NETWORKS FOR FUN AND PROFIT

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Networks are pervasive in our daily lives and essential to the functioning of our societies and economies.

Everywhere we look:

in business, science,
social systems, technology,
and education,

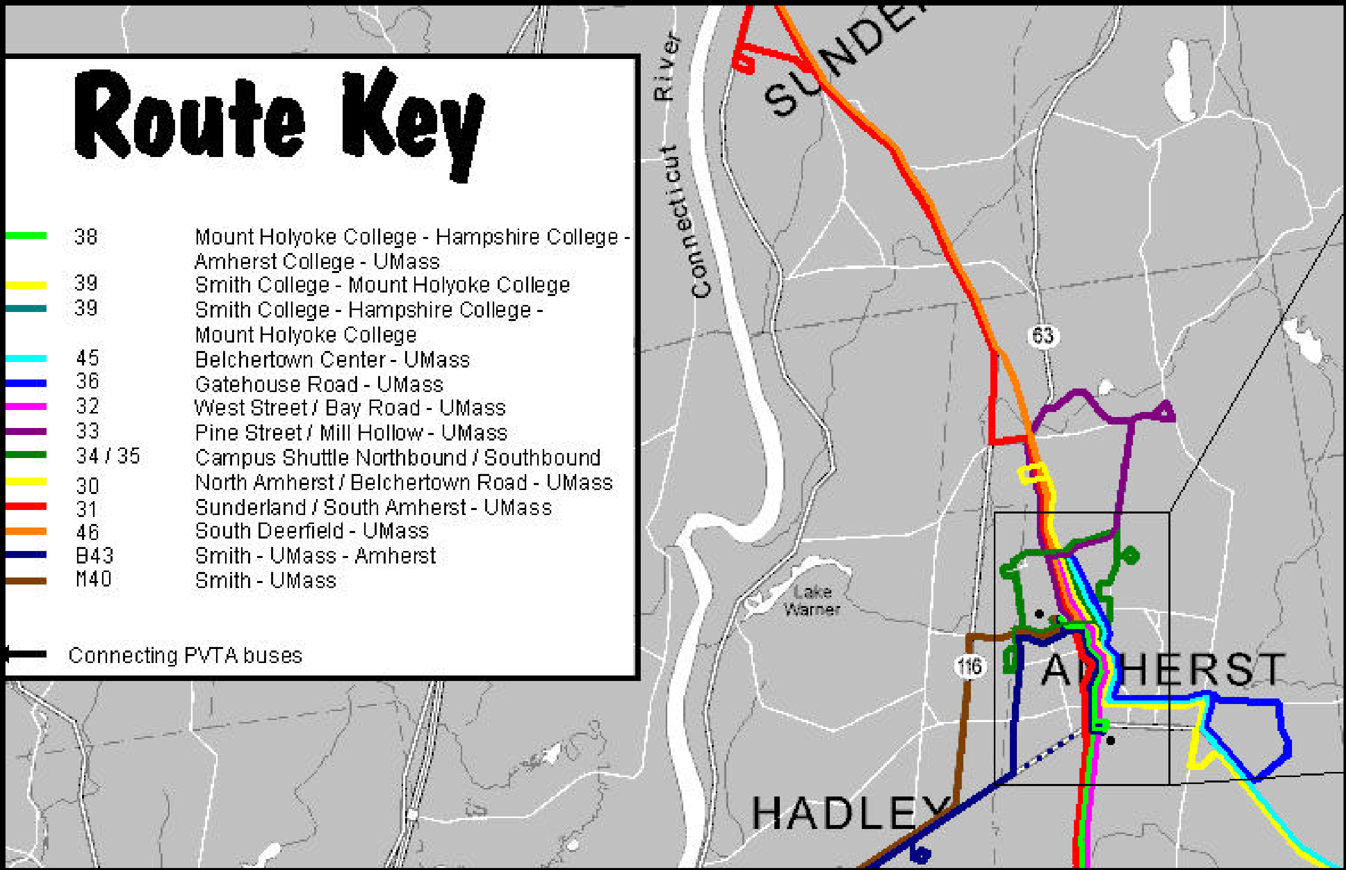
networks provide the infrastructure for communication, production, and transportation.

Transportation Networks

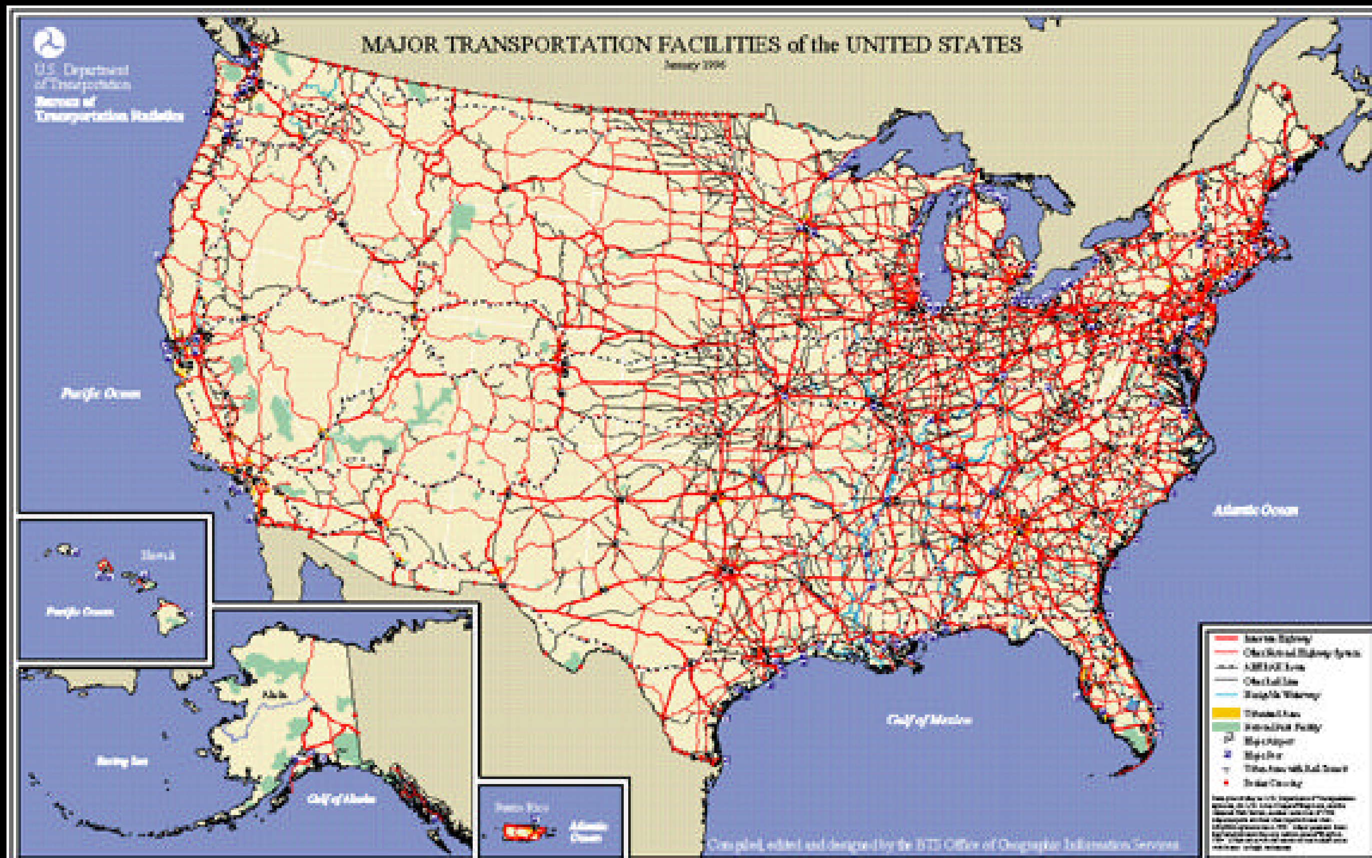
provide us with the means to cross distance in order to conduct our work, and to see our colleagues, students, clients, friends, and family members.

They provide us with access to food and consumer products.

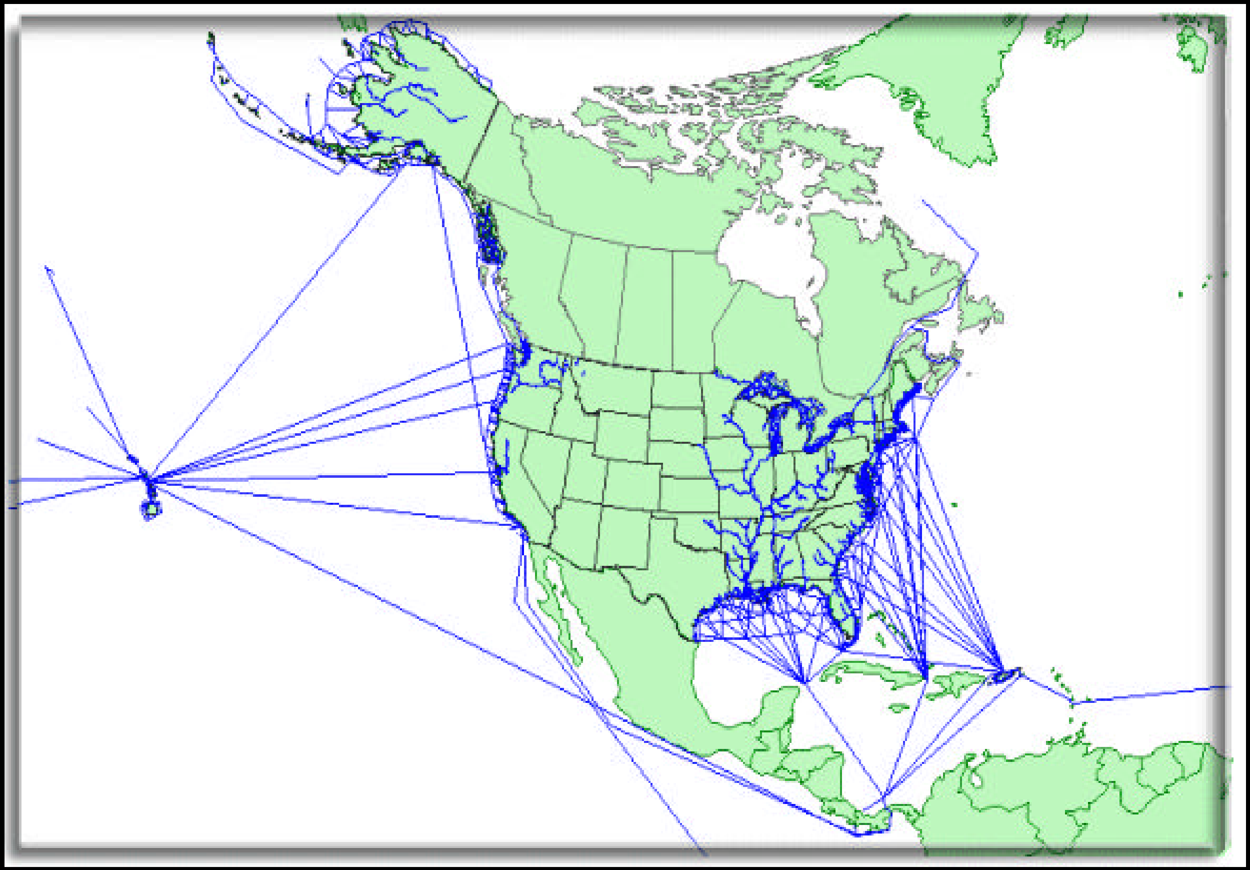
UMass Transit Route Network



Major Highway and Railroad Networks in the United States



Water Freight Transport Routes for the United States

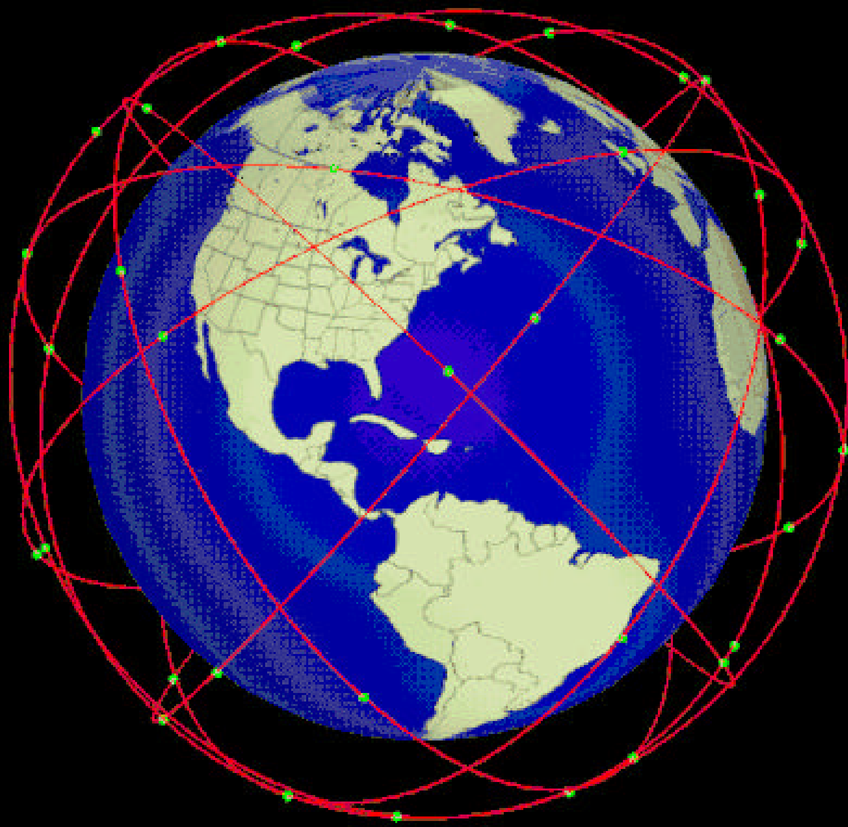


Communication Networks

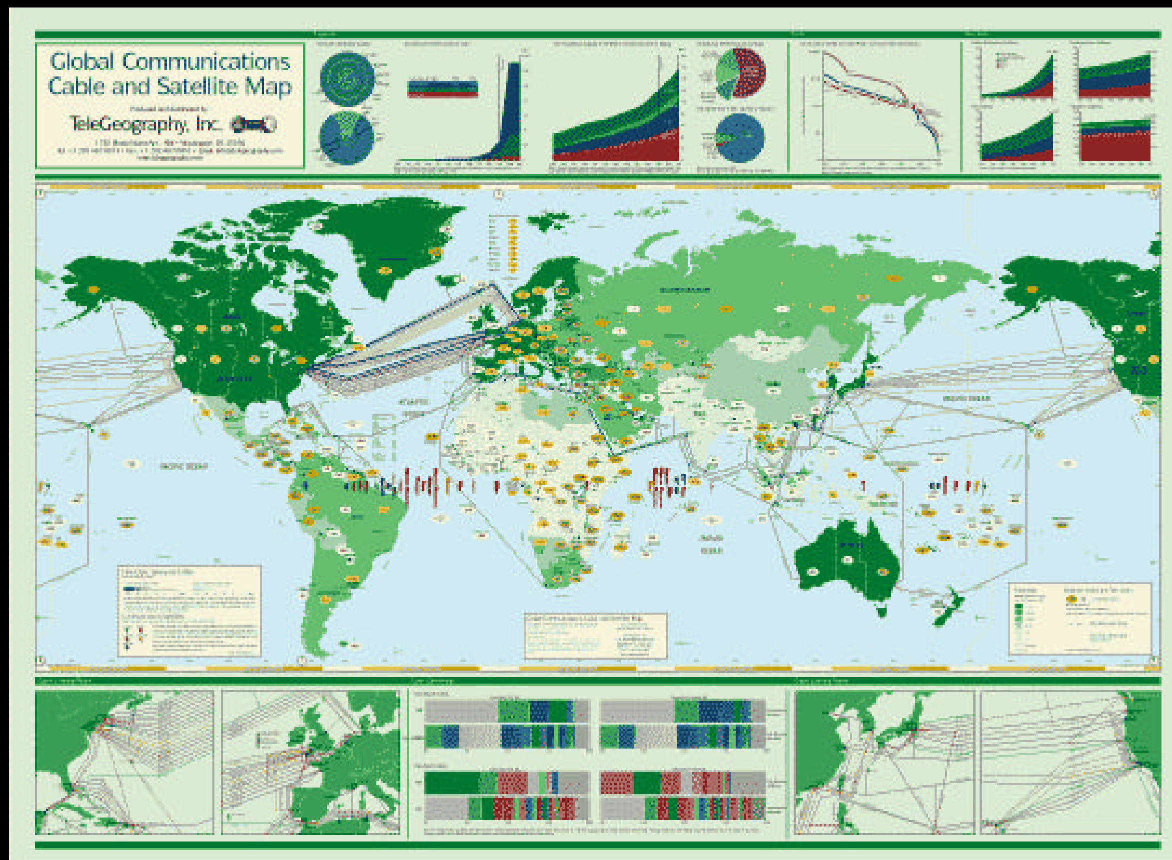
allow us to communicate within our own communities and across regions and national boundaries,

and have transformed the way we live, work, and conduct business.

Globstar Satellite Constellation Communication Network



Satellite and Undersea Cable Networks



Energy Networks

provide the energy for our homes, schools, and businesses, and to run our vehicles.

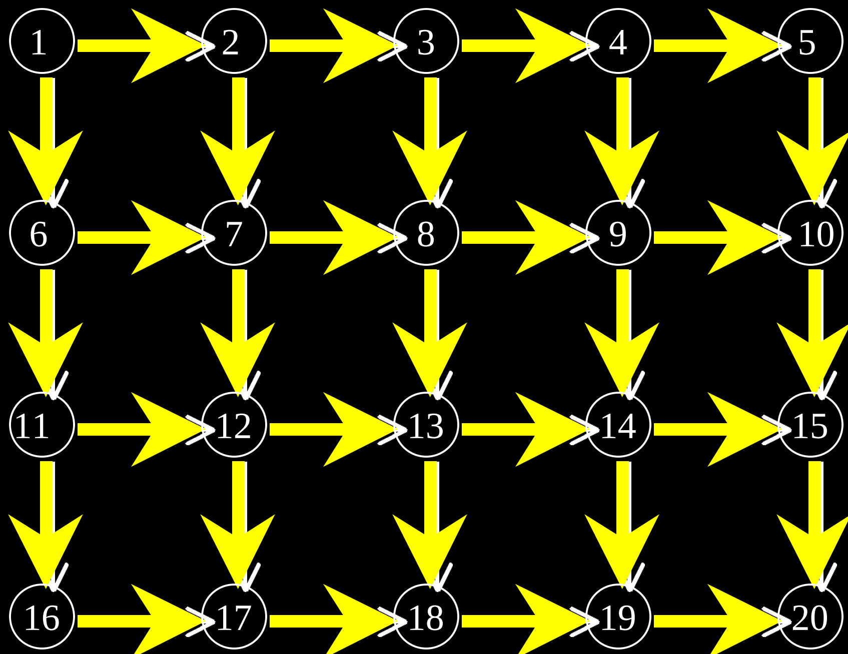
New England Electric Power Network

Duke Energy
Gas Pipeline Network

The basic components of networks are:

- nodes
- links or arcs
- flows.

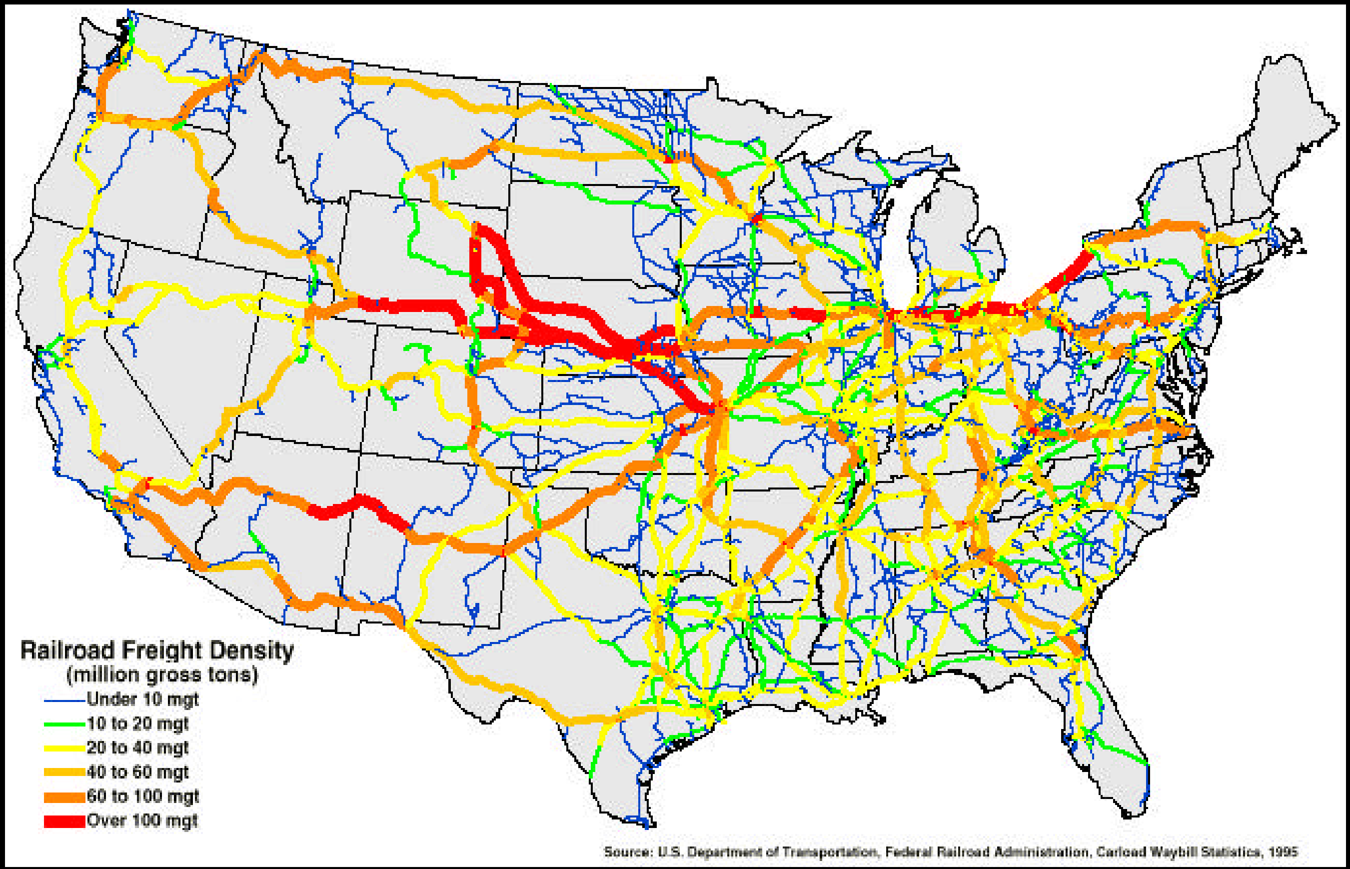
Nodes Links Flows



Components of Some Common Physical Networks

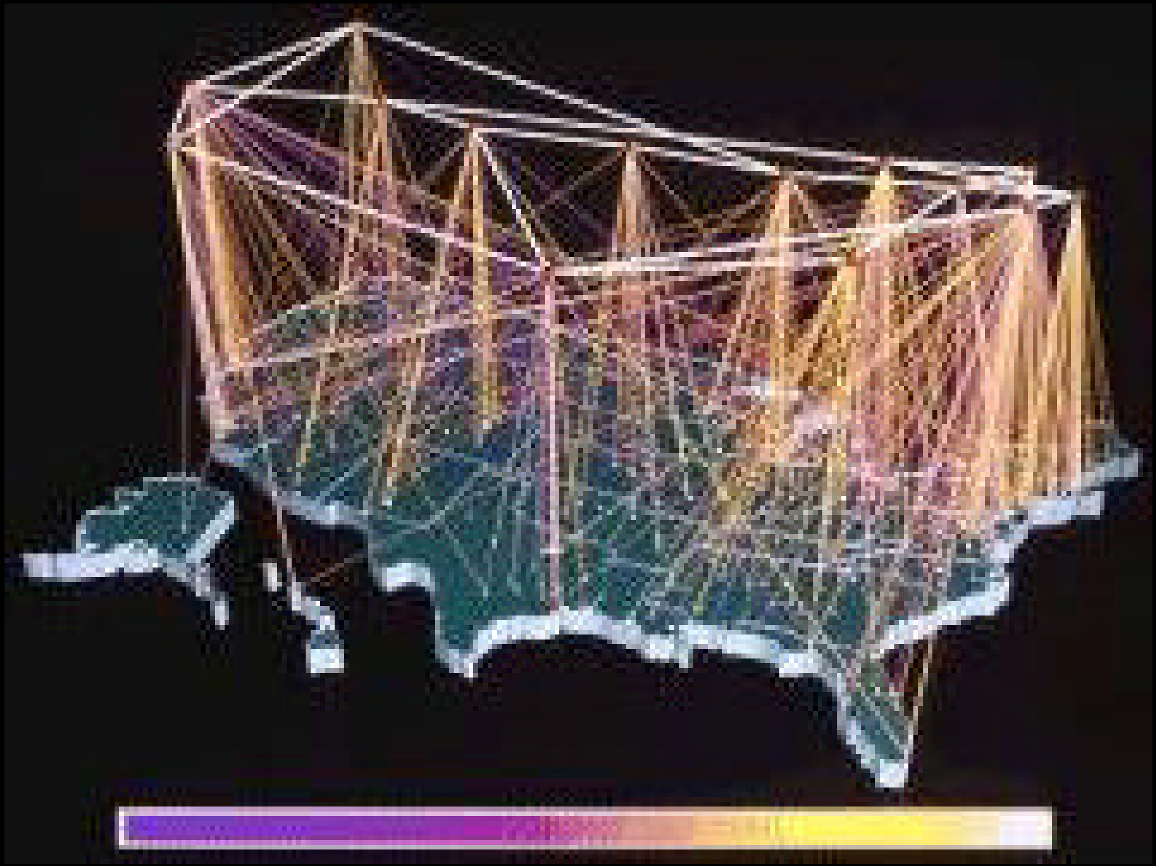
Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil

U.S. Railroad Freight Flows



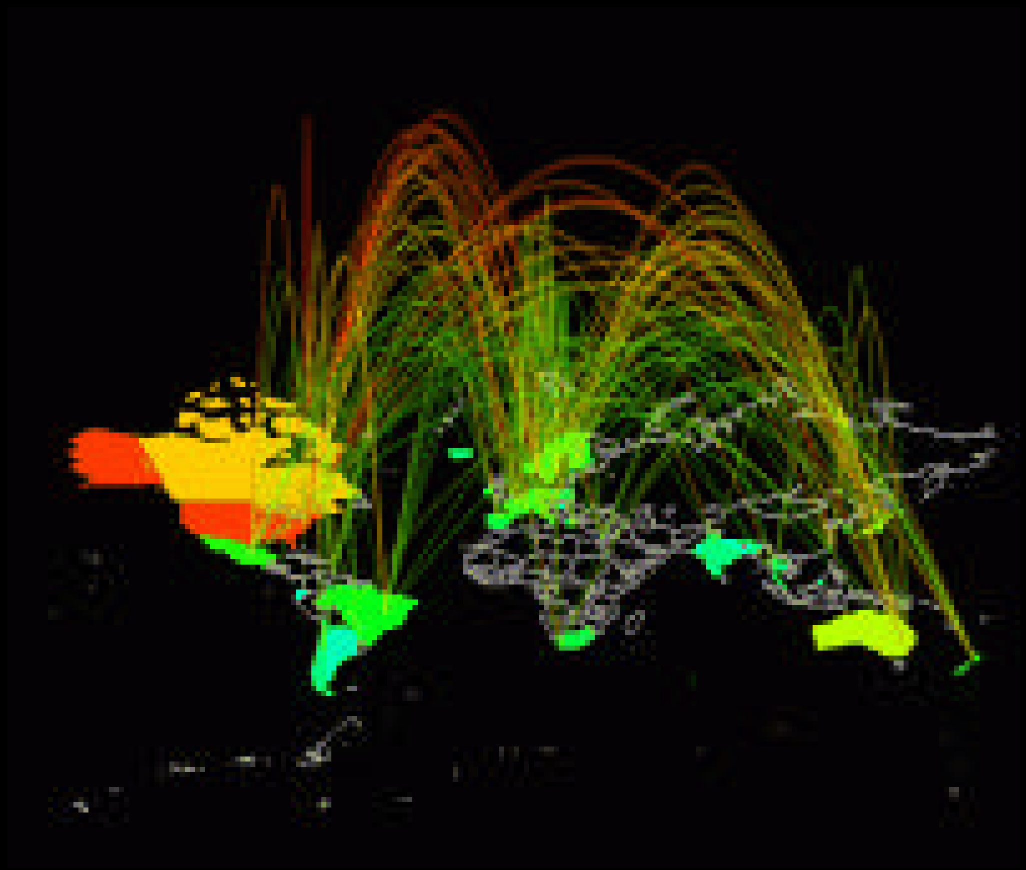
Traffic Flow on NSFNET T1 Backbone Network, September 1991

from Donna Cox and Robert Patterson, NCSA, 1992



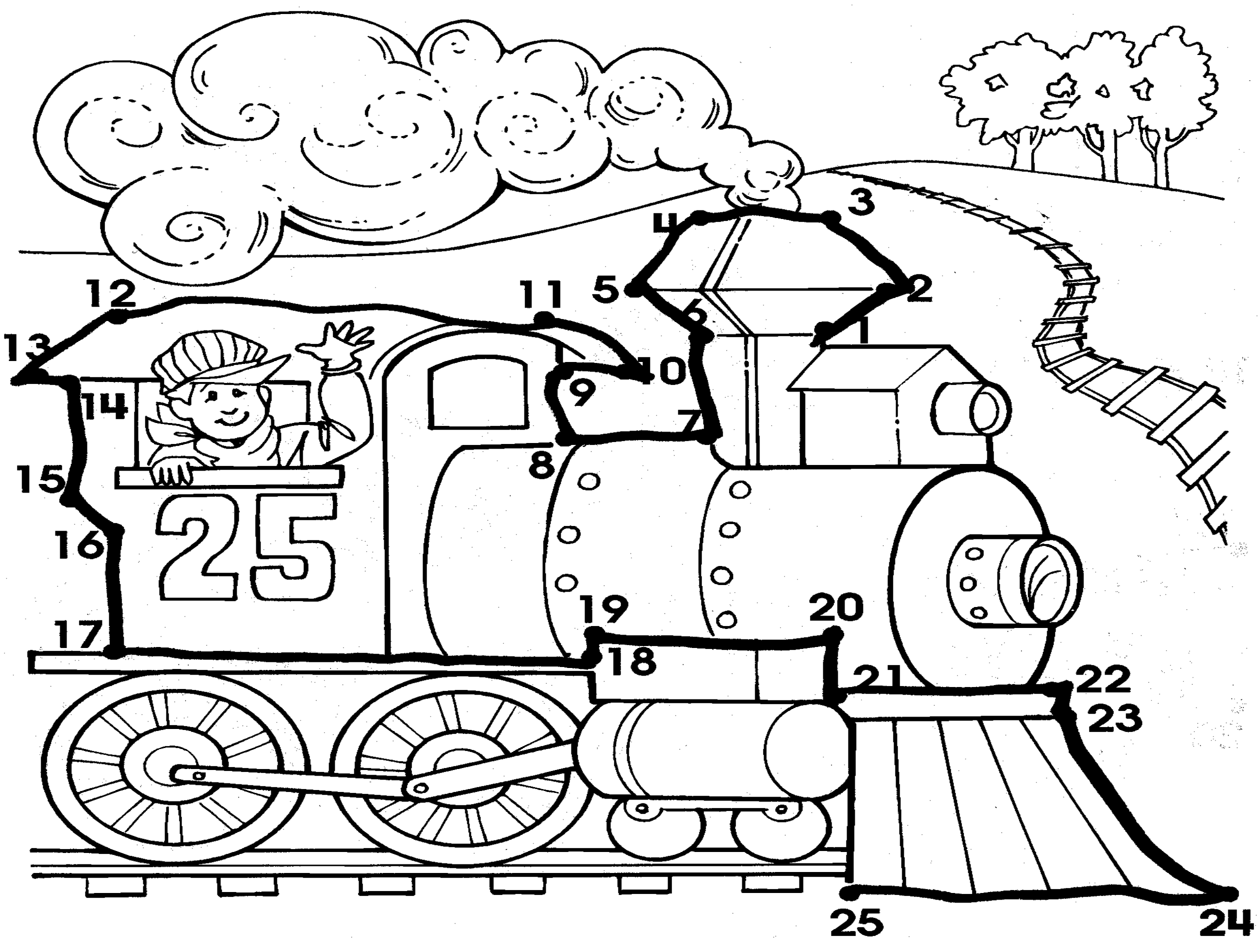
IP Traffic Flows Over One 2 Hour Period

from Stephen Eick, Visual Insights



Train

Connect the dots from **1** to **25**. Color.



Trains take people all over the country.

In a basic network problem domain:

one wishes to move the flow from one node to another in a way that is as efficient as possible.

The study of networks involves:

- how to **model** such applications - as **mathematical entities**,
- how to **study the models** qualitatively,
- and how to design **algorithms** to solve the resulting models.

Classic Examples of Network Problems Are:

- **The Shortest Path Problem**
- **The Maximum Flow Problem**
- **The Minimum Cost Flow Problem.**

Applications of the Shortest Path Problem

Arise in **transportation** and **telecommunications.**

Other applications include:

- simple building evacuation models
- DNA sequence alignment
- dynamic lot-sizing with backorders
- assembly line balancing
- compact book storage in libraries.

Applications of the Maximum Flow Problem

- machine scheduling
- network reliability testing
- building evacuation.

Applications of the Minimum Cost Flow Problem

- warehousing and distribution
- vehicle fleet planning
- cash management
- automatic chromosome classification
- satellite scheduling.

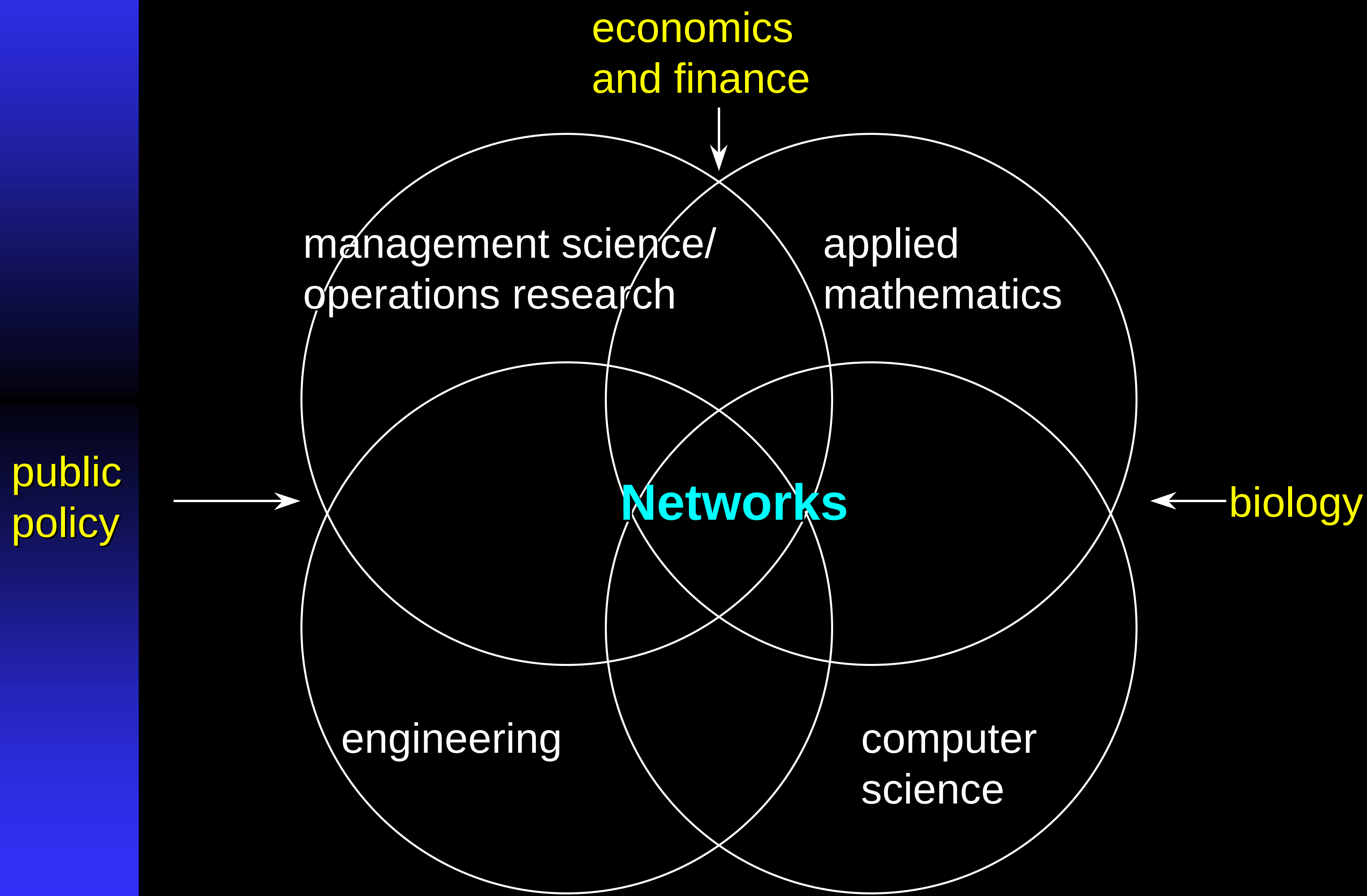
Network problems also arise in other ***surprising and fascinating*** ways for problems, which at first glance and on the surface, may not appear to involve networks at all. Hence, ***the study of networks is not limited to only physical networks*** but also to *abstract* networks in which nodes do not coincide to locations in space.

The advantages of a network formalism:

- many present-day problems are concerned with flows (material, human, capital, informational, etc.) over space and time and, hence, ideally suited as an application domain for network theory;
- provides a graphical or visual depiction of different problems;
- helps to identify similarities and differences in distinct problems through their underlying network structure;
- enables the application of efficient network algorithms;
- allows for the study of disparate problems through a unifying methodology.

One of the primary purposes of scholarly and scientific investigation is to ***structure*** the world around us and to ***discover patterns*** that cut across boundaries and, hence, help to ***unify diverse applications***.

Network theory provides us with a ***powerful methodology*** to establish connections with different disciplines and to ***break down boundaries***.


economics
and finance
management science/
operations research
applied
mathematics
public
policy
Networks
biology
engineering
computer
science

Interdisciplinary Impact of Networks

Finance and Economics

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

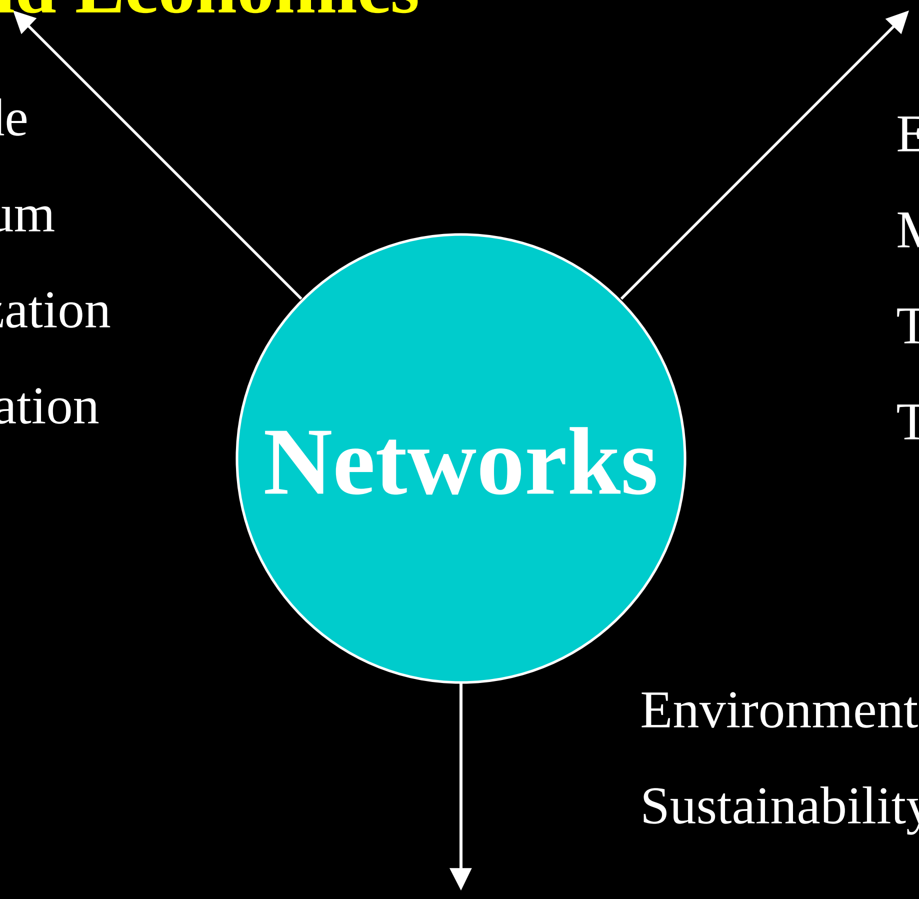
Engineering

Energy

Manufacturing

Telecommunications

Transportation



Environmental Networks

Sustainability Issues

Environmental Science

Reality of Many of Today's Networks

- large-scale nature and complexity of network topology
  - In Chicago’s Regional Network, there are 12,982 nodes, 39,018 links, and 2,297,945 O/D pairs.
  - AT&T’s domestic network has 100,000 O/D pairs. In their call detail graph applications (nodes are phone numbers, edges are calls) - 300 million nodes and 4 billion edges

Reality of Many of Today's Networks

- congestion is playing an increasing role in transportation networks:

  For example in the United States alone, congestion results in $100 billion in lost productivity annually, whereas the figure in Europe is estimated to be $150 billion.

  The number of cars is expected to increase by 50% by 2010 and to double by 2030.

Congestion

Courtesy: Pioneer Valley Planning Commission

Wasting Away in Traffic

Area	Annual delay in hours per driver
Los Angeles	82
Washington, DC	76
Seattle - Everett	69
Atlanta	68
Boston	66
Detroit	62
San Francisco - Oakland	58
Houston	58
Dallas	58
Miami - Hialeah	57

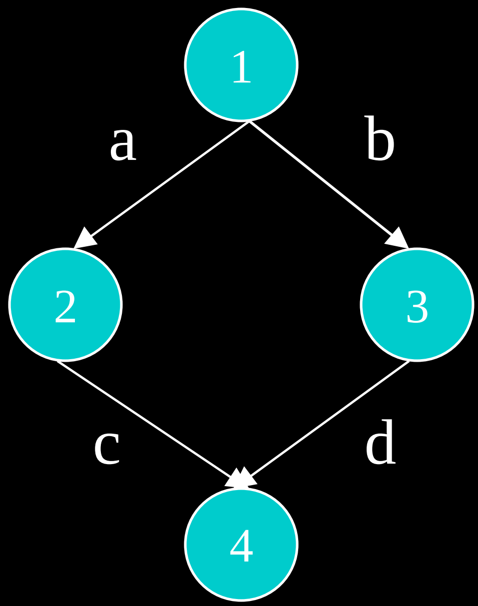
Source: Texas Transportation Institute data as reported in the LA Times, November 17, 1999

Reality of Many of Today's Networks

- alternative behaviors of the users of the network
  - system-optimized versus
  - user-optimized, which may lead to paradoxical phenomena.

Braess' Paradox (1968)

- Assume a network with a single O/D pair (1,4) . There are 2 paths available to travelers: p_1=(a,c) and p_2=(b,d).
- For a travel demand of 6, the equilibrium path flows are $x_{p_1}{}^* = x_{p_2}{}^* = 3$ and
- The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83$.

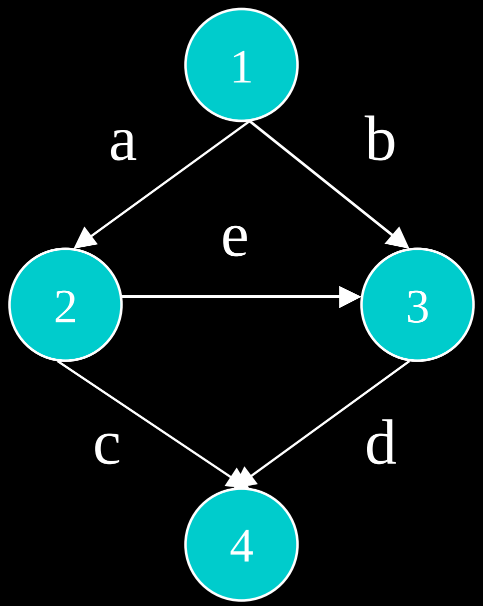


$c_a(f_a)=10\ f_a \quad c_b(f_b) = 10f_b+50$

$c_c(f_c) = f_c+50 \quad c_d(f_d) = 10f_d$

Adding a Link
Increased Travel Cost for All!

- Adding a new link creates a new path p_3=(a,e,d).
- The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, C_{p_3}=70.
- The new equilibrium flow pattern network is $x_{p_1}^* = x_{p_2}^* = x_{p_3}^*=2$.
- The equilibrium path travel costs:
- $C_{p_1} = C_{p_2} = C_{p_3} = 92$.



Reality of Many of Today's Networks

- interactions among users of the network such as different modes of transportation

plus

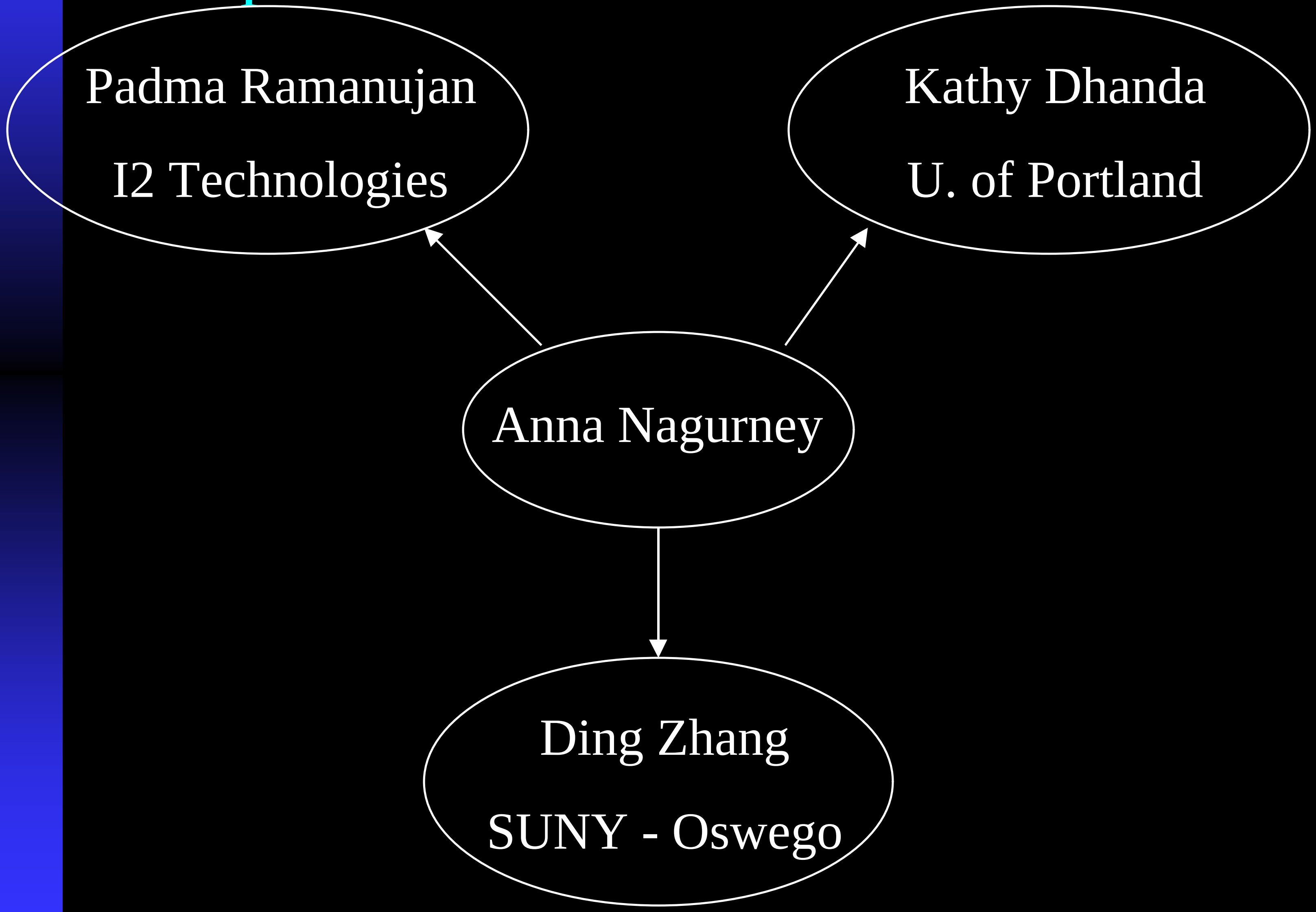
- interactions among networks themselves such as in transportation versus telecommunications.

Courtesy of Pioneer Valley Planning Commission

Reality of Many of Today’s Networks

- there are important policy issues surrounding networks in today's societies.

My Collaborative Network in Transportation/Environmental Science



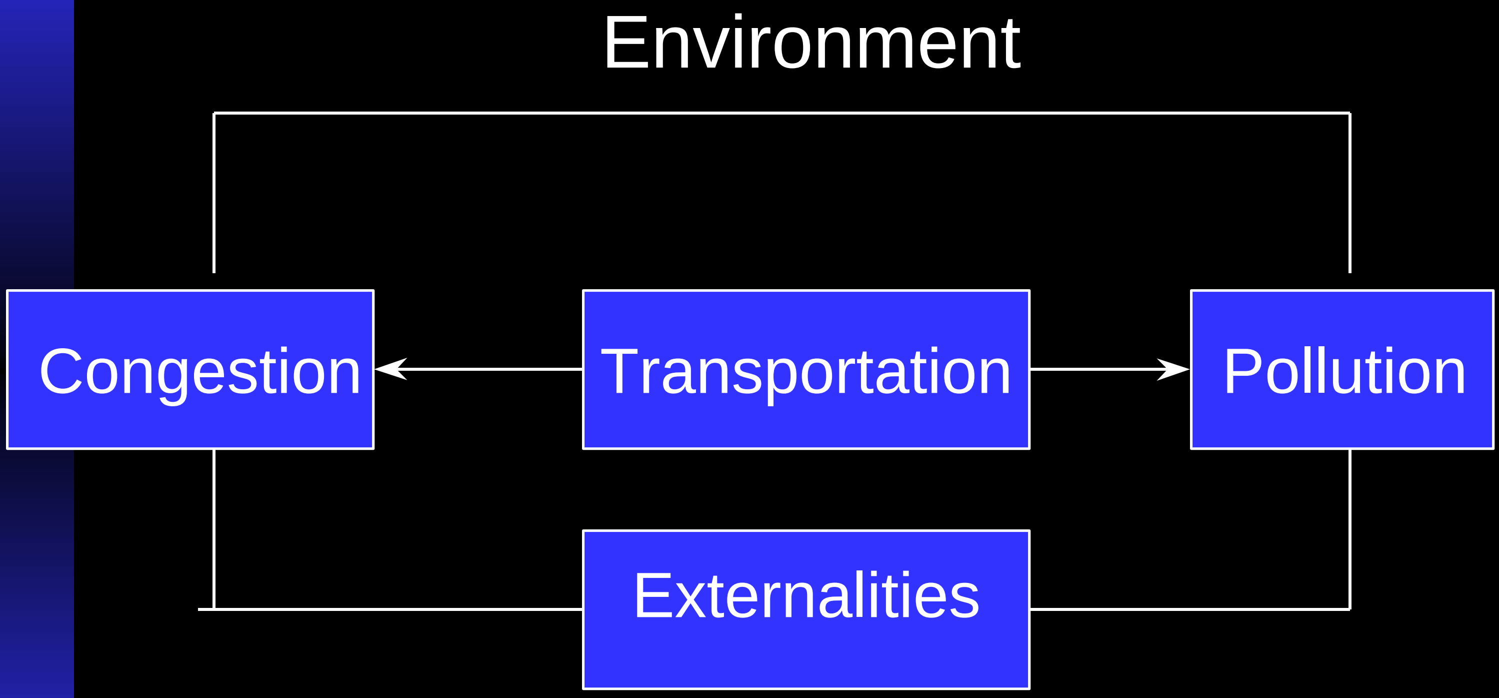
TRANSPORTATION AND THE ENVIRONMENT

The study of the efficient operation on transportation networks dates to ancient Rome with a classical example being the publicly provided Roman road network and the time of day chariot policy, whereby chariots were banned from the ancient city of Rome at particular times of day.

- From an economic perspective, some of the earliest contributions to the subject date to Pigou (1920), who in his book **The Economics of Welfare** suggested to use road pricing as a means of regulating congestion in a simple two-node, two-link network.
- Earlier Dupuit (1844) had used a bridge as an illustration of efficient pricing of public goods.

- Recently, there has been a growing interest in exploring road pricing not only as a tool in congestion management but also as a tool for environmental management since there has occurred an increase in environmental concern and awareness on the part of both citizens and government officials.

Some Externality Relationships



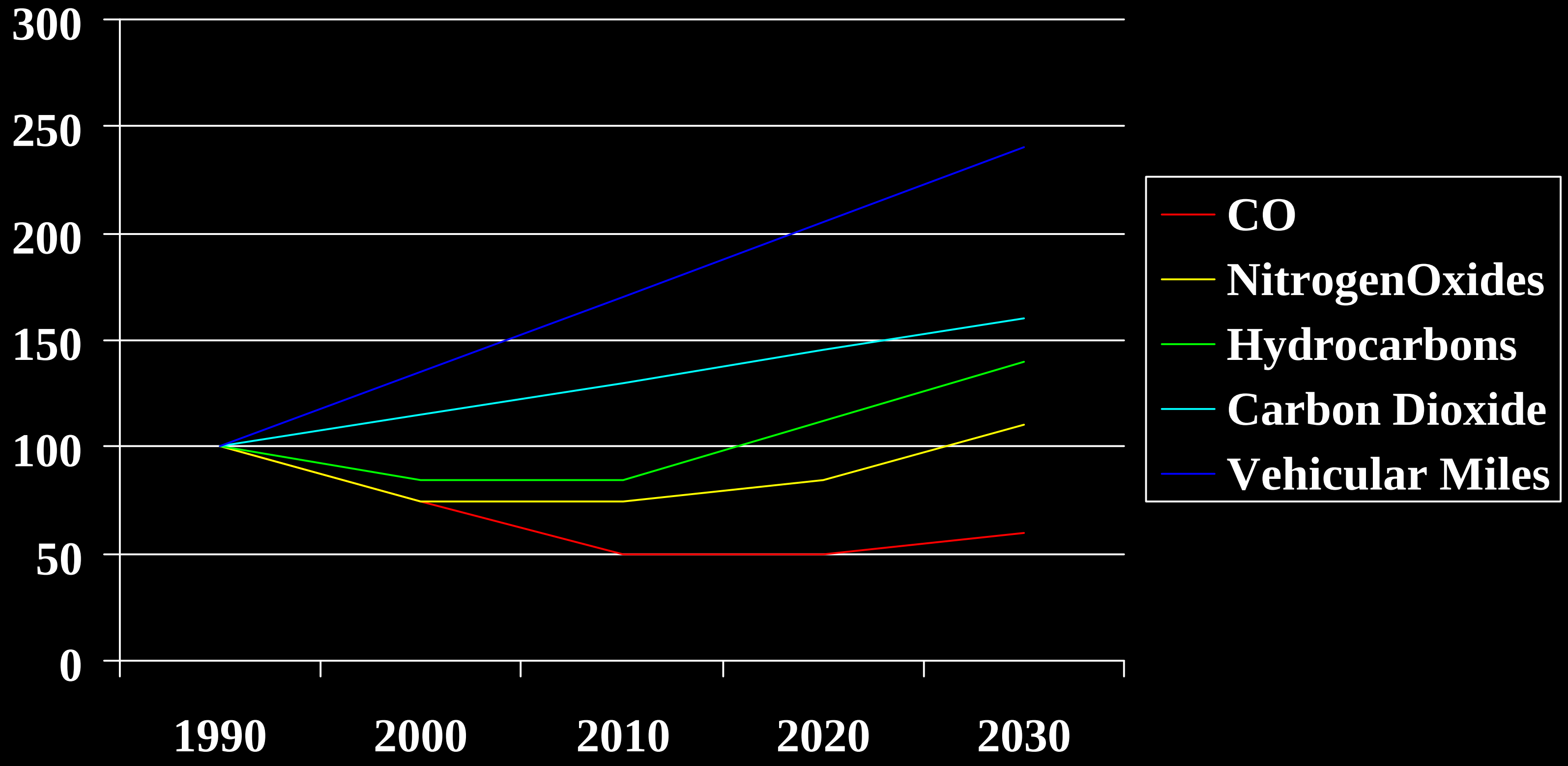
Motor Vehicles Generate:

- 15% of the world's emissions of carbon dioxide, the principle global warming gas,
- 50% of the emissions of nitrogen oxide, which in combination with other gasses falls to earth as acid rain, and
- 90% of the carbon monoxide.
- They are responsible for at least 50% of the air pollution in urban areas.

- The World Health Organization (WHO) has estimated that only about 20% of the world's town dwellers enjoy good enough air quality and people residing in about one half of the world's urban centers are breathing more carbon monoxide than is good for them.

- Given that the number of cars is expected to rise by 50% by 2010 and to double by 2030, and the emission forecasts, one may ask whether or not existing transportation systems are *sustainable*, that is, can they last?

Travel and Emission Forecasts



Boston - Clear Day June 16, 1999

view from www.hazecam.net

Boston - Haze June 7, 1999

view from http://www.hazecam.net

Boston - Brown - High Fine Particles
January 21, 1999

view from http://www.hazecam.net

Emissions Paradoxes

Paradox 1

Addition of a road with no change in demand — total emissions can rise.

Paradox 2

A decrease in travel demand — total emission can rise.

Paradox 3

A road improvement with no change in demand — total emissions can rise.

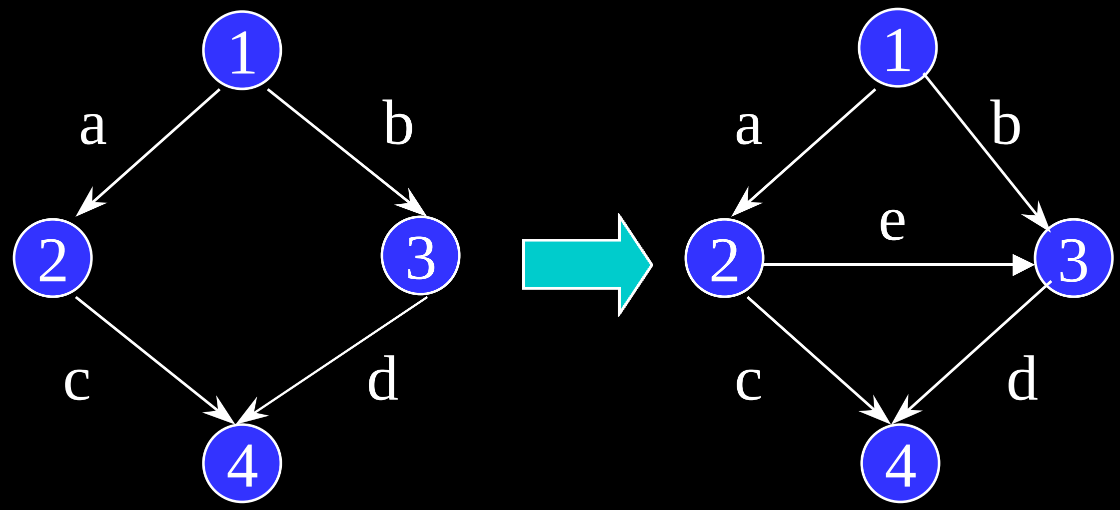
Paradox 4

A mode transfer from a higher emitting mode to a lower one — total emissions can rise.

Paradox 5

Making travel less attractive for an O/D pair — total emissions can rise.

Paradox 1-Braess Paradox Analogue



The emission factors on the links are: $h_a = h_b = h_c = h_d = h_e = 1$.

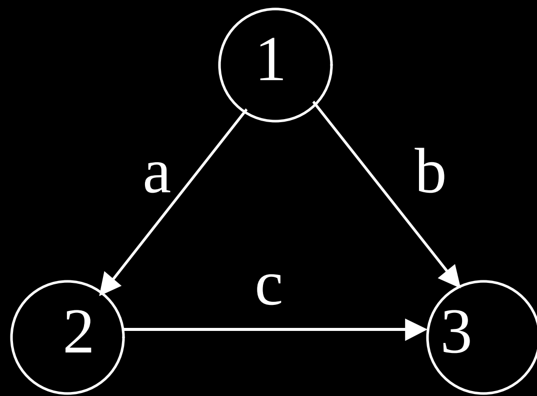
The total emissions generated are, hence, equal to:

$$h_a f_a^* + h_b f_b^* + h_c f_c^* + h_d f_d^* = .3 + .3 + .3 + .3 = \mathbf{1.2.}$$

The total emissions generated in the new network are equal to

$$h_a f_a^* + h_b f_b^* + h_c f_c^* + h_d f_d^* + h_e f_e^* = .4 + .2 + .2 + .4 + .2 = \mathbf{1.4}.$$

Paradox 2 - Decreased Demand Results in Increased Emissions



There are two origin/destination pairs: $w_1=(1,2)$ and $w_2=(1,3)$, with $d_{w_1}=1$ and $d_{w_2}=2$. The path connecting O/D pair w_1, p_1, consists of the single link a. The paths connecting O/D pair w_2 are: $p_2=(a,c)$ and $p_3=b$.

The user link travel cost functions are:

$c_a(f_a)=f_a+1$, $c_b(f_b)=f_b+4$, $c_c(f_c)=f_c+1$.

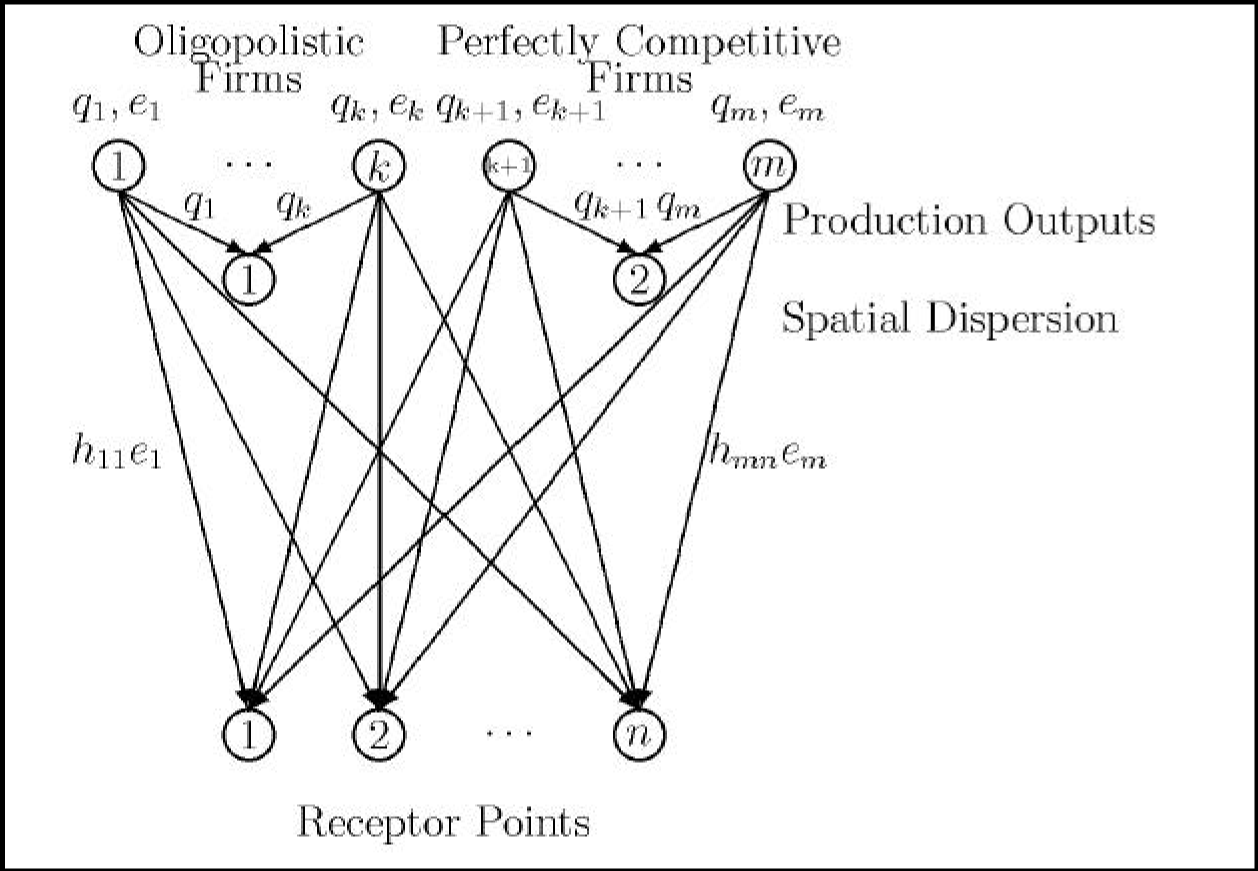
The emission factors on the links are: $h_a=.01$, $h_b=.01$, and $h_c=.5$.

The total emissions generated are equal to **.53**.

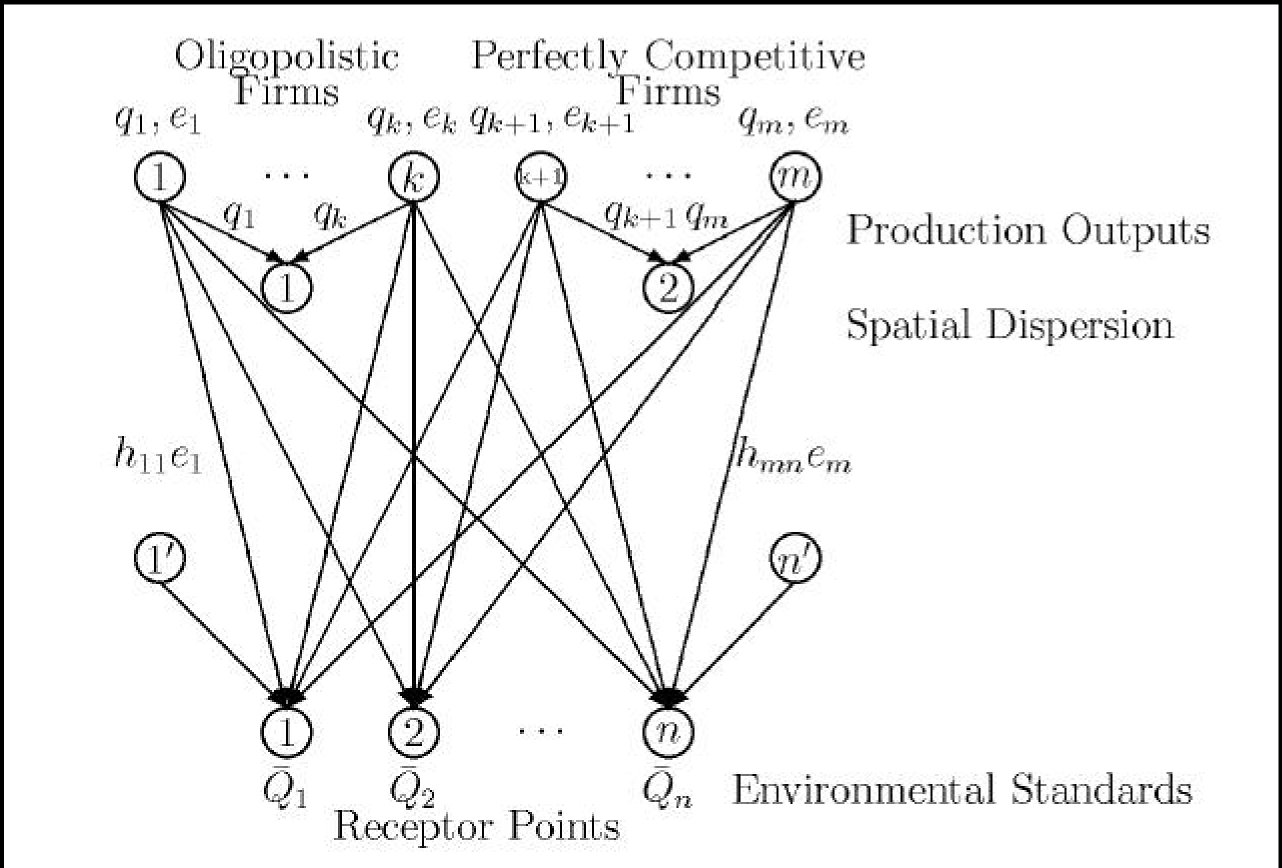
Consider now a decrease in travel demand associated with O/D pair w_1 with the new demand d_{w_1}=.5 and all other data remain the same.

The total emissions now generated by the new equilibrium pattern are equal to **.6079**, which exceeds the original of .53.

Firms and Environmental Networks: The Environmental Network for Single-Pollutant Problem



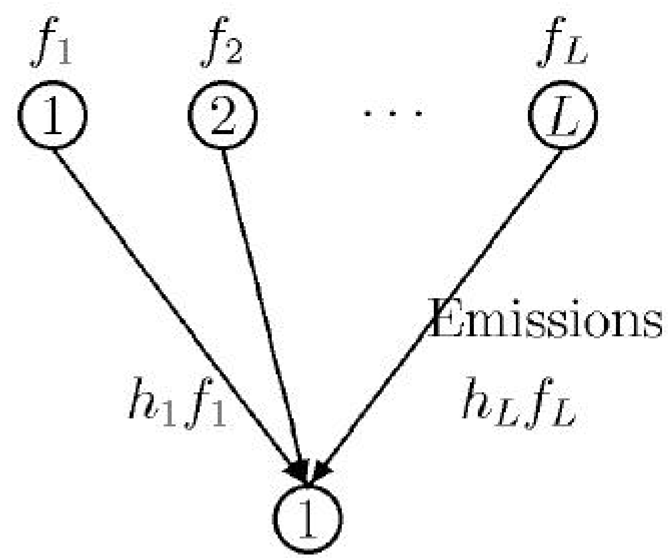
Firms and Environmental Networks: The Environmental Standards Network



Transportation and Environmental Networks: The Environmental Network for the Single-Modal, Single Receptor Point Traffic Network Equilibrium Problem

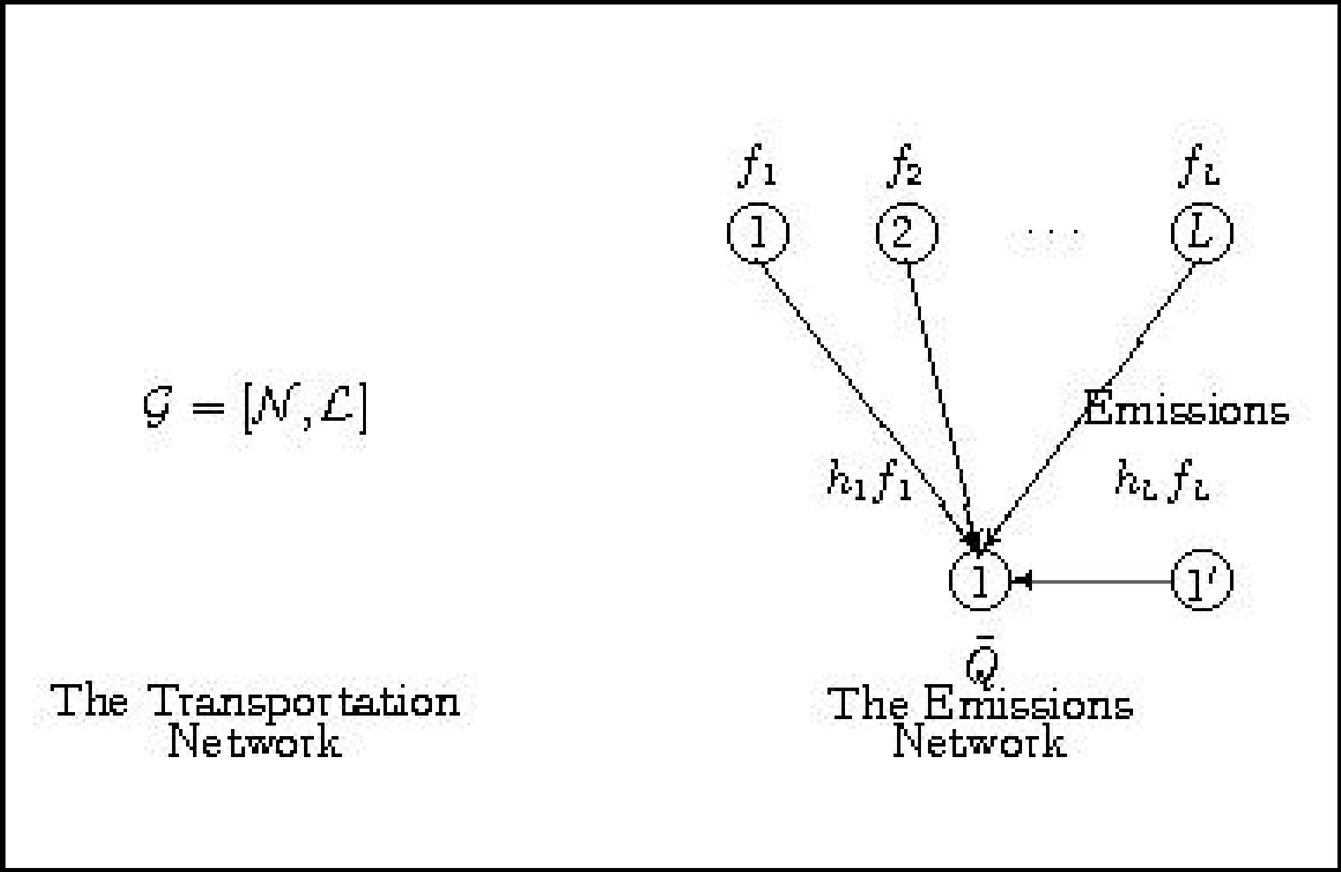
$$\mathcal{G} = [\mathcal{N}, \mathcal{L}]$$

The Transportation Network



The Emissions Network

The Environmental Standards Network:



Definition: Sustainable Transportation Network

A transportation network is said to be sustainable if the emissions generated by the flow pattern do not exceed the imposed environmental quality standard, subject to the operating principle(s) of the network.

Viability Appropriate Policy

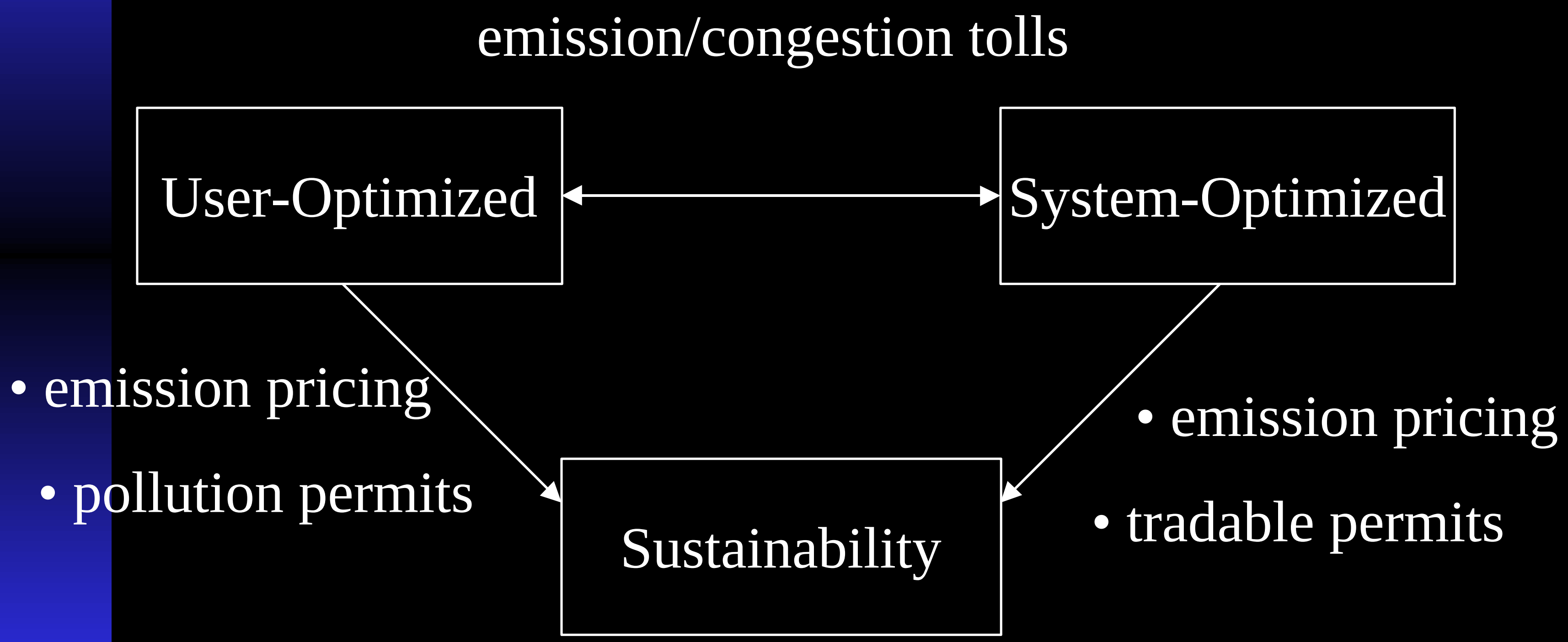
\+ → Sustainability

Behavioral Principle

where

Viability means that there exists a path flow pattern meeting the travel demand and such that the emissions generated don't exceed the imposed environmental quality standard.

Policy Instruments for Sustainability of Transportation Networks



Tradable Permits for System-Optimized Networks

- Marketable pollution permits can be traced to Crocker (1966) whose work was on air pollution and to Dales (1968) whose work was on water permits.
- Montgomery (1972) proposed two systems of permits: *emission-based* and *ambient-based* and proved that markets in equilibrium can achieve environmental quality standards. He explicitly recognized the spatial nature of pollution dispersion.

Trade in Pollution Permits

Jurisdiction 1	Jurisdiction 2 …	Jurisdiction I
Network 1	Network 2 …	Network I
$G_1 = [N_1, L_1]$	$G_2 = [N_2, L_2]$ …	$G_I = [N_I, L_I]$
Initial: $\sum_{a \in L_1} l_a^0$	Initial: $\sum_{a \in L_2} l_a^0$ …	Initial: $\sum_{a \in L_I} l_a^0$
	$\Downarrow$	
$\Rightarrow$	Trade	$\Leftarrow$
	$\Downarrow$	
Final: $\sum_{a \in L_1} l_a^*$	Final: $\sum_{a \in L_2} l_a^*$…	Final: $\sum_{a \in L_I} l_a^*$

A Model with Tradable Pollution Permits

Assume that there are I transportation jurisdictions, with a typical jurisdiction denoted by i. For each jurisdiction, i, consider a transportation network $G_i=[N_i,L_i]$ consisting of the set of nodes, N_i, and a set of directed links L_i.

For each network i ∈ I, one has the following system-optimization problem:

$$\text{Minimize} \quad \sum_{p\in P_i} S_i(x_i) - \sum_{a\in L_i} \rho^*(l_a^0 - l_a) = \text{Minimize} \quad \sum_{p\in P_i} \hat{C}_p - \sum_{a\in L_i} \rho^*(l_a^0 - l_a)$$

subject to:

$$h_a f_a \leq l_a, \quad \forall a \in L_i, \quad \sum_{p\in P_w} x_p = d_w, \quad \forall w \in W_i,$$

$$x_p \geq 0, \quad \forall p \in P_i, \qquad l_a \geq 0, \quad \forall a \in L_i.$$

The Optimality Conditions

For network i and all O/D pairs $w \in W_i$ and each path $p \in P_w$:

$$\tilde{C}'_p(f_i^*, \tau^*) = \tilde{C}'_p(x_i^*) + \sum_{a \in L_i} h_a \tau_a^* \delta_{ap} \begin{cases} = & \mu_w, \quad \text{if } x_p^* > 0 \\ \geq & \mu_w, \quad \text{if } x_p^* = 0. \end{cases}$$

Also, for each link *a* Î L_i, the equilibrium marginal cost of emission abatement τ_a* must satisfy:

$$h_a f_a^* \begin{cases} = & l_a^*, \quad \text{if } \tau_a^* > 0 \\ \leq & l_a^*, \quad \text{if } \tau_a^* = 0. \end{cases}$$

The following condition must also hold: For each link *a* Î $L_{i:}$

$$\tau_a^* \begin{cases} = & \rho^*, \quad \text{if } l_a^* > 0 \\ \leq & \rho^*, \quad \text{if } l_a^* = 0. \end{cases}$$

Market Equilibrium Condition for Licenses

$$\sum_{i \in I} \sum_{a \in L_i} (l_a^0 - l_a^*) \begin{cases} = & 0, \quad \text{if } \rho^* > 0 \\ \geq & 0, \quad \text{if } \rho^* = 0. \end{cases}$$

Definition (Sustainable System-Optimum with Tradable Pollution Permits) A vector $(f^*, \tau^*, l^*, \rho^*) \in K$ is an equilibrium of the sustainable system-optimal tradable pollution permits model if and only if it satisfies the systems of equalities and inequalities above for all networks $i \in I$.

Theorem (Variational Inequality Formulation of Tradable Pollution Permit System Traffic Network Equilibrium with System-Optimized Behavior)

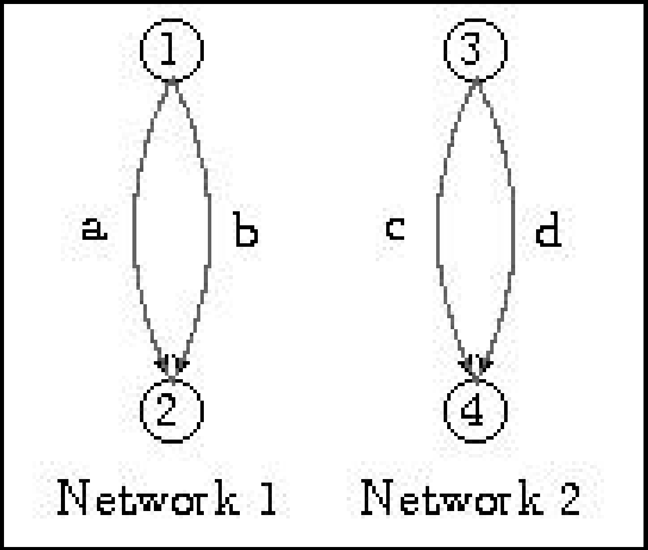
A vector of link loads, marginal costs of emission abatement, licenses, and license price, $(f^*, t^*, l^*, r^*) \in K$, is an equilibrium of the tradable pollution permit market equilibrium model in the case of sustainable system-optimized networks if and only if it is a solution to the variational inequality problem:

$$\sum_{i\in I}\sum_{a\in L_i}(\tilde{c}'_a(f_i^*)+h_a\tau_a^*)\times(f_a-f_a^*)+\sum_{i\in I}\sum_{a\in L_i}(l_a^*-h_af_a^*)\times(\tau_a-\tau_a^*)$$

$$+\sum_{i\in I}\sum_{a\in L_i}(\rho^*-\tau_a^*)\times(l_a-l_a^*)+\sum_{i\in I}\sum_{a\in L_i}(l_a^0-l_a^*)\times(\rho-\rho^*)\geq 0,$$

$$\forall (f,\tau,l,\rho)\in\mathcal{K}.$$

Example



Consider the two networks depicted above, each of which is the responsibility of a separate transportation authority. The O/D pair for Network 1, denoted by w_1, is (1,2), whereas the O/D pair for Network 2 is (3,4).

The user link travel cost functions are:

$c_a(f_a)=f_a+5$, $c_b(f_b)=f_b+10$, $c_c(fc)=f_c+5$, $c_d(fd)=f_d+5$.

The travel demands are: $d_{w_1}=10$ and $d_{w_2}=10$. Denote the paths as follows: $p_1=a$, $p_2=b$, $p_3=c$, and $p_4=d$.

Assume that the emission factors are: $h_a=h_b=1$, and $h_c=h_d=0.2$ with the initial license allocation given by: $l_a^0=l_b^0=l_c^0=l_d^0=3$. Hence, Network 1 has a total license allocation of 6 for its network links as does Network 2. Moreover, the environmental quality standard = 12.

Network 2 can sell several of its licenses to Network 1, which needs to purchase licenses since it cannot sustain its demand with the given allocation.

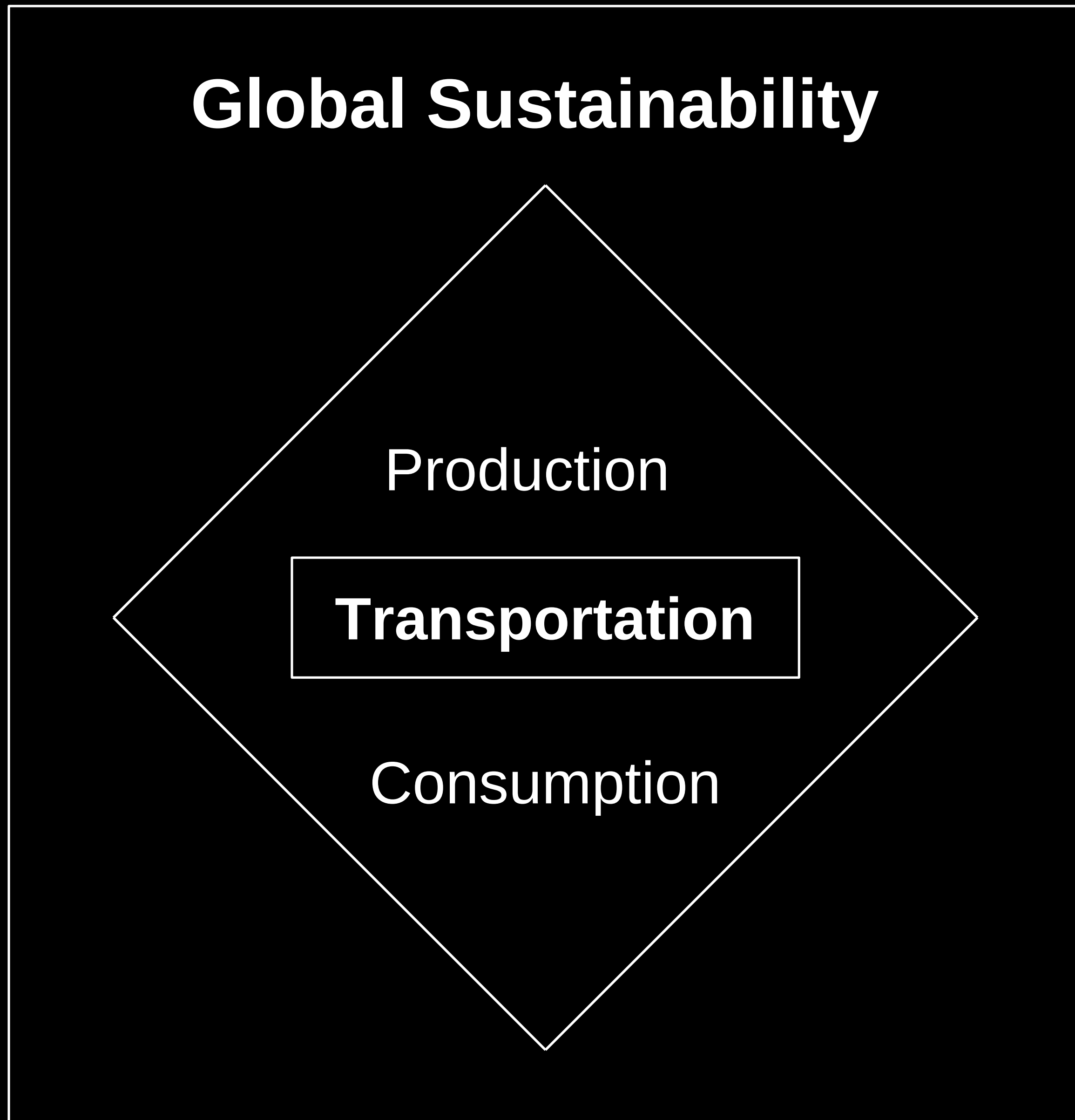
The flows are:

$x_{p_1}^*=f_a^*=6.35$, $x_{p_2}^*=f_b^*=3.75$, $x_{p_3}^*=f_c^*=5.00$, $x_{p_4}^*=f_d^*=5.00$,

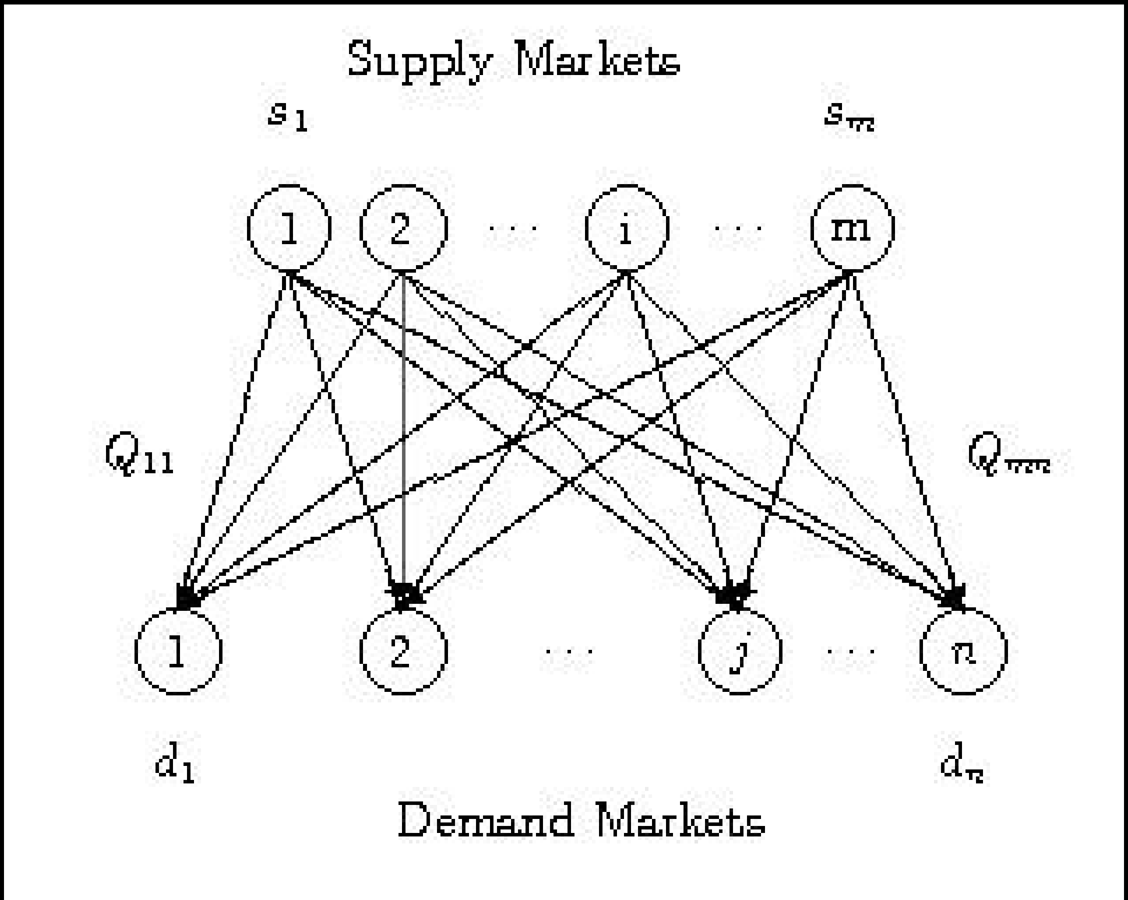
the marginal costs of emission abatement are:

$\tau_a^* = \tau_b^* = \tau_c^* = \tau_d^* = 1.94$, which is also the price ρ^*;

the licenses are: $l_a^*=6.25$, $l_b^*=3.75$, $l_c^*=1.00$, $l_d^*=1.00$.


Global Sustainability
Production
Transportation
Consumption

The bipartite network structure of the spatial price equilibrium problem



Spatial Price Equilibrium Conditions with Emission Pricing for Multiple Pollutants

The spatial price equilibrium conditions, in the presence of an emission pricing policy in the case of multiple pollutants, as described above, now take the form: For each pair of supply and demand markets *(i,j):*

$$\pi_i(s_i^*)+h_i^s\tau^{s*}+c_{ij}(Q_{ij}^*)+h_{ij}\tau^*+h_j^d\tau^{d*}\begin{cases} = & \rho_j(d_j^*), \quad \text{if } Q_{ij}^*>0 \\ \geq & \rho_j(d_j^*), \quad \text{if } Q_{ij}^*=0. \end{cases}$$

In addition, we must have:

$$\bar{Q}^s-\sum_{i=1}^m h_i^s s_i^*\begin{cases} = & 0, \quad \text{if } \tau^{s*}>0 \\ \geq & 0, \quad \text{if } \tau^{s*}=0. \end{cases}$$

$$\bar{Q}-\sum_{i=1}^m\sum_{j=1}^n h_{ij}Q_{ij}^*\begin{cases} = & 0, \quad \text{if } \tau^*>0 \\ \geq & 0, \quad \text{if } \tau^*=0. \end{cases}$$

$$\bar{Q}^d-\sum_{j=1}^n h_j^d d_j^*\begin{cases} = & 0, \quad \text{if } \tau^{d*}>0 \\ \geq & 0, \quad \text{if } \tau^{d*}=0. \end{cases}$$

Theorem (Variational Inequality Formulation - Multipollutant Spatial Price Emission Policy Model)

A vector $(s^*,Q^*,d^*,t^{s*}, t\ ^*,t^{d*})$ Î K^1 is an equilibrium of the multipollutant spatial price network equilibrium problem with the emission pricing policy given above if and only if it is a solution to the variational inequality problem:

$$\sum_{i=1}^{m}(\pi_i(s^*)+h_i^s\tau^{s*})\times(s_i-s_i^*)+\sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}(Q^*)+h_{ij}\tau^*)\times(Q_{ij}-Q_{ij}^*)$$

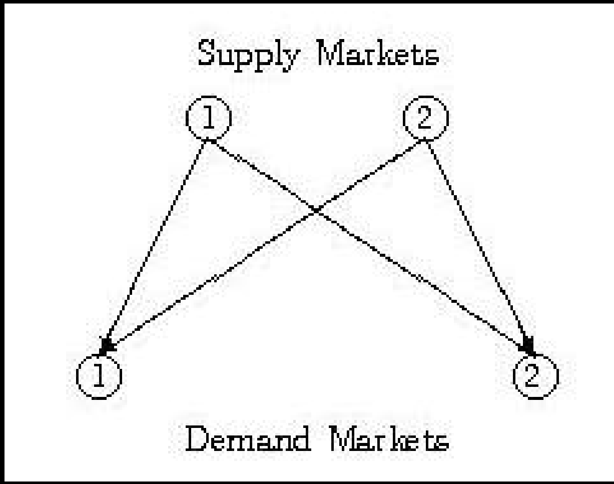
$$-(\sum_{j=1}^{n}\rho_j(d^*)-h_j^d\tau^{d*})\times(d_j-d_j^*)$$

$$+(\bar{Q}^s-\sum_{i=1}^{m}h_i^s s_i^*)\times(\tau^s-\tau^{s*})+(\bar{Q}-\sum_{i=1}^{m}\sum_{j=1}^{n}h_{ij}Q_{ij}^*)\times(\tau-\tau^*)$$

$$+(\bar{Q}^d-\sum_{j=1}^{n}h_j^d d_j^*)\times(\tau^d-\tau^{d*})\geq 0,\quad \forall(s,Q,d,\tau^s,\tau,\tau^d)\in\mathcal{K}^1,$$

where $\mathcal{K}^1\equiv K\times R_+^3$.

A Numerical Example



The data was: The supply price functions were:

$\pi_1(s)=5s_1+1s_2+2$, $\pi_2(s)=2s_2+1.5s_1+1.5$,

the unit transportation cost functions were:

$c_{11}(Q)=Q_{11}$, $c_{12}(Q)=2Q_{12}+0.035$,

$c_{21}(Q)=3Q_{21}+0.162$, $c_{22}(Q)=2Q_{22}+0.115$,

and the demand price functions were:

$\rho_1(d)=-2d_1-1.5d_2+28.75$, $\rho_2(d)=-4d_2-d_1+41$.

The emission parameters were: $h_1^s=0.5$, $h_2^s=1$; $h_{11}=0.5$, $h_{12}=1.$, $h_{21}=.5$, $h_{22}=1$; and $h_1^d=.5$, $h_2^d=1$.

I set the environmental standards to be 10, 5, 10.

The modified projection method yielded the following equilibrium pattern:

$s_1^*=1.9326$, $s_2^*=4.0477$,

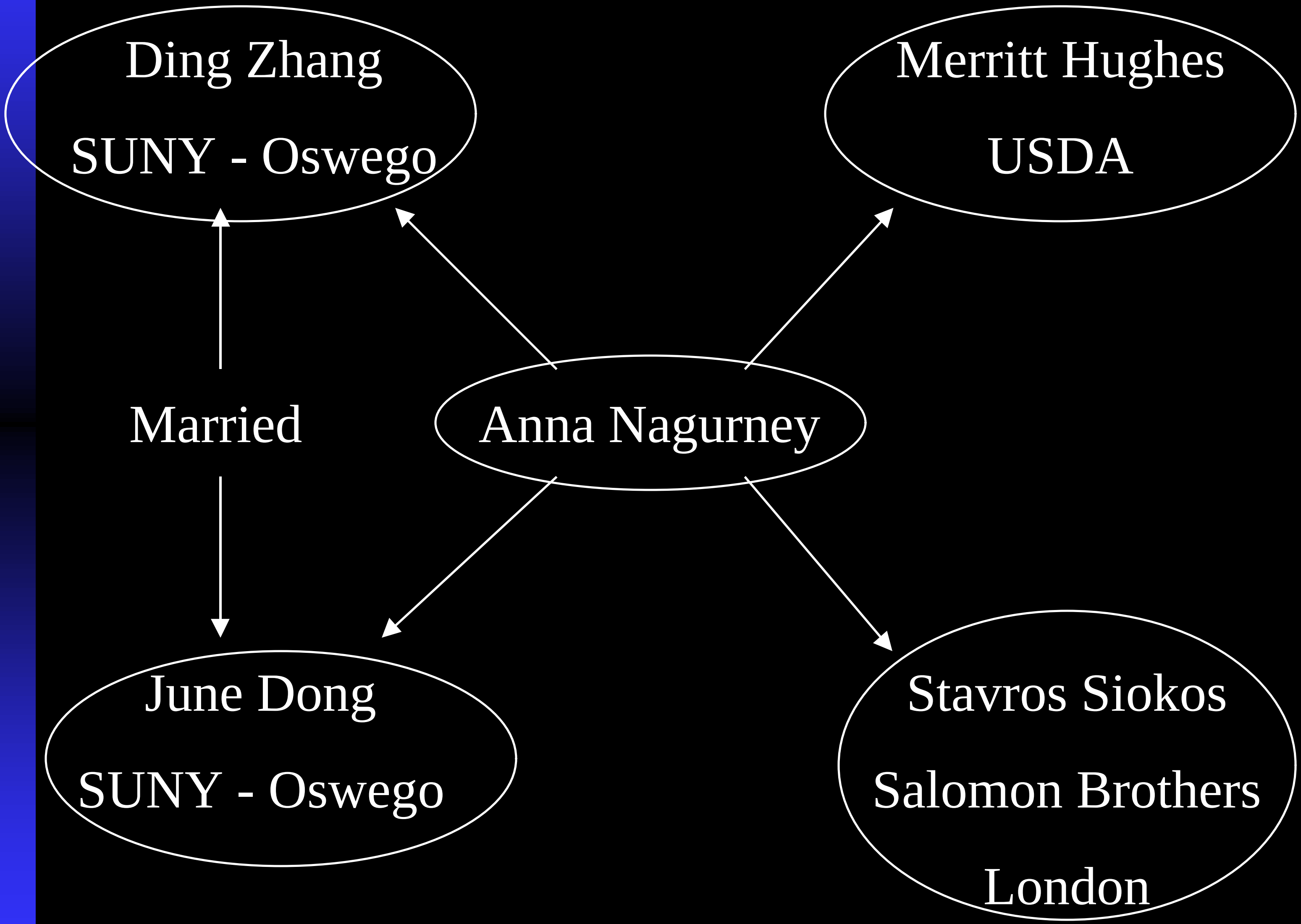
$Q_{11}^*=0.7069$, $Q_{12}^*=1.2257$, $Q_{21}^*=1.2536$,

$Q_{22}^*=2.7940$,

$d_1^*=1.9605$, $d_2^*=4.0197$,

$\tau^{s*}=0.0000$, $\tau^*=4.7632$, $\tau^{d*}=0.0000$.

My Collaborative Network in Finance



FINANCE

- Finance is concerned with the study of capital flows over space and time and, hence, it is ideally suited as an application domain of network theory.
- Importantly, the depiction of financial problems as network problems adds a graphic dimension to the understanding of the fundamental economic structure and its evolution over time.

Financial Networks - Economic Systems

- Financial networks date to Quesnay (1758), who in his **Tableau Economique** conceptualized the circular flow of funds in an economy as a network.
- Copeland in his (1952) book **A Study of Moneyflows in the United States** raised the question, "*Does money flow like water or electricity?*"
- Fei in 1960 proposed a linear graph for the study if the credit system in a paper in *The Review of Economics and Statistics*.
- Thore in 1980 published the book **Programming the Network of Financial Intermediation**.

Financial Networks - Optimization Problems

- The portfolio optimization problem of Markowitz (1952) has a network structure.

Single Country Model

Assume that there are I instruments with a typical instrument denoted by i and J sectors with a typical sector denoted by j.

The optimization problem faced by a sector j is given by:

$$\text{Maximize} \quad U^j(X^j, Y^j, r)$$

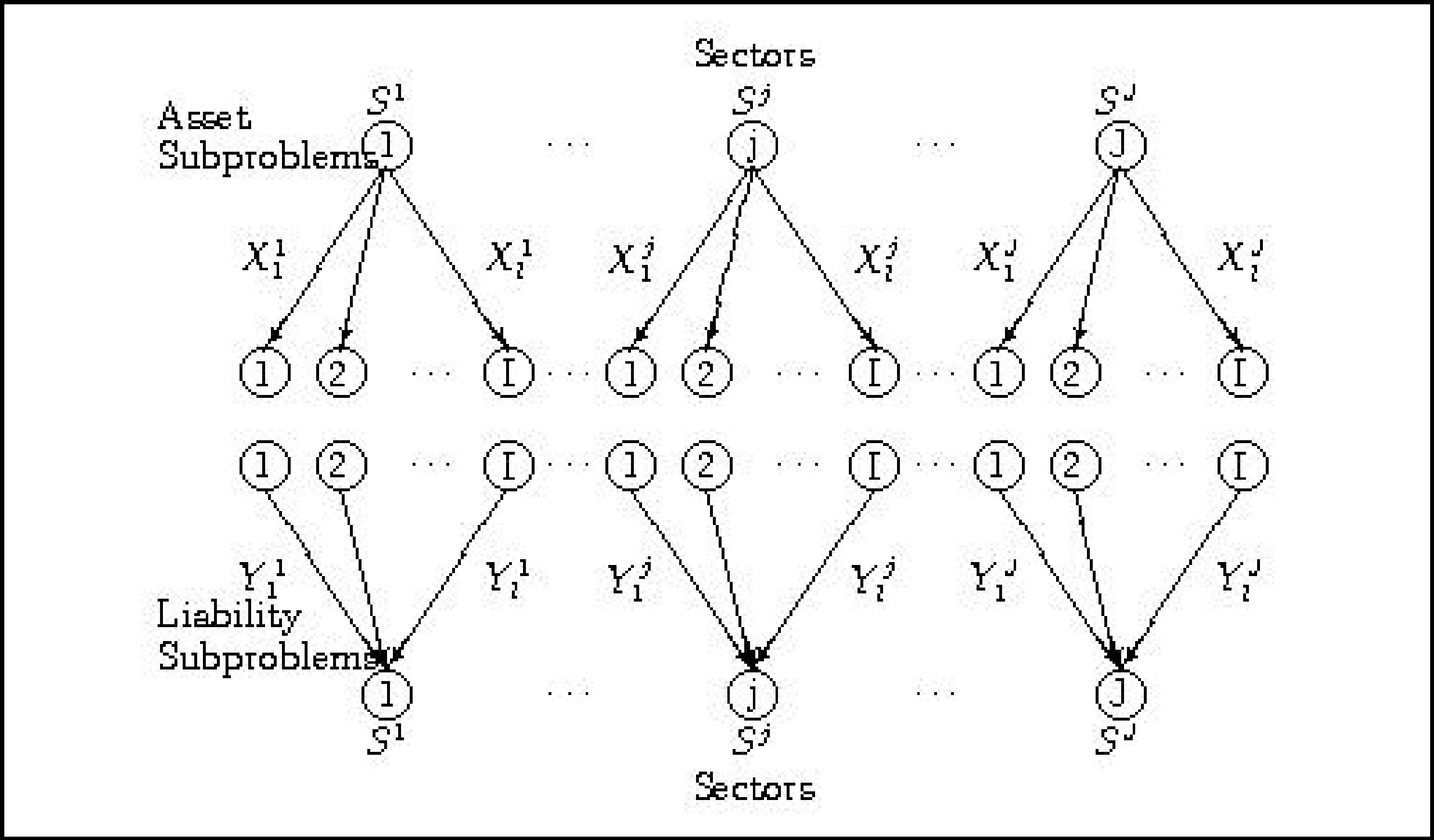
subject to:

$$\sum_{i=1}^{I} X_i^j = S^j \qquad \sum_{i=1}^{I} Y_i^j = S^j$$

$$X_i^j \geq 0, \; Y_i^j \geq 0, \qquad i = 1, 2, \ldots, I,$$

where the price vector r is an exogenous vector in the optimization problem of every sector j; $j=1,\ldots,J$.

Network Structure of the Sectors' Optimizaton Problems



Economic System Conditions

The economic system conditions that ensure market clearance at a positive instrument price (and a possible excess supply of the instrument at a zero price) are:

For each instrument i; i=1,…,I, we must have that

$$\sum_{j=1}^{J}(X_i^{j*} - Y_i^{j*}) \begin{cases} = 0, & \text{if } r_i^* > 0 \\ \geq 0, & \text{if } r_i^* = 0. \end{cases}$$

Definition (Market Equilibrium)

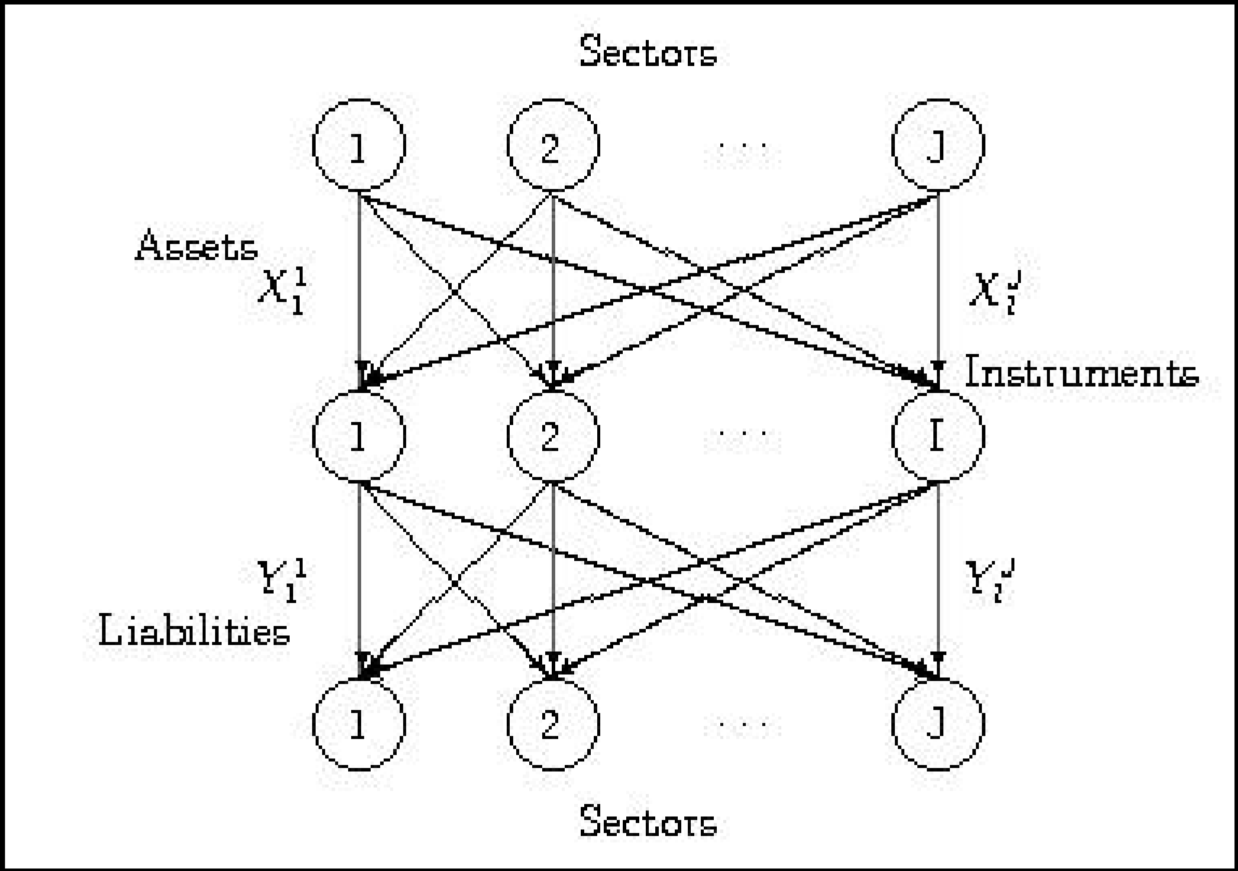
A vector $(X^*, Y^*, r^*) \in K$ is an equilibrium of the single country, multi-sector, multi-instrument financial model if and only if it satisfies the optimality conditions and the economic system conditions, for all sectors j; $j = 1, \ldots, J$, and for all instruments i; $i = 1, \ldots, I$, simultaneously.

Theorem (Variational Inequality Formulation)

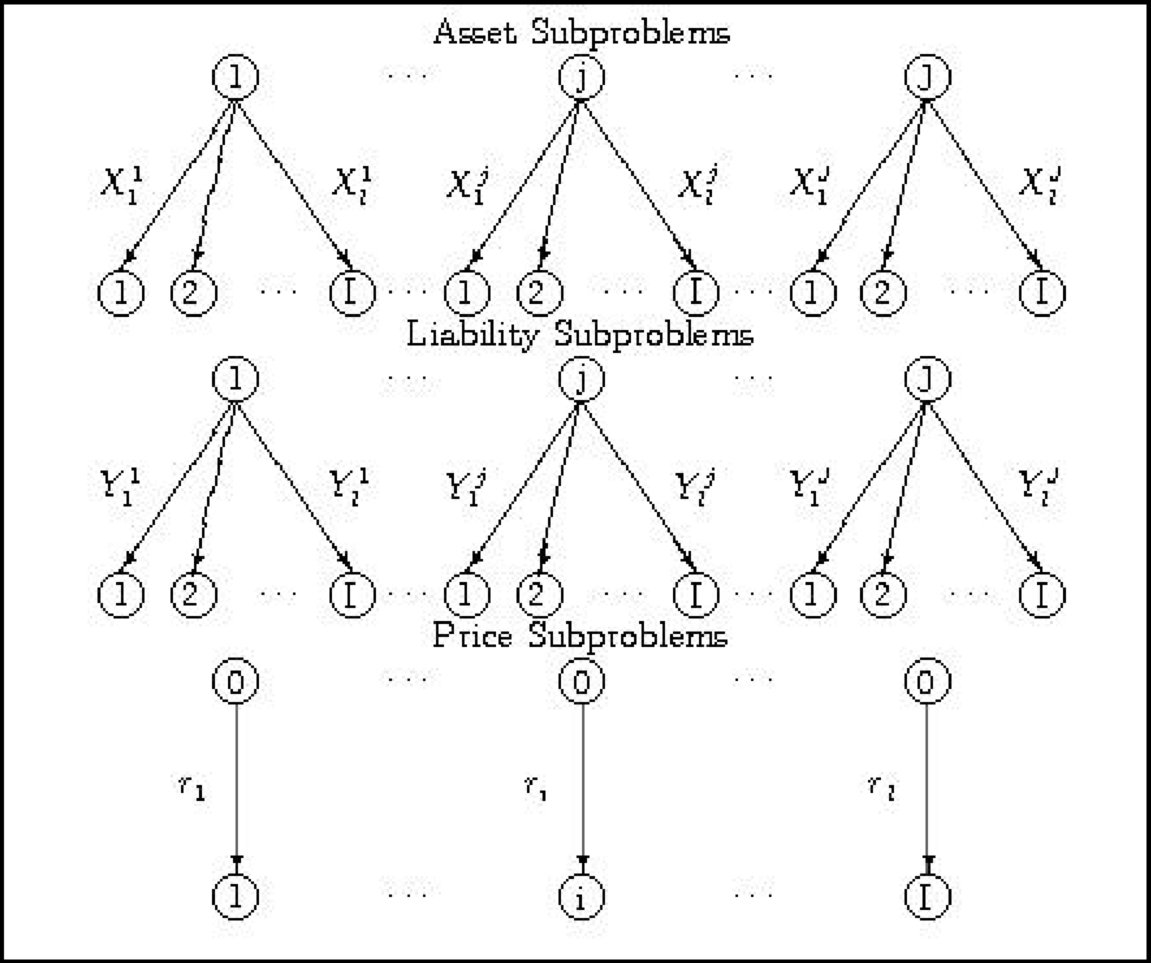
A vector of assets and liabilities of the sectors, and instrument prices, $(X^*, Y^*, r^*) \in K$, is a financial equilibrium if and only if it satisfies the variational inequality problem:

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\left[-\frac{\partial U^j(X^{j*},Y^{j*},r^*)}{\partial X_i^j}\right]\times\left[X_i^j - X_i^{j*}\right]$$
$$+\sum_{j=1}^{J}\sum_{i=1}^{I}\left[-\frac{\partial U^j(X^{j*},Y^{j*},r^*)}{\partial Y_i^j}\right]\times\left[Y_i^j - Y_i^{j*}\right]$$
$$+\sum_{i=1}^{I}\sum_{j=1}^{J}\left[X_i^{j*} - Y_i^{j*}\right]\times\left[r_i - r_i^*\right] \geq 0, \quad \forall (X,Y,r) \in K.$$

Network Structure of the Equilibrium



Financial Network Subproblems induced by the modified projection method



International Financial Models

Challenges:

- International investors must now be concerned with the exchange rate risk, which stems from the fact that the exchange prices between currencies change through time, in addition to the risk that they wish to minimize in a single country setting.
- Investors from different countries make decisions based on their own nations' different financial environments, and, therefore, they all hold distinct preferences and expectations.
- Recent advances in technology, along with the liberalization of international trade and the establishment of international agreements and treaties, have led many investors to diversify their portfolios internationally.

It has become clear that investing internationally is becoming increasingly common, and new international financial markets have been established that offer a greater variety of new financial products.

An International Financial Model

Consider an economy consisting of L countries, with a typical country denoted by l. Each country, in turn, has J sectors, with a typical sector denoted by j. We also consider K currencies, with a typical currency denoted by k. In each currency there are I instruments, with a typical instrument denoted by i.

The portfolio optimization problem of sector j in country l can be expressed as:

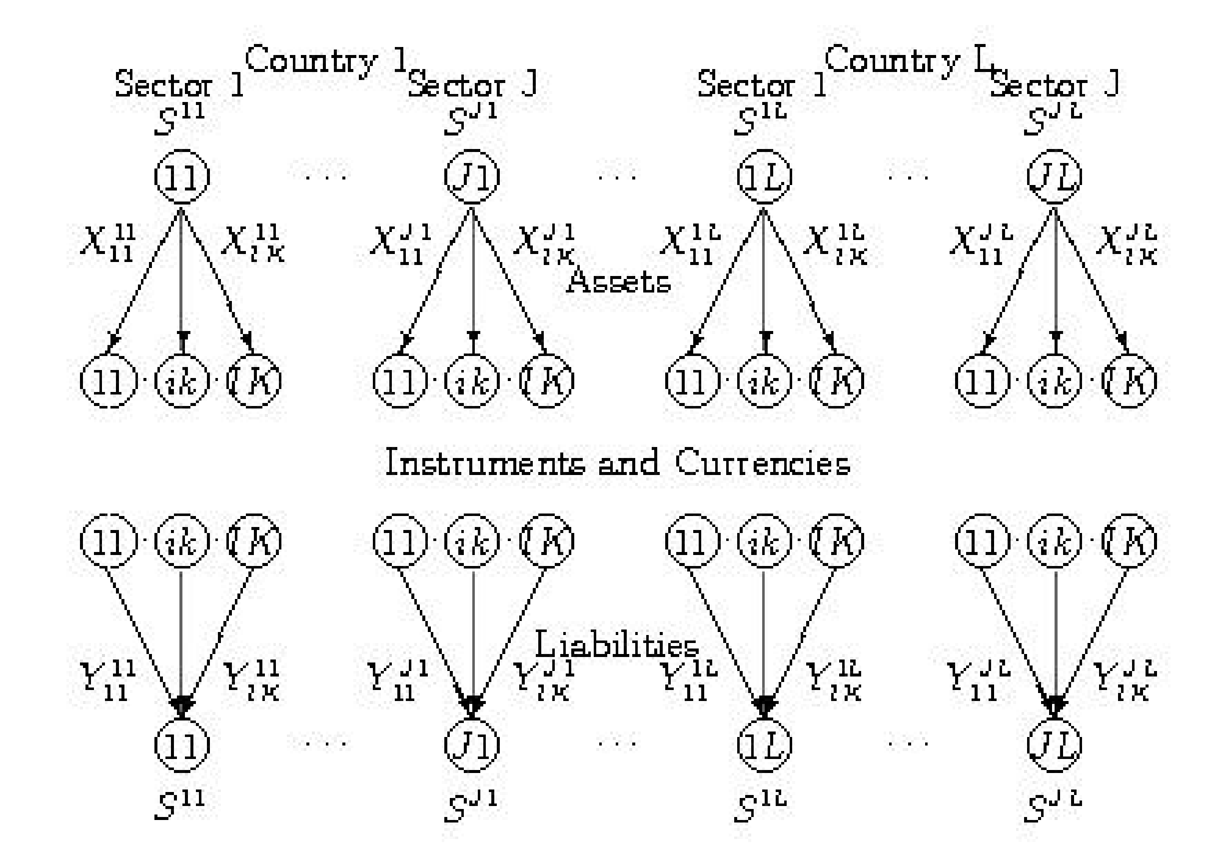
$$\text{Maximize } U^{jl}(X^{jl}, Y^{jl}, r, e)$$

subject to:

$$\sum_{i=1}^{I}\sum_{k=1}^{K} X_{ik}^{jl} = S^{jl} \qquad \sum_{i=1}^{I}\sum_{k=1}^{K} Y_{ik}^{jl} = S^{jl}$$

$$X_{ik}^{jl}, Y_{ik}^{jl} \geq 0, \quad i = 1, \ldots, I; \quad k = 1, \ldots, K,$$

Network Structure of Sectors' Optimization Problems



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Portfolio Optimality

Under the assumption that the utility function for each sector of each country is concave with respect to the instrument variables, continuous, and continuously differentiable, the necessary and sufficient conditions for an optimal portfolio for a sector *j* of country l, where, $(X^{jl*}, Y^{jl*}) \in \kappa^{jl}$, is that it satisfies the following inequality:

$$-\left[\nabla_{X^{jl}} U^{jl}(X^{jl*}, Y^{jl*}, r^*, e^*)^T\right] \cdot \left[X^{jl} - X^{jl*}\right]$$

$$-\left[\nabla_{Y^{jl}} U^{jl}(X^{jl*}, Y^{jl*}, r^*, e^*)^T\right] \cdot \left[Y^{jl} - Y^{jl*}\right] \geq 0, \forall (X^{jl}, Y^{jl}) \in \kappa^{jl}.$$

Furthermore, the economic system conditions for the instrument and currency prices are as follows:

Instrument Market Equilibrium Conditions

For each instrument *i; i=1,...,I*, and currency *k; k=1,...,K*, we must have that

$$\sum_{j=1}^{J}\sum_{l=1}^{L}(X_{ik}^{jl^*} - Y_{ik}^{jl^*}) \begin{cases} = 0, & \text{if } r_{ik}^* > 0 \\ \geq 0, & \text{if } r_{ik}^* = 0. \end{cases}$$

Currency Market Equilibrium Conditions

Also, for each currency *k; k=1,...,K*, we must have that

$$\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{i=1}^{I}(X_{ik}^{jl^*} - Y_{ik}^{jl^*}) \begin{cases} = 0, & \text{if } e_{k}^* > 0 \\ \geq 0, & \text{if } e_{k}^* = 0. \end{cases}$$

Definition (An International Financial Equilibrium)

A vector $(X^*,Y^*,r^*,e^*) \in K$ is an international financial equilibrium if and only if it satisfies the three system of equalities and inequalities above for all sectors $j;\ j=1,\ldots,J$, all countries $l;\ l=1,\ldots,L$, all instruments $i;\ i=1,\ldots,I$, and all currencies $k;\ k=1,\ldots,K$.

The equilibrium pattern can be obtained as a solution to:

Theorem (Variational Inequality Formulation)

A vector of assets and liabilities of the sectors, and instrument prices, $(X^*,Y^*,r^*,e^*) \in K$, is a perfect market international financial equilibrium if and only if it satisfies the variational inequality problem:

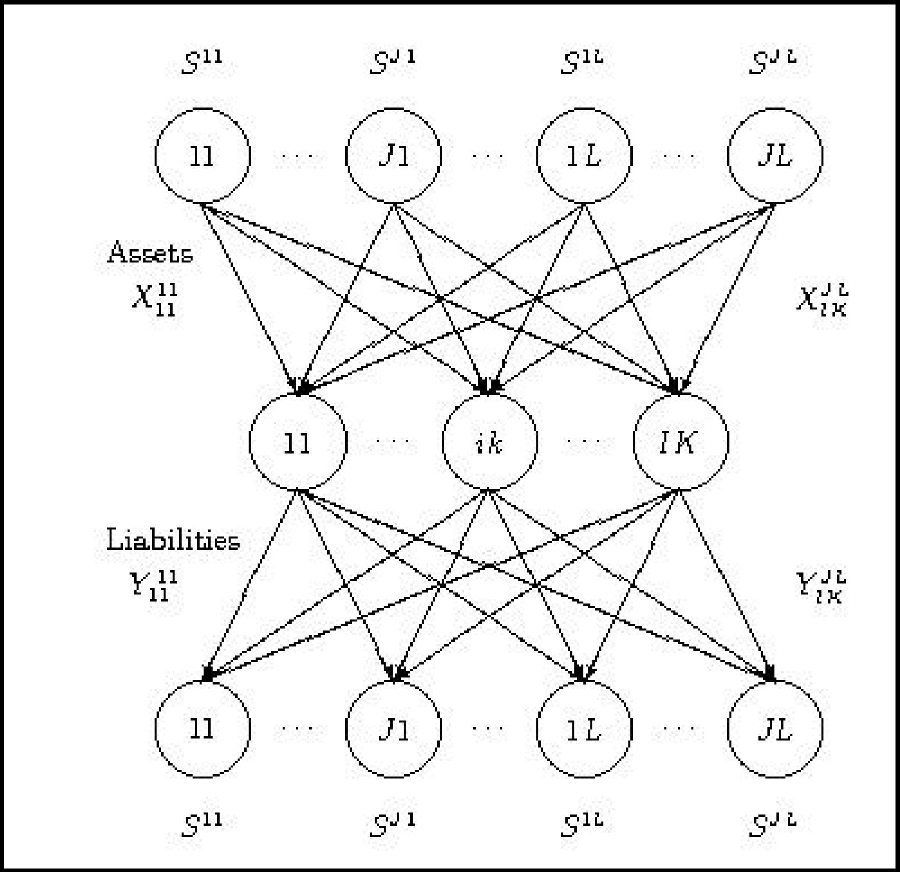
$$\sum_{j=1}^{J}\sum_{l=1}^{L}\left[-\nabla_{X^{jl}} U^{jl}(X^{jl*},Y^{jl*},r^*,e^*)^T\right]\cdot\left[X^{jl}-X^{jl*}\right]$$

$$+\sum_{j=1}^{J}\sum_{l=1}^{L}\left[-\nabla_{Y^{jl}} U^{jl}(X^{jl*},Y^{jl*},r^*,e^*)^T\right]\cdot\left[Y^{jl}-Y^{jl*}\right]$$

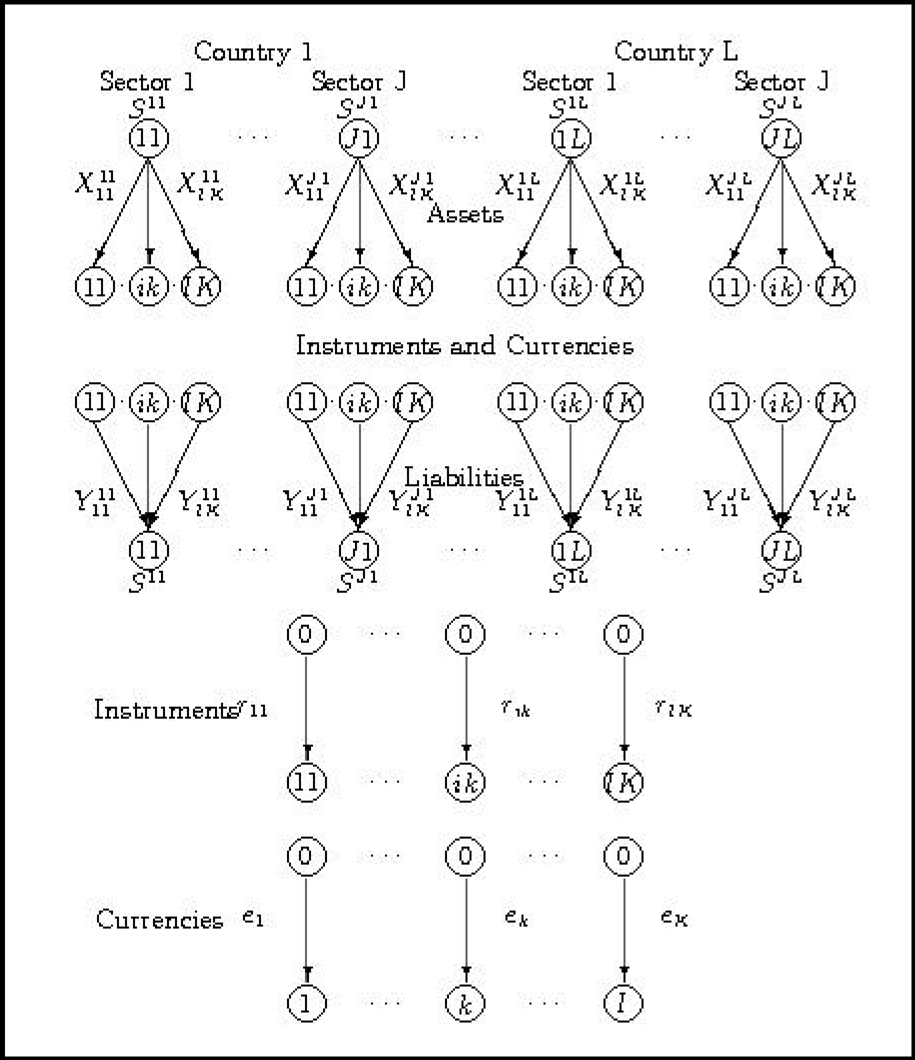
$$+\sum_{k=1}^{K}\sum_{i=1}^{I}\left[\sum_{j=1}^{J}\sum_{l=1}^{L} X_{ik}^{jl*}-Y_{ik}^{jl*}\right]\times\left[r_{ik}-r_{ik}^*\right]$$

$$+\sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\left(X_{ik}^{jl*}-Y_{ik}^{jl*}\right)\right]\times\left[e_k-e_k^*\right]\geq 0, \quad \forall (X,Y,r,e)\in K.$$

The Network Structure of International Financial Equilibrium



Structure of Network Problems Induced by the Modified Projection Method



An International Model with Hedging

Consider an economy consisting of *L* countries, with a typical country denoted by *l*, and with *J* sectors in each country, with a typical sector denoted by *j*. Moreover, we assume that there are *K* currencies, with a typical currency represented by *k*, where in each currency there are *I* instruments available, with a typical instrument denoted by *i*.

We assume that every sector of each country can either hedge his assets and liabilities through futures and/or options contracts, or be exposed to the risk incorporated in an internationally diversified unhedged portfolio.

The portfolio optimization problem of sector *j* of country *l* can be expressed as:

$$\text{Maximize} \quad U^{jl}(\mathcal{X}^{jl}, \mathcal{Y}^{jl}, r, e, \lambda, \delta; \mu, \pi, \eta)$$

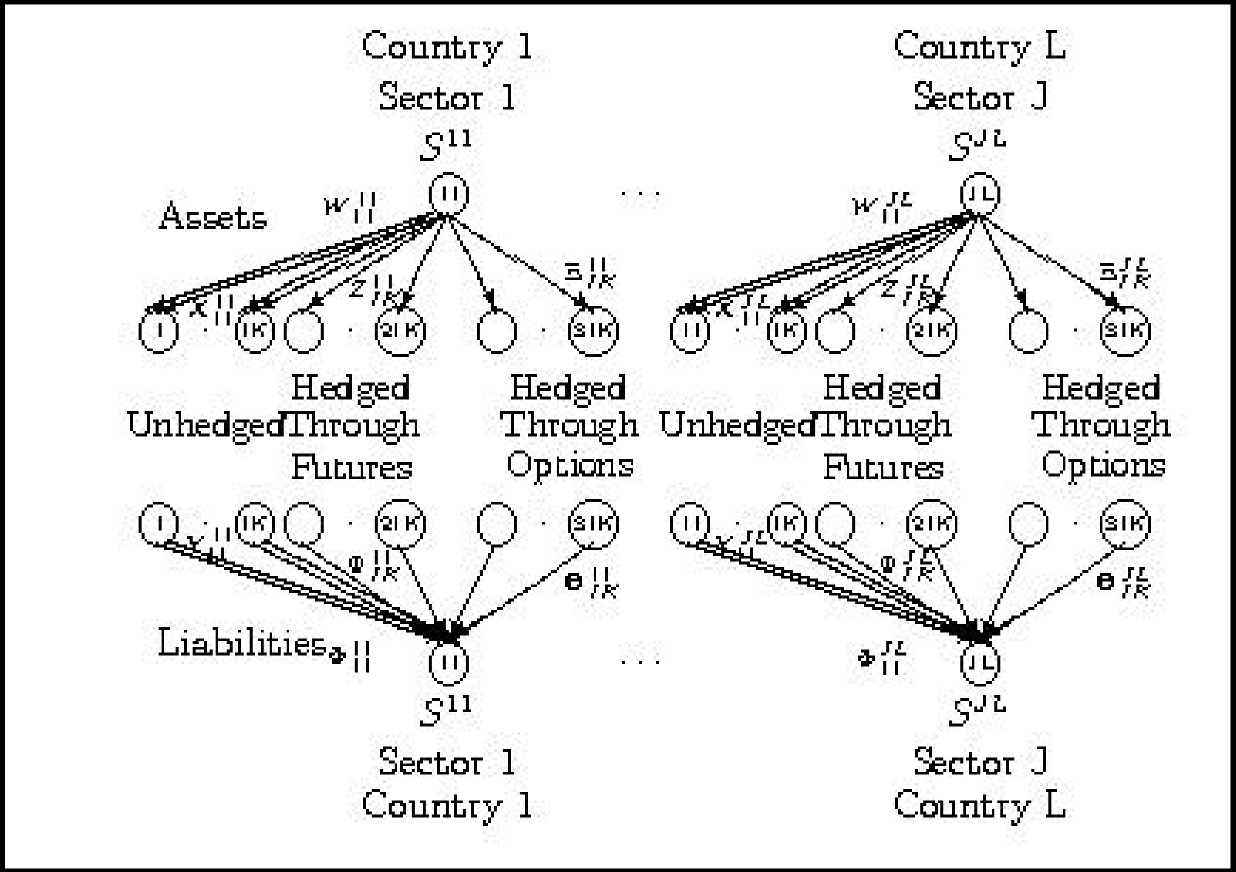
subject to:

$$\sum_{i=1}^{I}\sum_{k=1}^{K} X_{ik}^{jl} + W_{ik}^{jl} + Z_{ik}^{jl} + \Xi_{ik}^{jl} = S^{jl},$$

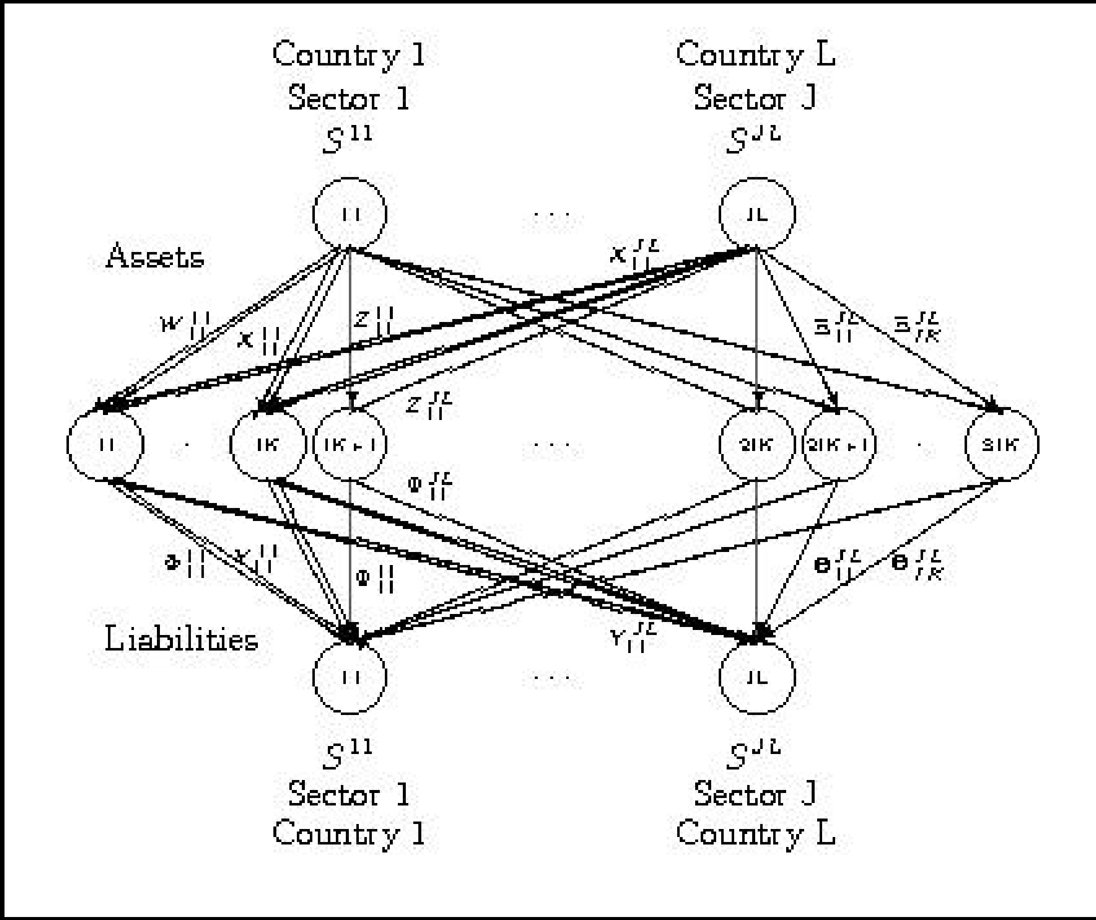
$$\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{ik}^{jl} + \Phi_{ik}^{jl} + \Psi_{ik}^{jl} + \Theta_{ik}^{jl} = S^{jl},$$

$$\mathcal{X}^{jl}, \mathcal{Y}^{jl} \geq 0.$$

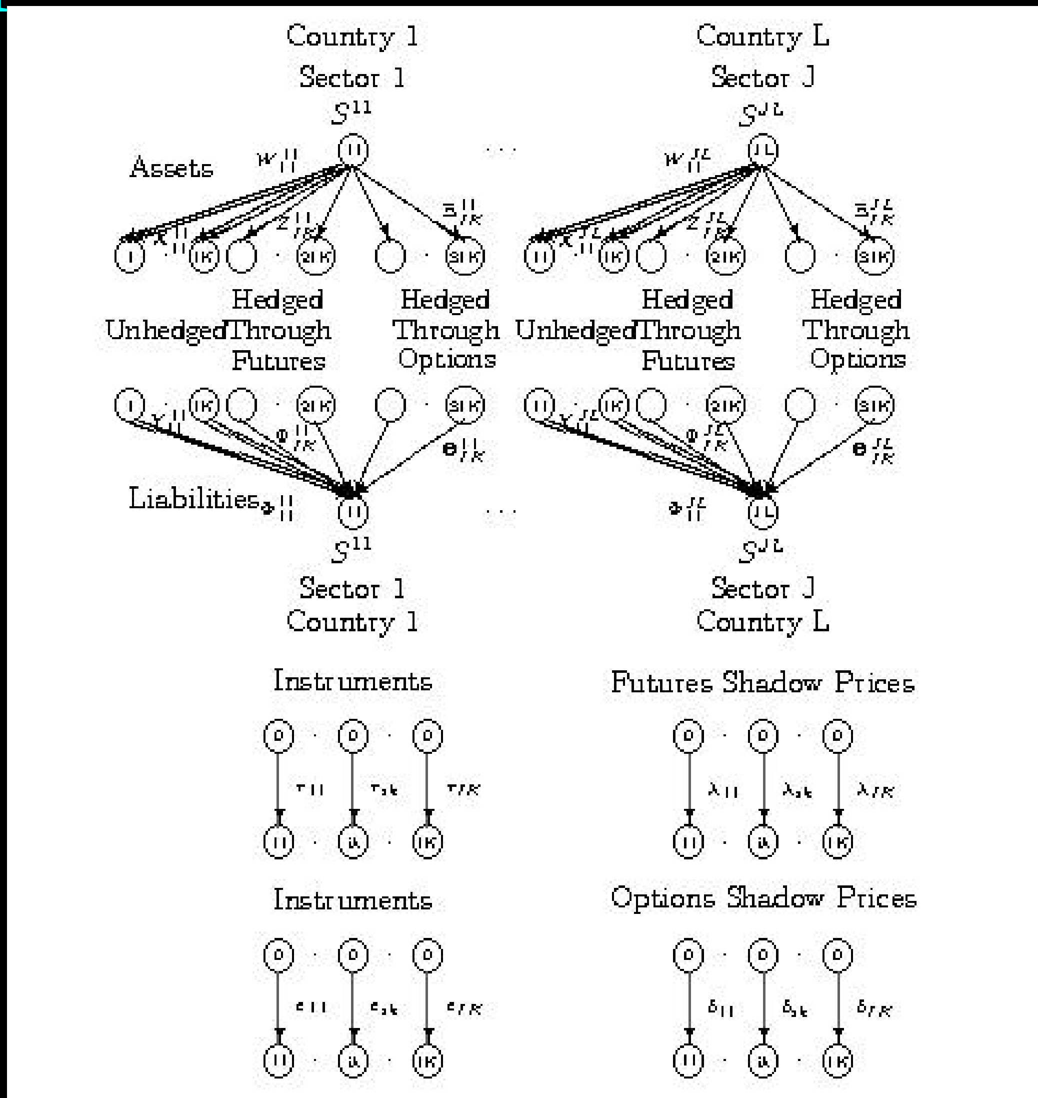
Network Structure of the Sectors' Optimization Problems



The Network Structure at Equilibrium



Network Structure of the Financial Subproblems



KNOWLEDGE NETWORKS

Knowledge networks as noted by Beckmann (1995) is a concept invented and utilized by Swedish economists in an atmosphere of growing international competition, which has led Sweden to focus on high technology industries which are knowledge intensive.

In today's *Network Economy*, the existence of highly skilled workers is essential for

- innovation;
- research and development;
- and for increasing the competitive position of regions and nations.

Knowledge is unarguably the most important factor in today’s economy.

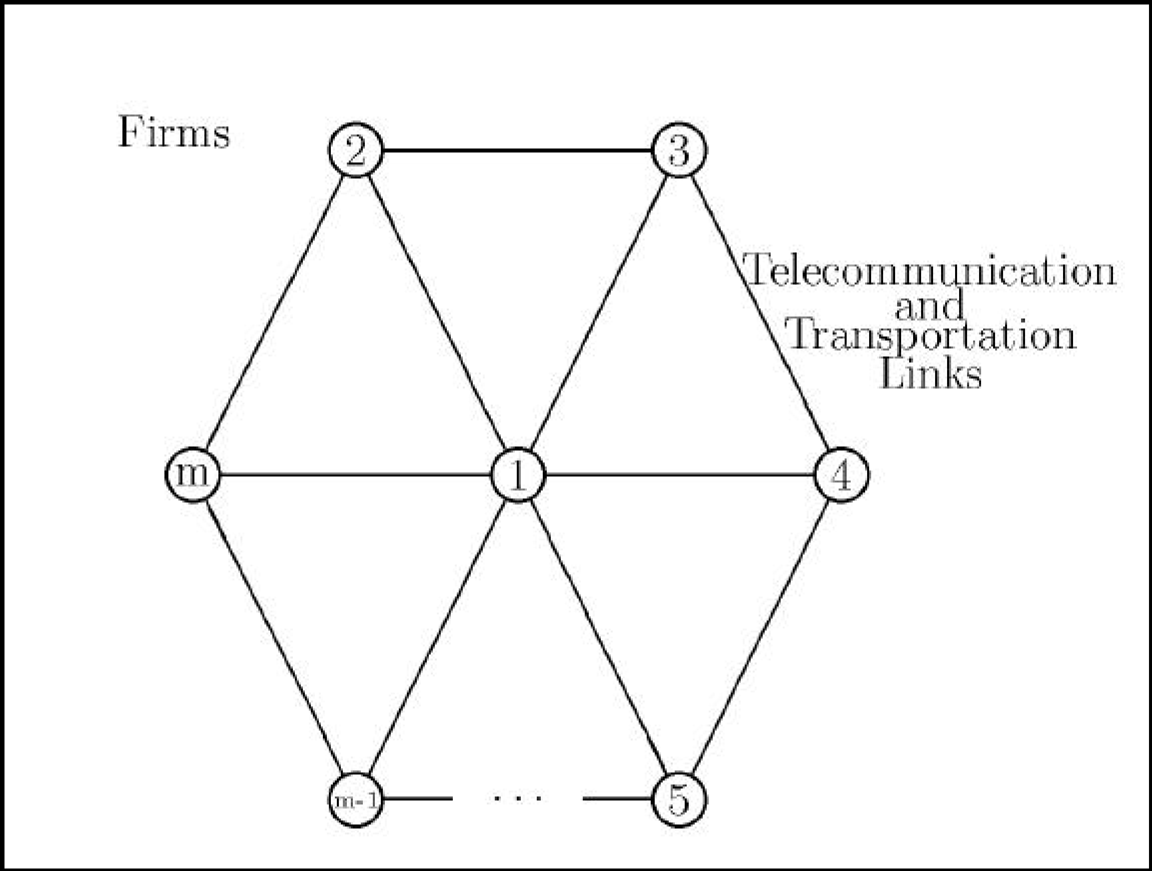
Knowledge is not information, which implies a set of uncomplicated messages and routinized data, which, to a large extent, can be subdivided and stored.

- Knowledge production involves both the creation of new knowledge and the search for new understanding from old knowledge.

- In order to understand the regional division of production, knowledge, and capital, as well as, their evolution, it is necessary to introduce network dimensions to the analysis of interactions between firms, individuals, and organizations.

Differences in creativity, labor skills, and innovation diffusion can explain quite convincingly why some economies have prospered while others have declined.

Hypothetical Knowledge Network Topology



A Knowledge Network Equilibrium Model

We assume that there are two distinct types of workers, *knowledge* workers and *goods* workers.

A Firm's Production Function

Each firm's production function:

output = knowledge production x goods production

$$q_i = g_i(D_i,G)\, f_i(K_i,L_i),$$

where q_i denotes the quantity produced by firm i, D_i is the capacity of the information systems, $G=(G_1,\ldots,G_m) \in R_+^m$ is the column vector of knowledge workers at the nodes, K_i denotes the amount of capital held by firm $_i$, and L_i denotes the amount of goods workers at firm i.

Transportation distance plays an important role in impeding the movement of individuals for purposes of information and knowledge exchange.

Distance, in terms of knowledge exchange on telecommunication networks, on the other hand, plays a less critical role.

The production function is quite general and can also incorporate different measures of *knowledge accessibility* which depend on the telecommunication and transportation networks.

For example, one may define a telecommunication accessibility measure, which we denote by TC_i for firm i, and which is defined as follows:

$$TC_i = \sum_{j=1}^{m} \sigma_1 f_{ij1} G_j^{\gamma_1}, \quad \forall i,$$

where σ_1 and γ_1 are parameters, $f_{ij1} = e^{-\beta\, d_{ij1}}$, where β denotes the distance friction associated with knowledge exchange across the telecommunication network between firms i and j and d_{ij1} is the distance between firms i and j.

Also, one may define a transportation accessibility measure, which we denote by TR_i for firm *i*, and which is defined as:

$$TR_i = \sum_{j=1}^{m} (\sigma_2 f_{ij2} W_j^{\gamma_2} + \sigma_3 f_{ij2} G_j^{\gamma_3}), \quad \forall i,$$

where σ_2, σ_3, γ_2, and γ_3 are parameters, W_j is the scale of firm *j*'s public research & development units, assumed to be fixed at each node, and f_{ij2} is the distance friction for knowledge exchange on the transportation network between firms *i* and *j*.

We assume that the firms are perfectly competitive in that they take the price of the good produced as fixed.

We assume also that the firms compete for the knowledge workers in a noncooperative fashion.

The utility function facing a firm *i* can be expressed as:

$$u_i(D_i, G, K_i, L_i) = \bar{p}_i g(D_i, G) f(K_i, L_i) - \omega_i K_i - \theta_i L_i - \eta D_i - \zeta G_i.$$

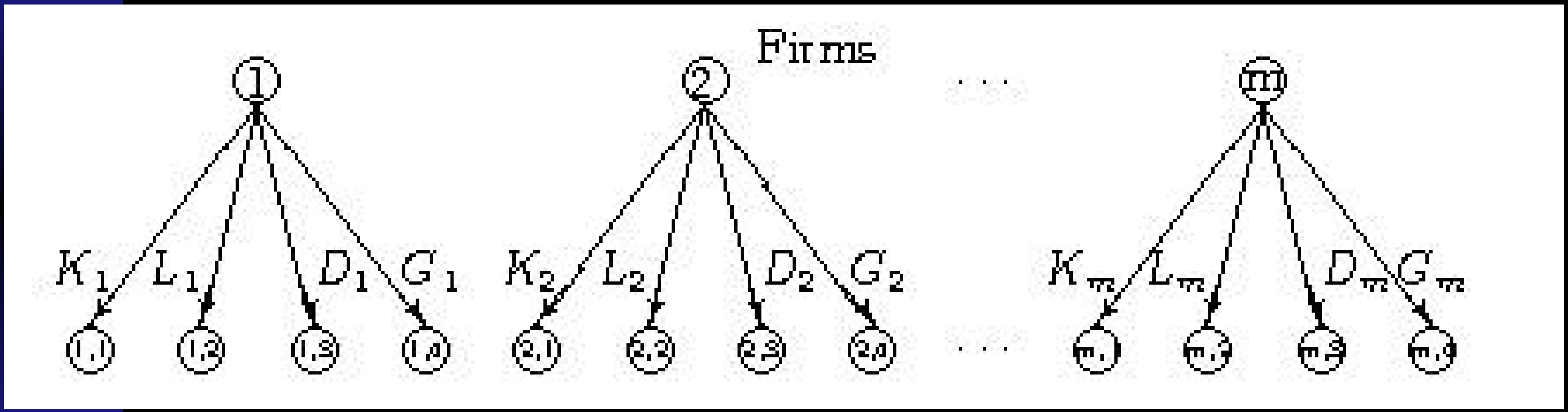
Hence, the objective function facing such a profit-maximizing firm is:

$$\text{Maximize} \quad u_i(D_i, G, K_i, L_i)$$

subject to:

$$D_i \geq 0, \quad G_i \geq 0, \quad K_i \geq 0, \quad L_i \geq 0.$$

Network Structure of the Firm's Decisions



Definition (Knowledge Network Equilibrium)

A knowledge network equilibrium is a vector of information system capacities, amounts of knowledge workers, capital, and goods workers $(D^*, G^*, K^*, L^*) \in R_+^{4m}$:

$$u_i(D_i^*, G_i^*, \widehat{G}_i^*, K_i^*, L_i^*) \geq u_i(D_i, G_i, \widehat{G}_i^*, K_i, L_i),$$

$$\forall D_i \in R_+, \quad \forall G_i \in R_+, \quad \forall K_i \in R_+, \quad \forall L_i \in R_+,$$

where $\widehat{G}_i \equiv (G_1, \ldots, G_{i-1}, G_{i+1}, \ldots, G_m).$

Theorem (Variational Inequality Formulation)

Assume that each u_i is continuously differentiable on R_+^4 and concave with respect to its arguments. Then *(D*,G*,K*,L*)* is a knowledge network equilibrium if and only if it satisfies the variational inequality problem

$$\sum_{i=1}^{m}(\eta - \bar{p}_i f_i(K_i^*, L_i^*)\frac{\partial g_i(D_i^*, G^*)}{\partial D_i}) \times (D_i - D_i^*)$$

$$+\sum_{i=1}^{m}(\zeta - \bar{p}_i f_i(K_i^*, L_i^*)\frac{\partial g_i(D_i^*, G^*)}{\partial G_i}) \times (G_i - G_i^*)$$

$$+\sum_{i=1}^{m}(\omega_i - \bar{p}_i g_i(D_i^*, G^*)\frac{\partial f_i(K_i^*, L_i^*)}{\partial K_i}) \times (K_i - K_i^*)$$

$$+\sum_{i=1}^{m}(\theta_i - \bar{p}_i g_i(D_i^*, G^*)\frac{\partial f_i(K_i^*, L_i^*)}{\partial L_i}) \times (L_i - L_i^*) \geq 0,$$

$$\forall (D, G, K, L) \in R_+^{4m}.$$

Sensitivity analysis results.

Theorem: Assume that the negative of the gradient of the utility functions is monotone.

Consider a change $\Delta\omega_i$ to the rent of capital for firm *i*, keeping all other data as before. Let ΔK_i denote the subsequent change in the equilibrium amount of capital held by firm *i*. Then

$$\Delta\omega_i \times \Delta K_i \leq 0.$$

Similarly, consider a change $\Delta\theta_i$ to the wage rate for the goods workers at firm *i*, keeping all other data as before. Let ΔL_i denote the subsequent change in the equilibrium amount of goods workers at firm *i*. Then

$$\Delta\theta_i \times \Delta L_i \leq 0.$$

Theorem: Assume that the negative of the gradient of the utility functions is monotone.

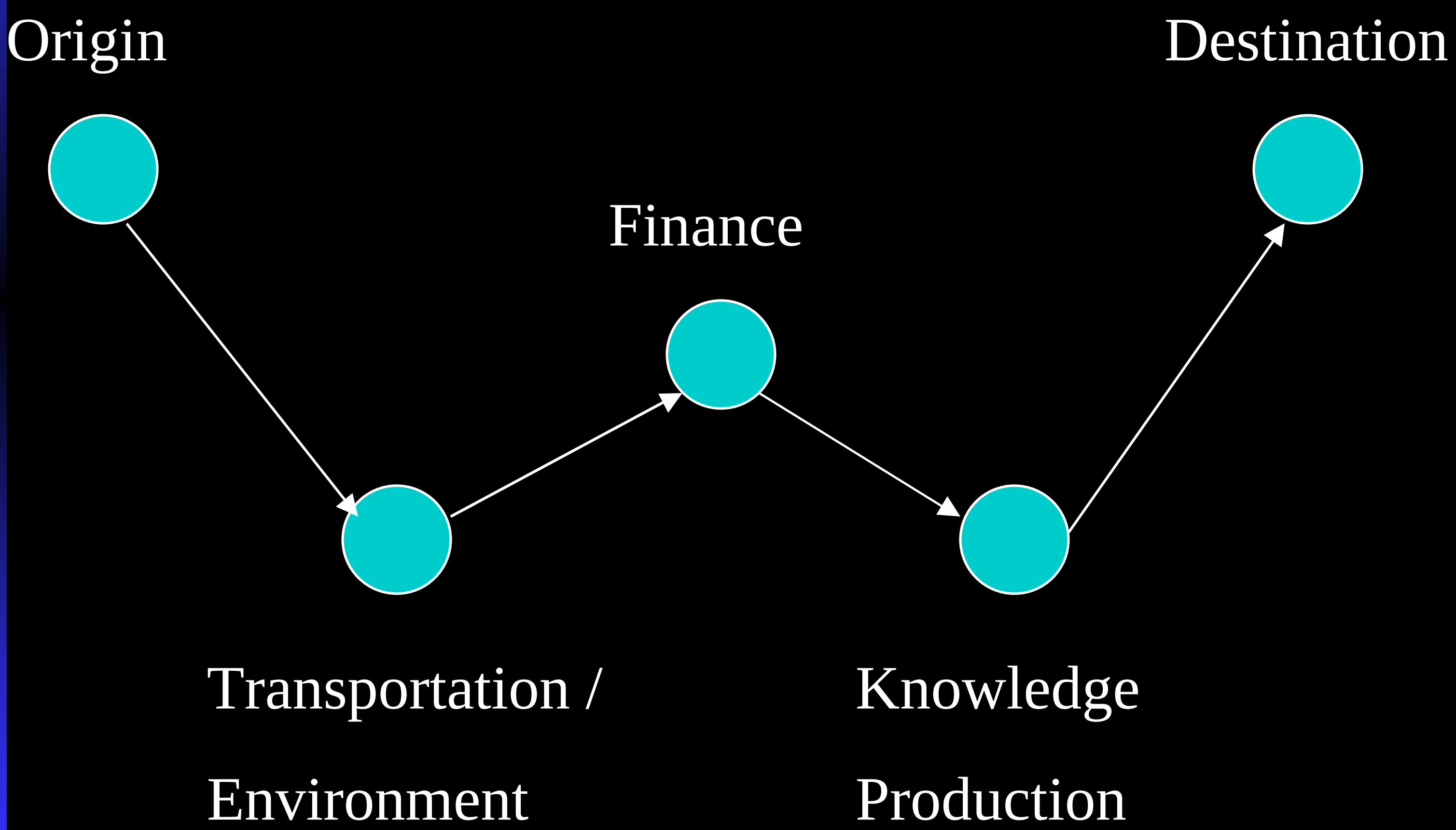
Consider a change $\Delta\eta$ to the rent of information systems, keeping all other data as before. Let ΔD_i denote the subsequent change in the equilibrium capacity of information. Then

$$\Delta\eta \times \sum_{i=1}^{m} \Delta D_i \leq 0.$$

Similarly, consider a change $\Delta\zeta$ to the wage rate of knowledge workers, keeping all other data as before. Let ΔG_i denote the subsequent change in equilibrium amounts of knowledge workers. Then

$$\Delta\zeta \times \sum_{i=1}^{m} \Delta G_i \leq 0.$$

This Intellectual Journey



Ongoing Research Problems:

- Interactions Between Transportation and Telecommunication Networks
- Dynamics of Evolving Networks
- Construction of Complex Networks
- Visualization of Network Phenomena
- New Applications