Supply Chain Supernetworks with Random Demands

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In Honor of Professor David Boyce

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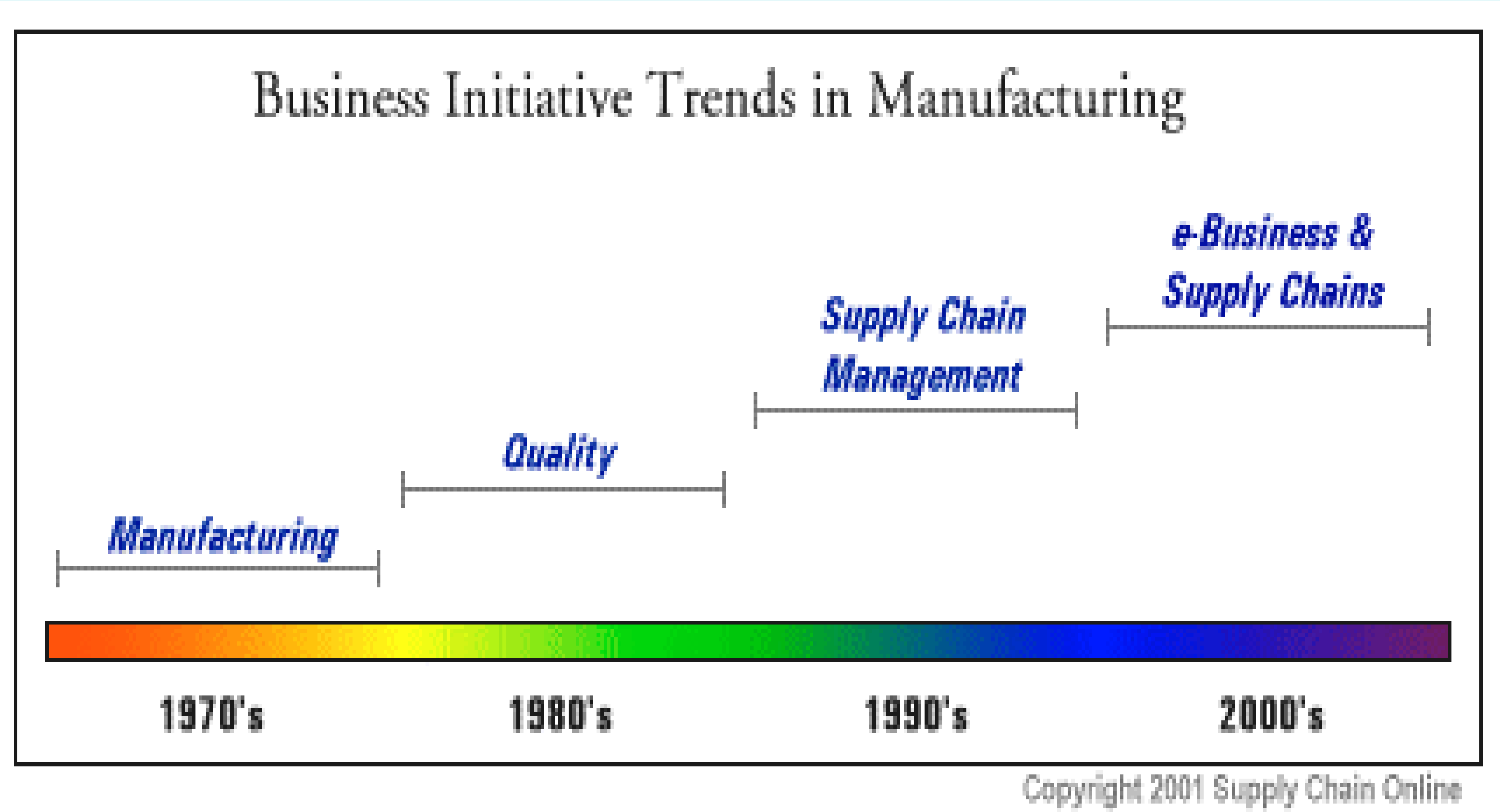
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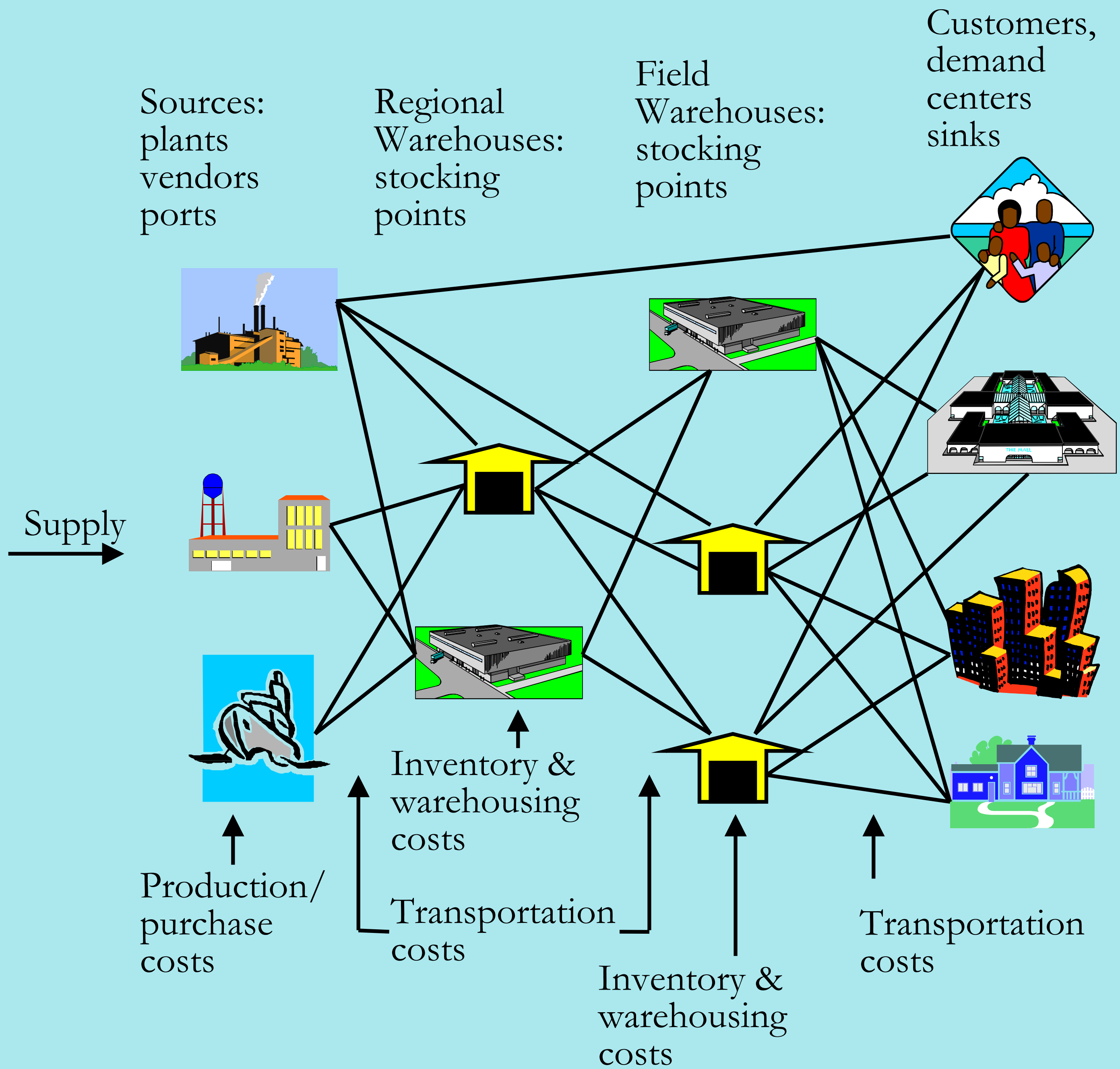

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Warehouses:
stocking
points
Field
Warehouses:
stocking
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Customers,
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Supply
Production/
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costs
Inventory &
warehousing
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Inventory &
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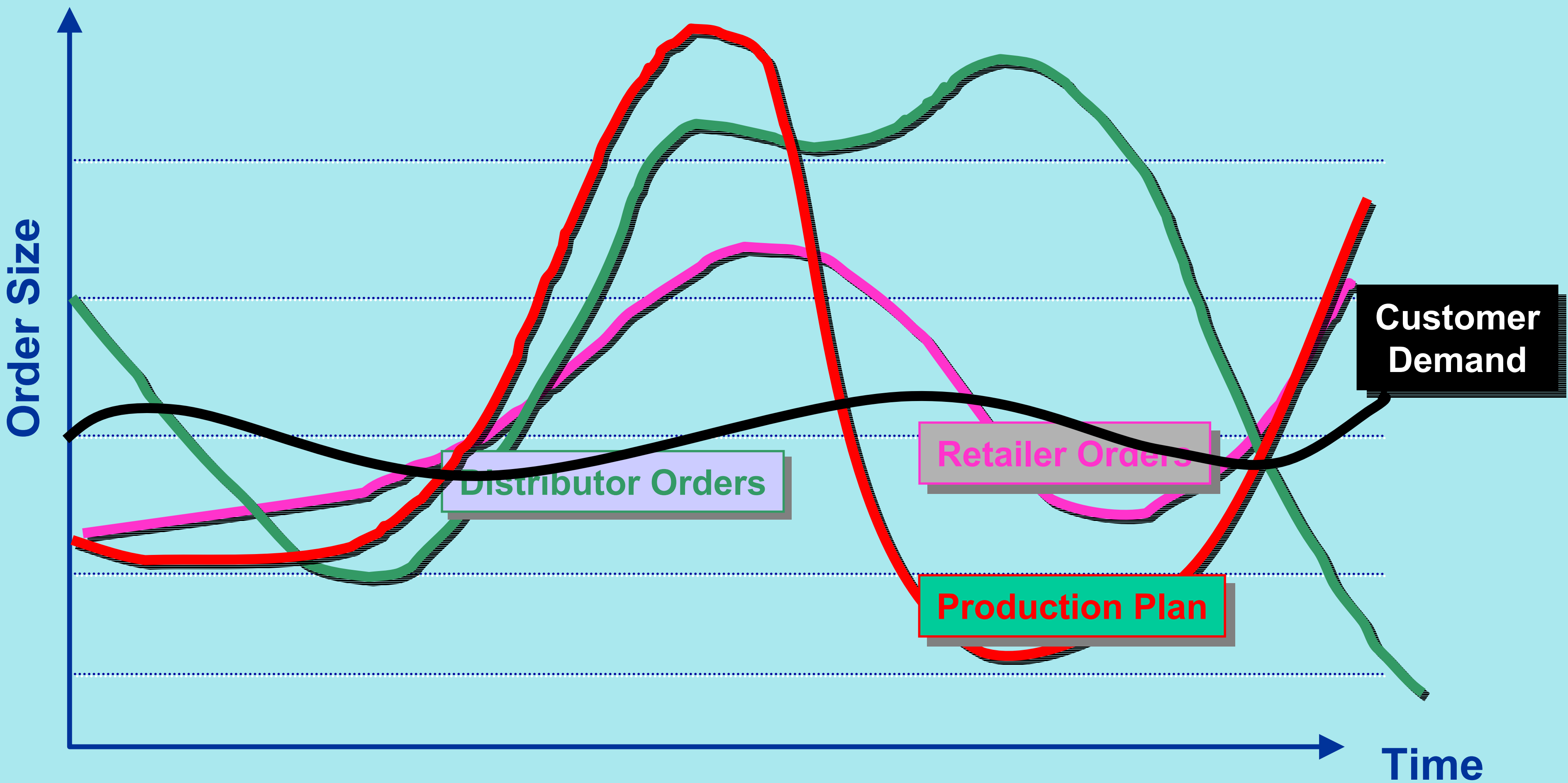
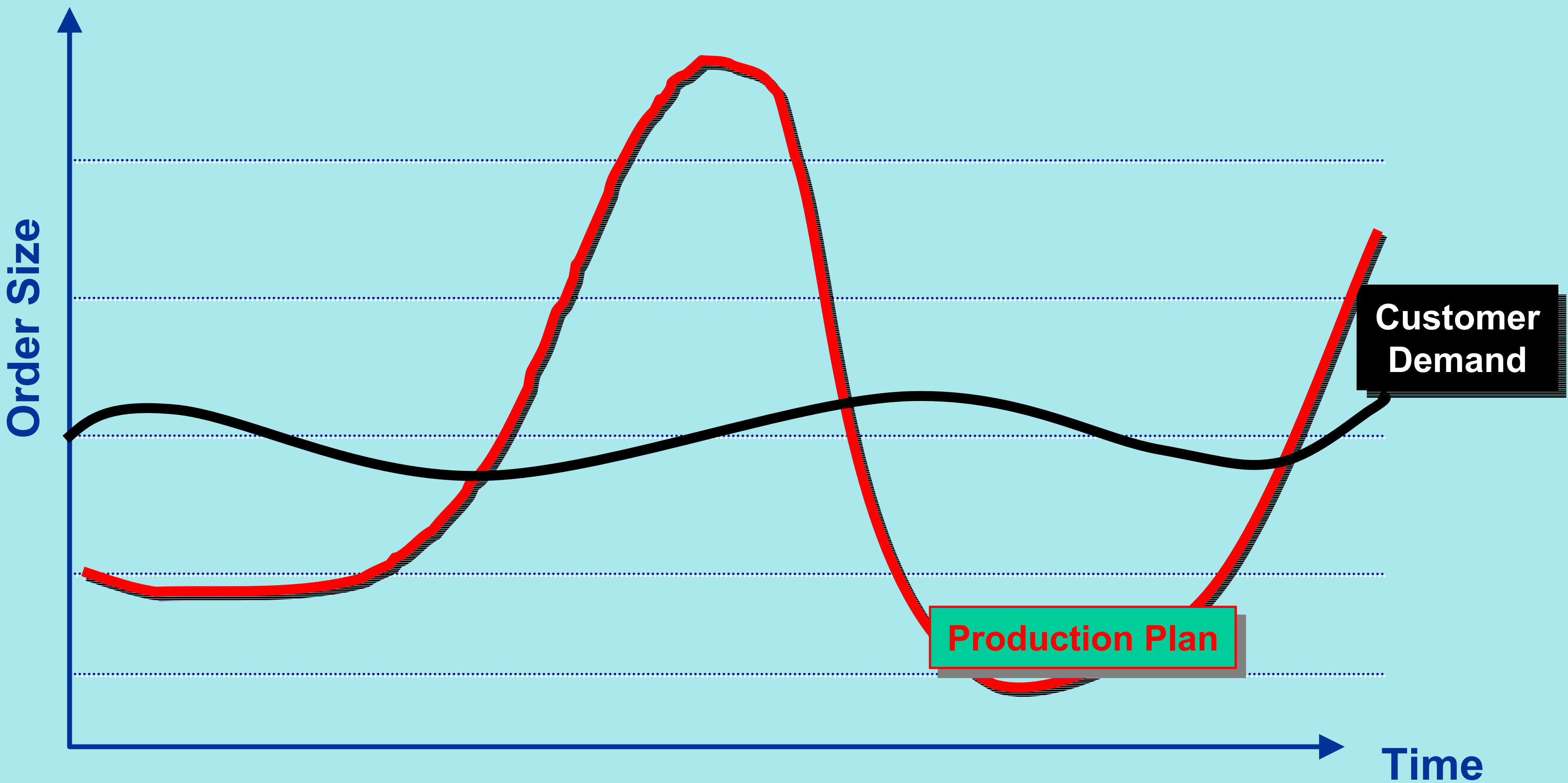
The Dynamics of the Supply Chain



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

What Management Gets...



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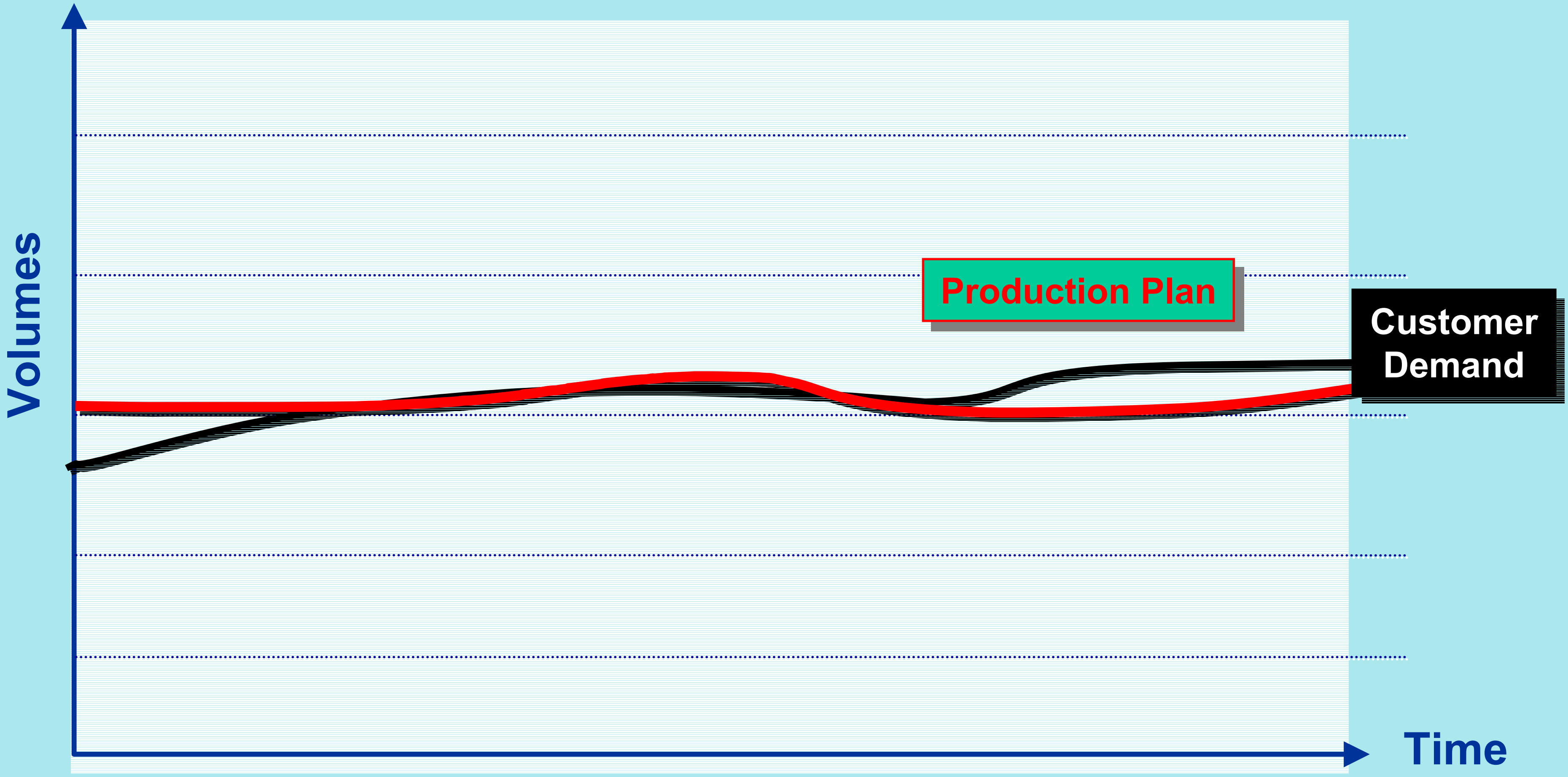
What Management Wants…



Source: Tom Mc Guffry, Electronic Commerce and Value Chain Management, 1998

The overall Supply Chain Planning & Collaboration (SCP&C) market exceeded $1,903 million in 2003 despite the weak macroeconomic environment over the past two years.

This is particularly interesting since the recession in the United States had substantial reductions in capital spending, especially spending on information technology.

The industrial manufacturing sector, at $1,615 million, accounted for 85 percent of this market.

DEDHAM, Mass.--(BUSINESS WIRE)--Nov. 18, 2003

The Supernetwork Supply Chain Model with Random Demands

Captures
- **competition and coordination**

Determines
- **production quantities**
- **shipments**
- **prices**
- **expected demands**

Includes
- **physical transactions**
- **electronic transactions**

Supernetwork Structure

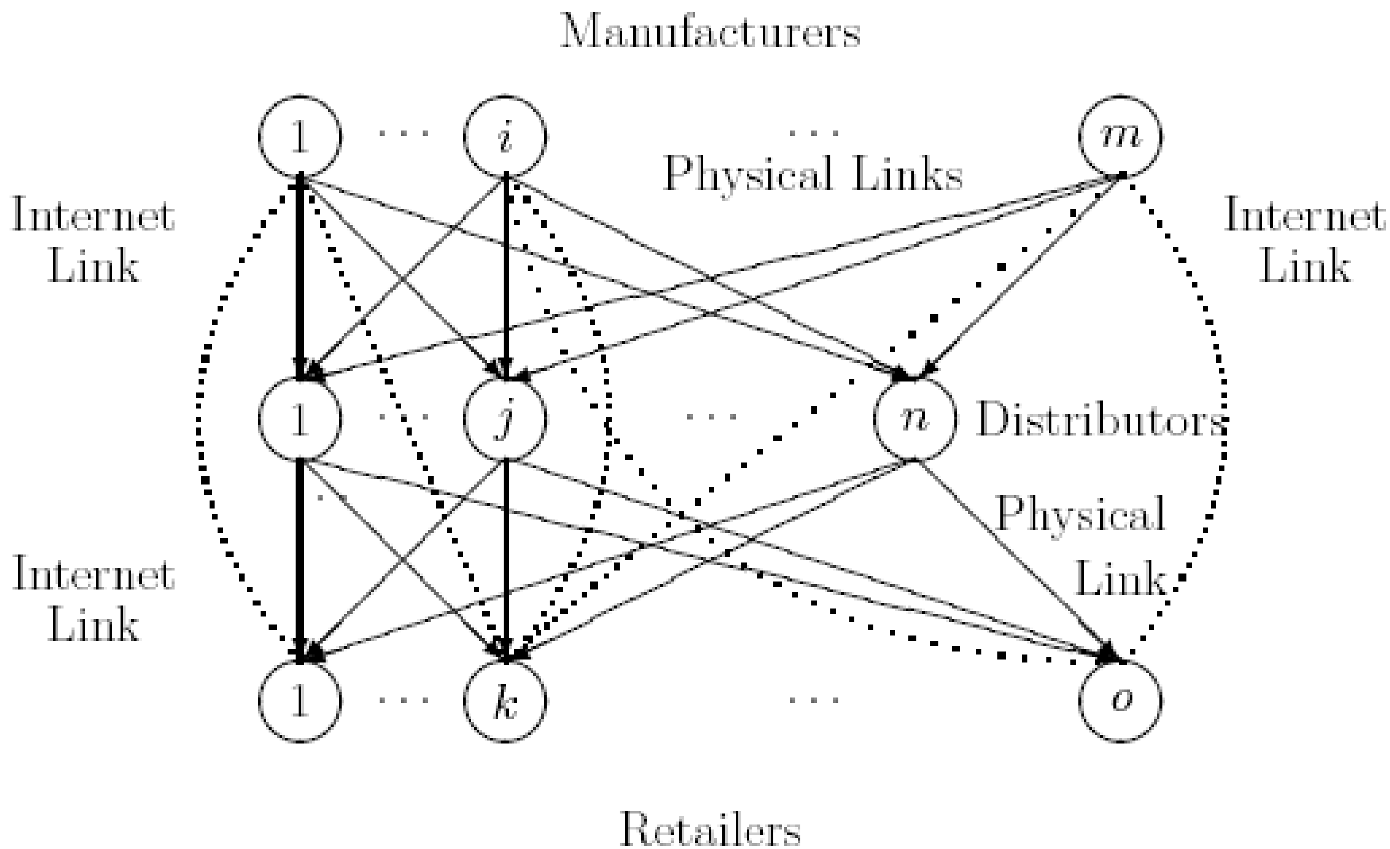


Figure 1: The Supernetwork Structure of the Supply Chain

Behavior of the Manufacturers

Profit maximization problem for manufacturer ***i***

$$\text{Maximize} \sum_{j=1}^{n} \rho_{1ij}^{*} q_{ij} + \sum_{k=1}^{o} \rho_{1ik}^{*} q_{ik} - f_i(Q^1, Q^2) - \sum_{j=1}^{n} c_{ij}(q_{ij}) - \sum_{k=1}^{o} c_{ik}(q_{ik}),$$

subject to $q_{ij} \geq 0$, for all j, and $q_{ik} \geq 0$, for all k.

q_i : production output

q_{ij}: shipment from i to j

q_{ik} : shipment from i to j via e-link

ρ_{1ij}^{*} : price charged by manufacturer i to distributor j

ρ_{1ik}^{*} : price charged by manufacturer i to retailer k

The Optimality Condition for Manufacturers

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}}-\rho_{1ij}^*\right]\times\left[q_{ij}-q_{ij}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1^*},Q^{2^*})}{\partial q_{ik}}+\frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}}-\rho_{1ik}^*\right]\times\left[q_{ik}-q_{ik}^*\right]\geq 0,\quad \forall (Q^1,Q^2)\in R_+^{mn+mo}. \quad (5)$$

Behavior of the Distributors

Profit maximization problem for distributor ***j***

$$\text{Maximize} \quad \gamma_j^* \sum_{k=1}^{o} q_{jk} - c_j(Q^1, Q^3) - \sum_{i=1}^{m} \rho_{1ij}^* q_{ij}$$

subject to:

$$\sum_{k=1}^{o} q_{jk} \leq \sum_{i=1}^{m} q_{ij},$$

q_{ij}: shipment from i to j

q_{jk} : shipment from j to j

γ_j^* : price charged by distributor j

Optimality Conditions for the Distributors

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_j(Q^{1*},Q^{3*})}{\partial q_{ij}}+\rho_{1ij}^{*}-\rho_{2j}^{*}\right]\times\left[q_{ij}-q_{ij}^{*}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[-\gamma_j^{*}+\frac{\partial c_j(Q^{1*},Q^{3*})}{\partial q_{jk}}+\rho_{2j}^{*}\right]\times\left[q_{jk}-q_{jk}^{*}\right]+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}q_{ij}^{*}-\sum_{k=1}^{o}q_{jk}^{*}\right]\times\left[\rho_{2j}-\rho_{2j}^{*}\right]\geq 0$$

$$\forall Q^1\in R_+^{mn},\forall Q^3\in R_+^{no},\forall \rho_2\in R_+^{n}, \quad (9)$$

The Retailers

$$s_k = \sum_{i=1}^{m} q_{ik} + \sum_{j=1}^{n} q_{jk}$$

ρ_{3k} : demand price at retailer k

$\hat{d}_k(\rho_{3k})$: random demand at retailer k

$\square_k(x, \rho_{3k})$: density function

$P_k(x, \rho_{3k})$: probability function of $\hat{d}_k(\rho_{3k})$

$$P_k(x, \rho_{3k}) = P_k(\hat{d}_k \leq x) = \int_0^x \square_k(x, \rho_{3k}) dx$$

$min\{s_k, \hat{d}_k\}$: the actual sale of k cannot exceed this amount

$$\Delta_k^+ \equiv \max\{0, s_k - \hat{d}_k\} \tag{11}$$

and

$$\Delta_k^- \equiv \max\{0, \hat{d}_k - s_k\}, \tag{12}$$

where Δ_k^+ is a random variable representing the excess supply (inventory), whereas Δ_k^- is a random variable representing the excess demand (shortage).

Note that the expected values of excess supply and excess demand of retailer k are scalar functions of s_k and ρ_{3k}. In particular, let e_k^+ and e_k^- denote, respectively, the expected values: $E(\Delta_k^+)$ and $E(\Delta_k^-)$, that is,

$$e_k^+(s_k, \rho_{3k}) \equiv E(\Delta_k^+) = \int_0^{s_k} (s_k - x)\mathcal{F}_k(x, \rho_{3k})dx, \tag{13}$$

$$e_k^-(s_k, \rho_{3k}) \equiv E(\Delta_k^-) = \int_{s_k}^{\infty} (x - s_k)\mathcal{F}_k(x, \rho_{3k})dx. \tag{14}$$

λ_k^+ : unit penalty of having excess supply at retailer k

λ_k^- : unit penalty of having excess demand at retailer k

The expected total penalty of retailer k is given by

$$E(\lambda_k^+ \Delta_k^+ + \lambda_k^- \Delta_k^-) = \lambda_k^+ e_k^+(s_k, \rho_{3k}) + \lambda_k^- e_k^-(s_k, \rho_{3k}).$$

Behavior of the Retailers

Profit maximization problem for retailer *k*

$$\text{Maximize } E(\rho_{3k} \min\{s_k, \hat{d}_k\}) - E(\lambda_k^+ \Delta_k^+ + \lambda_k^- \Delta_k^-) - c_k(Q^2, Q^3) - \sum_{i=1}^{m} \rho_{1ik}^* q_{ik} - \sum_{j=1}^{n} \gamma_j^* q_{jk}$$

subject to:

$$q_{ik} \geq 0, \quad q_{jk} \geq 0, \text{ for all } i \text{ and } j.$$

Applying the definitions of Δ_k^+, Δ_k^-,

$$\text{Maximize} \quad \rho_{3k} d_k(\rho_{3k}) - (\rho_{3k} + \lambda_k^-) e_k^-(s_k, \rho_{3k}) - \lambda_k^+ e_k^+(s_k, \rho_{3k}) - c_k(Q^2, Q^3) - \sum_{i=1}^{m} \rho_{1ik}^* q_{ik} - \sum_{j=1}^{n} \gamma_j^* q_{jk}$$

where $d_j(\rho_{3k}) \equiv E(\hat{d}_k)$ is a scalar function of ρ_{3k}.

Optimality Conditions for the Retailers

Assuming that the handling cost for each retailer is continuous and convex, then the optimality conditions for all the retailers satisfy the variational inequality: determine $(Q^{2^*}, Q^{3^*}) \in R_+^{mo+no}$, satisfying:

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\lambda_k^+ P_k(s_k^*, \rho_{3k}) - (\lambda_k^- + \rho_{3k})(1 - P_k(s_k^*, \rho_{3k})) + \frac{\partial c_k(Q^{2^*}, Q^{3^*})}{\partial q_{ik}} + \rho_{1ik}^*\right] \times [q_{ik} - q_{ik}^*]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\lambda_k^+ P_k(s_k^*, \rho_{3k}) - (\lambda_k^- + \rho_{3k})(1 - P_k(s_k^*, \rho_{3k})) + \frac{\partial c_k(Q^{2^*}, Q^{3^*})}{\partial q_{jk}} + \gamma_j^*\right] \times \left[q_{jk} - q_{jk}^*\right] \geq 0,$$

$$\forall (Q^2, Q^3) \in R_+^{mo+no}. \qquad (20)$$

The Stochastic Market Equilibrium Conditions

For any retailer *k*

$$\hat{d}_k(\rho_{3k}^*) \begin{cases} \leq \sum_{i=1}^{m} q_{ik}^* + \sum_{j=1}^{o} q_{jk}^* & \text{a.e., if } \rho_{3k}^* = 0 \\ = \sum_{i=1}^{m} q_{ik}^* + \sum_{j=1}^{o} q_{jk}^* & \text{a.e., if } \rho_{3k}^* > 0, \end{cases}$$

where a.e. means that the corresponding equality or inequality holds almost everywhere.

$$\sum_{k=1}^{o}\left(\sum_{i=1}^{m} q_{ik}^* + \sum_{j=1}^{n} q_{jk}^* - d_j(\rho_{3k}^*)\right) \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \qquad \forall \rho_3 \in R_+^o, \qquad (22)$$

where ρ_3 is the o-dimensional column vector with components: $\{\rho_{31}, \ldots, \rho_{3o}\}$.

Supply Chain Network Equilibrium with Random Demands

Definition:

The equilibrium state of the supply chain with random demands is one where the product flows between the tiers of the decision-makers coincide and the product shipments and prices satisfy the sum of the optimality conditions (5), (9), and (20), and the conditions (20).

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation

A product shipment and price pattern $(Q^{1*}, Q^{2*}, Q^{3*}, \rho_2^*, \rho_3^*) \in \mathcal{K}$ *is an equilibrium pattern of the supply chain model according to Definition 1 if and only if it satisfies the variational inequality problem:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ij}} + \frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}} + \frac{\partial c_j(Q^{1*},Q^{3*})}{\partial q_{ij}} - \rho_{2j}^*\right] \times \left[q_{ij} - q_{ij}^*\right]$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q_{ik}^*)}{\partial q_{ik}} + \frac{\partial c_k(Q^{2*},Q^{3*})}{\partial q_{ik}}\right.$$

$$\left. + \lambda_k^+ P_k(s_k^*, \rho_{3k}^*) - (\lambda_k^- + \rho_{3k}^*)(1 - P_k(s_k^*, \rho_{3k}^*))\right] \times [q_{ik} - q_{ik}^*]$$

$$+ \sum_{j=1}^{n}\sum_{k=1}^{o}\left[\lambda_k^+ P_k(s_k^*, \rho_{3k}^*) - (\lambda_k^- + \rho_{3k}^*)(1 - P_k(s_k^*, \rho_{3k}^*)) + \frac{\partial c_j(Q^{1*},Q^{3*})}{\partial q_{jk}}\right.$$

$$\left. + \frac{\partial c_k(Q^{2*},Q^{3*})}{\partial q_{jk}} + \rho_{2j}^*\right] \times \left[q_{jk} - q_{jk}^*\right] + \sum_{j=1}^{n}\left[\sum_{i=1}^{m} q_{ij}^* - \sum_{k=1}^{o} q_{jk}^*\right] \times \left[\rho_{2j} - \rho_{2j}^*\right]$$

$$+ \sum_{k=1}^{o}\left[\sum_{j=1}^{n} q_{jk}^* + \sum_{i=1}^{m} q_{ik}^* - d_k(\rho_3^*)\right] \times [\rho_{3k} - \rho_{3k}^*] \geq 0, \quad \forall (Q^1, Q^2, Q^3, \rho_2, \rho_3) \in \mathcal{K}, \qquad (23)$$

Qualitative Properties

Under certain conditions we proved the existence of the solution and the uniqueness of the solution to the VIP.

Numerical Examples

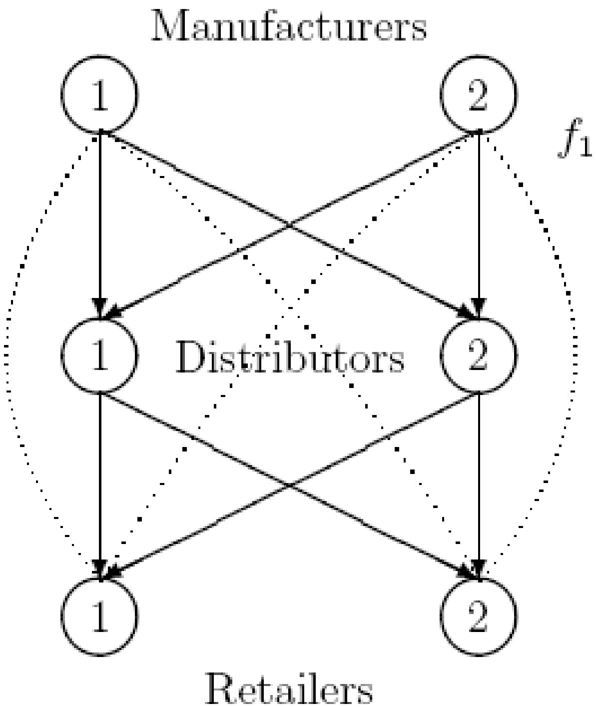
In all the examples, we assumed that the demands associated with the retail outlets followed a uniform distribution. Hence, we assumed that the random demand, $\hat{d}_k(\rho_{3k})$, of retailer k, is uniformly distributed in $[0, \frac{b_k}{\rho_{3k}}]$, $b_k > 0$; $k = 1, \ldots, o$. Therefore,

$$P_k(x, \rho_{3k}) = \frac{x\rho_{3k}}{b_k}, \tag{51}$$

$$\mathcal{F}_j(x, \rho_{3k}) = \frac{\rho_{3k}}{b_k}, \tag{52}$$

$$d_k(\rho_{3k}) = E(\hat{d}_k) = \frac{1}{2}\frac{b_k}{\rho_{3k}}; \quad k = 1, \ldots, o. \tag{53}$$

It is easy to verify that the expected demand function $d_k(\rho_{3k})$ associated with retailer k is a decreasing function of the price at the demand market.



$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2.$$

$$c_{ij}(q_{ij}) = .5q_{ij}^2 + 3.5q_{ij}, \quad \text{for } i = 1, 2;\ j = 1, 2.$$

$$c_{ik}(q_{ik}) = .5q_{ik}^2 + 5q_{ik}, \quad \text{for } i = 1, 2;\ k = 1, 2.$$

$$c_j(Q^1, Q^3) = .5(\sum_{i=1}^{2} q_{ij})^2, \quad \text{for } j = 1, 2,$$

$$c_k(Q^2, Q^3) = .5(\sum_{j=1} q_{jk})^2, \quad \text{for } k = 1, 2.$$

Example 1

$\lambda_k^+ = \lambda_k^- = 1, k = 1, 2$

$b_k = 100, k = 1, 2.$

$q_{ij}^* = .3697, i = 1, 2; j = 1, 2.$

$q_{ik}^* = .3487, i = 1, 2; k = 1, 2.$

$q_{jk}^* = .3697, j = 1, 2; k = 1, 2.$

$\rho_{2j}^* = 15.2301, j = 1, 2.$

$\rho_{3k}^* = 34.5573, k = 1, 2.$

$d_1(\rho_{31}^*) = d_2(\rho_{32}^*) = 1.4469.$

Example 2

$\lambda_k^+ = \lambda_k^- = 1, k = 1, 2$

$b_k = 1000, k = 1, 2.$

$q_{ij}^* = .6974, i = 1, 2; j = 1, 2.$

$q_{ik}^* = 1.9870, i = 1, 2; k = 1, 2.$

$q_{jk}^* = .6973, j = 1, 2; k = 1, 2.$

$\rho_{2j}^* = 39.8051, j = 1, 2.$

$\rho_{3k}^* = 92.9553, k = 1, 2.$

$d_1(\rho_{31}^*) = d_2(\rho_{32}^*) = 5.3789.$

Thank you!

More information about Supernetworks can be found at

http://supernet.som.umass.edu