A Cybersecurity Investment Supply Chain Game Theory Model

Patrizia Daniele A. Maugeri A. Nagurney

Department of Mathematics and Computer Science
University of Catania - Italy

ODS 2017 - Sorrento, September 4-7, 2017

Session: Game Theory

Cyber Attacks

Verizon's 2016 Data Breach Investigations Report

2,260 confirmed data breaches at organizations in 82 countries

Recent Cyber Attacks

Late summer 2014: 76 million customers

Recent Cyber Attacks

Late 2013: 40 million payment cards stolen and upwards of 70 million other personal records compromised

Recent Cyber Attacks

Late Fall 2014: catastrophic and a public relations nightmare

Data

Center for Strategic and International Sudies (2014)

It has been estimated that the world economy sustained $445 billion in losses from cyberattacks in 2014.
The estimated annual cost to the global economy from cybercrime is more than $400 billion with a conservative estimate being $375 billion in losses, a number that exceeds the national income of most countries.

Data

Center for Strategic and International Sudies (2014)

It has been estimated that the world economy sustained $445 billion in losses from cyberattacks in 2014.
The estimated annual cost to the global economy from cybercrime is more than $400 billion with a conservative estimate being $375 billion in losses, a number that exceeds the national income of most countries.

Pricewaterhouse Coopers (2014)

The number of cybersecurity incidents that were detected by respondents to their survey increased by 48% to 42.8 million in 2014

Data

Center for Strategic and International Sudies (2014)

It has been estimated that the world economy sustained $445 billion in losses from cyberattacks in 2014.
The estimated annual cost to the global economy from cybercrime is more than $400 billion with a conservative estimate being $375 billion in losses, a number that exceeds the national income of most countries.

Pricewaterhouse Coopers (2014)

The number of cybersecurity incidents that were detected by respondents to their survey increased by 48% to 42.8 million in 2014

No industrial sector is immune to cyber attacks with sectors such as financial services, insurance, pharmaceuticals, healthcare, high technology, energy, automotive and governments being especially attractive targets.

Data

Kaspersky Lab (2015)

A multinational gang of cybercriminals, known as *Carbanak*, infiltrated more than 100 banks across 30 countries and extracted as much as one billion dollars over a period of roughly two years

Data

Kaspersky Lab (2015)

A multinational gang of cybercriminals, known as *Carbanak*, infiltrated more than 100 banks across 30 countries and extracted as much as one billion dollars over a period of roughly two years

Forbes (2015)

Cyberattacks can result not only in direct financial losses and/or the loss of data, but also in an organization's highly valued asset - its reputation

World-wide spending on cybersecurity was approximately \$75 billion in 2015, with the expectation that, by 2020, companies around the globe will be spending around \$170 billion annually

Data

Kaspersky Lab (2015)

A multinational gang of cybercriminals, known as *Carbanak*, infiltrated more than 100 banks across 30 countries and extracted as much as one billion dollars over a period of roughly two years

Forbes (2015)

Cyberattacks can result not only in direct financial losses and/or the loss of data, but also in an organization's highly valued asset - its reputation

World-wide spending on cybersecurity was approximately $75 billion in 2015, with the expectation that, by 2020, companies around the globe will be spending around $170 billion annually

Numerous companies and organizations have now realized that investing in cybersecurity is an imperative

Some Bibliography

- A. Nagurney, *Service Science* (2015): developed a multiproduct network economic model of cybercrime with a focus on financial services, since that industrial sector is a major target of cyberattacks
- A. Nagurney, L.S. Nagurney, *Netnomics* (2015): constructed a supply chain game theory model in which sellers maximize their expected profits while determining both their product transactions with consumers as well as their cybersecurity investments
- A. Nagurney, L.S. Nagurney, S. Shukla, in **Computation, Cryptography, and Network Security** (2015): extended the model to quantify and compute network vulnerability

Some Bibliography

A. Nagurney, P. D., S. Shukla, *Ann. Oper. Res.* (2017): introduced a novel game theory model in which the budget constraints for cybersecurity investments of retailers, which are nonlinear, are explicitly included, and conducted a spectrum of sensitivity analysis exercises

P.D., A. Maugeri, A. Nagurney, in **Operations Research, Engineering, and Cyber Security** (2017): provided an alternative formulation of the variational inequality and a deeper qualitative and economic analysis with a focus on the Lagrange multipliers associated with the constraints

The Model

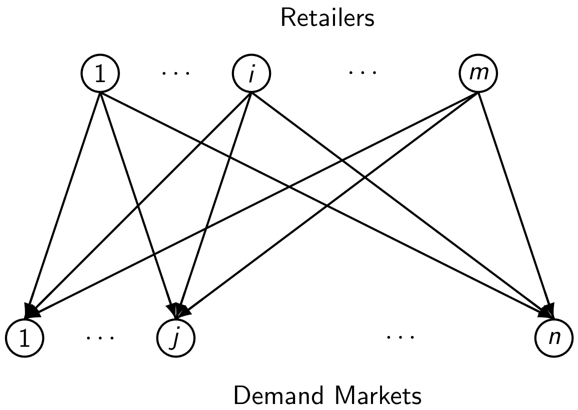

Figure: The Bipartite Structure of the Supply Chain Network Game Theory Model

The Model

Aim:

The retailers seek to maximize their individual expected utilities, consisting of expected profits, and compete in a noncooperative game in terms of strategies consisting of their respective product transactions and security levels

Conservation Law:

$$d_j = \sum_{i=1}^{m} Q_{ij}, \quad j = 1, \dots, n$$

The Model

Upper Bounds for Production Transactions

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad i = 1, \ldots, m; j = 1, \ldots, n$$

The Model

Upper Bounds for Production Transactions

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad i = 1, \dots, m; j = 1, \dots, n$$

Upper Bounds for Cybersecurity Levels

$$0 \leq s_i \leq u_{s_i}, \quad i = 1, \dots, m, \quad \text{where } u_{s_i} < 1$$

The Model

Upper Bounds for Production Transactions

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad i = 1, \ldots, m; j = 1, \ldots, n$$

Upper Bounds for Cybersecurity Levels

$$0 \leq s_i \leq u_{s_i}, \quad i = 1, \ldots, m, \quad \text{where } u_{s_i} < 1$$

Demand Price of the Product at Demand Market j

$$\rho_j(d, \bar{s}) \equiv \hat{\rho}_j(Q, \bar{s}); \quad j = 1, \ldots, n$$

The Model

Upper Bounds for Production Transactions

$$0 \leq Q_{ij} \leq \bar{Q}_{ij}, \quad i = 1, \ldots, m; j = 1, \ldots, n$$

Upper Bounds for Cybersecurity Levels

$$0 \leq s_i \leq u_{s_i}, \quad i = 1, \ldots, m, \quad \text{where } u_{s_i} < 1$$

Demand Price of the Product at Demand Market j

$$\rho_j(d, \bar{s}) \equiv \hat{\rho}_j(Q, \bar{s}); \quad j = 1, \ldots, n$$

Investment Cost Function Associated with Achieving a Security Level s_i

$$h_i(s_i) = \alpha_i \left(\frac{1}{\sqrt{1 - s_i}} - 1 \right) \text{ with } \alpha_i > 0$$

The Model

Budget Constraint

$$\alpha_i \left(\frac{1}{\sqrt{(1-s_i)}} - 1 \right) \leq B_i; \quad i = 1, \ldots, m,$$

The Model

Budget Constraint

$$\alpha_i \left(\frac{1}{\sqrt{(1-s_i)}} - 1 \right) \leq B_i; \quad i = 1, \ldots, m,$$

Profit of Retailer i

$$f_i(Q, s) = \sum_{j=1}^{n} \hat{\rho}_j(Q, s) Q_{ij} - c_i \sum_{j=1}^{n} Q_{ij} - \sum_{j=1}^{n} c_{ij}(Q_{ij})$$

Utility Optimization

Each retailer seeks to maximize his expected utility

$$\max E(U_i) = (1 - p_i) f_i(Q, s) + p_i (f_i(Q, s) - D_i) - h_i(s_i)$$
$$= f_i(Q, s) - p_i D_i - h_i(s_i)$$

where:

- D_i : damage incurred by retailer i
- $p_i = (1 - s_i)(1 - \bar{s}), \quad i = 1, \ldots, m$: probability of a successful cyberattack on retailer i

Nash Equilibrium

Definition (A Supply Chain Nash Equilibrium in Product Transactions and Security Levels)

A product transaction and security level pattern $(Q^*, s^*) \in \mathbb{K}$ is said to constitute a supply chain Nash equilibrium if for each retailer $i; i = 1, \dots, m$,

$$E(U_i(Q_i^*, s_i^*, \hat{Q}_i^*, \hat{s}_i^*)) \geq E(U_i(Q_i, s_i, \hat{Q}_i^*, \hat{s}_i^*)), \quad \forall (Q_i, s_i) \in \mathbb{K}^i,$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \dots, Q_{i-1}^*, Q_{i+1}^*, \dots, Q_m^*); \quad \text{and}$$

$$\hat{s_i^*} \equiv (s_1^*, \dots, s_{i-1}^*, s_{i+1}^*, \dots, s_m^*)$$

Nash Equilibrium

Definition (A Supply Chain Nash Equilibrium in Product Transactions and Security Levels)

A product transaction and security level pattern $(Q^*, s^*) \in \mathbb{K}$ is said to constitute a supply chain Nash equilibrium if for each retailer i; $i = 1, \ldots, m$,

$$E(U_i(Q_i^*, s_i^*, \hat{Q}_i^*, \hat{s}_i^*)) \geq E(U_i(Q_i, s_i, \hat{Q}_i^*, \hat{s}_i^*)), \quad \forall (Q_i, s_i) \in \mathbb{K}^i,$$

where

$$\hat{Q}_i^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_m^*); \quad \text{and}$$

$$\hat{s}_i^* \equiv (s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_m^*)$$

A supply chain Nash equilibrium is established if no retailer can unilaterally improve upon his expected utility (expected profit) by choosing an alternative vector of product transactions and security level.

Variational Inequality Formulation

Theorem

Assume that $E(U_i(Q,s))$, $i = 1, \dots, m$ is concave and continuously differentiable. Then $(Q^*, s^*) \in \mathbb{K}$ is a supply chain Nash equilibrium $\Longleftrightarrow$ if it satisfies variational inequality

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}} \times (Q_{ij} - Q_{ij}^*)$$

$$-\sum_{i=1}^{m}\frac{\partial E(U_i(Q^*,s^*))}{\partial s_i} \times (s_i - s_i^*) \geq 0, \quad \forall (Q,s) \in \mathbb{K}$$

Variational Inequality Formulation

Feasible Set

$$\mathbb{K} = \Big\{(Q,s) \in \mathbb{R}^{mn+n} : -Q_{ij} \leq 0,\ Q_{ij} - \overline{Q}_{ij} \leq 0,\ -s_i \leq 0,$$
$$s_i - u_{s_i} \leq 0, \quad h_i(s_i) - B_i \leq 0,\ i = 1, \ldots, m,\ j = 1, \ldots, n\Big\}$$

Variational Inequality Formulation

Feasible Set

$$\mathbb{K} = \Big\{(Q,s) \in \mathbb{R}^{mn+n} : -Q_{ij} \leq 0,\ Q_{ij} - \overline{Q}_{ij} \leq 0,\ -s_i \leq 0,$$

$$s_i - u_{s_i} \leq 0, \quad h_i(s_i) - B_i \leq 0,\ i = 1, \dots, m,\ j = 1, \dots, n\Big\}$$

Minimization Problem

$V(Q,s) \geq 0$ in $\mathbb{K}$ and $\min_{\mathbb{K}} V(Q,s) = V(Q^*, s^*) = 0$, where

$$V(Q,s) = -\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}} \left(Q_{ij} - Q^*_{ij}\right) - \sum_{i=1}^{m} \frac{\partial E(U_i(Q^*,s^*))}{\partial s_i} \left(s_i - s^*_i\right)$$

The Lagrange Theory

Lagrange Function

$$
\begin{aligned}
\mathcal{L}(Q, s, \lambda^1, \lambda^2, \mu^1, \mu^2, \lambda) = & -\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} (Q_{ij} - Q_{ij}^*) \\
& -\sum_{i=1}^{m} \frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} (s_i - s_i^*) + \sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_{ij}^1(-Q_{ij}) \\
& +\sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_{ij}^2(Q_{ij} - \overline{Q}_{ij}) + \sum_{i=1}^{m} \mu_i^1(-s_i) \\
& +\sum_{i=1}^{m} \mu_i^2(s_i - u_{s_i}) + \sum_{i=1}^{m} \lambda_i(h_i(s_i) - B_i),
\end{aligned}
$$

where $(Q, s) \in \mathbb{R}^{mn+n}$, $\lambda^1, \lambda^2 \in \mathbb{R}^{mn}_+$, $\mu^1, \mu^2 \in \mathbb{R}^m_+$,

The Lagrange Theory

Theorem (Saddle Point)

There exist $\overline{\lambda}^1, \overline{\lambda}^2 \in \mathbb{R}^{mn}_+$, $\overline{\mu}^1, \overline{\mu}^2, \overline{\lambda} \in \mathbb{R}^m_+$ *such that the vector* $(Q^*, s^*, \overline{\lambda}^1, \overline{\lambda}^2, \overline{\mu}^1, \overline{\mu}^2, \overline{\lambda})$ *is a saddle point of the Lagrange function; namely,*

$$\begin{aligned} \mathcal{L}(Q^*, s^*, \lambda^1, \lambda^2, \mu^1, \mu^2, \lambda) &\leq \mathcal{L}(Q^*, s^*, \overline{\lambda}^1, \overline{\lambda}^2, \overline{\mu}^1, \overline{\mu}^2, \overline{\lambda}) \\ &\leq \mathcal{L}(Q, s, \overline{\lambda}^1, \overline{\lambda}^2, \overline{\mu}^1, \overline{\mu}^2, \overline{\lambda}) \end{aligned}$$

$\forall (Q, s) \in \mathbb{K}$, $\forall \lambda^1, \lambda^2 \in \mathbb{R}^{mn}_+$, $\forall \mu^1, \mu^2, \lambda \in \mathbb{R}^m_+$ *and*

$$\overline{\lambda}^1_{ij}(-Q^*_{ij}) = 0, \quad \overline{\lambda}^2_{ij}(Q^*_{ij} - \overline{Q}_{ij}) = 0, \quad \forall i, \ \forall j$$

$$\overline{\mu}^1_i(-s^*_i) = 0, \quad \overline{\mu}^2_i(s^*_i - u_{s_i}) = 0, \quad \overline{\lambda}_i(h_i(s^*_i) - B_i) = 0, \quad \forall i$$

The Lagrange Theory

It follows that $(Q^*, s^*) \in \mathbb{R}_+^{mn+n}$ is a minimal point of $\mathcal{L}(Q, s, \overline{\lambda}^1, \overline{\lambda}^2, \overline{\mu}^1, \overline{\mu}^2, \overline{\lambda})$ in the whole space $\mathbb{R}^{mn+n}$ and, hence, for all $i = 1, \ldots, m$, and $j = 1, \ldots, n$, we get:

$$\frac{\partial \mathcal{L}(Q^*, s^*, \overline{\lambda}^1, \overline{\lambda}^2, \overline{\mu}^1, \overline{\mu}^2, \overline{\lambda})}{\partial Q_{ij}} = -\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} - \overline{\lambda}_{ij}^1 + \overline{\lambda}_{ij}^2 = 0$$

$$\frac{\partial \mathcal{L}(Q^*, s^*, \overline{\lambda}^1, \overline{\lambda}^2, \overline{\mu}^1, \overline{\mu}^2, \overline{\lambda})}{\partial s_i} = -\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} - \overline{\mu}_i^1 + \overline{\mu}_i^2 + \overline{\lambda}_i \frac{\partial h_i(s_i^*)}{\partial s_i} = 0$$

which represent an equivalent formulation of the variational inequality

Expected Utilities

We define:

- $\dfrac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}}$: the *marginal expected transaction utility*, $i = 1, \ldots, m$, $j = 1, \ldots, n$,

Expected Utilities

We define:

- $\dfrac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}}$: the *marginal expected transaction utility*, $i = 1, \dots, m$, $j = 1, \dots, n$,
- $\dfrac{\partial E(U_i(Q^*, s^*))}{\partial s_i}$: the *marginal expected cybersecurity investment utility*, $i = 1, \dots, m$

Expected Utilities

We define:

- $\dfrac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}}$: the *marginal expected transaction utility*, $i = 1, \ldots, m$, $j = 1, \ldots, n$,
- $\dfrac{\partial E(U_i(Q^*, s^*))}{\partial s_i}$: the *marginal expected cybersecurity investment utility*, $i = 1, \ldots, m$

$\overline{\lambda}_{ij}^1$, $\overline{\lambda}_{ij}^2$ give a precise evaluation of the behavior of the market with respect to the supply chain product transactions as well as $\overline{\mu}_i^1$, $\overline{\mu}_i^2$ describe the effects of the marginal expected cybersecurity investment utilities.

We get

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} - \overline{\lambda}_{ij}^1 + \overline{\lambda}_{ij}^2 = 0, \quad i = 1, \ldots, m, \ j = 1, \ldots, n.$$

So, if $0 < Q_{ij}^* < \overline{Q}_{ij}$, then we get $\forall i = 1, \ldots, m, \ j = 1, \ldots, n:$

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} = c_i + \frac{\partial c_{ij}(Q_{ij}^*)}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k}{\partial Q_{ij}} \times Q_{ik}^* = 0,$$

We get

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} - \overline{\lambda}^1_{ij} + \overline{\lambda}^2_{ij} = 0, \quad i = 1, \dots, m, \ j = 1, \dots, n.$$

So, if $0 < Q^*_{ij} < \overline{Q}_{ij}$, then we get $\forall i = 1, \dots, m, \ j = 1, \dots, n:$

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} = c_i + \frac{\partial c_{ij}(Q^*_{ij})}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k}{\partial Q_{ij}} \times Q^*_{ik} = 0,$$

whereas if $\overline{\lambda}^1_{ij} > 0$, and, hence, $Q^*_{ij} = 0$, and $\overline{\lambda}^2_{ij} = 0$, we get

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial Q_{ij}} = c_i + \frac{\partial c_{ij}(Q^*_{ij})}{\partial Q_{ij}} - \hat{\rho}_j(Q^*, s^*) - \sum_{\substack{k=1 \\ k \neq i}}^{m} \frac{\partial \hat{\rho}_k}{\partial Q_{ij}} \times Q^*_{ik} = \overline{\lambda}^1_{ij},$$

We get

$$-\frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}}-\overline{\lambda}^1_{ij}+\overline{\lambda}^2_{ij}=0,\quad i=1,\dots,m,\ j=1,\dots,n.$$

So, if $0<Q^*_{ij}<\overline{Q}_{ij}$, then we get $\forall i=1,\dots,m,\ j=1,\dots,n:$

$$-\frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}}=c_i+\frac{\partial c_{ij}(Q^*_{ij})}{\partial Q_{ij}}-\hat{\rho}_j(Q^*,s^*)-\sum_{k=1}^{m}\frac{\partial\hat{\rho}_k}{\partial Q_{ij}}\times Q^*_{ik}=0,$$

whereas if $\overline{\lambda}^1_{ij}>0$, and, hence, $Q^*_{ij}=0$, and $\overline{\lambda}^2_{ij}=0$, we get

$$-\frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}}=c_i+\frac{\partial c_{ij}(Q^*_{ij})}{\partial Q_{ij}}-\hat{\rho}_j(Q^*,s^*)-\sum_{\substack{k=1\\k\neq i}}^{m}\frac{\partial\hat{\rho}_k}{\partial Q_{ij}}\times Q^*_{ik}=\overline{\lambda}^1_{ij},$$

and if $\overline{\lambda}^2_{ij}>0$, and, hence, $Q^*_{ij}=\overline{Q}_{ij}$, and $\overline{\lambda}^1_{ij}=0$, we have

$$-\frac{\partial E(U_i(Q^*,s^*))}{\partial Q_{ij}}=c_i+\frac{\partial c_{ij}(Q^*_{ij})}{\partial Q_{ij}}-\hat{\rho}_j(Q^*,s^*)-\sum_{\substack{k=1\\k\neq i}}^{m}\frac{\partial\hat{\rho}_k}{\partial Q_{ij}}\times Q^*_{ik}=-\overline{\lambda}^2_{ij},$$

Analysis of Marginal Expected Cybersecurity Investment Utilities

We have $\forall i = 1, \ldots, m$:

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} - \overline{\mu}_i^1 + \overline{\mu}_i^2 + \overline{\lambda}_i \frac{\partial h_i(s^*)}{\partial s_i} = 0,$$

If $0 < s_i^* < u_{s_i}$, then $\overline{\mu}_i^1 = \overline{\mu}_i^2 = 0$ and we have

$$\frac{\partial h_i(s_i^*)}{\partial s_i} + \overline{\lambda}_i \frac{\partial h_i(s_i^*)}{\partial s_i}$$
$$= \left(1 - \sum_{k=1}^{m} \frac{s_k^*}{m} + \frac{1 - s_i^*}{m}\right) D_i + \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} \times Q_{ik}^*.$$

Analysis of Marginal Expected Cybersecurity Investment Utilities

We have $\forall i = 1, \dots, m$:

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i} - \overline{\mu}_i^1 + \overline{\mu}_i^2 + \overline{\lambda}_i \frac{\partial h_i(s^*)}{\partial s_i} = 0,$$

If $0 < s_i^* < u_{s_i}$, then $\overline{\mu}_i^1 = \overline{\mu}_i^2 = 0$ and we have

$$\frac{\partial h_i(s_i^*)}{\partial s_i} + \overline{\lambda}_i \frac{\partial h_i(s_i^*)}{\partial s_i}$$
$$= \left(1 - \sum_{k=1}^{m} \frac{s_k^*}{m} + \frac{1 - s_i^*}{m}\right) D_i + \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} \times Q_{ik}^*.$$

Since $0 < s_i^* < u_{s_i}$, $h(s_i^*)$ cannot be the upper bound B_i; hence, $\overline{\lambda}_i$ is zero and hence

$$\frac{\partial h_i(s_i^*)}{\partial s_i} = \left(1 - \sum_{k=1}^{m} \frac{s_k^*}{m} + \frac{1 - s_i^*}{m}\right) D_i + \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} \times Q_{ik}^*$$

Analysis of Marginal Expected Cybersecurity Investment Utilities

If $\overline{\mu}_i^1 > 0$ and, hence, $s_i^* = 0$, and $\overline{\mu}_i^2 = 0$, we get:

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i}$$

$$= \frac{\partial h_i(0)}{\partial s_i} - \left(1 - \sum_{\substack{k=1 \\ k \neq i}}^{m} \frac{s_k^*}{m} + \frac{1 - s_i^*}{m} \right) D_i - \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} Q_{ik}^* = \overline{\mu}_i^1 .$$

Analysis of Marginal Expected Cybersecurity Investment Utilities

Patrizia Daniele, A. Maugeri, A. Nagurney

If $\overline{\mu}_i^1 > 0$ and, hence, $s_i^* = 0$, and $\overline{\mu}_i^2 = 0$, we get:

$$-\frac{\partial E(U_i(Q^*, s^*))}{\partial s_i}$$

$$= \frac{\partial h_i(0)}{\partial s_i} - \left(1 - \sum_{\substack{k=1\\k\neq i}}^{m} \frac{s_k^*}{m} + \frac{1 - s_i^*}{m}\right) D_i - \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} Q_{ik}^* = \overline{\mu}_i^1.$$

In contrast, if $\overline{\mu}_i^2 > 0$ and, hence, $s_i^* = u_{s_i}$, retailer j has a marginal gain given by $\overline{\mu}_i^2$, because

$$-\frac{\partial E(U_i(Q^*, u_{s_i}))}{\partial s_i} = -\left(1 - \sum_{\substack{k=1\\k\neq i}}^{m} \frac{u_{s_k}}{m} + \frac{1 - u_{s_i}}{m}\right) D_i$$

$$- \sum_{k=1}^{m} \frac{\partial \hat{\rho}_k(Q^*, s^*)}{\partial s_i} \times Q_{ik}^* + \frac{\partial h_i(u_{s_i})}{\partial s_i} + \overline{\lambda}_i \frac{\partial h_i(u_{s_i})}{\partial s_i} = -\overline{\mu}_i^2.$$

A Numerical Example

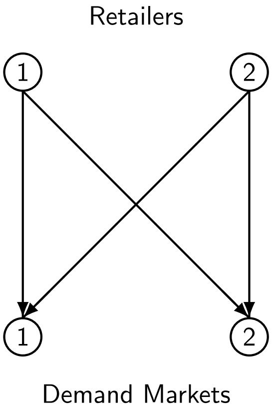

Figure: Network Topology

A Numerical Example

Cost Functions

$$
\begin{aligned}
c_1 &= 5, & c_2 &= 10,\\
c_{11}(Q_{11}) &= .5Q_{11}^2 + Q_{11}, & c_{12}(Q_{12}) &= .25Q_{12}^2 + Q_{12},\\
c_{21}(Q_{21}) &= .5Q_{21}^2 + Q_{21}, & c_{22}(Q_{22}) &= .25Q_{22}^2 + Q_{22}
\end{aligned}
$$

A Numerical Example

Cost Functions

$$
\begin{aligned}
c_1 &= 5, & c_2 &= 10,\\
c_{11}(Q_{11}) &= .5Q_{11}^2 + Q_{11}, & c_{12}(Q_{12}) &= .25Q_{12}^2 + Q_{12},\\
c_{21}(Q_{21}) &= .5Q_{21}^2 + Q_{21}, & c_{22}(Q_{22}) &= .25Q_{22}^2 + Q_{22}
\end{aligned}
$$

Demand Price Functions

$$\rho_1(d,\bar{s}) = -d_1 + .1\frac{s_1+s_2}{2} + 100, \quad \rho_2(d,\bar{s}) = -.5d_2 + .2\frac{s_1+s_2}{2} + 200$$

A Numerical Example

Patrizia Daniele, A. Maugeri, A. Nagurney

Cost Functions

$$
\begin{aligned}
c_1 &= 5, & c_2 &= 10,\\
c_{11}(Q_{11}) &= .5Q_{11}^2 + Q_{11}, & c_{12}(Q_{12}) &= .25Q_{12}^2 + Q_{12},\\
c_{21}(Q_{21}) &= .5Q_{21}^2 + Q_{21}, & c_{22}(Q_{22}) &= .25Q_{22}^2 + Q_{22}
\end{aligned}
$$

Demand Price Functions

$$\rho_1(d,\bar{s}) = -d_1 + .1\frac{s_1+s_2}{2} + 100, \quad \rho_2(d,\bar{s}) = -.5d_2 + .2\frac{s_1+s_2}{2} + 200$$

Damage Parameters and Budgets

$D_1 = 200$ and $D_2 = 210$, $B_1 = B_2 = 2.5$ in millions of \$

A Numerical Example

Equilibrium Solution

$$Q^*_{11} = 24.148, \quad Q^*_{21} = 21.586, \quad Q^*_{12} = 99.16, \quad Q^*_{22} = 94.16,$$

$$\overline{\mu}^2_1 = 19.6055, \quad \overline{\mu}^2_2 = 20.3273,$$

where $\overline{\mu}^2_1$ and μ^2_2 are the positive marginal expected gains.

A Numerical Example

Equilibrium Solution

$$Q^*_{11} = 24.148, \quad Q^*_{21} = 21.586, \quad Q^*_{12} = 99.16, \quad Q^*_{22} = 94.16,$$

$$\overline{\mu}^2_1 = 19.6055, \quad \overline{\mu}^2_2 = 20.3273,$$

where $\overline{\mu}^2_1$ and μ^2_2 are the positive marginal expected gains.

If we double the value of the damage for the first retailer and assume now $D_1 = 400$, then the new value of the Lagrange multiplier is $\overline{\mu}^2_1 = 46.6055$.

Conclusions

- Cyberattacks are negatively globally impacting numerous sectors of economies

Conclusions

- Cyberattacks are negatively globally impacting numerous sectors of economies
- Organizations are investing in cybersecurity

Conclusions

- Cyberattacks are negatively globally impacting numerous sectors of economies
- Organizations are investing in cybersecurity
- Retailers compete in both product transactions and cybersecurity levels seeking to maximize their expected utilities

Conclusions

- Cyberattacks are negatively globally impacting numerous sectors of economies
- Organizations are investing in cybersecurity
- Retailers compete in both product transactions and cybersecurity levels seeking to maximize their expected utilities
- The governing equilibrium concept is that of Nash equilibrium

Conclusions

- Cyberattacks are negatively globally impacting numerous sectors of economies
- Organizations are investing in cybersecurity
- Retailers compete in both product transactions and cybersecurity levels seeking to maximize their expected utilities
- The governing equilibrium concept is that of Nash equilibrium
- We perform an analysis of both the marginal expected transaction utilities and the marginal expected cybersecurity investment utilities of the retailers