Supernetworks: Management Science for the 21st Century

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Eugene M. Isenberg School of Management

The Virtual Center for Supernetworks
http://supernet.som.umass.edu

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我非常荣幸有机会在这次论坛做讲演

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Outline of Presentation:

- Background
- History of the Science of Networks
- Interdisciplinary Impact of Networks
- Characteristics of Networks Today
- The Braess Paradox
- Supernetworks and New Tools
- Applications of Supernetworks
- The Time-Dependent Braess Paradox, the Internet, and Evolutionary Variational Inequalities
- New Challenges and Opportunities: Unification of EVIs and PDSs

We are in a New Era of Decision-Making Characterized by:

- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- constraints on resources: natural, human, financial, time, etc.;
- global reach of many decisions;
- high impact of many decisions;
- increasing risk and uncertainty, and
- the *importance of dynamics* and realizing a fast and sound response to evolving events.

The 21st Century is Network-Based with the Internet providing critical infrastructure along with transportation/logistical networks and energy networks.

This era is ideal for Management Science!

We must harness the best and most powerful methodologies for the modeling, analysis, and solution of complex decision-making problems.

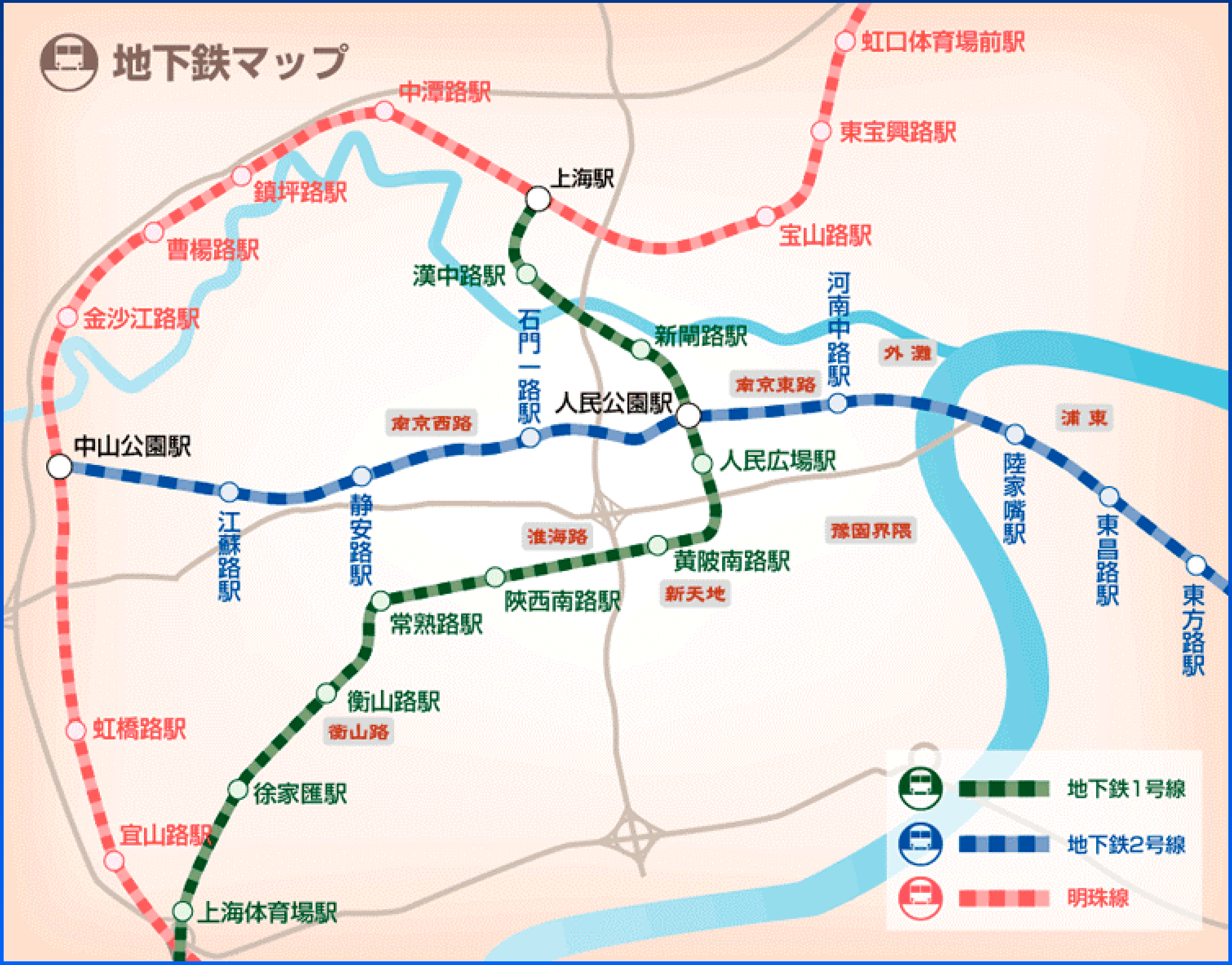
Examples of Physical Networks

Transportation Networks

provide us with the means to cross distance in order to conduct our work, and to see our colleagues, students, friends, and family members.

They provide us with access to food and consumer products.

Subway Network



Railroad Networks in China

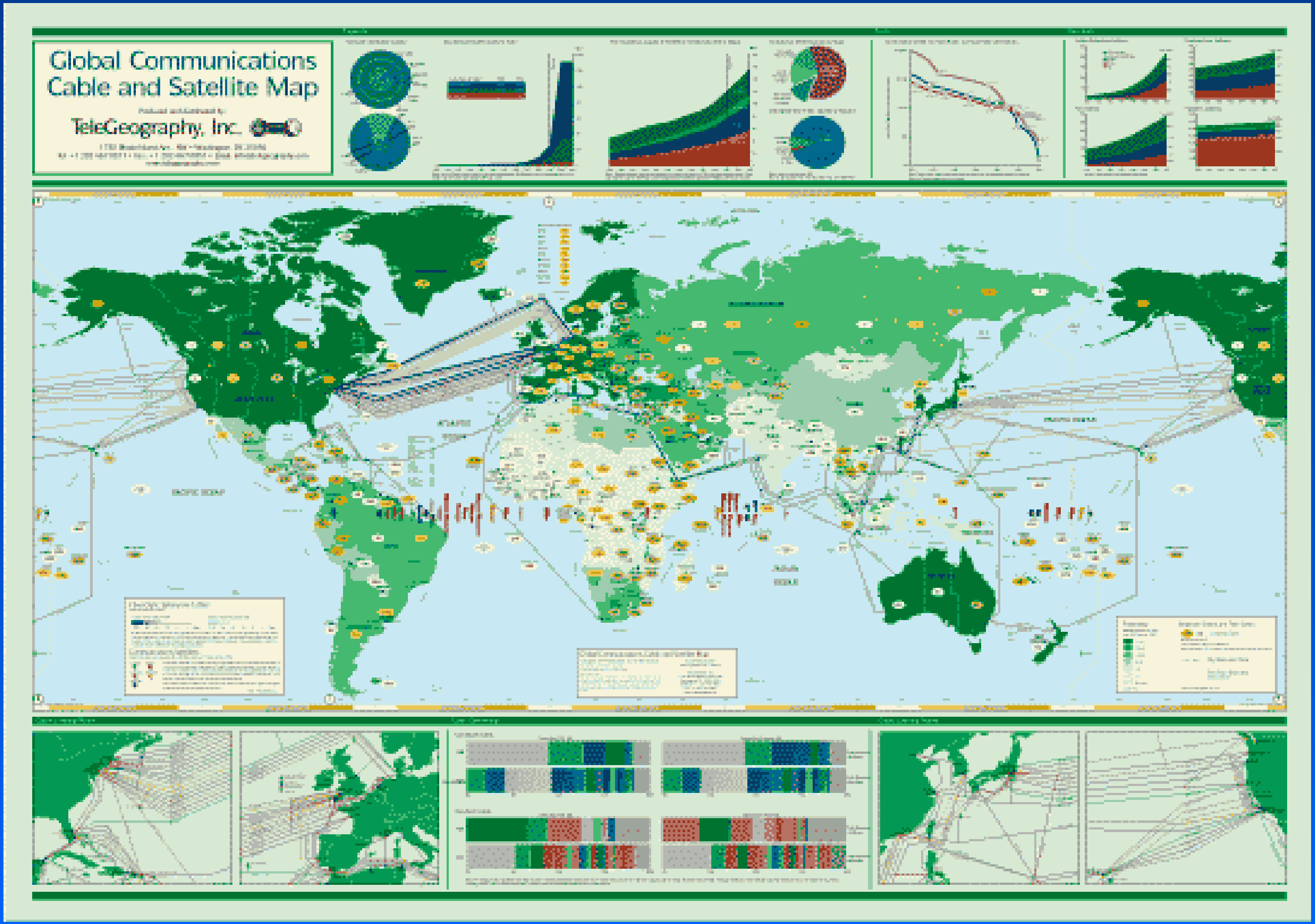
Communication Networks

allow us to communicate within our own communities and across regions and national boundaries,

and have transformed the way we live, work, and conduct business.

Iridium Satellite Constellation Network

Satellite and Undersea Cable Networks



Energy Networks

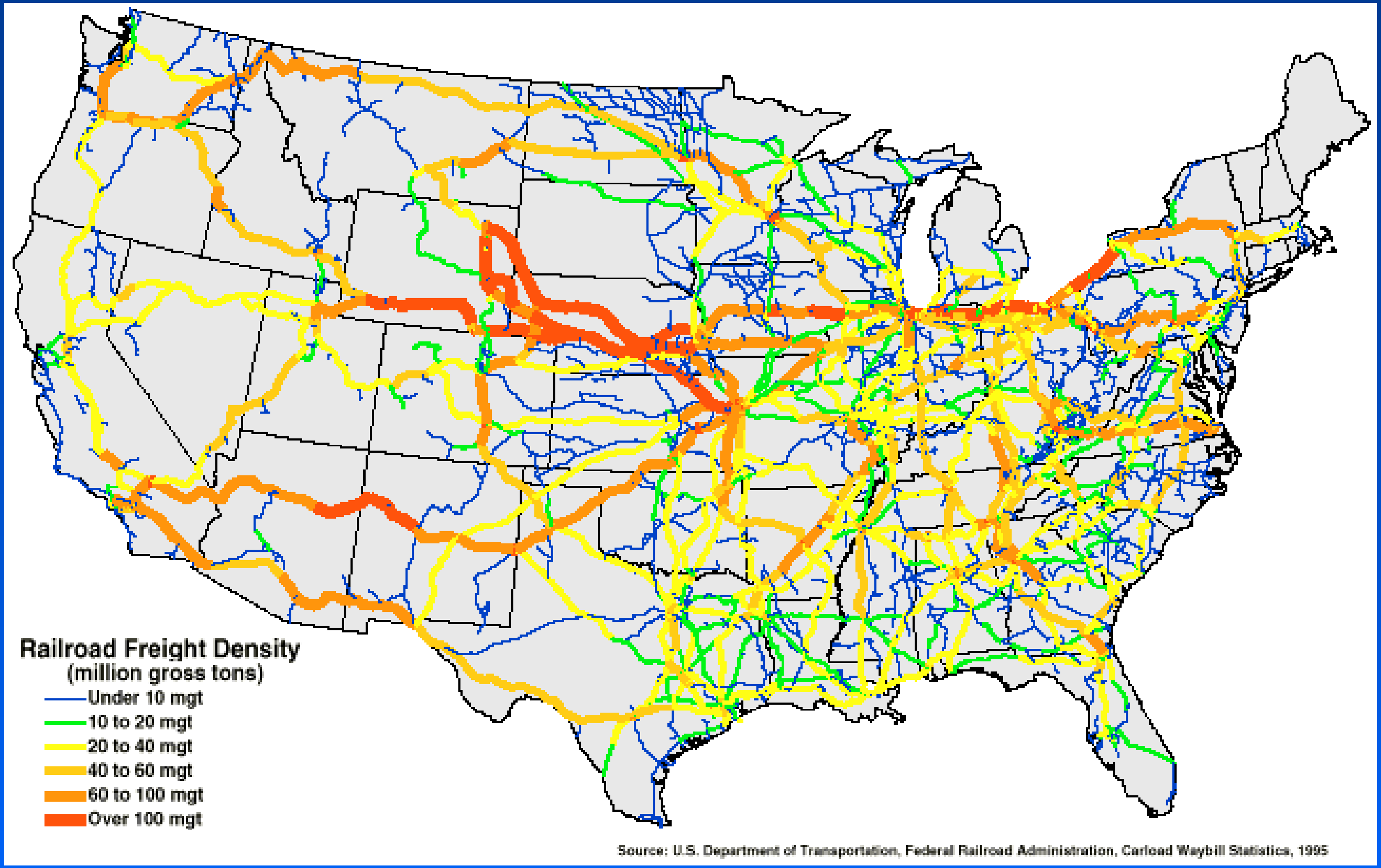
provide the energy for our homes, schools, and businesses, and to run our vehicles.

Duke Energy Gas Pipeline Network

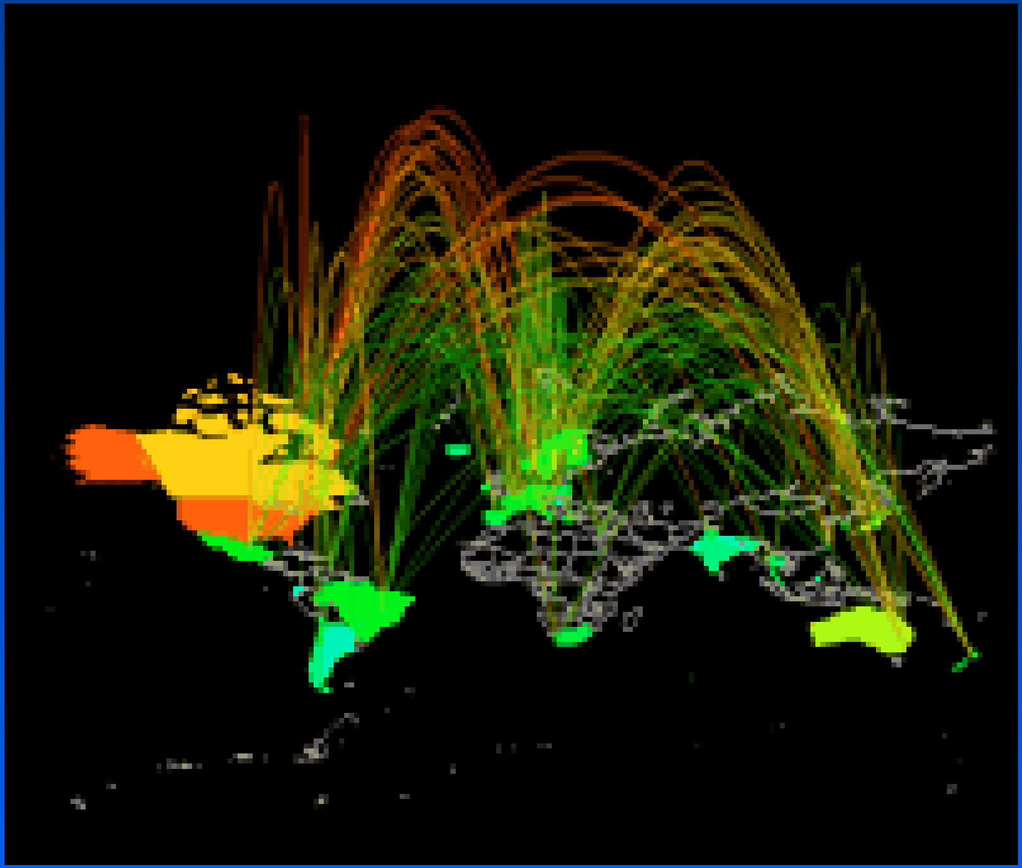
Components of Common Physical Networks

Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity

US Railroad Freight Flows

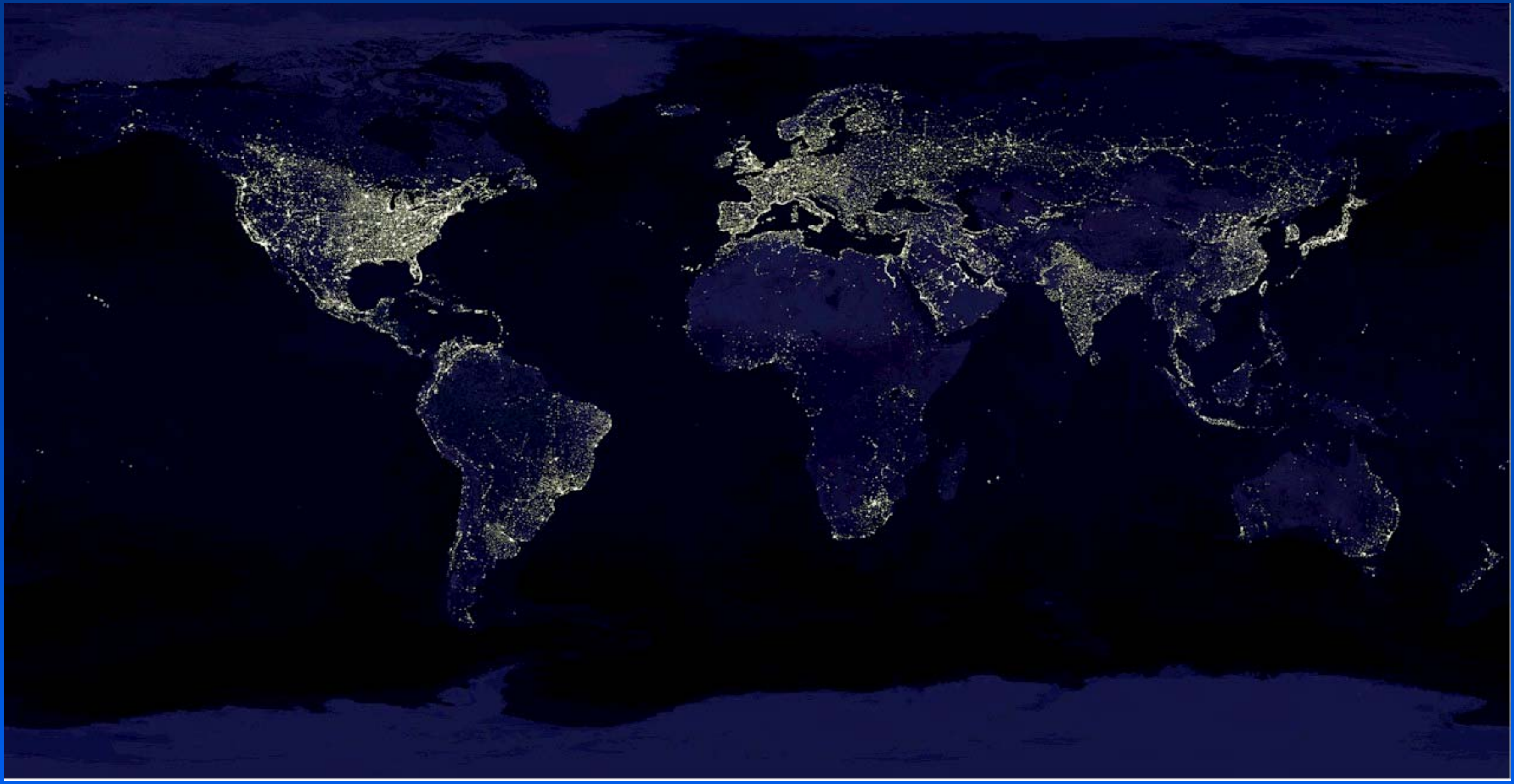


Internet Traffic Flows Over One 2 Hour Period



from Stephen Eick, Visual Insights

Electricity is Modernity



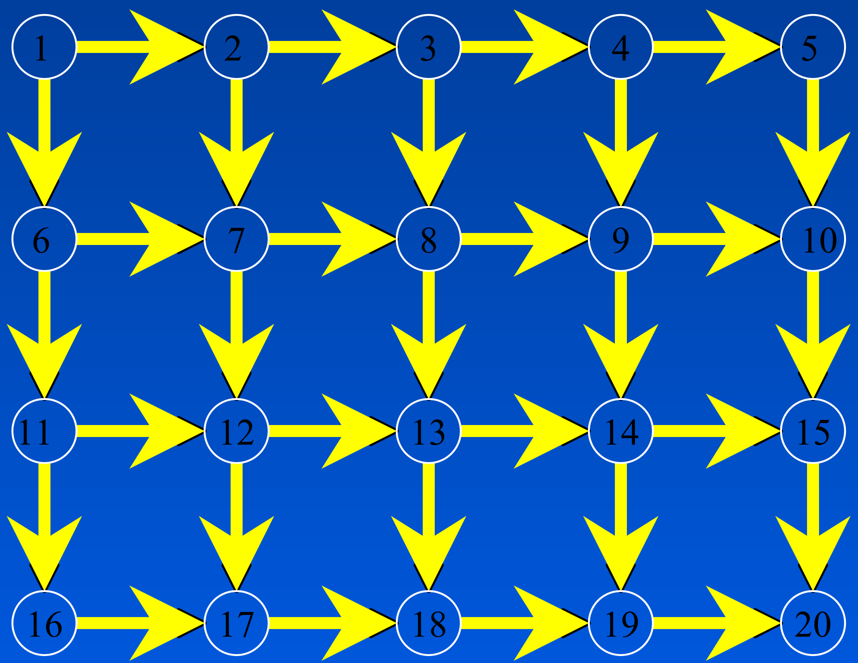
The scientific study of networks involves:

- how to **model** such applications as **mathematical entities**,
- how to **study the models** qualitatively,
- how to design **algorithms** to solve the resulting models.

The basic components of networks are:

- Nodes
- Links or arcs
- Flows

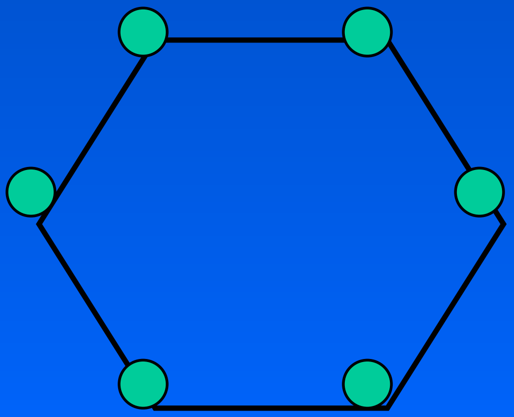
Nodes   Links   Flows



Brief History of the Science of Networks

1736 - Euler - the earliest paper on graph theory - Konigsberg bridges problem.

1758 - Quesnay in his *Tableau Economique* introduced a graph to depict the circular flow of financial funds in an economy.



1781 - Monge, who had worked under Napoleon Bonaparte, publishes what is probably the first paper on transportation in minimizing cost.

1838 - Cournot states that a competitive price is determined by the intersection of supply and demand curves in the context of spatially separate markets in which transportation costs are included.

1841 - Kohl considered a two node, two route transportation network problem.

1845 - Kirchhoff wrote *Laws of Closed Electric Circuits.*

1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

1936 - Konig published the first book on graph theory.

1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

1952 - Copeland in his book asked, *Does money flow like water or electricity?*

1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

1969 - Dafermos and Sparrow coined the terms *user-optimization* and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

In a basic network problem domain:

one wishes to move the flow from one node to another in a way that is as efficient as possible.

Classic Examples of Network Problems Are:

- The Shortest Path Problem
- The Maximum Flow Problem
- The Minimum Cost Flow Problem.

These are optimization problems.

Network problems arise in other *surprising and fascinating* ways for problems, which at first glance and on the surface, may not appear to involve networks at all.

The study of networks is not limited to only physical networks but also to *abstract* networks in which nodes do not coincide to locations in space.

Moreover, networks can be used to model *complex decision-making processes.*

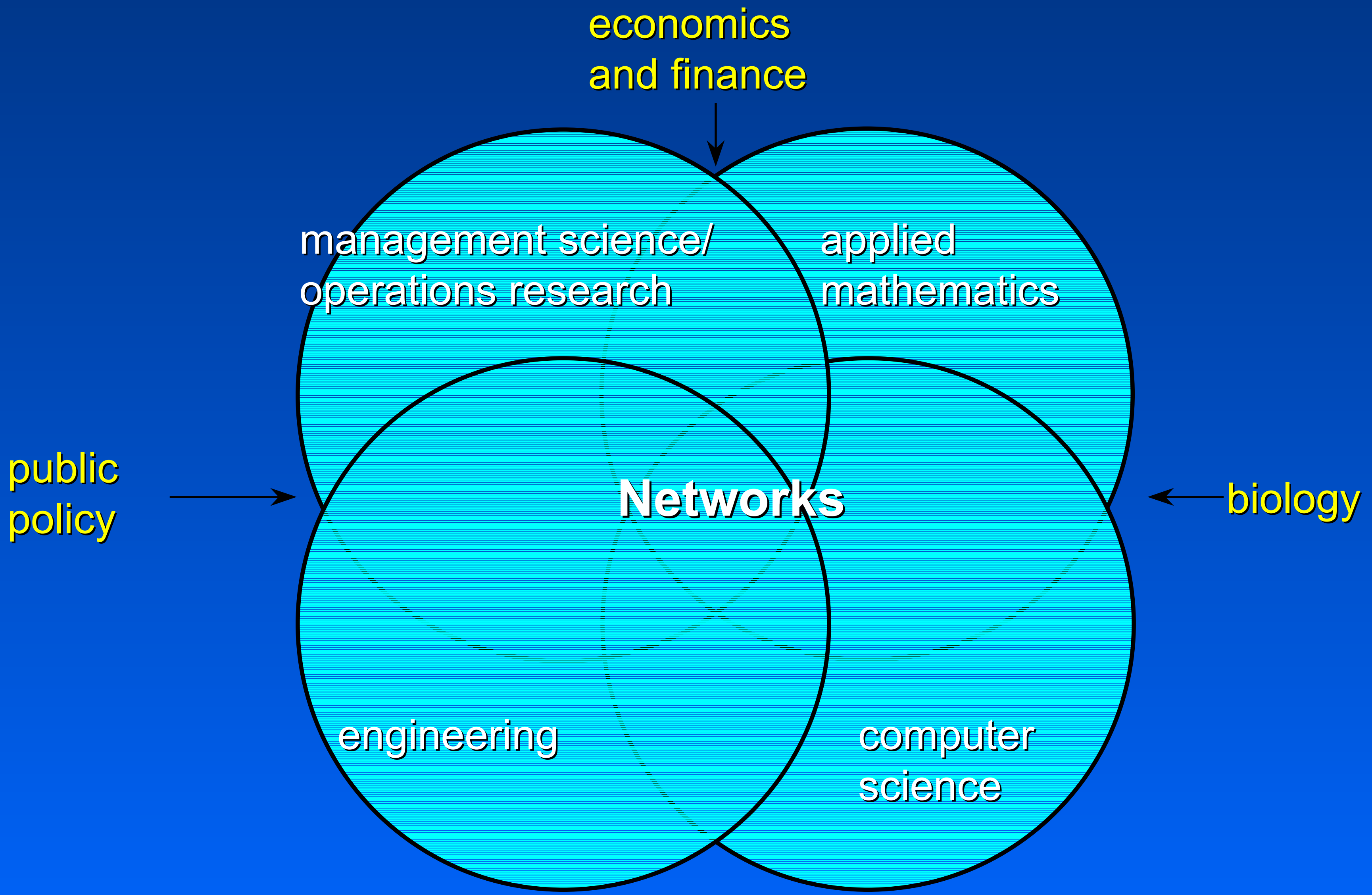
The advantages of a scientific network formalism:

- many present-day problems are concerned with flows over space and time and, hence, ideally suited as an application domain for network theory;
- provides a graphical or visual depiction of different problems;

- helps to identify similarities and differences in distinct problems through their underlying network structure;

- enables the application of efficient network algorithms;

- allows for the study of disparate problems through a unifying methodology.


economics
and finance
management science/
operations research
applied
mathematics
public
policy
Networks
biology
engineering
computer
science

Interdisciplinary Impact of Networks

Management Science

Networks

Economics

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

Engineering

Energy

Manufacturing

Telecommunications

Transportation

Sociology

Social Networks

Organizational Theory

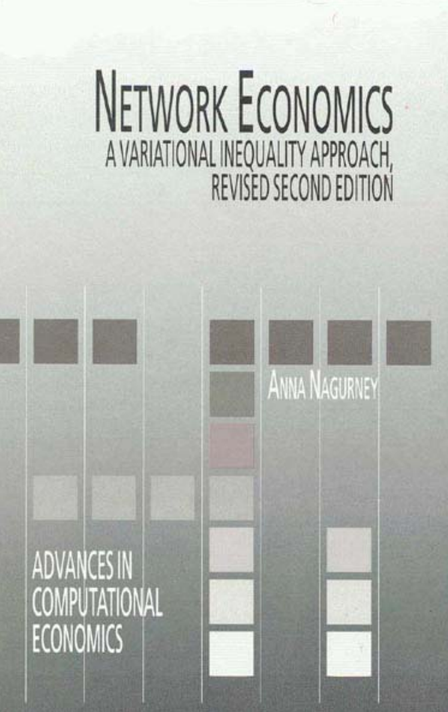
Computer Science

Routing Algorithms

Biology

DNA Sequencing

Targeted Cancer Therapy


NETWORK ECONOMICS
A VARIATIONAL INEQUALITY APPROACH,
REVISED SECOND EDITION
ANNA NAGURNEY
ADVANCES IN
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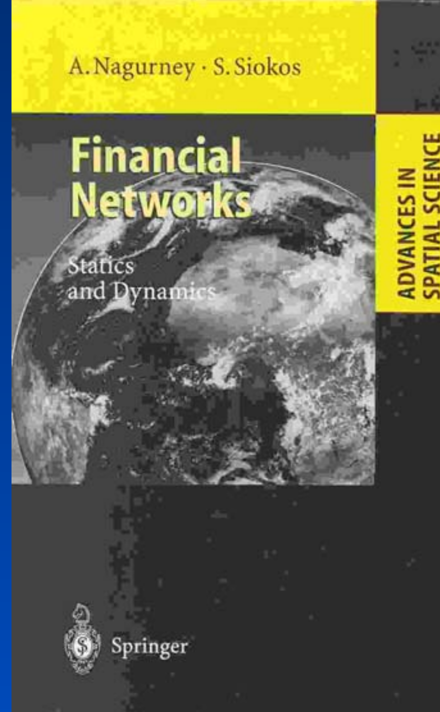
A. Nagurney · S. Siokos
Financial Networks
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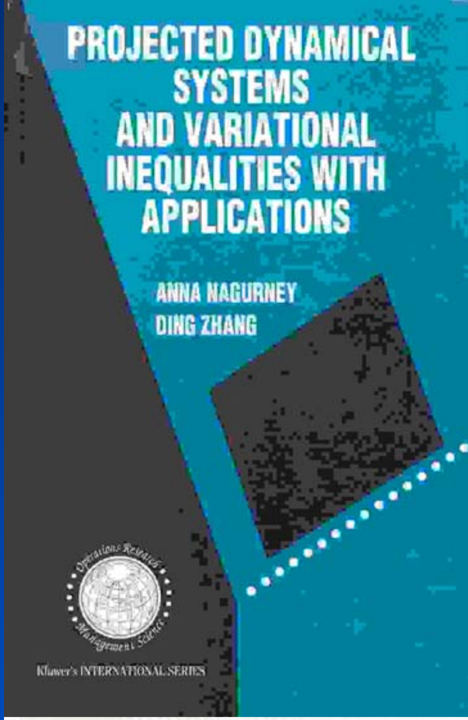
PROJECTED DYNAMICAL SYSTEMS AND VARIATIONAL INEQUALITIES WITH APPLICATIONS
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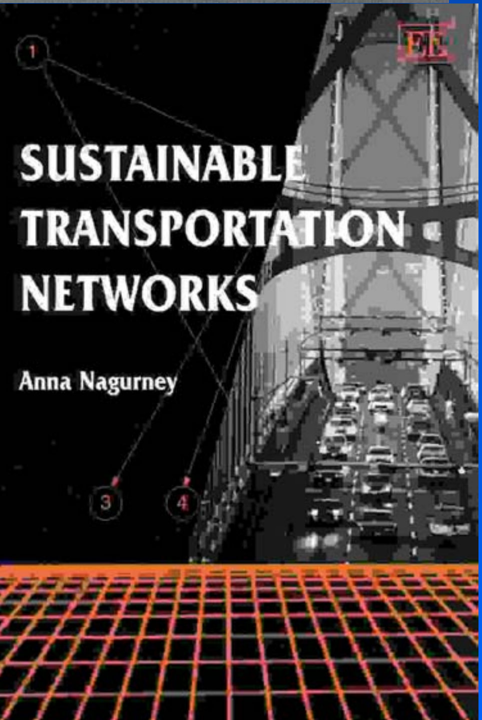
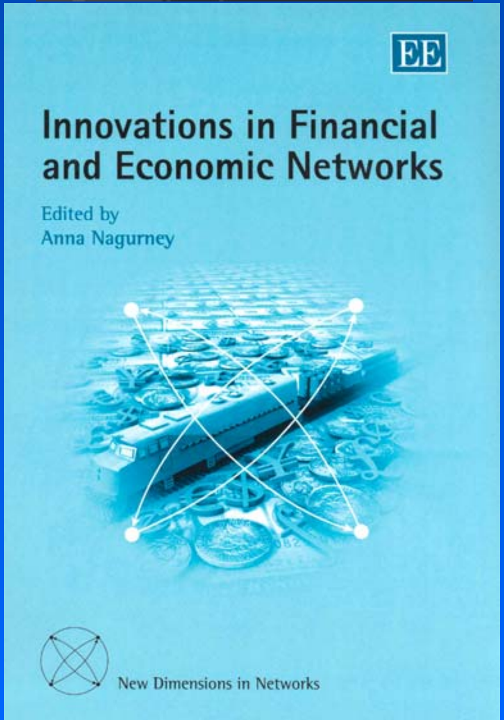
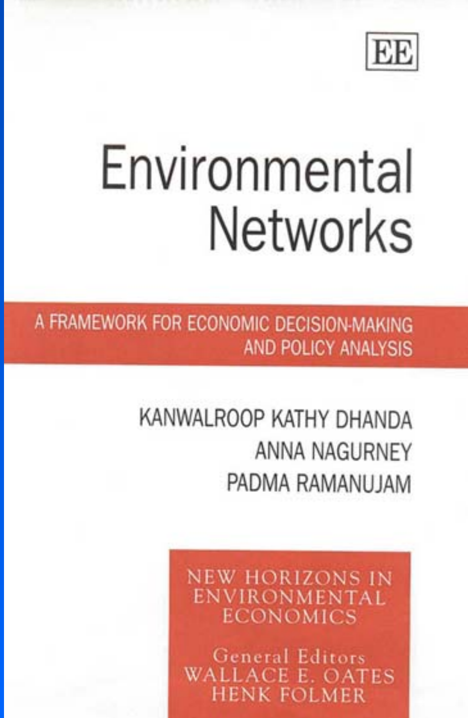

EE
SUSTAINABLE TRANSPORTATION NETWORKS
Anna Nagurney


EE
Innovations in Financial and Economic Networks
Edited by
Anna Nagurney
New Dimensions in Networks


EE
Environmental Networks
A FRAMEWORK FOR ECONOMIC DECISION-MAKING AND POLICY ANALYSIS
KANWALROOP KATHY DHANDA
ANNA NAGURNEY
PADMA RAMANUJAM
NEW HORIZONS IN ENVIRONMENTAL ECONOMICS
General Editors
WALLACE E. OATES
HENK FOLMER

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion;*
- alternative behavior of users of the network, which may lead to *paradoxical phenomena;*
- the *interactions among networks* themselves such as in transportation versus telecommunications networks;
- *policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.

- *alternative behaviors of the users of the network*
  - system-optimized versus
  - user-optimized (network equilibrium),

which may lead to

paradoxical phenomena.

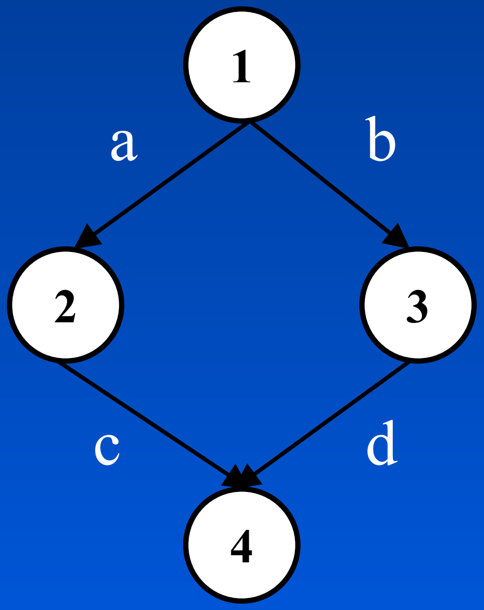
The Braess Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}^* = x_{p_2}^* = 3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83}$.



$\mathbf{c_a(f_a)=10\ f_a}$ $\mathbf{c_b(f_b) = f_b+50}$

$\mathbf{c_c(f_c) = f_c+50}$ $\mathbf{c_d(f_d) = 10\ f_d}$

Adding a Link
Increases Travel Cost for All!

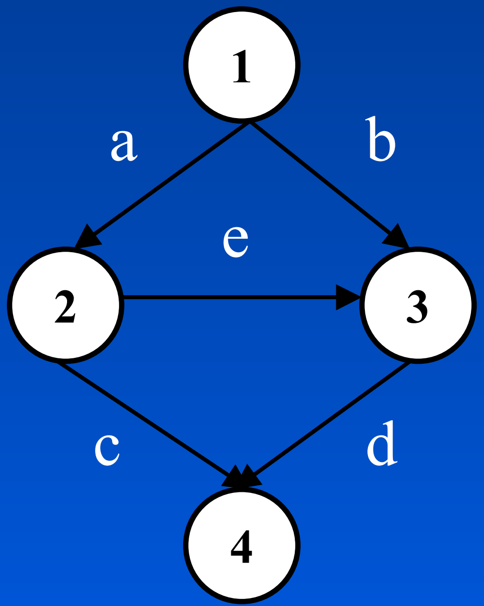
Adding a new link creates a new path $\mathbf{p_3=(a,e,d)}$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path $\mathbf{p_3}$, $\mathbf{C_{p_3}=70}$.

The new equilibrium flow pattern network is

$$\mathbf{x_{p_1}{}^* = x_{p_2}{}^* = x_{p_3}{}^*=2.}$$

The equilibrium path travel costs:
$\mathbf{C_{p_1} = C_{p_2} = C_{p_3} = 92.}$



$\mathbf{c_e(f_e) = f_e + 10}$

This phenomenon is relevant to telecommunications networks and the Internet which is another example of a

noncooperative network.

The Price of Anarchy!!!

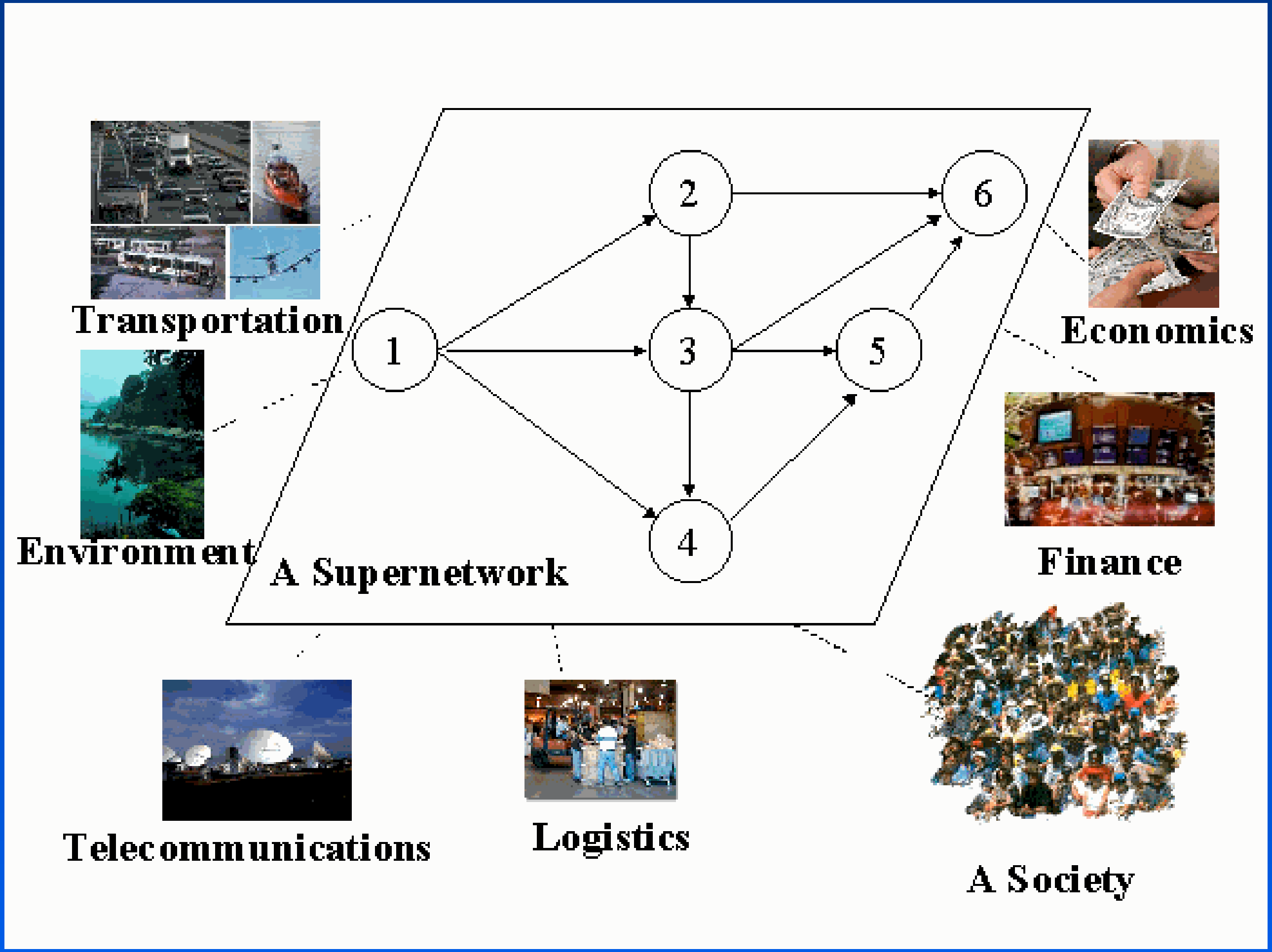
The 1968 Braess article has been translated from German to English and appears as

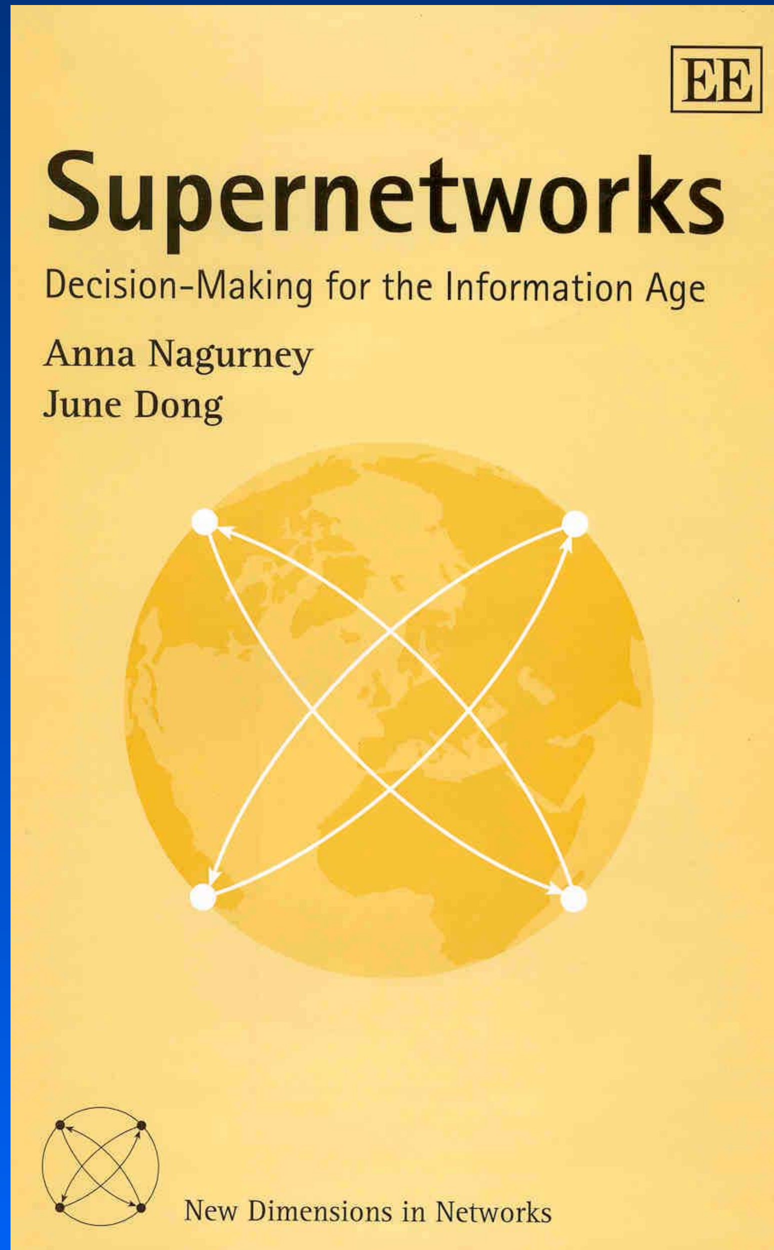
On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science.*

Supernetworks: A New Paradigm




EE
Supernetworks
Decision-Making for the Information Age
Anna Nagurney
June Dong
New Dimensions in Networks

Supernetworks

- Supernetworks may be comprised of such networks as transportation, telecommunication, logistical, and/or financial networks.

- They may be *multilevel* as when they formalize the study of supply chain networks or *multitiered* as in the case of financial networks with intermediation.

- Decision-makers may be faced with multiple criteria; thus, the study of supernetworks also includes the study of *multicriteria decision-making*.

The Supernetwork Team 2005 - 2006





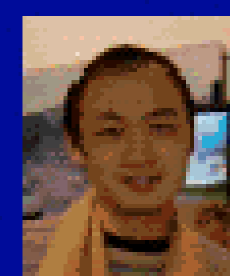












New Tools

The tools that we are using in our Supernetwork research include:

- network theory
- optimization theory
- game theory
- variational inequality theory
- evolutionary variational inequality theory
- projected dynamical systems theory
- double-layered dynamics theory
- network visualization tools.

Transportation science has historically been the discipline that has *pushed the frontiers* of methodological developments for network problems.

- Dafermos (1980) showed that the traffic network equilibrium (also referred to as user-optimization) conditions as formulated by Smith (1979) were a finite-dimensional variational inequality.
- In 1993, Dupuis and Nagurney proved that the set of solutions to a variational inequality problem coincided with the set of solutions to a projected dynamical system (PDS) in R^n.
- In 1996, Nagurney and Zhang published **Projected Dynamical Systems and Variational Inequalities**.
- In 2002, Cojocaru proved the 1993 result for Hilbert Spaces.

EQUILIBRIA of PDS and VARIATIONAL INEQUALITIES

An important feature of any PDS is that it is intimately related to a variational inequality problem (VI).

Theorem

The equilibria of a PDS:

$$\frac{\partial}{\partial t}(x(t)) = \Pi_K(x(t), -F(x(t)))$$

$$= \lim_{\delta \to 0} \frac{P_K(x(t) - \delta F(x(t))) - x(t)}{\delta}, \quad x(0) = x_0,$$

that is, $x^* \in K$ *such that*

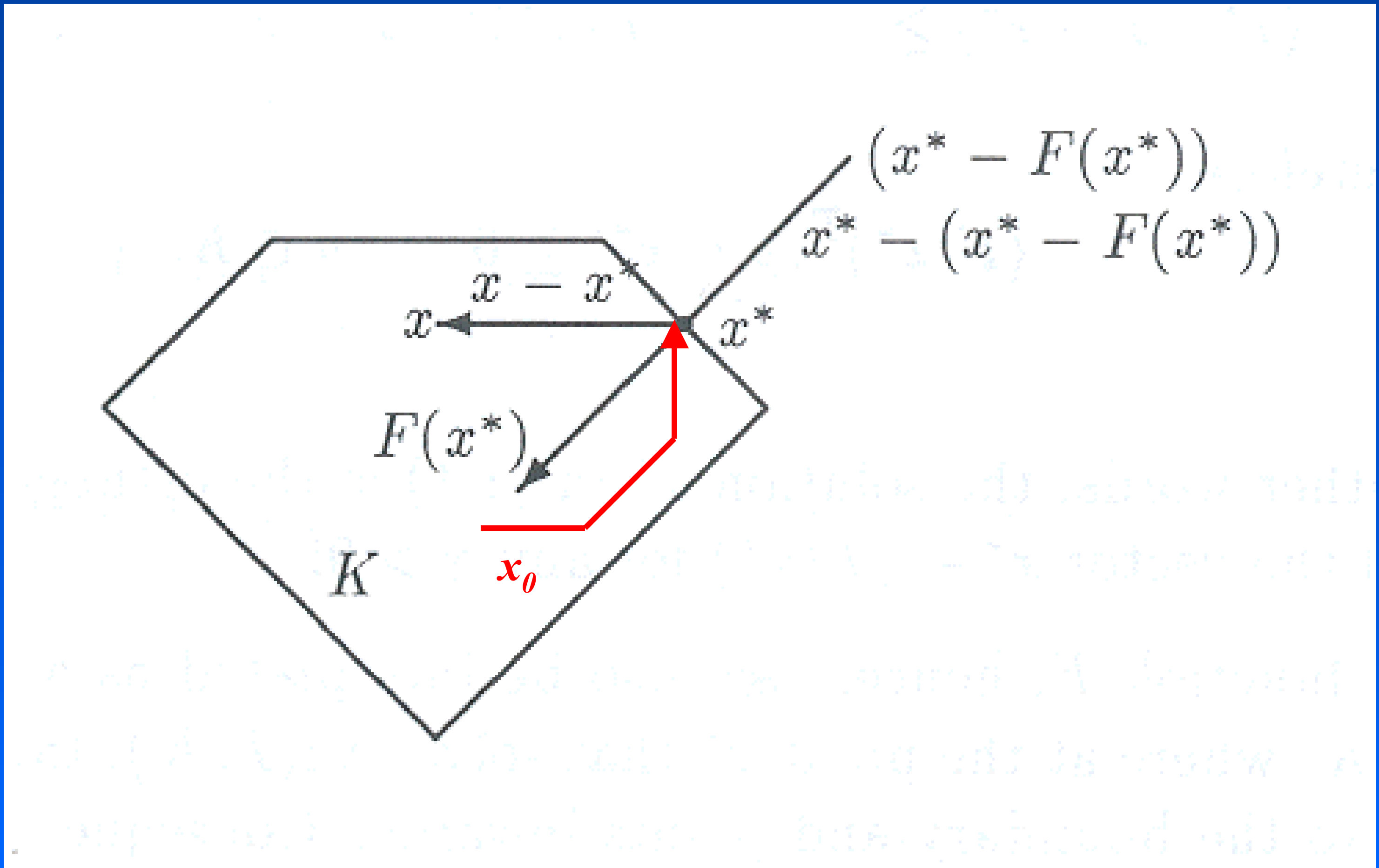
$$\Pi_K(x^*, -F(x^*)) = 0$$

are solutions to the $\mathrm{VI}(F, K)$*: find* $x^* \in K$ *such that*

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

and vice-versa, where $\langle \cdot, \cdot \rangle$ *denotes the inner product on* X*, where* X *is a Hilbert space.*

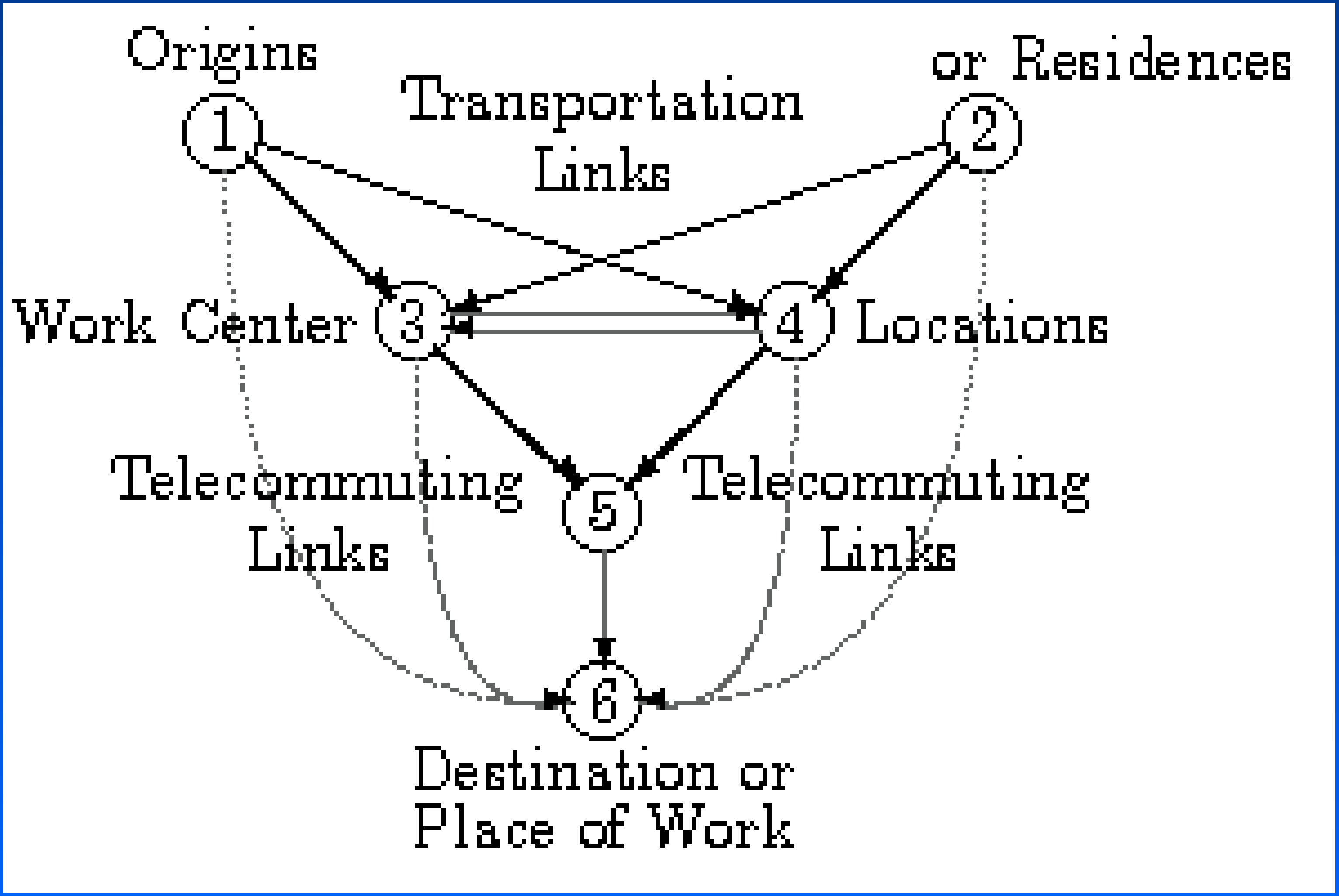
A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System



Applications of Supernetworks

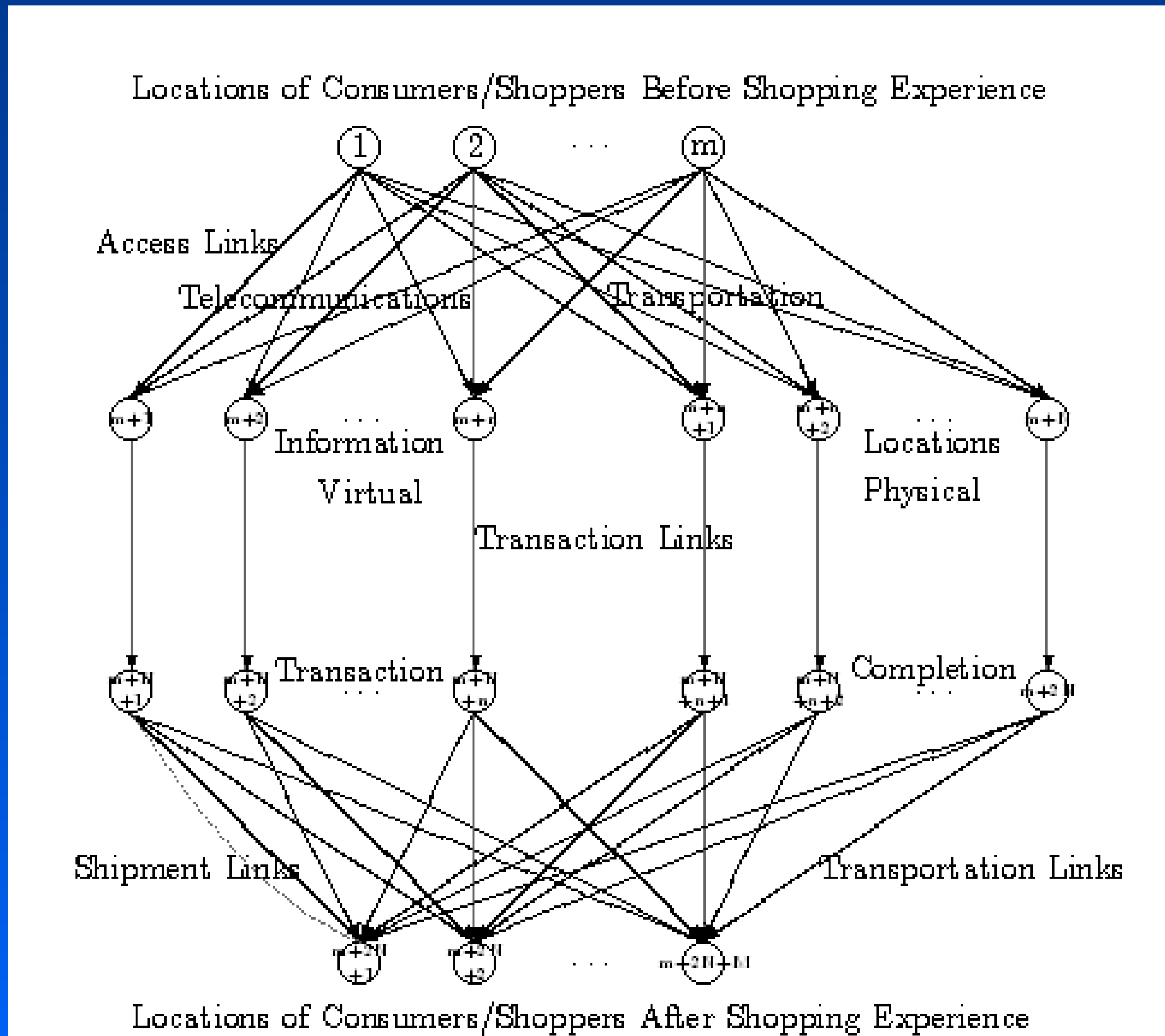
- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Energy Networks/Power Grids
- Social Networks integrated with Economic Networks

A Supernetwork Conceptualization of Commuting versus Telecommuting



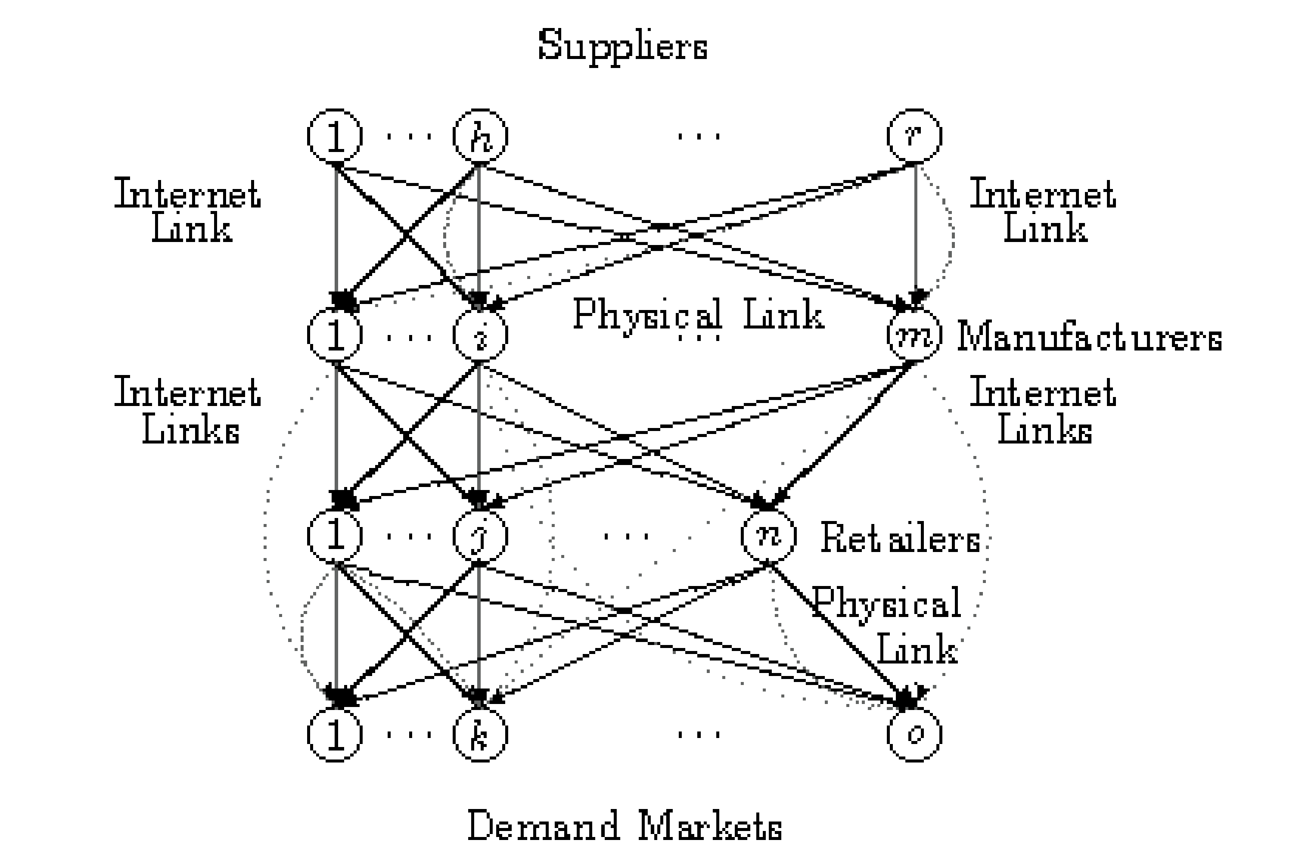
Nagurney, Dong, and Mokhtarian, *JEDC* (2002)

A Supernetwork Framework for Teleshopping versus Shopping



Nagurney, Dong, and Mokhtarian, *JEDC* (2002)

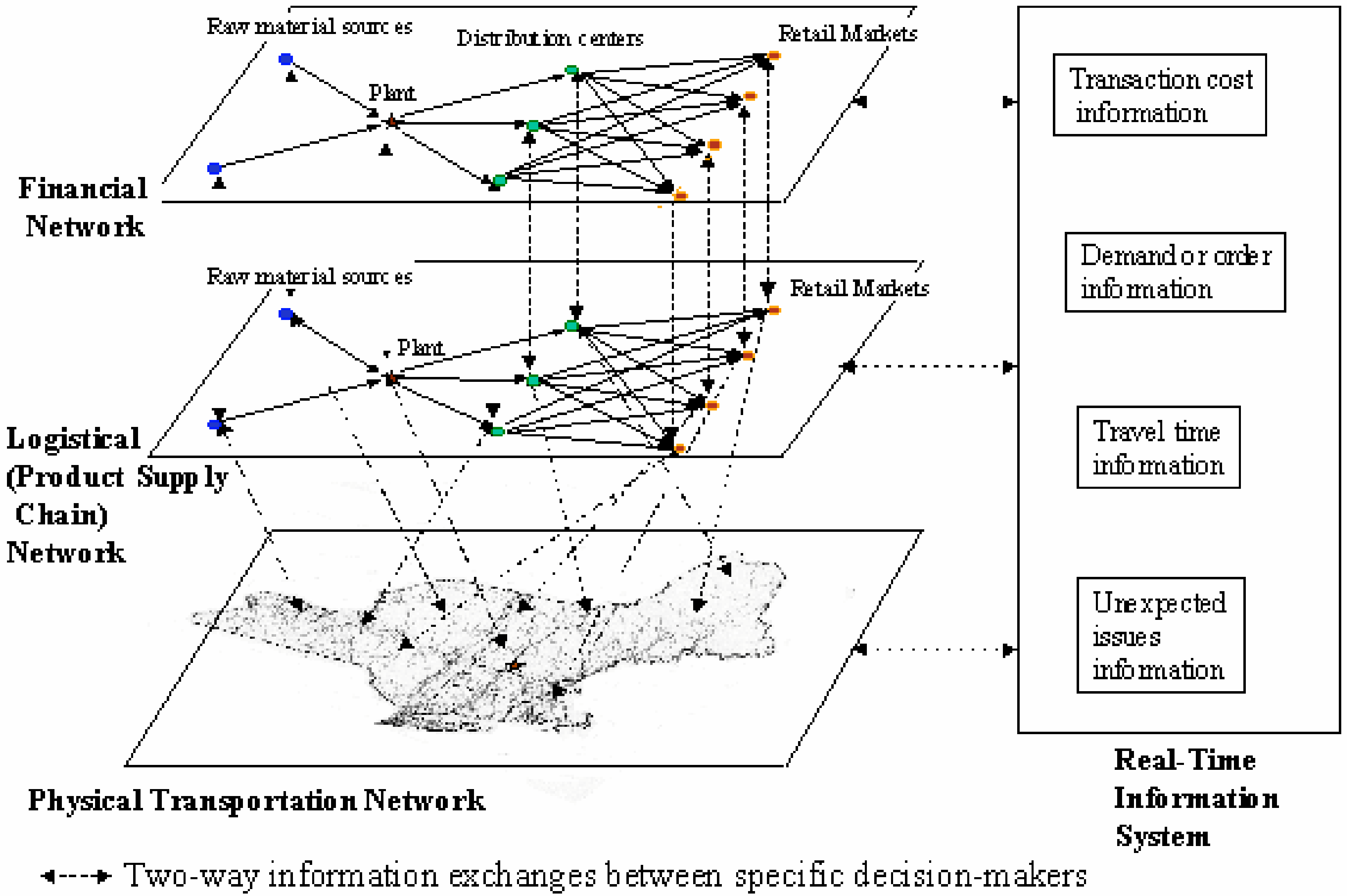
The Supernetwork Structure of a Supply Chain Network



Nagurney, Dong, and Zhang, *Transportation Research E* (2002)

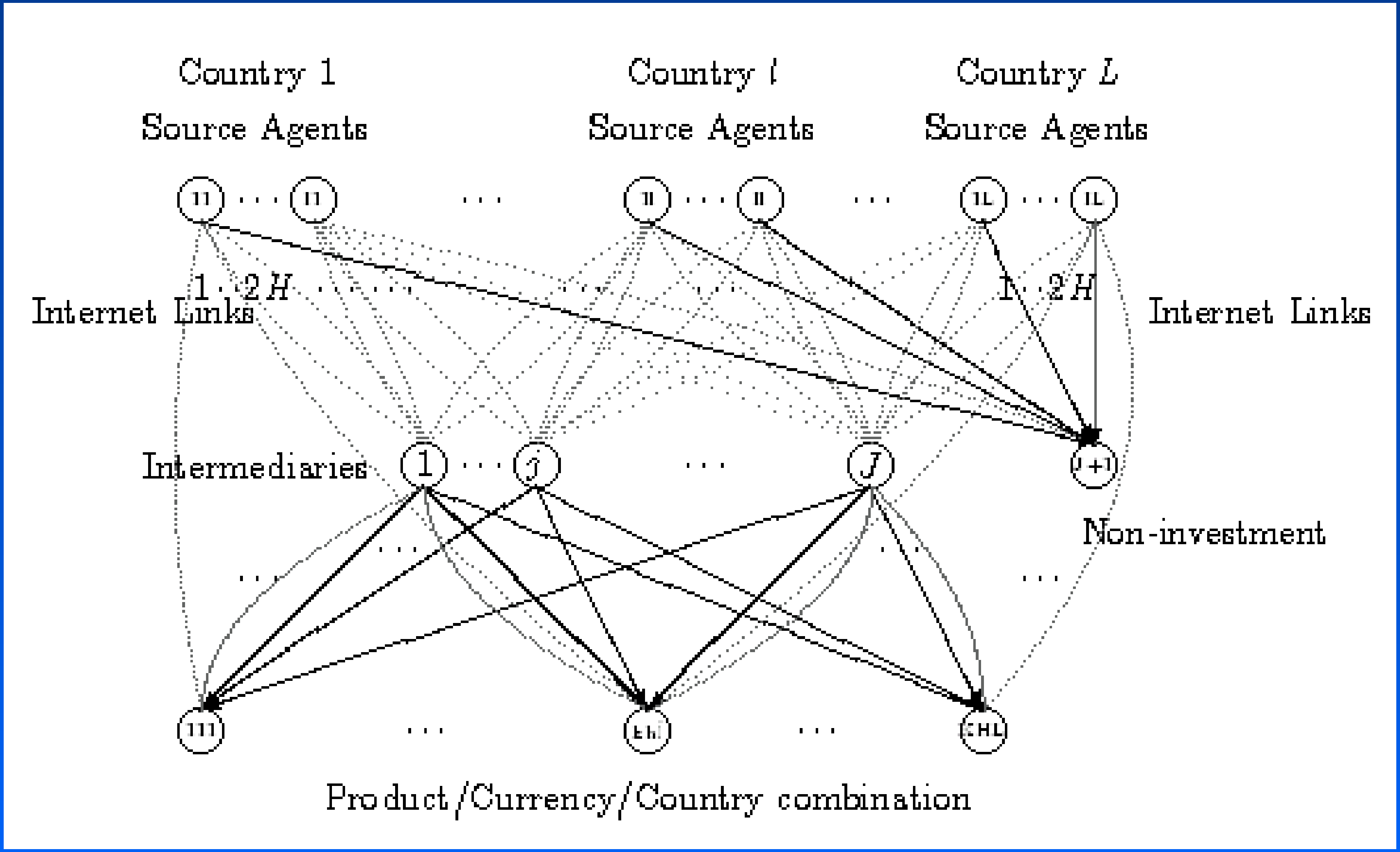
Nagurney, Loo, Dong, and Zhang, *Netnomics* (2002)

Supply Chain -Transportation Supernetwork Representation



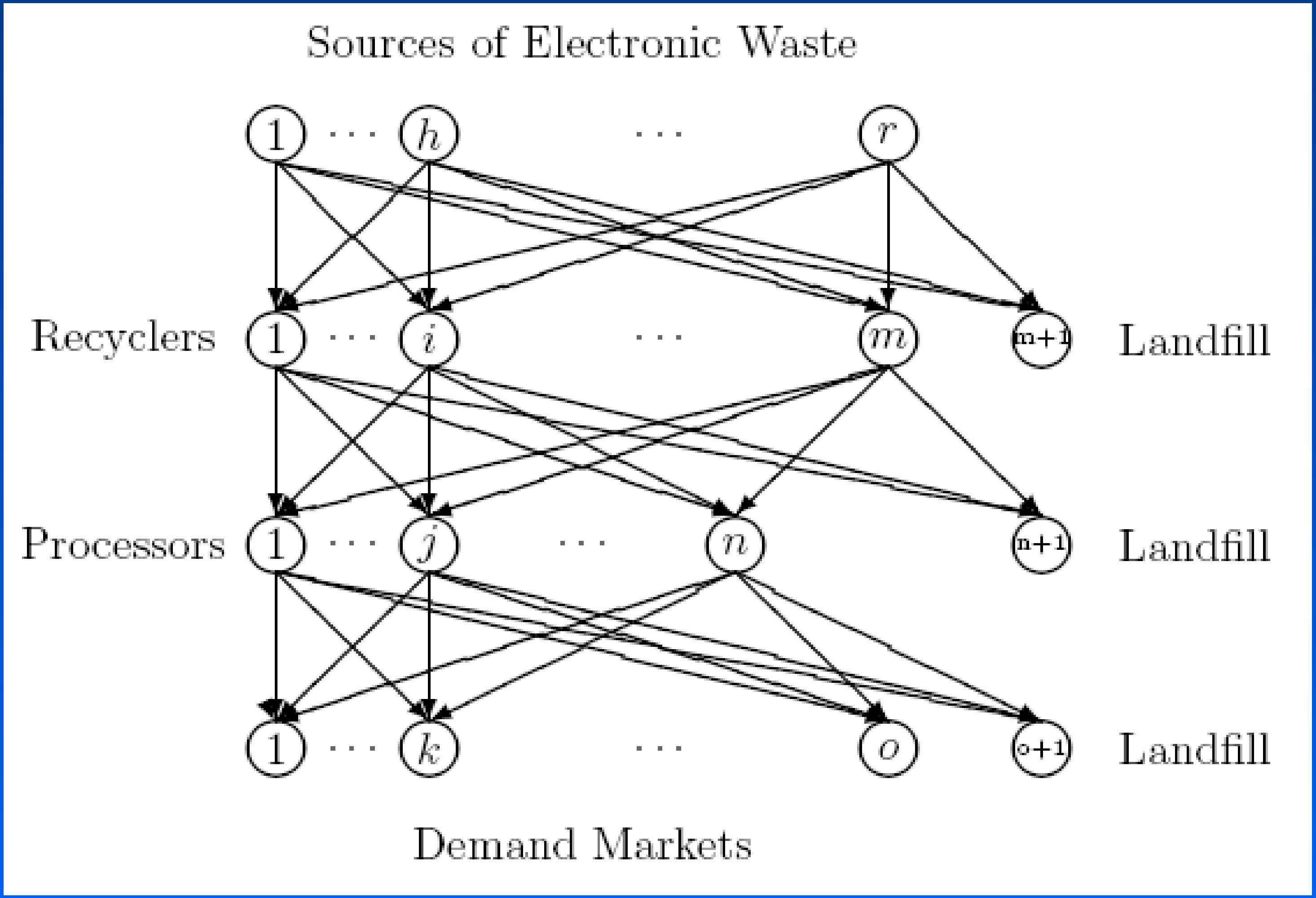
Nagurney, Ke, Cruz, Hancock, Southworth, *Environment and Planning B* (2002)

International Financial Networks with Electronic Transactions



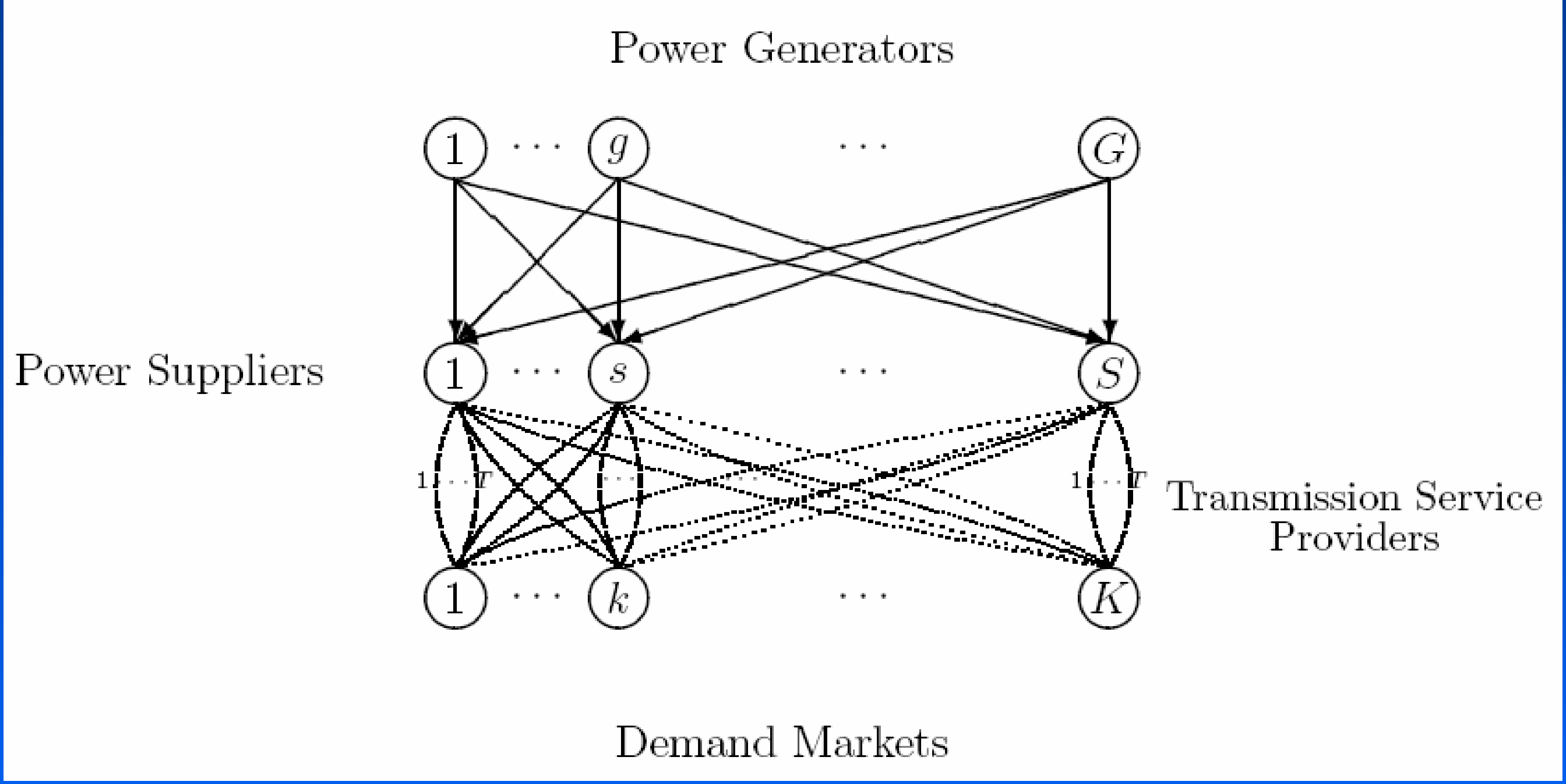
Nagurney and Cruz, in **Innovations in Financial and Economics Networks** (2003)

The 4-Tiered E-Cycling Network



Nagurney and Toyasaki, *Transportation Research E* (2005)

The Electric Power Supply Chain Network

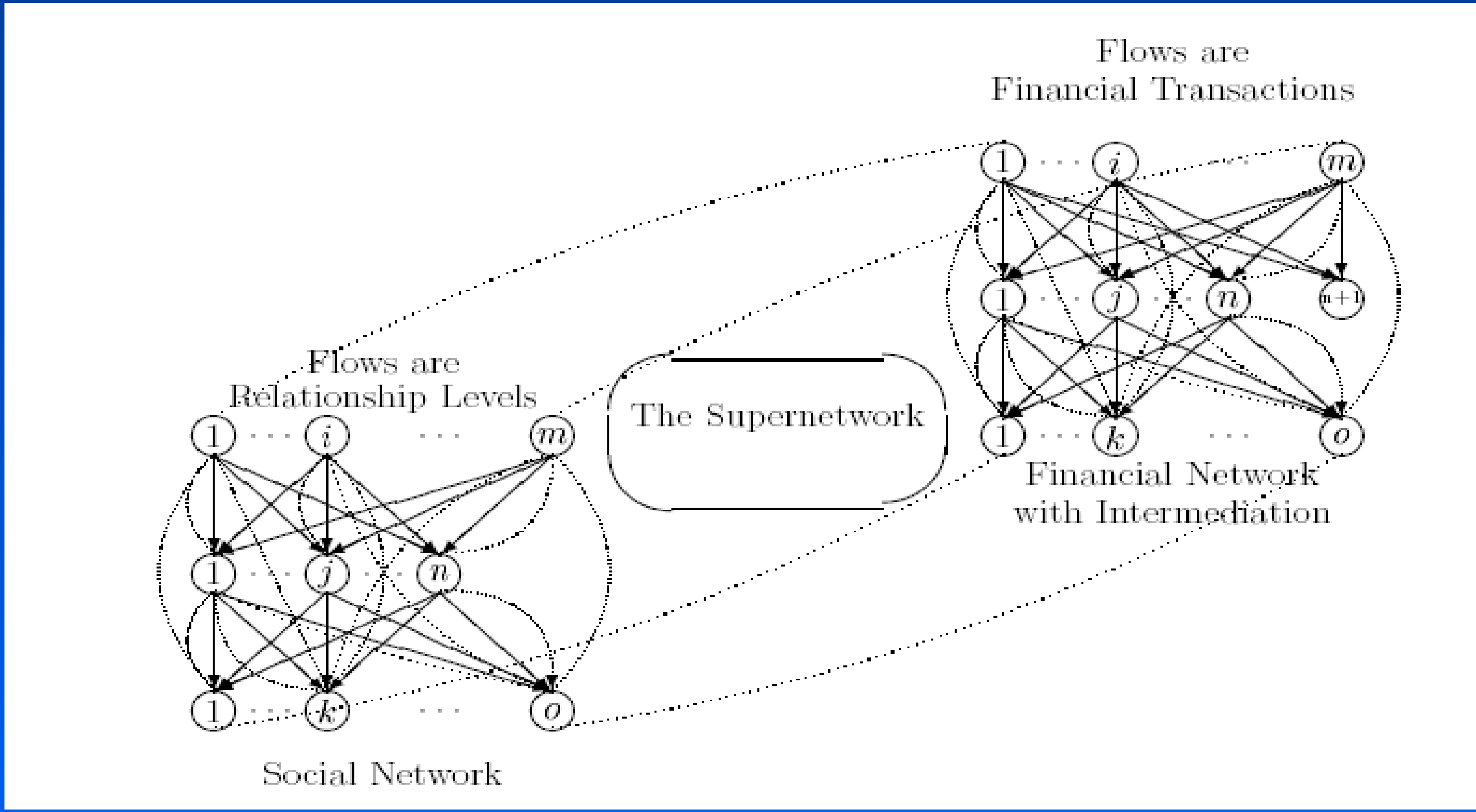


Nagurney and Matsypura, Proceedings of the *CCCT* (2004)

Research Motivation

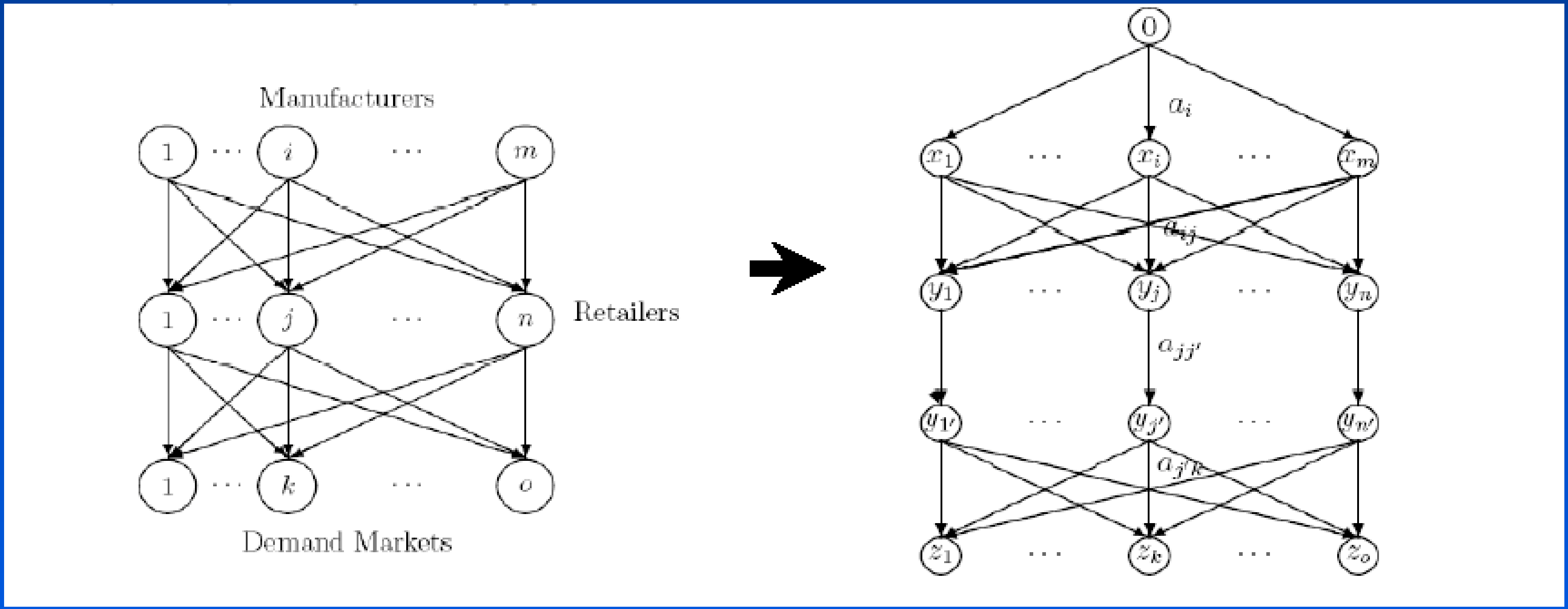
Can Social Networks and Financial Networks be Unified?

Supernetwork Structure: Integrated Financial/Social Network System



Cruz, Nagurney, and Wakolbinger, *Naval Research Logistics* (2006)

The Supernetwork Equivalence of Supply Chain Networks and Transportation Networks



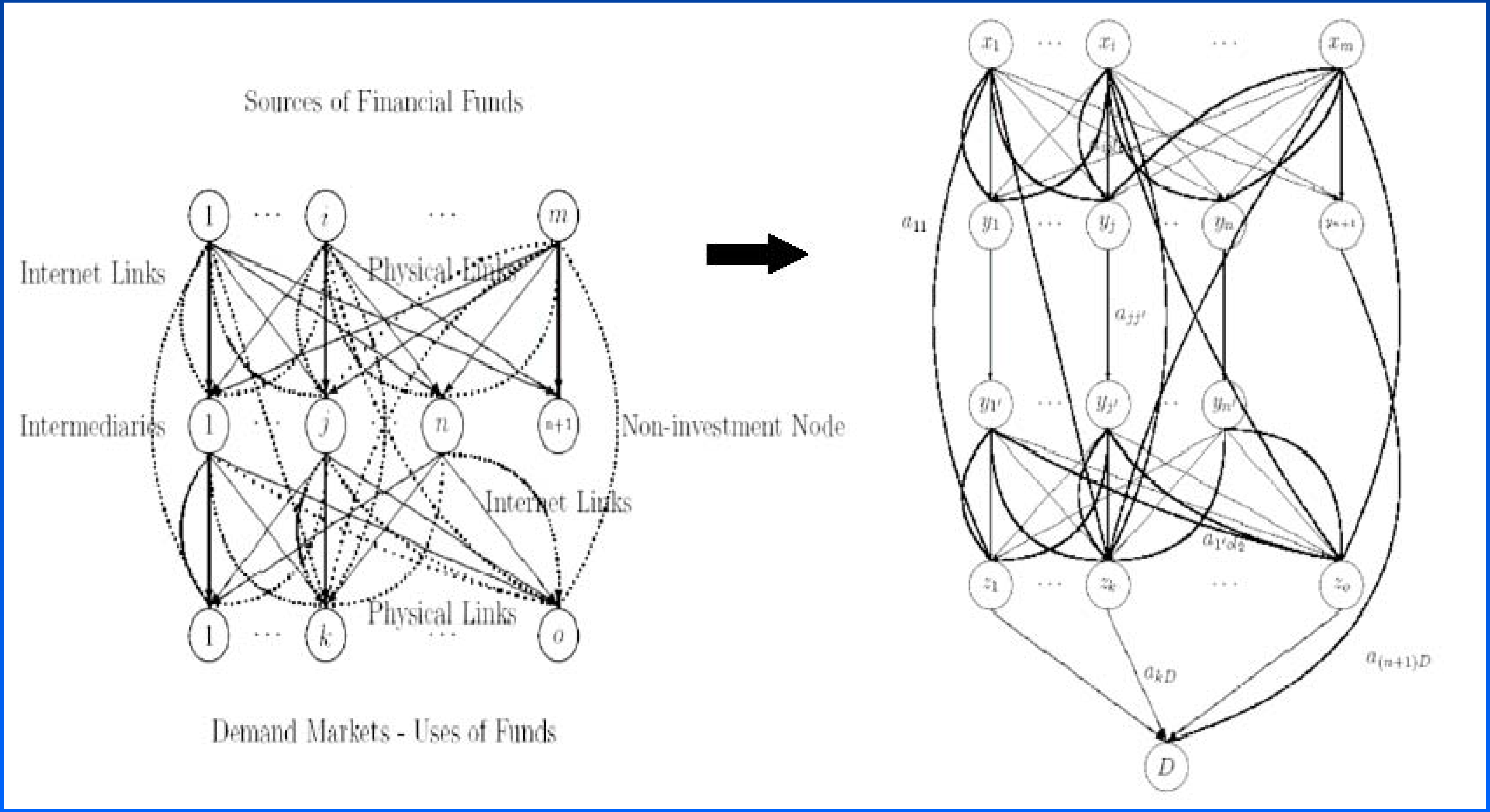
Nagurney, *Transportation Research E* (2006)

The Supernetwork Equivalence of Supply Chain Networks and Transportation Networks

Copeland (1952) wondered whether money flows like water or electricity.

Liu and Nagurney have shown that money and electricity flow like transportation network flows (*Computational Management Science* (2006)).

The Transportation Network Equilibrium Reformulation of the Financial Network Equilibrium Model with Intermediation

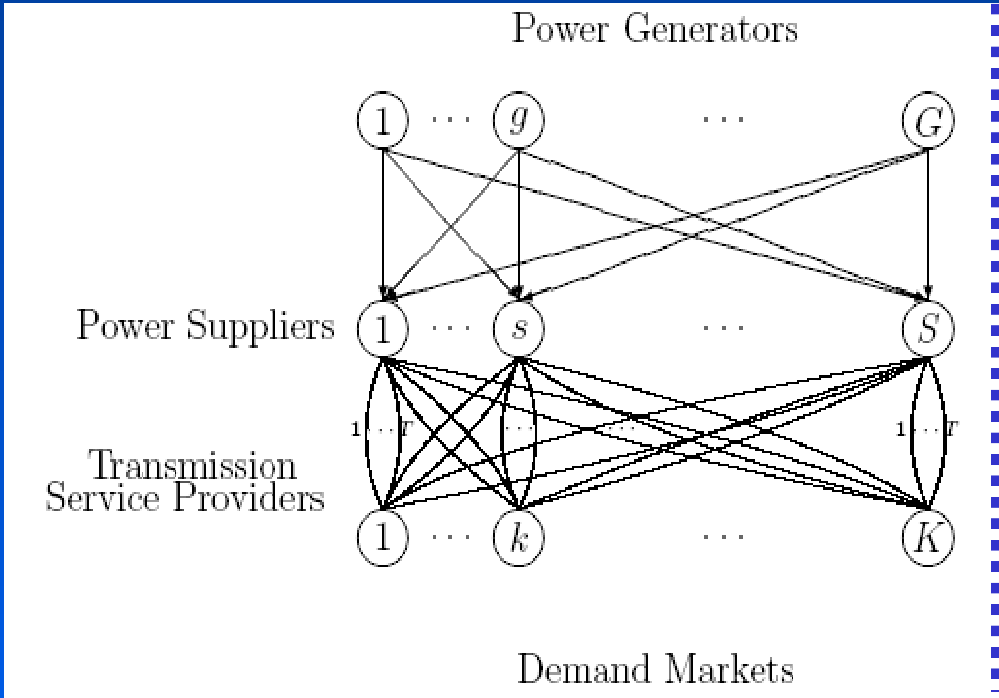


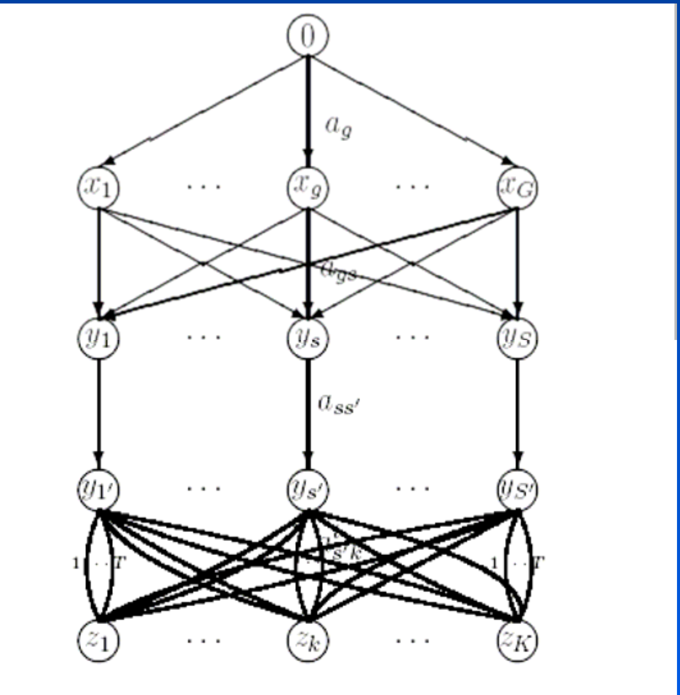
The Supernetwork Equivalence of Electric Power Supply Chain Networks and Transportation Networks

The fifth chapter of Beckmann, McGuire, and Winsten's book, **Studies in the Economics of Transportation** (1956) describes some *unsolved problems* including a single commodity network equilibrium problem that the authors imply could be generalized to capture electric power networks.

Nagurney, Liu, Cojoucaru, and Daniele took up this challenge of establishing the relationship and application of transportation network equilibrium models to electric power networks. (*Transportation Research D* (2006)).

The Supernetwork Equivalence of Electric Power Supply Chain Networks and Transportation Networks





Electric Power Supply Chain Network

Transportation Network

We have, hence, shown that money as well as electricity flow like *transportation* and have answered questions posed fifty years ago by Copeland and Beckmann, McGuire, and Winsten, respectively.

We are working with Cojocaru and Daniele on infinite-dimensional projected dynamical systems and evolutionary variational inequalities and their relationships and unification.

This allows us to model dynamic networks with:

- *dynamic (time-dependent)* supplies and demands
- *dynamic (time-dependent)* capacities
- *structural changes* in the networks themselves.

Such issues are important for robustness, resiliency, and reliability of networks (including supply chains).

What happens if the demand is varied in the Braess Network?

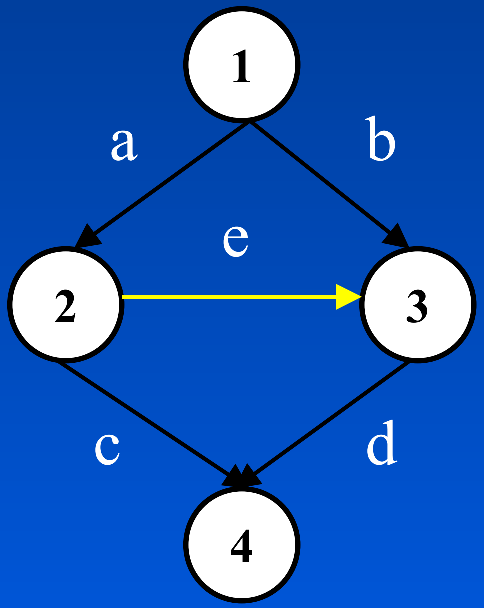
The answer lies in the solution of an Evolutionary (Time-Dependent) Variational Inequality.

Find $x^* \in K$, such that

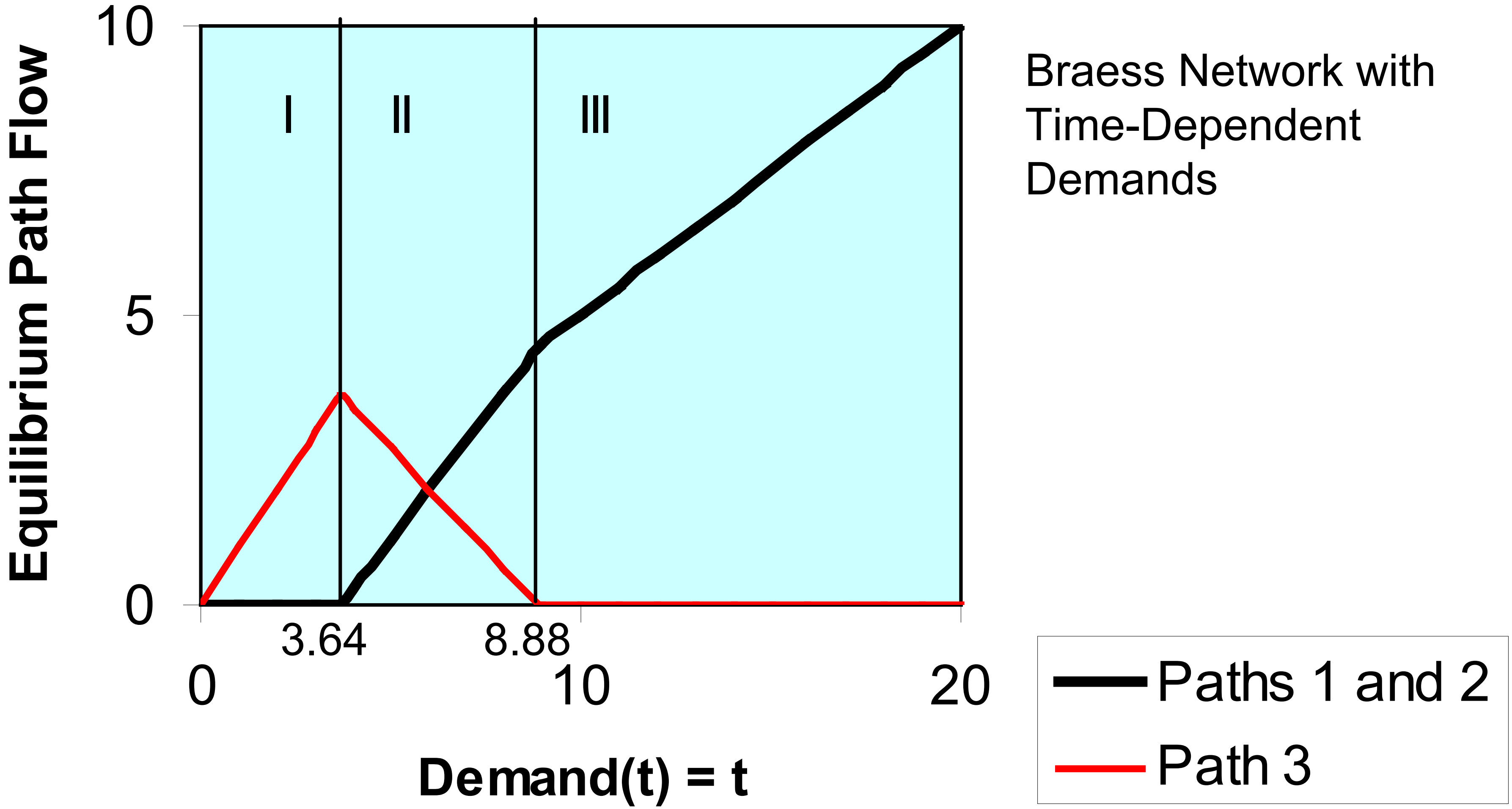
$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle \, dt \geq 0 \qquad \forall x \in K$$

Nagurney, Parkes, and Daniele, *Computational Management Science* (2006)

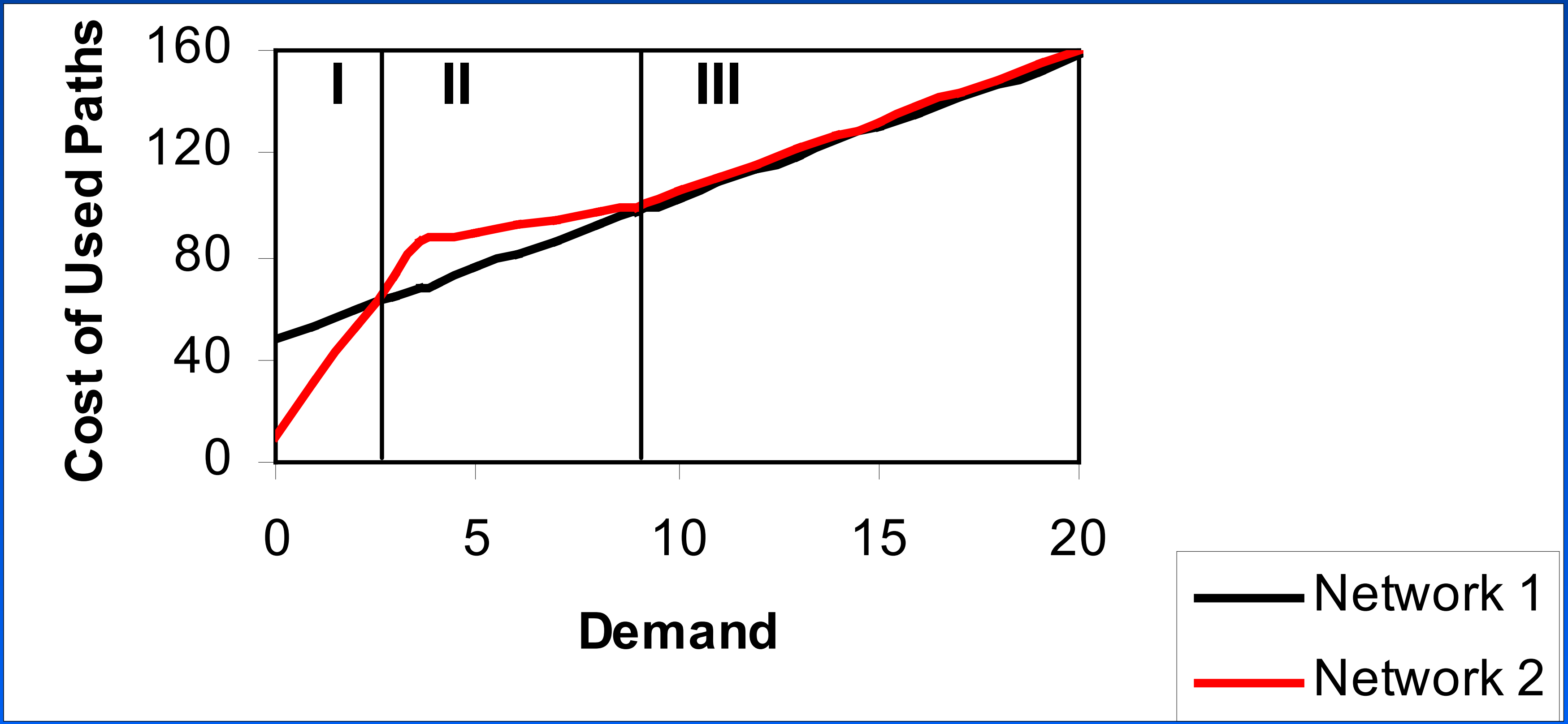
Recall the Braess Network where we add the link e.



The Solution of an Evolutionary (Time-Dependent) Variational Inequality for the Braess Network with Added Link (Path)



In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

New Challenges and Opportunities: The Unification of EVIs and PDSs

Double-Layered Dynamics

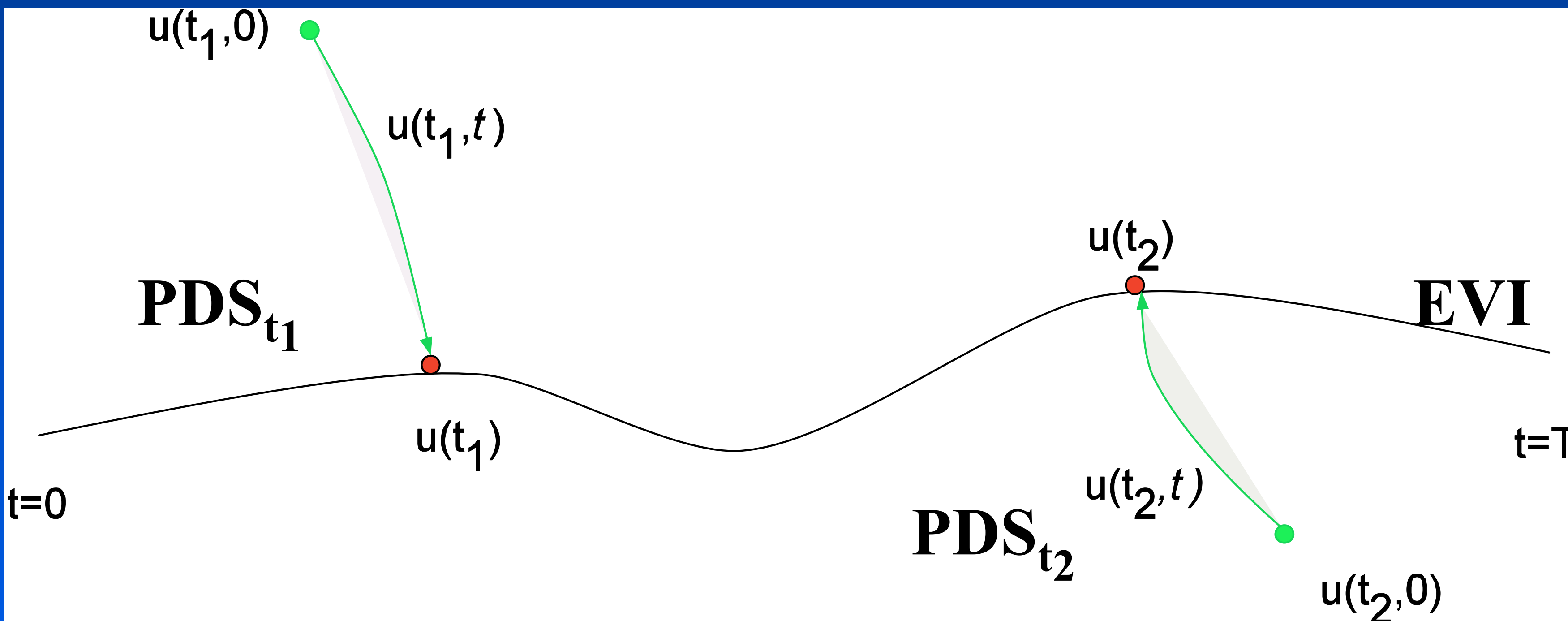
The unification of EVIs and PDSs allows the modeling of dynamic networks over *different time scales*.

Papers:

Projected Dynamical Systems and Evolutionary Variational Inequalities via Hilbert Spaces with Applications (Cojocaru, Daniele, and Nagurney), *Journal of Optimization Theory and Applications*, vol. 127, no. 3, pp. 1-15, December 2005.

Double-Layered Dynamics: A Unified Theory of Projected Dynamical Systems and Evolutionary Variational Inequalities (Cojocaru, Daniele, and Nagurney), *European Journal of Operational Research,* in press.

A Pictorial of the Double-Layered Dynamics



Theorem (Cojocaru, Daniele, and Nagurney (2005))

The solutions to the EVI problem are the same as the critical points of the PDS and vice versa, that is, the critical points of the PDS are the solutions to the EVI.

Hence, by choosing the Hilbert space to be $L^2([0,T], R^q)$, *we find that the solutions to the evolutionary variational inequality: find* $x^* \in K$ *such that*

$$\int_0^T \langle F(x^*(t)), x(t) - x^*(t) \rangle dt \geq 0, \quad \forall x \in K$$

are the same as the critical points of the equation:

$$\frac{\partial x(t,\tau)}{\partial \tau} = \Pi_K(x(t,\tau), -F(x(t,\tau))),$$

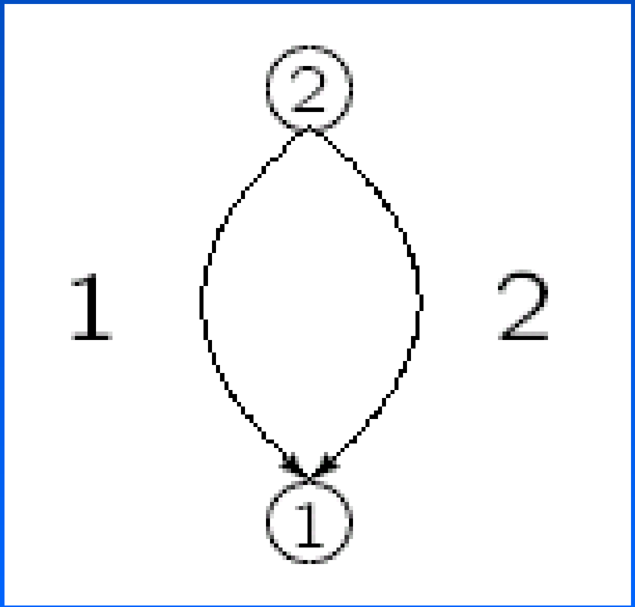
that is, the points such that

$$\Pi_K(x(t,\tau), -F(x(t,\tau))) \equiv 0 \quad a.e. \ in \ [0,T],$$

which are obviously stationary with respect to τ.

A Dynamic Network Example with Time-Varying Demand and Capacities

We consider a network consisting of a single origin/destination pair of nodes and two paths connecting these nodes.



Let cost on path **1** be: $\mathbf{2x_1(t)-1.5}$ and cost on path **2** be: $\mathbf{x_2(t)-1}$.

The demand is **t** in the interval **[0,2].**

Suppose that we also have capacities**:**

$\mathbf{(0,0) \le (x_1(t), x_2(t)) \le (t, 3/2\, t)}$**.**

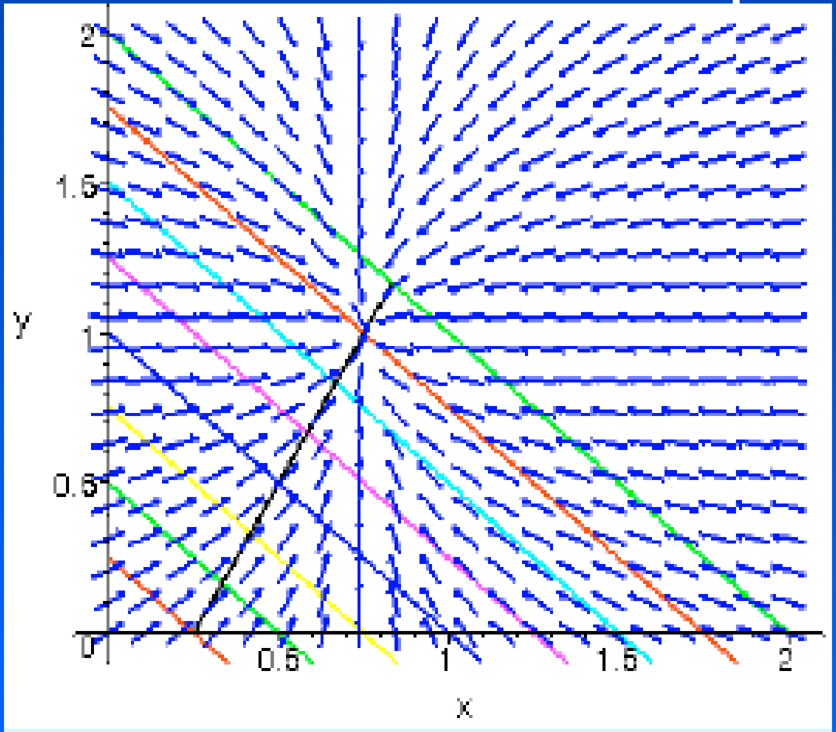
With the help of PDS theory, we can compute an approximate curve of equilibrium by choosing

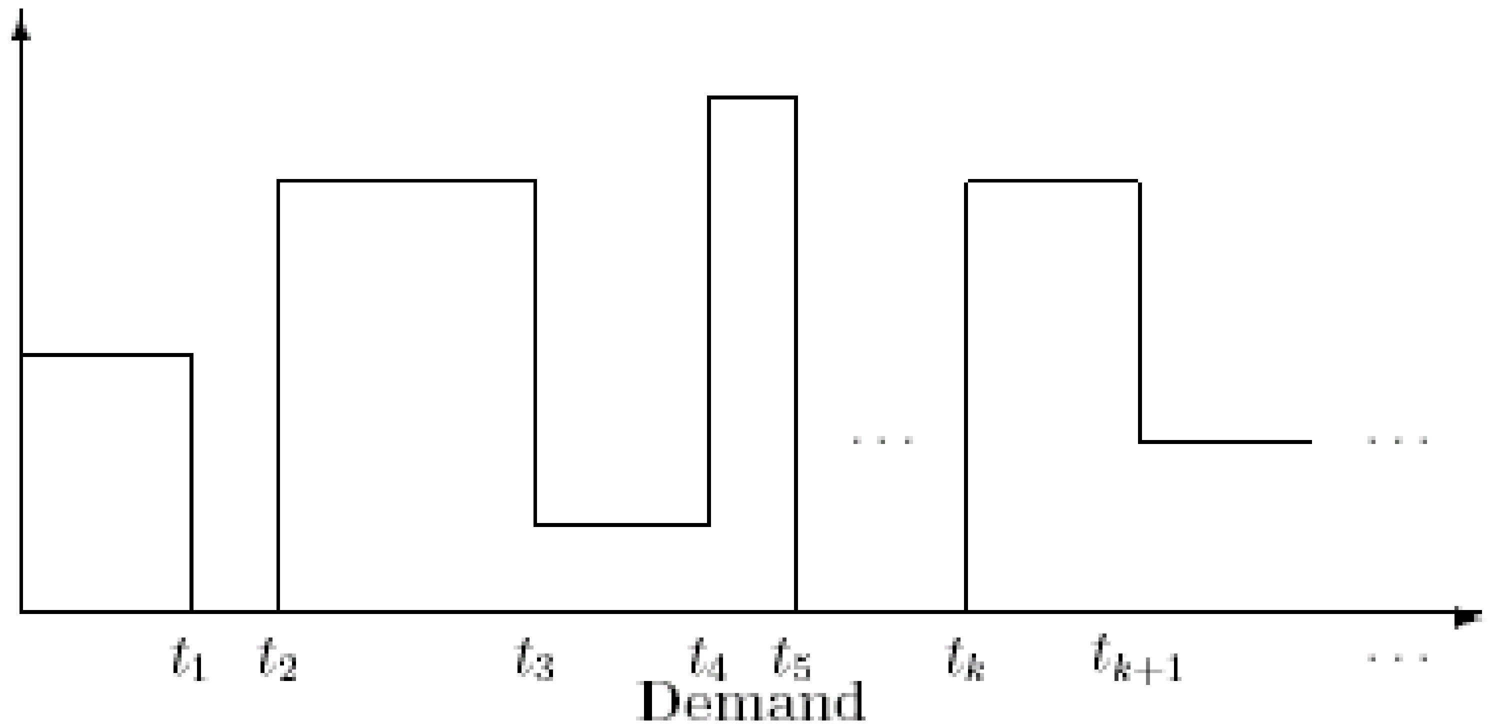
$$t_0 \in \left\{ \frac{k}{4} | k \in \{0, \ldots, 8\} \right\}.$$

Using a simple MAPLE computation, we obtain that the equilibria are the points:

$$\left\{(0,0),\left(\frac{1}{4},0\right),\left(\frac{1}{3},\frac{1}{6}\right),\left(\frac{5}{12},\frac{1}{3}\right),\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{7}{12},\frac{2}{3}\right),\left(\frac{2}{3},\frac{5}{6}\right),\left(\frac{3}{4},1\right),\left(\frac{5}{6},\frac{7}{6}\right)\right\}.$$

Interpolating these points, we obtain the approximate curve of network equilibria:





If the demand is a step function, the solution to the EVI has the structure:

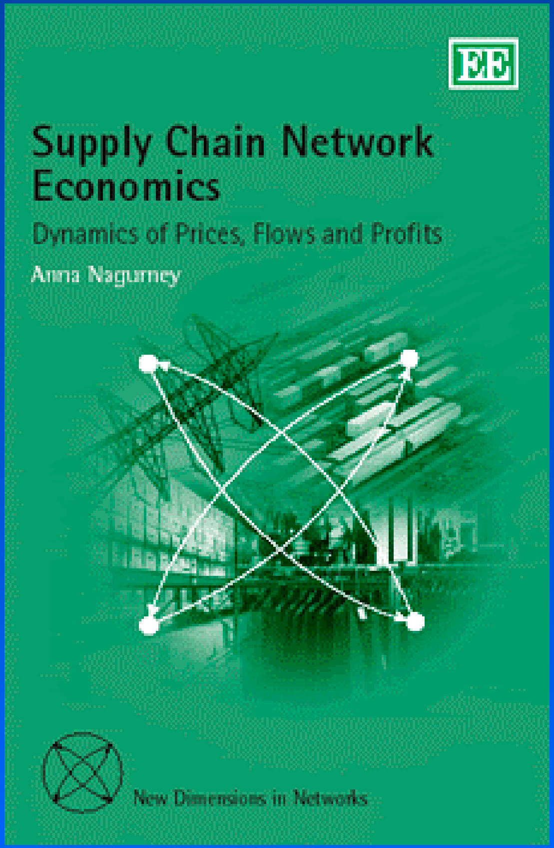
$$x^*(t) = \begin{cases} x_1^* & \text{if } 0 \leq t \leq t_1 \\ x_2^* & \text{if } t_1 < t \leq t_2 \\ \vdots & \vdots \\ x_{k+1}^* & \text{if } t_k < t \leq t_{k+1} \\ \vdots & \vdots \end{cases}$$

Evolutionary variational inequalities have now been used to model dynamic:

- *transportation networks,*
- *supply chains,*
- *financial networks*,
- *electric power supply chains, and the Internet*.

For additional background and new applications see:

Supply Chain Network Economics



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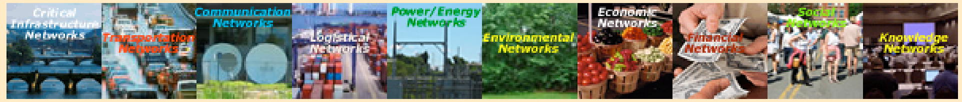

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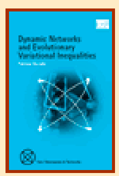

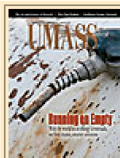
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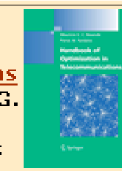

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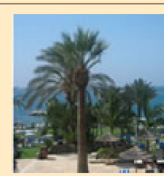

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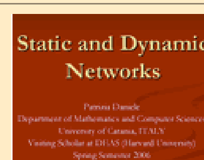

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Conclusion

As we face challenges in the 21st Century, we can see how complex decision-making can benefit from powerful

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