incorporate the dimension of the explicit dynamic behavior of the various financial agents as well as the price dynamics
- Inclusion of variable weights brings the model closer to the “reality” of financial transactions
Future Research

- **Empirical study**
  - Multilevel supply chain network model
  - Theoretical models of financial networks

- **Informational Networks**
  - Vary with respect to
    - Timelines
    - Quality of information
  - Different costs

- **Introduction of variable weights into current**
  - Supply chain network models
  - International financial network models

- **Additional criteria**
Acknowledgement

This research was supported, in part, by NSF Grant No.: CMS-0085720 and, in part, by NSF Grant No.: IIS-0002647. This support is gratefully acknowledged.

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