Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations

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9th INFORMS Computing Society Conference

January 5-7, 2005

Support

Support for this research has been provided by the National Science Foundation under Grant No.: IIS-0002647 under the

Management of Knowledge Intensive Dynamic Systems (MKIDS) Program.

This support is gratefully acknowledged.

This presentation is based on the paper:

"Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations,"

T. Wakolbinger and A. Nagurney, *Netnomics* 6 (2004): pp 153-185.

Outline of the Presentation

- The importance of social networks in economic transactions
- The framework of supernetworks
- The supernetwork model integrating social networks with supply chain networks.

Definition of a Social Network

- "A social network is a set of actors that may have relationships with one another. Networks can have few or many actors (nodes), and one or more kinds of relations (edges) between pairs of actors." (Hannemann, 2001)

Roles of Social Networks in Economic Transactions

- Examples from Sociology
  - Embeddedness theory
    - Granovetter (1985)
    - Uzzi (1996)
- Examples from Economics
  - Williamson (1983)
  - Joskow (1988)
  - Crawford (1990)
  - Vickers and Waterson (1991)
  - Muthoo (1998)

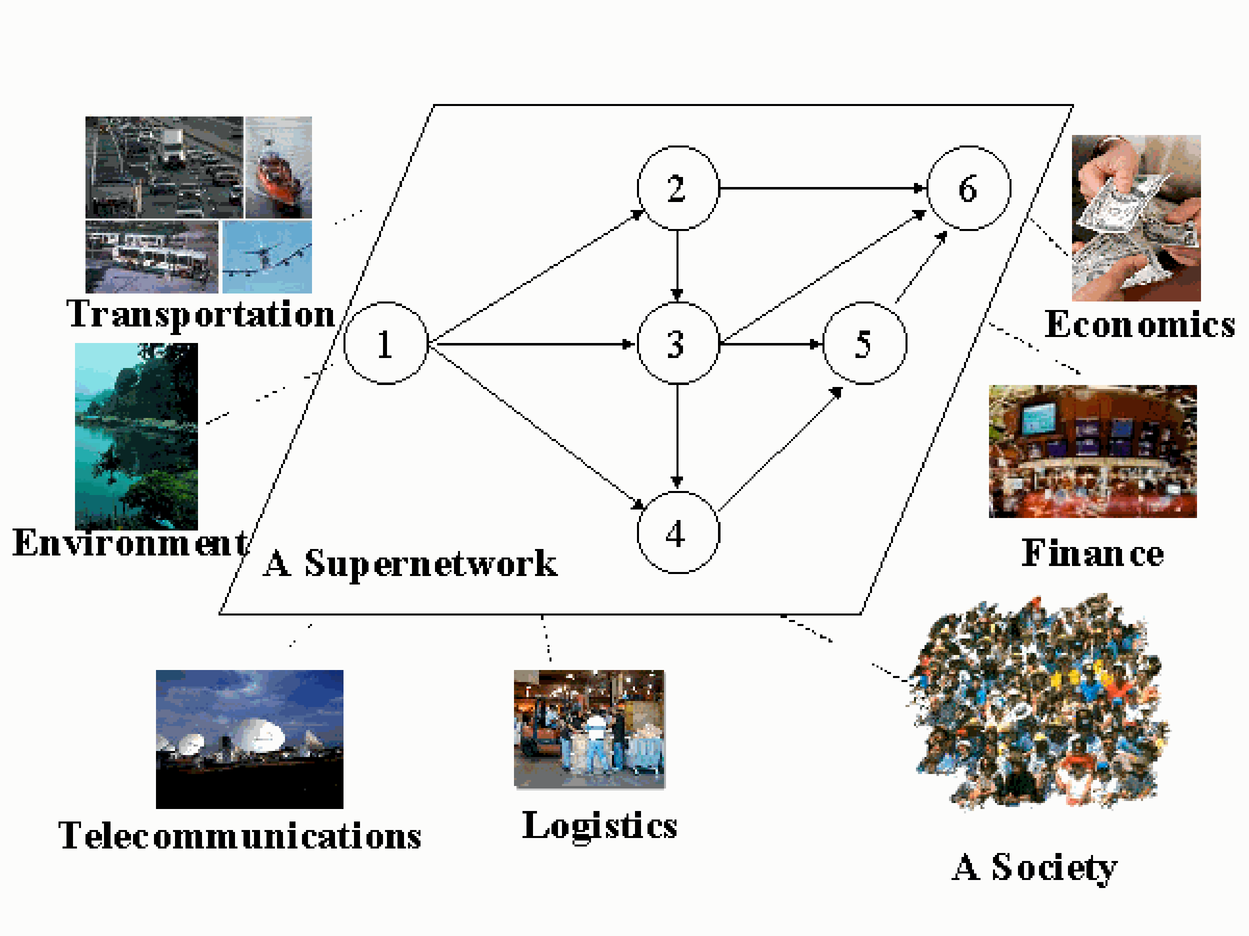
Roles of Social Networks in Economic Transactions

- Examples from Marketing
  - Relationship marketing
    - Ganesan (1994)
    - Bagozzi (1995)

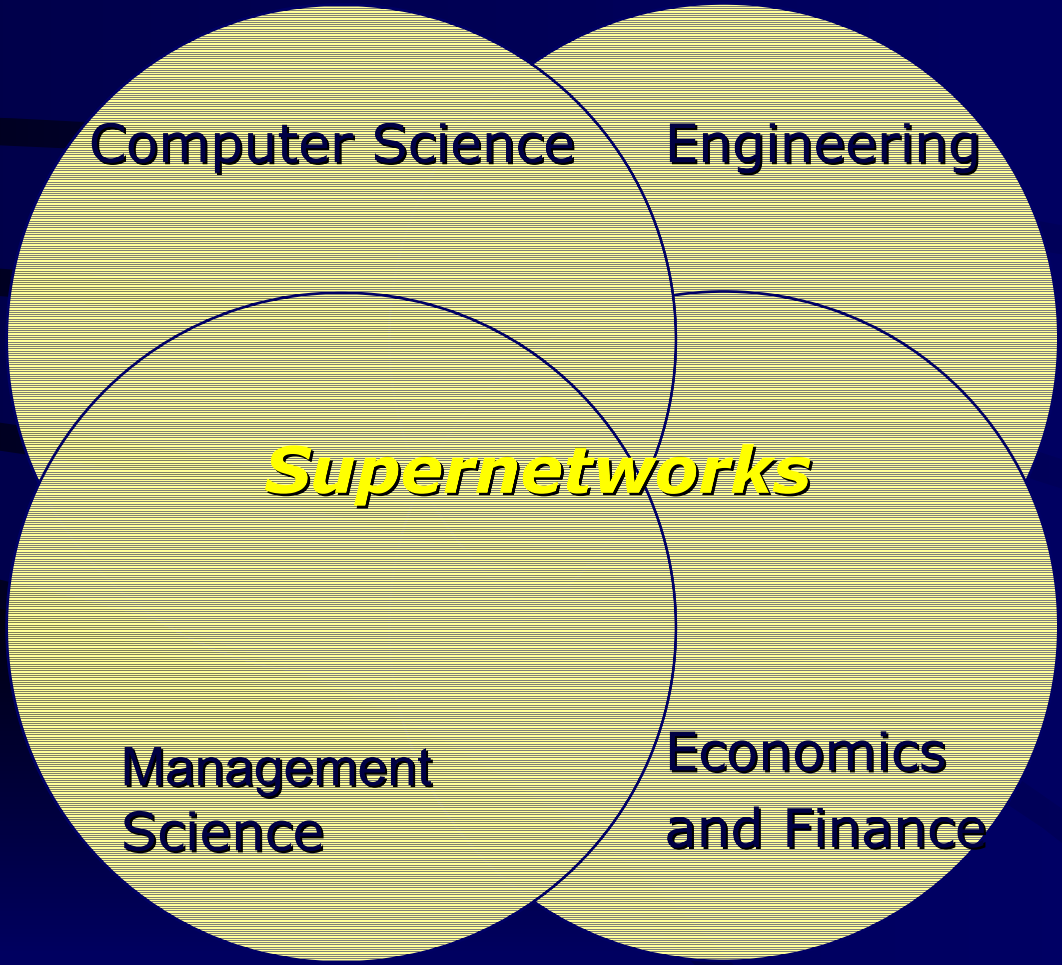
Novelty of Our Research

- Supernetworks show the dynamic co-evolution of economic (product, price and even informational) flows and the social network structure
- Economic flows and social network structure are interrelated
- Network of relations has a measurable economic value.

Supernetworks



A Multidisciplinary Approach



Tools That We Have Been Using

- Network theory
- Optimization theory
- Game theory
- Variational inequality theory
- Projected dynamical systems theory (which we have been instrumental in developing)
- Network visualization tools

Applications of Supernetworks

- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Energy Networks/Power Grids
- Knowledge Networks

The Supernetwork Team















The Supernetwork Integrating Social and Supply Chain Networks

- Decision-makers in the network can decide about the relationship levels [0,1] that they want to establish.
- Establishing relationship levels incurs some costs.
- Higher relationship levels
  - Reduce transaction costs
  - Reduce risk
  - Have some additional value (“relationship value”)

The Supernetwork Integrating Social and Supply Chain Networks

Dynamic evolution of :

- Product transactions and associated prices on the supply chain network
- Relationship levels on the social network.

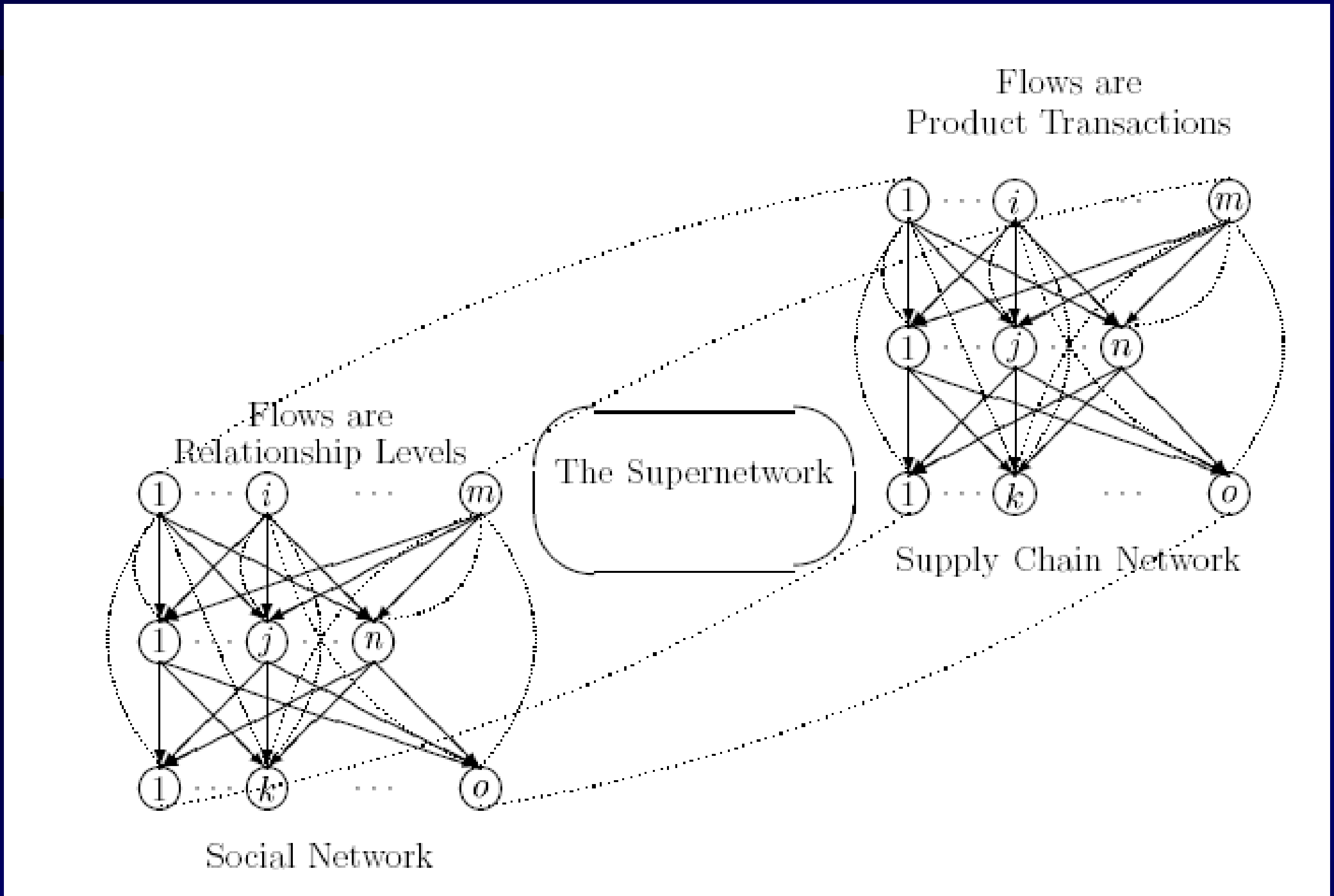
Features of the Model

- Models the interaction of supply chain and social networks
- Captures interactions among individual decision-makers
- Includes electronic transactions
- Incorporates transaction costs and risk.

Assumptions of the Model

- Manufacturers can transact either physically or electronically with the intermediaries.
- Manufacturers can transact directly with the demand markets via Internet links.
- Retailers can transact through physical links with demand markets.

The Supernetwork Structure: Integrated Supply Chain/Social Network System



Behavioral Assumptions: Multicriteria Decision-Makers

Manufacturers and Retailers try to:

- Maximize profit
- Minimize risk
- Maximize relationship value

with individual weights assigned to the different criteria.

A Manufacturer's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl} + \sum_{k=1}^{o}\rho^*_{1ik}q_{ik} - f_i(Q^1,Q^2) - \sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl},h_{ijl}) - \sum_{k=1}^{o}c_{ik}(q_{ik},h_{ik})$$

$$-\sum_{j=1}^{n}\sum_{l=1}^{2}b_{ijl}(h_{ijl}) - \sum_{k=1}^{o}b_{ik}(h_{ik})$$

$$-\alpha_i\left(\sum_{j=1}^{n}\sum_{l=1}^{2}r_{ijl}(q_{ijl},h_{ijl}) + \sum_{k=1}^{o}r_{ik}(q_{ik},h_{ik})\right) + \beta_i\left(\sum_{j=1}^{n}\sum_{l=1}^{2}v_{ijl}(h_{ijl}) + \sum_{k=1}^{o}v_{ik}(h_{ik})\right) \tag{19}$$

subject to:

$$q_{ijl} \geq 0, \quad \forall j,l, \quad q_{ik} \geq 0, \quad \forall k, \tag{20}$$

$$0 \leq h_{ijl} \leq 1, \quad \forall j,l, \quad 0 \leq h_{ik} \leq 1, \quad \forall k. \tag{21}$$

Optimality Conditions of the Manufacturers

determine $(Q^{1*}, Q^{2*}, h^{1*}, h^{2*}) \in \mathcal{K}_1$, such that

$(Q^{1*}, Q^{2*}, h^{1*}, h^{2*}) \in \mathcal{K}_1$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial q_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial q_{ijl}} - \rho^*_{1ijl}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q^*_{ik},h^*_{ik})}{\partial q_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik},h^*_{ik})}{\partial q_{ik}} - \rho^*_{1ik}\right] \times [q_{ik} - q^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik},h^*_{ik})}{\partial h_{ik}} - \beta_i \frac{\partial v_{ik}(h^*_{ik})}{\partial h_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik},h^*_{ik})}{\partial h_{ik}} + \frac{\partial b_{ik}(h^*_{ik})}{\partial h_{ik}}\right] \times [h_{ik} - h^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial h_{ijl}} - \beta_i \frac{\partial v_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial b_{ijl}(h^*_{ijl})}{\partial h_{ijl}}\right] \times \left[h_{ijl} - h^*_{ijl}\right] \geq 0,$$

$$\forall (Q^1, Q^2, h^1, h^2) \in \mathcal{K}_1, \quad (22)$$

where

$$\mathcal{K}_1 \equiv \left[(Q^1, Q^2, h^1, h^2) \mid q_{ijl} \geq 0,\ q_{ik} \geq 0,\ 0 \leq h_{ijl} \leq 1,\ 0 \leq h_{ik} \leq 1,\ \forall i, j, l, k\right]. \quad (23)$$

A Retailer's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad \rho^*_{2j}\sum_{k=1}^{o} q_{jk} - c_j(Q^1) - \sum_{i=1}^{m}\sum_{l=1}^{2}\hat{c}_{ijl}(q_{ijl},h_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} c_{jk}(q_{jk},h_{jk}) - \sum_{i=1}^{m}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl}$$

$$-\sum_{i=1}^{m}\sum_{l=1}^{2}\hat{b}_{ijl}(h_{ijl}) - \sum_{k=1}^{o} b_{jk}(h_{jk}) - \delta_j\Big(\sum_{i=1}^{m}\sum_{l=1}^{2}\hat{r}_{ijl}(q_{ijl},h_{ijl}) + \sum_{k=1}^{o} r_{jk}(q_{jk},h_{jk})\Big)$$

$$+\gamma_j\Big(\sum_{i=1}^{m}\sum_{l=1}^{2}\hat{v}_{ijl}(h_{ijl}) + \sum_{k=1}^{o} v_{jk}(h_{jk})\Big) \tag{42}$$

subject to:

$$\sum_{k=1}^{o} q_{jk} \le \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl} \tag{43}$$

$$q_{ijl} \ge 0 \quad \forall i,l, \quad q_{jk} \ge 0, \quad \forall k, \tag{44}$$

$$0 \le h_{ijl} \le 1, \quad \forall i,l, \quad 0 \le h_{jk} \le 1, \quad \forall k. \tag{45}$$

Optimality Conditions of the Retailers

determine $(Q^{1*}, Q^{3*}, h^{1*}, h^{3*}, \epsilon^*) \in \mathcal{K}_2$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} + \rho^*_{1ijl} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} - \epsilon^*_j\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_{jk}(q^*_{jk}, h^*_{jk})}{\partial q_{jk}} - \rho^*_{2j} + \epsilon^*_j + \delta_j \frac{\partial r_{jk}(q^*_{jk}, h^*_{jk})}{\partial q_{jk}}\right] \times \left[q_{jk} - q^*_{jk}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial \hat{c}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} - \gamma_j \frac{\partial \hat{v}_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial \hat{b}_{ijl}(h^*_{ijl})}{\partial h_{ijl}}\right] \times \left[h_{ijl} - h^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_{jk}(q^*_{jk}, h^*_{jk})}{\partial h_{jk}} - \gamma_j \frac{\partial v_{jk}(h^*_{jk})}{\partial h_{jk}} + \delta_j \frac{\partial r_{jk}(q^*_{jk}, h^*_{jk})}{\partial h_{jk}} + \frac{\partial b_{jk}(h^*_{jk})}{\partial h_{jk}}\right] \times \left[h_{jk} - h^*_{jk}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{o} q^*_{jk}\right] \times \left[\epsilon_j - \epsilon^*_j\right] \geq 0, \quad \forall (Q^1, Q^3, h^1, h^3, \epsilon) \in \mathcal{K}_2, \tag{46}$$

where

$$\mathcal{K}_2 \equiv \left[(Q^1, Q^3, h^1, h^3, \epsilon) \mid q_{ijl} \geq 0, q_{jk} \geq 0, 0 \leq h_{ijl} \leq 1, 0 \leq h_{jk} \leq 1, \epsilon_j \geq 0, \forall i, j, l, k\right]. \tag{47}$$

Equilibrium Conditions of the Demand Markets

for all retailers: j; $j = 1, ..., n$:

$$\rho_{2j}^* + \hat{c}_{jk}(q_{jk}^*, h_{jk}^*) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{jk}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{jk}^* = 0, \end{cases} \tag{51}$$

and for all manufacturers i; $i = 1, \ldots, m$:

$$\rho_{1ik}^* + \hat{c}_{ik}(q_{ik}^*, h_{ik}^*) \begin{cases} = \rho_{3k}^*, & \text{if} \quad q_{ik}^* > 0 \\ \geq \rho_{3k}^*, & \text{if} \quad q_{ik}^* = 0, \end{cases} \tag{52}$$

and

$$d_k(\rho_3^*) \begin{cases} = \sum_{j=1}^n q_{jk}^* + \sum_{i=1}^m q_{ik}^*, & \text{if} \quad \rho_{3k}^* > 0 \\ \leq \sum_{j=1}^n q_{jk}^* + \sum_{i=1}^m q_{ik}^*, & \text{if} \quad \rho_{3k}^* = 0. \end{cases} \tag{53}$$

VI Formulation of the Equilibrium Conditions for the Demand Markets

determine $(Q^{2*}, Q^{3*}, \rho_3^*) \in R_+^{mo+no+o}$, such that

$$\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\rho_{2j}^* + \hat{c}_{jk}(q_{jk}^*, h_{jk}^*) - \rho_{3k}^*\right] \times \left[q_{jk} - q_{jk}^*\right] + \sum_{i=1}^{m}\sum_{k=1}^{o}\left[\rho_{1ik}^* + \hat{c}_{ik}(q_{ik}^*, h_{ik}^*) - \rho_{3k}^*\right] \times \left[q_{ik} - q_{ik}^*\right]$$

$$+ \sum_{k=1}^{o}\left[\sum_{j=1}^{n} q_{jk}^* + \sum_{i=1}^{m} q_{ik}^* - d_k(\rho_3^*)\right] \times \left[\rho_{3k} - \rho_{3k}^*\right] \geq 0, \qquad (54)$$

$$\forall (Q^2, Q^3, \rho_3) \in R_+^{mo+no+o}. \qquad (55)$$

The Equilibrium State

Definition 1: The equilibrium state of the supernetwork is one where the flows between the tiers of the supernetwork coincide and the product transactions, relationship levels, and prices satisfy the sum of the optimality conditions and the equilibrium conditions.

The equilibrium state is equivalent to a VI of the form:

$$\text{determine } X^* \in \mathcal{K} \text{ satisfying}$$

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

VI Formulation

The equilibrium conditions governing the supernetwork model are equivalent to the solution of the variational inequality problem given by: determine $(Q^{1*}, Q^{2*}, Q^{3*}, h^{1*}, h^{2*}, h^{3*}, \epsilon^*, \rho_3^*) \in \mathcal{K}$ *satisfying*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\Bigg[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ijl}} + \frac{\partial c_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial q_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial q_{ijl}}$$

$$+\delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial q_{ijl}} - \epsilon^*_j\Bigg] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial f_i(Q^{1*},Q^{2*})}{\partial q_{ik}} + \frac{\partial c_{ik}(q^*_{ik},h^*_{ik})}{\partial q_{ik}} + \hat{c}_{ik}(q^*_{ik},h^*_{ik}) + \alpha_i \frac{\partial r_{ik}(q^*_{ik},h^*_{ik})}{\partial q_{ik}} - \rho^*_{3k}\right] \times [q_{ik} - q^*_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_{jk}(q^*_{jk},h^*_{jk})}{\partial q_{jk}} + \hat{c}_{jk}(q^*_{jk},h^*_{jk}) + \epsilon^*_j + \delta_j \frac{\partial r_{jk}(q^*_{jk},h^*_{jk})}{\partial q_{jk}} - \rho^*_{3k}\right] \times \left[q_{jk} - q^*_{jk}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_{jk}(q^*_{jk},h^*_{jk})}{\partial h_{jk}} - \gamma_j \frac{\partial v_{jk}(h^*_{jk})}{\partial h_{jk}} + \delta_j \frac{\partial r_{jk}(q^*_{jk},h^*_{jk})}{\partial h_{jk}} + \frac{\partial b_{jk}(h^*_{jk})}{\partial h_{jk}}\right] \times \left[h_{jk} - h^*_{jk}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik},h^*_{ik})}{\partial h_{ik}} - \beta_i \frac{\partial v_{ik}(h^*_{ik})}{\partial h_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik},h^*_{ik})}{\partial h_{ik}} + \frac{\partial b_{ik}(h^*_{ik})}{\partial h_{ik}}\right] \times [h_{ik} - h^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\Bigg[\frac{\partial c_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial h_{ijl}} - \beta_i \frac{\partial v_{ijl}(h^*_{ijl})}{\partial h_{ijl}} - \gamma_j \frac{\partial \hat{v}_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial h_{ijl}}$$

$$+\delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl},h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial b_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial \hat{b}_{ijl}(h^*_{ijl})}{\partial h_{ijl}}\Bigg] \times \left[h_{ijl} - h^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{o} q^*_{jk}\right] \times \left[\epsilon_j - \epsilon^*_j\right] + \sum_{k=1}^{o}\left[\sum_{j=1}^{n} q^*_{jk} + \sum_{i=1}^{m} q^*_{ik} - d_k(\rho^*_3)\right] \times [\rho_{3k} - \rho^*_{3k}] \geq 0,$$

$$\forall (Q^1, Q^2, Q^3, h^1, h^2, h^3, \epsilon, \rho_3) \in \mathcal{K}, \qquad (56)$$

where

$$\mathcal{K} \equiv \Big[(Q^1, Q^2, Q^3, h^1, h^2, h^3, \epsilon, \rho_3) | q_{ijl} \geq 0, q_{ik} \geq 0, q_{jk} \geq 0, 0 \leq h_{ij} \leq 1, 0 \leq h_{ik} \leq 1,$$

$$0 \leq h_{jk} \leq 1, \epsilon_j \geq 0, \rho_{3k} \geq 0, \forall i,j,l,k\Big]. \qquad (57)$$

The Projected Dynamical System

- **The dynamic model can be formulated as a projected dynamical system (Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) defined by the initial value problem:**

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{59}$$

where $\Pi_{\mathcal{K}}$ denotes the projection of $-F(X)$ onto $\mathcal{K}$ at X and X_0 is equal to the point corresponding to the initial product transactions, relationship levels, shadow prices, and demand market prices.

- **The set of stationary points of the projected dynamical system coincides with the set of solutions of the variational inequality problem .**

The Disequilibrium Dynamics

- The trajectory of the PDS describes the dynamic evolution of:
  - the product transactions on the supply chain network
  - the relationship levels on the social network along with the demand market prices, and
  - the Lagrange multipliers or shadow prices associated with the retailers.

Dynamics of Demand Market Prices

- The demand market prices evolve according to the difference between the demand at the market (as a function of the prices at the demand markets at that time) and the amount of the product transactions.
- The projection operator guarantees that the prices do not take on negative values.

Dynamics of Shadow Prices

- The Lagrange multipliers/shadow prices associated with the retailers evolve according to the difference between the sum of the product transacted with the demand markets and that obtained from the manufacturers.
- The projection operator guarantees that these prices do not become negative.

Dynamics of Relationship Levels

- The relationship levels evolve on the social network level of the supernetwork according to the difference of the sum of the corresponding weighted value functions and the sum of the various marginal transaction cost and weighted marginal risk functions.
- The relationship levels are guaranteed to remain within the range zero to one.

Dynamics of Product Transactions

- The product transactions evolve on the supply chain network links according to the difference between the characteristic price and the marginal production cost and various marginal transaction and handling costs plus the weighted marginal risk cost functions.
- These flows are guaranteed to not assume negative values due to the projection operation.

The Computational Procedure

We use the Euler Method to solve the Variational Inequality in standard form:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

The notable feature of the algorithm in this application is that the induced subproblems can be solved exactly and in closed form due to the network structure and simplicity of the feasible set.

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \to 0$, as $T \to \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

$$y = P_{\mathcal{K}} X = \arg\min_{z \in \mathcal{K}} \|X - z\|.$$

Step 2: Convergence Verification

If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1.

Qualitative Properties

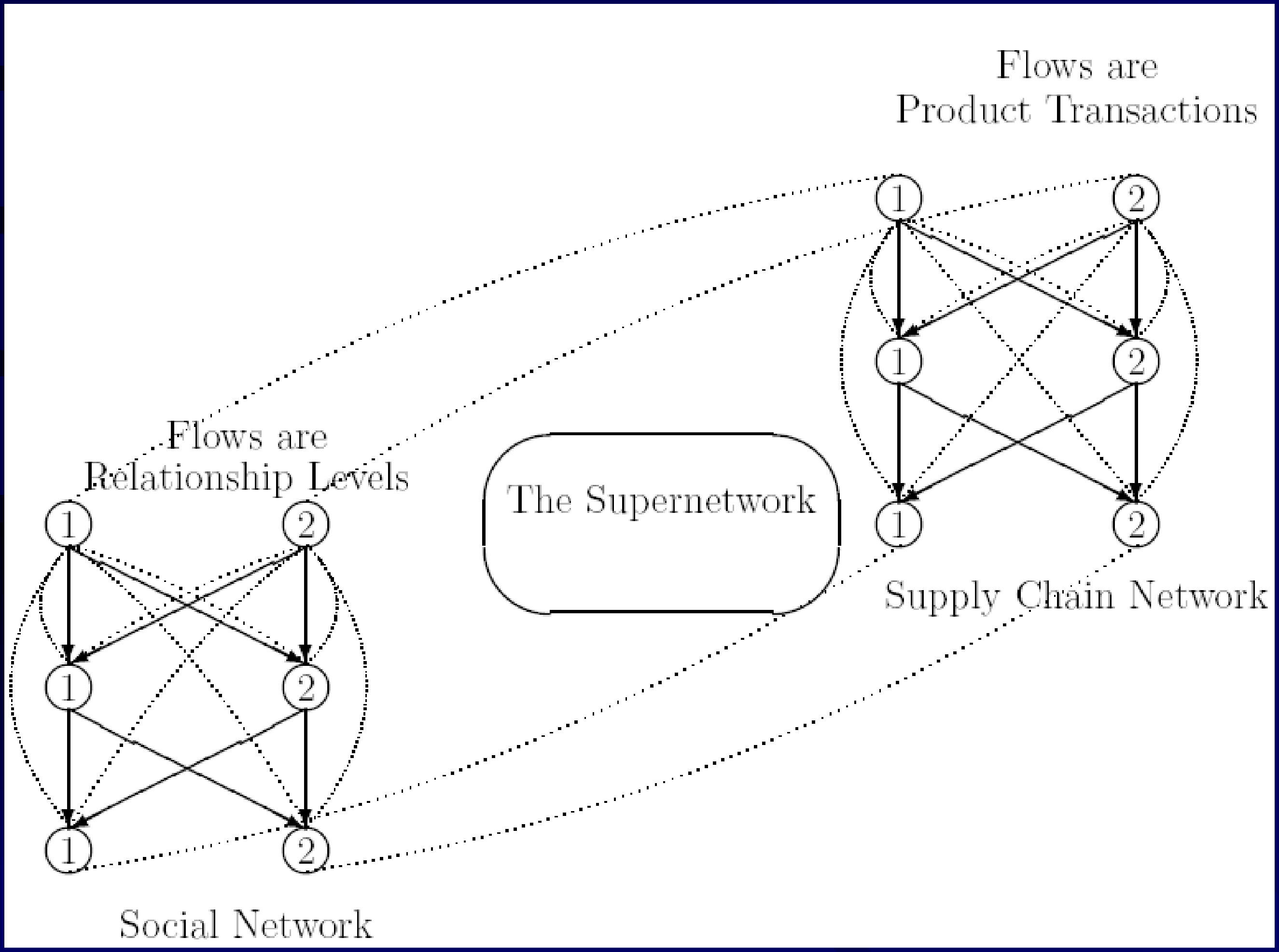
We have also established:

- **Existence of a solution to the VI**
- **Uniqueness of a solution to the VI**
- **Conditions for the existence of a unique trajectory to the projected dynamical system**
- **Convergence of the Euler method.**

Characteristics of the Numerical Examples

- 2 manufacturers
- 2 retailers
- 2 demand markets
- Physical and electronic transactions between manufacturers and retailers
- Electronic transactions between manufacturers and demand markets
- Physical transactions between retailers and demand markets

Network Structure of the Numerical Examples



Example 1-3: Manufacturer Information

- 2 manufacturers
  - Production cost functions

$$f_1(q) = 2.5q_1^2 + q_1q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1q_2 + 2q_2.$$

  - Transaction cost functions

$$c_{ij1}(q_{ij1}, h_{ij1}) = .5q_{ij1}^2 + 3.5q_{ij1} - h_{ij1}, \quad \forall i, j,$$

$$c_{ij2}(q_{ij2}, h_{ij2}) = 1.5q_{ij2}^2 + 3q_{ij2} - .5h_{ij2}, \quad \forall i, j.$$

$$c_{ik}(q_{ik}, h_{ik}) = q_{ik}^2 + 2q_{ik} - 2h_{ik}, \quad \forall i, k.$$

Example 1-3: Retailer Information

- 2 retailers
  - Handling cost functions

$$c_1(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i2})^2.$$

  - Transaction cost functions

$$\hat{c}_{ijl}(q_{ijl}, h_{ijl}) = 1.5q_{ijl}^2 + 3q_{ijl}, \quad \forall i, j, l.$$

Example 1-3: Demand Market Information

- 2 demand markets
  - Demand functions

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

  - Transaction cost functions

$$\hat{c}_{jk}(q_{jk}, h_{jk}) = q_{jk} - h_{jk} + 5, \quad \forall j, k,$$

$$\hat{c}_{ik}(q_{ik}, h_{ik}) = q_{ik} + 1, \quad \forall i, k.$$

Example 1-3: Relationship Functions

- Relationship value functions

$$v_{ijl}(h_{ijl}) = h_{ijl}, \quad \forall i, j, l; \quad v_{ik}(h_{ik}) = h_{ik}, \quad \forall i, k; \quad v_{jk}(h_{jk}) = h_{jk}, \quad \forall j, k.$$

- Relationship cost functions

$$b_{ijl}(h_{ijl}) = 2h_{ijl} + 1, \quad \forall i, j, l;$$

$$b_{ik}(h_{ik}) = h_{ik} + 1, \quad \forall i, k;$$

$$b_{jk}(h_{jk}) = h_{jk} + 1, \quad \forall j, k.$$

Differences between the Examples

- Example 1
  - All weights for relationship values are equal to 1.
- Example 2
  - The weights for relationship values for the two manufacturers increased from 1 to 10.
- Example 3
  - The weights for relationship values for the two manufacturers increased from 10 to 20.

Example 1-3: Equilibrium Relationship Levels

- **Example 1**

$$h^*_{ij1} = h^*_{jk} = h^*_{ik} = 0, \forall i, j, k \qquad h^*_{ij2} = 1 \quad \forall i, j$$

- **Example 2**

$$h^*_{ijl} = 1, \quad \forall i, j, l; \quad h^*_{jk} = .2179, \quad \forall j, k; \quad h^*_{ik} = 0, \quad \forall i, k.$$

- **Example 3**
  - Like example 2 except for

$$h^*_{jk} = .3700, \quad \forall j, k.$$

Example 1-3: Equilibrium Product Transactions

- **Example 1**

$$Q^{1*} := q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.4622; \quad q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.3914.$$

$$Q^{2*} := q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.3016.$$

$$Q^{3*} := q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 5.8521.$$

- **Example 2**

$$Q^{1*} := q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.4791; \quad q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.4027,$$

$$Q^{2*} := q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.2790.$$

- **Example 3**

$$Q^{1*} := q^*_{111} = q^*_{121} = q^*_{211} = q^*_{221} = 3.4904; \quad q^*_{112} = q^*_{122} = q^*_{212} = q^*_{222} = 2.4102;$$

$$Q^{2*} := q^*_{11} = q^*_{12} = q^*_{21} = q^*_{22} = 13.2790,$$

$$Q^{3*} := q^*_{11} = q^*_{21} = q^*_{21} = q^*_{22} = 5.8696.$$

Example 1-3: Equilibrium Shadow and Demand Prices

- **Example 1**

$$\epsilon_1^* = \epsilon_2^* = 263.9186,$$

$$\rho_{31}^* = \rho_{32}^* = 274.7686.$$

- **Example 2**

$$\epsilon_1^* = \epsilon_2^* = 264.1087,$$

$$\rho_{31}^* = \rho_{32}^* = 274.7666.$$

- **Example 3**

$$\epsilon_1^* = \epsilon_2^* = 264.2347$$

$$\rho_{31}^* = \rho_{32}^* = 274.7649.$$

Types of Simulations that can be Performed

We can simulate:

- Changes in production, transaction, handling, and relationship production cost functions
- Changes in demand and risk functions
- Changes in weights for relationship value and risk
- Addition and removal of actors
- Addition and removal of multiple transaction modes.

Summary

- We modeled the behavior of the decision-makers, their interactions, and the dynamic evolution of the associated variables.
- We introduced actual flows into social networks.
- We studied the problems qualitatively as well as computationally.
- We proposed an algorithm, implemented it, and used it to compute solutions in numerical examples.

To-date, we have studied ***good behavior***.

Fascinating questions arise when there may be: ***situations of instability, multiple equilibria, chaos, cycles***, etc.

Supernetworks Integrating Social Networks with Other Networks

- In addition, in other papers, we have modeled the integration of:
  - **Financial and social networks**
  - **International supply chain and social networks**
  - **International financial and social networks.**

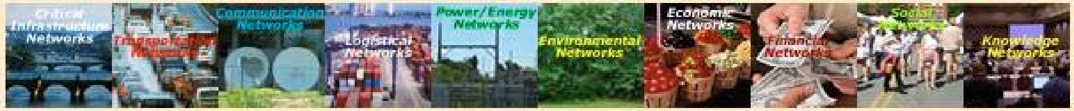
The full text of the papers can be found under Downloadable Articles at:

http://supernet.som.umass.edu

Eugene M. Isenberg School of Management

Virtual Center for Supernetworks

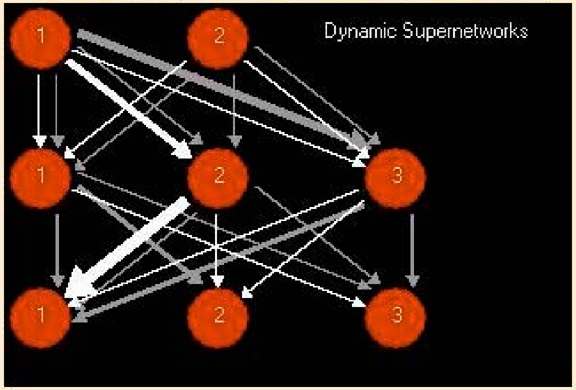
The Virtual Center for Supernetworks



Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center.

NEW! The Supernetwork Sentinel Fall 2004 Issue
NEW! Isenberg School of Management Website
NEW! Papers on Dynamic Supernetworks



NEW! INFORMS Student Chapter
NEW! Fall 2004 Seminar Series!!!

Top site users during the past month were from UMass, Cornell, Columbia, and Purdue University. Top foreign users were from China, Russia, Australia, Italy, France, and Morocco.

Friends of the Virtual Center for Supernetworks

Mission: The mission of the Virtual Center for Supernetworks is to foster the study and application of supernetworks and to serve as a resource to academia, industry, and government on networks ranging from transportation, logistical, telecommunication, and power networks to economic, environmental, financial, knowledge and social networks.

The applications of Supernetworks include: transportation, logistics, critical infrastructure, telecommunications, power and energy, electronic commerce, supply chain management, environment, economics, finance, knowledge and social networks, and decision-making.

Funding for the Center has been provided by:
The National Science Foundation
The AT&T Foundation
The Rockefeller Foundation
The John F. Smith Memorial Fund of the University of Massachusetts
The Isenberg School of Management - University of Massachusetts.

Thank you!

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