Workshop on Traffic Assignment with Equilibrium Methods

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Transportation Network Modeling Committee

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Contents of second part

- Variable demand equilibrium assignment
- Multi-modal models : equilibration
- Some large scale applications
- Stochastic user equilibrium assignment
Variable demand equilibrium assignment

- The demand for travel is assumed to be given by a direct demand function which has the property that as travel time (cost) increases the demand decreases.

- The network equilibrium is established when all used routes are of equal time (cost) and the travel time leads to a demand which yields the equilibrium link flows.
One Link – One Origin-Destination Pair

**Demand function**

- **Travel time** vs. **Demand**
  - The demand decreases as travel time increases, showing a decreasing demand function.

**Volume/delay function**

- **Travel time** vs. **Flow**
  - The volume increases as delay increases, following a non-linear pattern.

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![Graph of One Link – One Origin-Destination Pair](image-url)
Demand-performance equilibrium

Volume/delay function

Flow-Demand

Travel time

Inverse demand function

Equilibrium solution
A two link variable demand example

A volume delay function is associated with each link and a demand function is associated with the origin-destination pair.
Two link – graphical solution

Graphical Equilibrium Solution

Any horizontal line is an equilibrium solution
The two-link equilibrium solution

- The travel time on both links is the same
- The demand on each link is that which corresponds to the travel time of the two paths
- One computes the aggregate of the two volume/delay functions. It is the lower free flow time volume/delay function until it reaches the free flow time of the higher travel time function. Then for each equilibrium travel time
- One adds the corresponding flow on the two links.
- Then the intersection of the inverse demand function with the aggregate impedance curve is the equilibrium solution for the link that gives the minimum travel time
Two-link equilibrium solution

Graphical Equilibrium Solution

Travel time

Flow

Composite impedance function
Two-link equilibrium solution

Graphical Equilibrium Solution

Travel time

Flow-Demand

Composite impedance function

Equilibrium solution
Variable demand network equilibrium

This is a well solved problem

Numerous algorithms have been proposed to solve this problem

The applications are few since the demand is usually not specified by a direct demand function

Has been used for combined mode choice assignment equilibration
Variable demand equilibrium assignment

Multi-modal models: equilibration

Some large scale applications

Stochastic user equilibrium assignment
The demand is usually represented by a sequence of models that determine:

- The production and attraction of trips
- The destination choice
- The mode choice
- Classical demand models are aggregated by zone
- More modern demand modeling practice is based on disaggregate behavior, tours and activities
General comments on demand procedures

► All the approaches to demand modeling have the property that, as trip time increases for a mode the demand for that mode decreases, if all other factors influencing demand are kept constant.

► Hence, often the variable demand equilibration is not carried with a well defined mathematical model that can be analyzed rigorously.
The need for averaging or “feedback”

- In order to equilibrate the demand and network performance in complex models one needs to use an averaging or “feedback” method that would ensure the consistency of the modeling suite used.

- That is, ensure that the travel times that correspond to the link flows are indeed those that yield the demand that is assigned to the network.
Transportation Planning Models with “Feedback”

- The “Feedback Mechanism” may take different forms depending on the knowledge that one has of the underlying model.
  - If it is an optimization model the step sizes are computed according to a descent algorithm.
  - Otherwise one carries out “ad-hoc” feedback by some averaging scheme.
Combined models that have well understood mathematical formulations

- Combined trip distribution-assignment

- Combined trip distribution-mode choice-assignment

- Can be solved by methods which have well known convergence properties since they can be formulated as optimization models with the appropriate constraints

- Key point: starting from a feasible solution the method finds an auxiliary solution by generating a new solution and a step size to combine with the current solution by using the objective function of the problem

- New solution = Current solution \(*(1 - \text{step size}) + \text{Auxiliary Solution} * \text{step size})*
Complex models of the type that can not be formulated as optimization models

- Solved by using “ad-hoc” equilibration methods that are inspired from optimization models but do not have the same reassuring properties

- The Method of Successive Averages (MSA) is one of the more popular ones that is used in attempting to equilibrate complex models.

- The averaging scheme uses step sizes which are related to the number of iterations carried out but are not based on an objective function; for instance if $k$ is the iteration number

\[
\text{New solution} = \text{Current solution} \times (1 - \frac{1}{k}) + \frac{\text{Auxiliary Solution}}{k}
\]
Various convergence measures

The measures of gap that can be used in “ad-hoc” equilibration procedures can be related to link flow changes or changes in total travel time, VHT or VMT.

- Relative changes in link flows:
  \[
  \text{SUM} \left( \frac{|\text{current flows} - \text{previous flows}|}{\text{SUM (current flows)}} \right)
  \]

- Relative changes in total travel costs:
  \[
  \text{SUM} \left( \frac{\text{travel costs} \times (\text{current flows} - \text{prevs. flows})}{\text{SUM (travel costs} \times \text{current flows)}} \right)
  \]

This convergence measure is intuitively related to the notion of a relative or normalized gap.
Various convergence measures

- Some practitioners just use change in VMT (Vehicle Miles Traveled) from one iteration to the next:

  \[ \text{SUM} \left( \text{travel dist.} \times \text{current flows} - \text{travel dist.} \times \text{prevs. flows} \right) \]

- Another convergence measure may be the changes in VHT (Vehicle hours traveled):

  \[ \text{SUM} \left( \text{travel time} \times \text{current flows} - \text{travel time} \times \text{prevs. flows} \right) \]

  which is related to the normalized travel cost measure.
Various convergence measures

- These convergence measures are satisfactory since they all measure how close to a consistent solution are the resulting demands and flows.

- In the absence of a well formulated optimization model one has to resort to these types of convergence measures; the results are satisfactory as will be shown with some examples from practice.

- One also assumes that the equilibrium solution is unique which may or may not be the case.
Variable demand equilibrium assignment

Multi-modal models: equilibration

Some large scale applications

Stochastic user equilibrium assignment
Examples of Equilibration:
Some examples derived from practice
where “feedback” was used

Metro Portland, OR
Winnipeg, Manitoba
Santiago, Chile
SCAG, California

(None of these are current models)
The Metro Portland Model Study on Equilibration

► This part of the presentation is based on work done by Dick Walker, Cindy Pederson and Scott Higgins

► The paper is entitled “Equilibrating the Input and Output Impedances in the Demand Modeling Process” November 1995

► The paper is publicly available
The Metro Portland Model Study on Equilibration

The dimensions of the application are:

- 1,260 zones
- 8,034 nodes
- 22,546 links
- 7 trip purposes
- 2 time periods
Method of Successive Averages (MSA)  
Variant 1

**Model Application**

*Initial Model Run*
- Input: free flow skims
- Output: save skims
- save Vol1

*Destination Choice*

*Fixed Mode Split*

*Create New Trip Table by Time of Day*

*Auto Assignment*  
(save **link volumes**)  

*Link Volume Adjustment*

*Link Weighting Technique at Each Model Iteration*

**1st Iteration**
- Volume = Vol2
- $AVG_1 = \frac{1}{3}Vol_1 + \frac{2}{3}Vol_2 = \text{Averaged volume}$

**2nd Iteration**
- Volume = Vol3
- $AVG_2 = \frac{2}{3}AVG_1 + \frac{1}{3}Vol_3 = \text{Averaged volume}$

**3rd Iteration**
- Volume = Vol4
- $AVG_3 = \frac{3}{4}AVG_2 + \frac{1}{4}Vol_4 = \text{Averaged volume}$

**4th Iteration**
- Volume = Vol5
- $AVG_4 = \frac{4}{5}AVG_3 + \frac{1}{5}Vol_5 = \text{Averaged volume}$

**5th Iteration**
- Volume = Vol6
- $AVG_5 = \frac{5}{6}AVG_4 + \frac{1}{6}Vol_6 = \text{Averaged volume}$

and so on ...
The implied step size

- The step size used is in this variant of MSA is simply $1/\text{iteration number}$

- That is, the last volume obtained is given a decreasing weight as the number of iterations increase

- This is the most conventional form of the MSA method
Method of Successive Averages (MSA) Variant 2

**Model Application**

**Initial Model Run**
- input: free flow skims
- output: save skims
  - save Vol1

**Destination Choice**

**Fixed Mode Choice**

**Create New Trip Table by Time of Day**

**Auto Assignment (save link volumes)**

**Link Volume Adjustment**

**Auto Assignment (null trip table, Current volume, save skims)**

**Link Averaging Technique at Each Model Iteration**

1. **1st Iteration**
   - volume = Vol2
   - $AVG_1 = Vol1 + Vol2 = \text{Averaged volume}$

2. **2nd Iteration**
   - volume = Vol3
   - $AVG_2 = AVG_1 + Vol3 = \text{Averaged volume}$

3. **3rd Iteration**
   - volume = Vol4
   - $AVG_3 = AVG_2 + Vol4 = \text{Averaged volume}$

4. **4th Iteration**
   - volume = Vol5
   - $AVG_4 = AVG_3 + Vol5 = \text{Averaged volume}$

5. **5th Iteration**
   - volume = Vol6
   - $AVG_5 = AVG_4 + Vol6 = \text{Averaged volume}$

and so on…
The implied step size

► In this variant the step size is always $\frac{1}{2}$.

► That is, the last volume is always given more weight. Previous iteration volumes are given implicit weights of $\frac{1}{2}^{**}$number of iterations

► This is a less common version of the MSA method
Travel Costs Averaging Method

Model Application

Initial Model Run
input skims: free flow
output skims: **Skims1**

Destination Choice

Fixed Mode Choice

Create New Trip Table by Time of Day

Auto Assignment (save Skims)

Prepare Skims for Destination Choice

Skim Preparation Technique at Each Model Iteration

1st Iteration
output skims = Skims2
**AVG1** = \((\text{Skims1} + \text{Skims2})/2\)

2nd Iteration
output skims = Skims3
**AVG2** = \((\text{AVG1} + \text{Skims3})/2\)

3rd Iteration
output skims = Skims4
**AVG3** = \((\text{AVG2} + \text{Skims4})/2\)

4th Iteration
output skims = Skims5
**AVG4** = \((\text{AVG3} + \text{Skims5})/2\)

5th Iteration
output skims = Skims6
**AVG5** = \((\text{AVG4} + \text{Skims6})/2\)

and so on ….
Impedance averaging

► This variant of MSA is less common but in some circumstances it may be useful

► The results obtained resemble those obtained with link volume averaging but require more iterations

► The next two slides show the convergence results for VHT and VMT. Link flow differences were not given
Convergence of VHT

Convergence - MSA variants Portland

- Skim
- MSA2
- MSA1

Iteration

VHT
Convergence of VMT

Convergence MSA variants Portland

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<th>MSA2</th>
<th>MSA3</th>
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The Winnipeg, Canada Model

- This application is based on a rather old data base. The purpose here is to demonstrate the equilibration mechanism.

- The model is a rather conventional four step model where the equilibration involves the trip distribution, the mode choice and the assignment on both road and transit networks.

- The problem size is modest: 154 zones, 1,100 nodes, 3000 links, 67 transit lines, 2 modes, 1 trip purpose.
The Winnipeg base network
The Winnipeg, Canada Model

- The "Feedback Mechanism" that was used here is the conventional MSA method.
- The trip distribution model is a three dimensional model.
- The mode choice is a binomial logit model.
- There are only two modes: bus and car.

Diagram:

- Generation
- Attraction
- Trip Distribution
- Mode Choice
- Trip Assignment: road and transit
- Impedances
- Conventional MSA
The Winnipeg, Canada Model

- The next two slides give the convergence results for the link difference and normalized gap measures for two variants of the trip distribution model: two-dimensional and three-dimensional models.
Winnipeg Convergence Results

WPG Model Convergence with 2-D Balancing

Relative link difference %
Normalized gap %

Iterations
Winnipeg Convergence Results

3-D Trip Distribution Model

WPG Model Convergence with 3-D Balancing

Iterations

% relative difference %

% relative gap %
Santiago, Chile - Base network (2001)

Base Network of Santiago, Chile (Year 2001)

Plot generated by Enif 2002-01-23 10:30:20 at INRO
Santiago, Chile Strategic Planning Model
- developed by Fernandez and DeCea (ESTRAUS)

• Base network
  • 409 centroids including 49 parking locations
  • 1808 nodes, 11,331 directional links
  • 1116 transit lines and 52468 line segments
  • 11 modes, including 4 combined modes
    (bus-metro, txc-metro, auto-metro and auto passenger-metro)

• The demand
  • subdivided into 13 socio-economic classes
  • 3 trip purposes (work, study, other)
  • driving license holders can access to 11 modes
  • no license holders can access to 9 modes
The mode choice function is a simple multinomial logit function:
Solution Procedure

Start and Initialize

Trip Distribution and Mode Choice

Auto Assignment

MSA Auto Volume

Auto Skims

Convergence?

end

Multiple Transit Assignments

Standard Transit Assignment for buses

Transit Assignment with adjusted headway for metro

Auto Impedance

Bus Impedance

Metro MSA Impedance

Park-and-Ride Model for auto-metro and bus-metro

All Impedances
The next slide shows the convergence of the MSA algorithm that uses link flow averaging for the car network and travel time averaging for the transit network.

The convergence of both the car demand and link flows are given for two variants: uncongested transit assignment and equilibrium transit assignment.
Convergence of equilibration

Normalized Gap vs. Iterations
congested vs. non-congested metro assignment with metro capacity

Normalized gap (%)

Iteration

Auto demand
Auto link volume

cong. Link vol.  cong. demand  non-cong. Link vol.  non-cong. demand
The SCAG California Model

- This is a very large model of the aggregate variety

- The model dimensions are impressive:
  - 15 modes
  - 4 time periods
  - 13 trip purposes – trip distribution models
  - 5 mode choice models
The SCAG California Model (1997)

- There are 3325 zones (3217 SCAG region and externals; 108 zones for parking lots)
- 26290 Nodes and 108897 directional links
- 672 transit lines for 21 operators and 60926 transit line segments
The SCAG Highway Network
with parking lots
THE SCAG MODEL COMPUTATIONS
FLOW CHART:

- Run times on AIX RS/6000:
  - Auto skim for one mode: 9 min
  - Transit skims for one mode: 1-2 hr (10 matrices per mode)
  - Trip distribution for one mode: 3 min
  - Transit assignment for one mode: 9 min
  - Auto assignment for 6 classes: 1 iteration: 18 min
The SCAG California Model (1997)

The next slide shows the convergence of both inner iterations of the car assignment (in blue) and the outer iterations (in red). Only 5 iterations were carried out.
SCAG MSA Convergence results

SCAG Model Convergence (AM peak)

link relative difference

loops

inner-loop
outer-loop
Comments on the SCAG results

- Each inner loop is so time consuming that the number of iterations are limited by the total computational time.

- Depending on which initial solution is chosen there may be 2%-5% difference in the resulting VMT values.

- Convergence is probably very approximate.
Conclusions on “ad-hoc” equilibration

- The “ad-hoc” equilibration methods do converge empirically.
- If the network is congested one may have to carry out at least 15 iterations and the results may still exhibit 2%–5% variation.
- This raises issues about the necessity of very fine solutions in the inner loop.
- A lot is left to the judgment of the analyst who carries out the study.
- Variable demand equilibrium assignment
- Multi-modal models: equilibration
- Some large scale applications
- Stochastic user equilibrium assignment
Stochastic user equilibrium

For each origin-destination pair of zones, all used routes have equal perceived travel times, and no unused route has a lower perceived travel time.

The models that are used are based on various distribution for the perception errors: logit, probit, uniform.
The computational methods used

- The logit path choice combined with the user optimal principle may be formulated as an optimization problem (In the case of constant travel times STOCH by Dial deserves special mention).

- Hence it may be solved by using a descent algorithm with well defined convergence properties.

- The probit path choice requires a solution by Monte Carlo simulation and the use of the MSA method. In this case a convergence proof exists.
Uses of Stochastic assignment

► In relatively un-congested conditions.

► When congestion dominates the stochastic and user equilibrium yield similar results.

► In mildly congested conditions the results may reflect better the paths chosen.

► A nice property of stochastic assignment is that the path flows are unique, which is not the case with deterministic equilibrium assignment.
The use of stochastic assignment enriches the methods used for determining path choice on lightly congested networks.

There is no hard and fast rule which may be used to determine when stochastic assignment should be used. It is up to the analyst that carries out the study.