An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England

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Source: http://www.nasa.gov

Outline

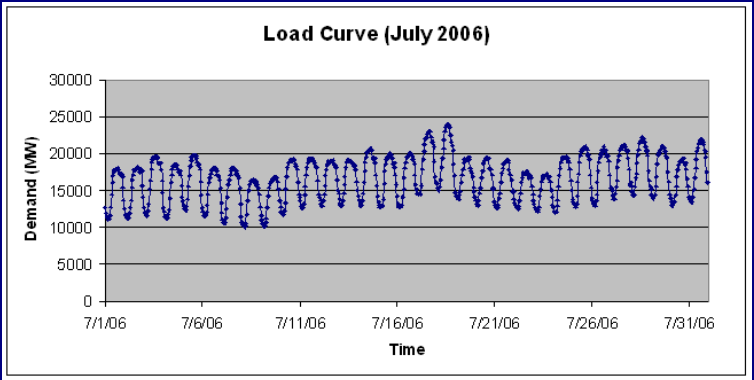
- Introduction
- Literature review
- An integrated electric power supply chain and fuel market network framework
- Empirical case study and examples
- Conclusions.

Electric Power Supply Chains and Fuel Suppliers

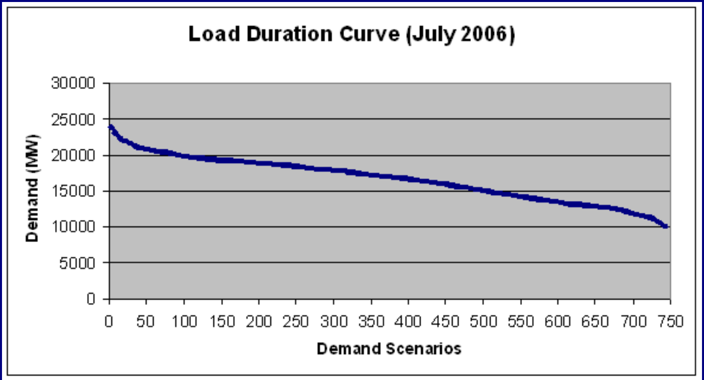
Electric Power Supply Chains (Cont'd)

- The U.S. electric power industry: Half a trillion dollars of net assets, $220 billion annual sales, 40% of domestic primary energy (Energy Information Administration (2000, 2005))
- Deregulation
  - Wholesale market
  - Bilateral contract
  - Power pool.
- Electric power supply chain networks
  - Various generation technologies
  - Insensitive demands
  - Transmission congestion
  - In 2007, the total transmission congestion cost in New England was about $130 million (ISO New England Annual Market Report, 2007).

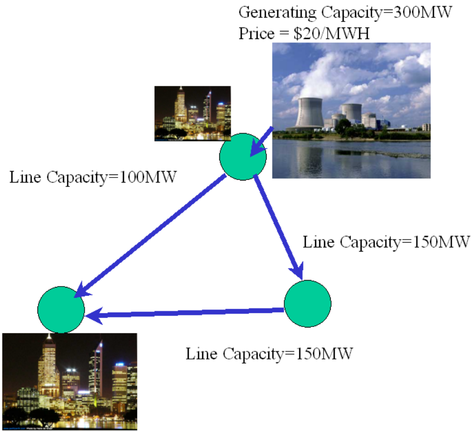
Load Curve


Load Curve (July 2006)
Demand (MW)
30000
25000
20000
15000
10000
5000
0
7/1/06
7/6/06
7/11/06
7/16/06
7/21/06
7/26/06
7/31/06
Time

Load Duration Curve

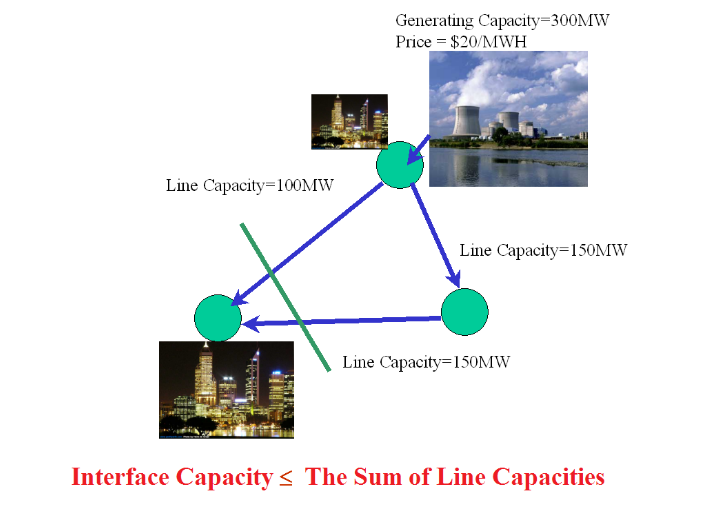


A Simple Example of Transmission Congestion

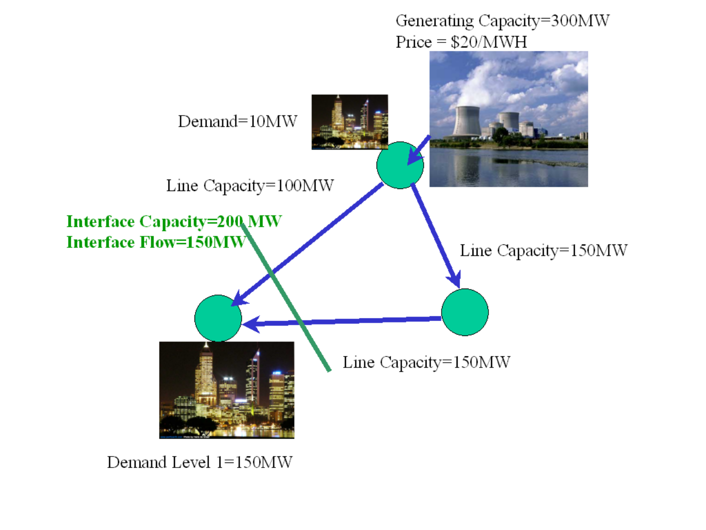


Power Flow is determined by Kirchoff's Laws.

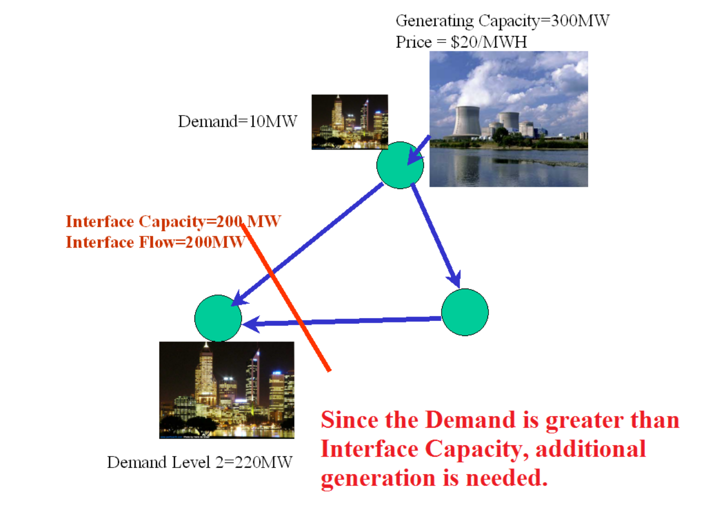
A Simple Example of Transmission Congestion



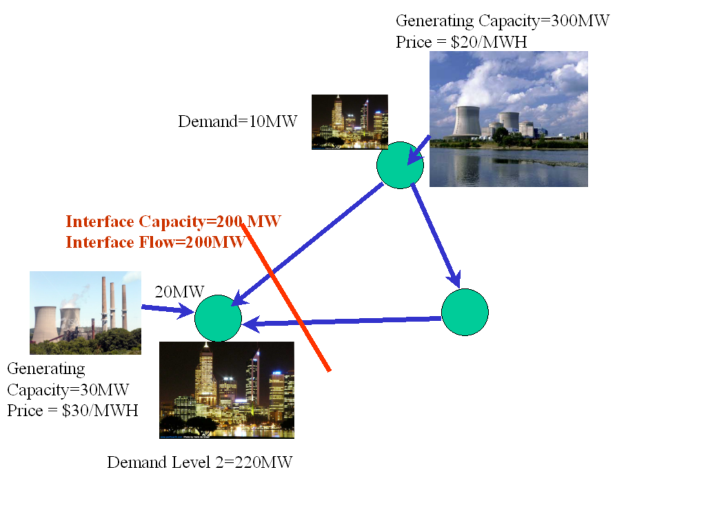
A Simple Example of Transmission Congestion



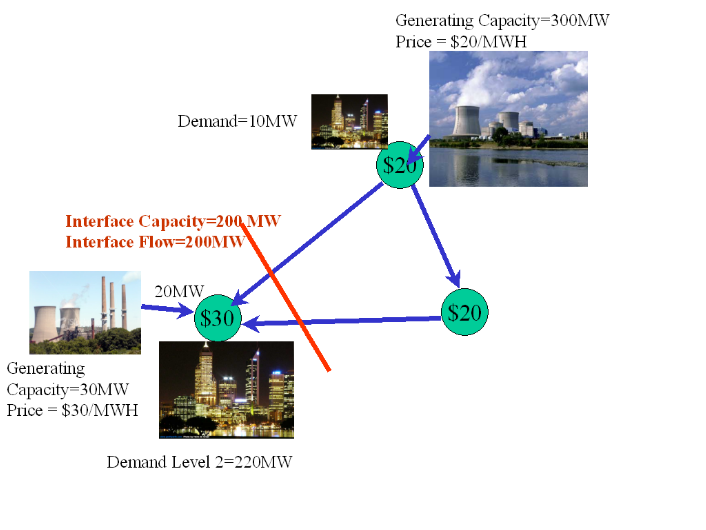
A Simple Example of Transmission Congestion



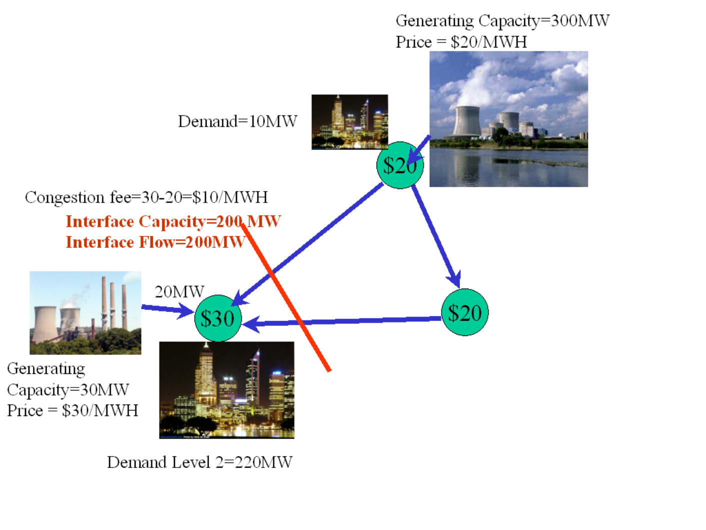
A Simple Example of Transmission Congestion



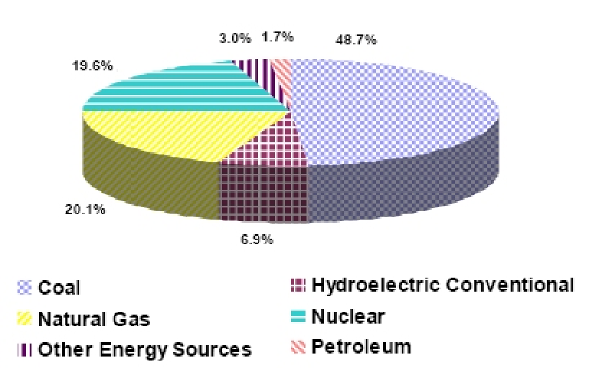
A Simple Example of Transmission Congestion



A Simple Example of Transmission Congestion



Sources of Electricity in the U.S. in 2007



Source: http://www.eia.doe.gov

Electric Power Supply Chains and Fuel Markets

- In the U.S., electric power generation accounts for significant portions of fuel demands
  - 30% of the natural gas demand (over 50% in the summer)
  - 90% of the coal demand
  - over 45% of the residual fuel oil demand.

Electric Power Supply Chains and Fuel Markets (Cont'd)

The interactions between electric power supply chains and fuel markets affect demands and prices of electric power and fuels.

- From December 1, 2005 to April 1, 2006, the wholesale electricity price in New England decreased by 38% mainly because the delivered natural gas price declined by 45%.
- In August, 2006, the natural gas price jumped 14% because hot weather across the U.S. led to elevated demand for electricity. This high electricity demand also caused the crude oil price to rise by 1.6%.

Electric Power Supply Chains and Fuel Markets (Cont'd)

The availability and the reliability of diversified fuel supplies also affect national security.

- In January 2004, over 7000MW of electric power generation, which accounts for almost one fourth of the total capacity of New England, was unavailable during the electric system peak due to the limited natural gas supply.
- The American Association of Railroads has requested that the Federal Energy Regulatory Commission (FERC) investigate the reliability of the energy supply chain with a focus on electric power and coal transportation.

Literature Review

- Beckmann, McGuire, and Winsten (1956): How are electric power flows related to transportation flows?
- Electric power wholesale and retail markets
  - Smeers (1997), Hogan (1992), Chao and Peck (1996), Casazza and Delea (2003), Hobbs and Pang (2003), Borenstein and Holland (2003), and Garcia, Campos, and Reitzes (2005), etc.
- Electric power markets and fuel markets
  - Emery and Liu (2001), Bessembinder and Lemmon (2002), Huntington and Schuler (1997), Brown and Yucel (2007), etc.

Literature Review (Cont'd)

- A. Nagurney and D. Matsypura, "A Supply Chain Network Perspective for Electric Power Generation, Supply, Transmission, and Consumption," in **Optimisation, Econometric and Financial Analysis**, E. J. Kontoghiorghes and C. Gatu, Editors (2006) Springer, Berlin, Germany, pp 3-27
- A. Nagurney, Z. Liu, M. G. Cojocaru, and P. Daniele, "Dynamic electric power supply chains and transportation networks: An evolutionary variational inequality formulation," *Transportation Research E* 43 (2007), 624-646
- D. Matsypura, A. Nagurney, and Z. Liu, "Modeling of electric power supply chain networks with fuel suppliers via variational inequalities," *International Journal of Emerging Electric Power Systems* 8 (2007), 1, Article 5.

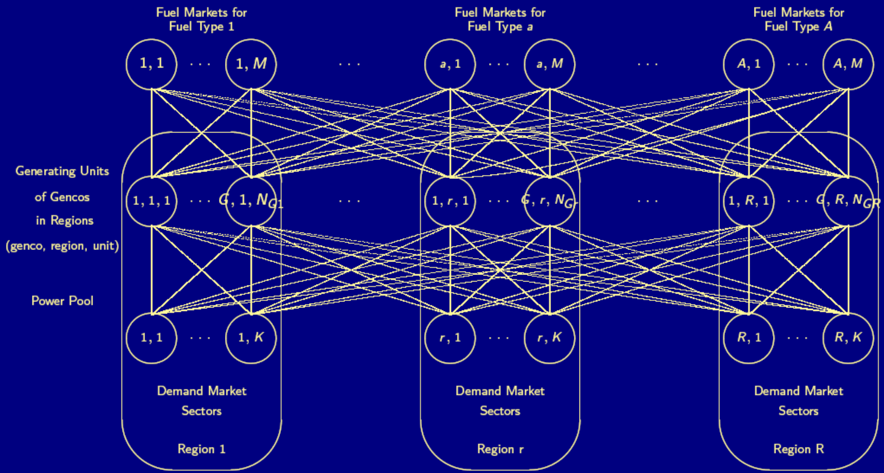
"An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England", Zugang Liu and Anna Nagurney, 2007

This paper can be downloaded at: http://supernet.som.umass.edu/dart.html.

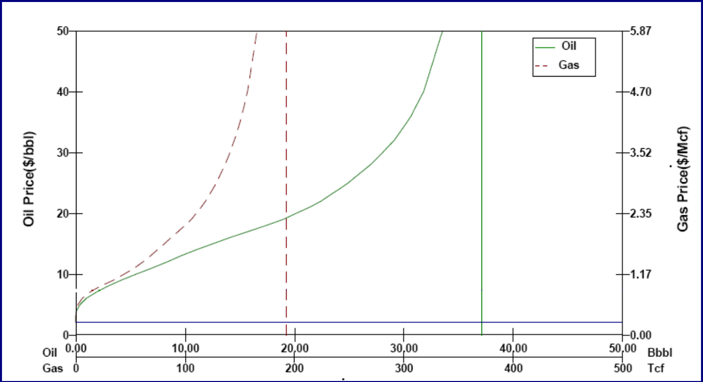
Contributions

- The model captures both economic transactions and physical transmission constraints.
- The model considers the behaviors of all major decision makers including gencos, consumers and the independent system operator (ISO).
- The model considers multiple fuel markets, electricity wholesale markets, and operating reserve markets.
- The model is applied to the New England electric power supply chain consisting of 6 states, 5 fuel types, 82 power generators, with a total of 573 generating units, and 10 demand markets.

The Electric Power Supply Chain Network with Fuel Supply Markets



Energy Fuel Supply Curves



Source: Minerals Management Service, Gulf of Mexico Region

The Equilibrium Conditions for the Fuel Supply Markets

Assume that the following conservation of flow equations must hold for all fuel supply markets $a = 1, \dots, A$; $m = 1, \dots, M$:

$$\sum_{w=1}^{W} \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} q^{am}_{gr_1uw} + \bar{q}_{am} = h_{am}.$$

The (spatial price) equilibrium conditions (cf. Nagurney (1999)) for suppliers at fuel supply market am; $a = 1, ..., A$; $m = 1, ..., M$, take the form: for each generating unit gr_1u; $g = 1, ..., G$; $r_1 = 1, ..., R$; $u = 1, ..., N_{gr_1}$, and at each demand level w:

$$\pi_{am}(h^*) + c^{am}_{gr_1uw} \begin{cases} = \rho^{am*}_{gr_1uw}, & \text{if} \quad q^{am*}_{gr_1uw} > 0, \\ \geq \rho^{am*}_{gr_1uw}, & \text{if} \quad q^{am*}_{gr_1uw} = 0. \end{cases}$$

Power Generator's Maximization Problem

- Multiple power plants
- Dual-fuel power plants
- Revenue
  - Bilateral contracts
  - Power pool
  - Operating reserve markets.
- Cost
  - Fuel cost
  - Operating cost
  - Transaction cost
  - Congestion cost.

Power Generator's Maximization Problem (Cont'd)

$$\text{Maximize} \quad \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} \rho_{r_2kw}^{gr_1u*} q_{r_2kw}^{gr_1u}$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \rho_{r_2w}^{*} y_{r_2w}^{gr_1u} + \sum_{w=1}^{W} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} L_w \varphi_{r_1w}^{*} z_{gr_1uw}$$

$$-\sum_{w=1}^{W} \sum_{a=1}^{A} \sum_{m=1}^{M} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \rho_{gr_1uw}^{am*} q_{gr_1uw}^{am}$$

$$-\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} f_{gr_1uw}(q_{gr_1uw}) - \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{k=1}^{K} c_{r_2kw}^{gr_1u}(q_{r_2kw}^{gr_1u})$$

$$-\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} c_{r_2w}^{gr_1u}(y_{r_2w}^{gr_1u}) - \sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} c_{gr_1uw}(z_{gr_1uw})$$

$$-\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} \sum_{r_2=1}^{R} \sum_{b=1}^{B} \mu_{bw}^{*} \alpha_{r_1r_2b} [\sum_{k=1}^{K} q_{r_2kw}^{gr_1u} + y_{r_2w}^{gr_1u}]$$

Power Generator's Maximization Problem (Cont'd)

subject to:

$$\sum_{r_2=1}^{R}\sum_{k=1}^{K} q^{gr_1u}_{r_2kw} + \sum_{r_2=1}^{R} y^{gr_1u}_{r_2w} = q_{gr_1uw}, \quad r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; \; w = 1, ..., W,$$

$$\sum_{a=1}^{A} \beta_{gr_1ua} \sum_{m=1}^{M} q^{am}_{gr_1uw} + L_w \beta_{gr_1u0} q_{gr_1uw} = L_w q_{gr_1uw}, \quad r_1 = 1, ..., R;$$

$$u = 1, ..., N_{gr_1}; \; w = 1, ..., W,$$

$$q_{gr_1uw} + z_{gr_1uw} \leq Cap_{gr_1u}, \quad r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; \; w = 1, ..., W,$$

$$z_{gr_1uw} \leq OP_{gr_1u}, \quad r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; \; w = 1, ..., W,$$

$$q^{gr_1u}_{r_2kw} \geq 0, \quad r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; \; r_2 = 1, ..., R; \; k = 1, ..., K; \; w = 1, ..., W,$$

$$q^{am}_{gr_1uw} \geq 0, \quad a = 1, ..., A; \; m = 1, ..., M; \; r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; w = 1, \ldots, W,$$

$$y^{gr_1u}_{r_2w} \geq 0, \quad r_1 = 1, ..., R; \; u = 1, ..., N_{gr_1}; \; r_2 = 1, ..., R; \; w = 1, .., W,$$

$$z_{gr_1uw} \geq 0, \quad r_1 = 1, ..., R; \; u = N_{gr_1}; \; w = 1, ..., W$$

The ISO's Role

- Manages the power pool.
- Schedules transmission.
- Manages congestion.
- Ensures system reliability.

The ISO's Role

The ISO ensures that the regional electricity markets $r = 1, \ldots, R$ clear at each demand level $w = 1, \ldots, W$, that is,

$$\sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} y_{rw}^{gr_1 u*} \begin{cases} = \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2 kw}^{r*}, & \text{if} \quad \rho_{rw}^* > 0, \\ \geq \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2 kw}^{r*}, & \text{if} \quad \rho_{rw}^* = 0. \end{cases}$$

The ISO also ensures that the regional operating reserve markets; hence, $r_1 = 1, \ldots, R$ clear at each demand level $w = 1, \ldots, W$, that is,

$$\sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} z_{gr_1 uw}^* \begin{cases} = OPR_{r_1 w}, & \text{if} \quad \varphi_{r_1 w}^* > 0, \\ \geq OPR_{r_1 w}, & \text{if} \quad \varphi_{r_1 w}^* = 0. \end{cases}$$

The following conditions must hold for each interface b and at each demand level w, where $b = 1, \ldots, B$; $w = 1, \ldots, W$:

$$\sum_{r_1=1}^{R} \sum_{r_2=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2 kw}^{gr_1 u*} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2 w}^{gr_1 u*} + \sum_{k=1}^{K} y_{r_2 kw}^{r_1 *}] \alpha_{r_1 r_2 b} \begin{cases} = TCap_b, & \text{if} \quad \mu_{bw}^* > 0, \\ \leq TCap_b, & \text{if} \quad \mu_{bw}^* = 0. \end{cases}$$

The Equilibrium Conditions for the Demand Markets

We assume that all demand markets have fixed and known demands. and the following conservation of flow equations, hence, must hold for all demand markets $k = 1, \ldots, K_{r_2}$, all regions $r_2 = 1, \ldots, R$, and at all demand levels $w = 1, \ldots, W$:

$$\sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} q_{r_2 kw}^{gr_1 u*} + \sum_{r_1=1}^{R} y_{r_2 kw}^{r_1 *} = (1 + \kappa_{r_2 w}) d_{r_2 kw}.$$

The equilibrium conditions for consumers at demand market k in region r_2 take the form: for each power plant u; $u = 1, ..., U_{r_1 g}$; each generator $g = 1, ..., G$; each region $r_1 = 1, ..., R$, and each demand level w; $w = 1, \ldots, W$:

$$\rho_{r_2 kw}^{gr_1 u*} + \hat{c}_{r_2 kw}^{gr_1 u}(Q_w^{2*}) \begin{cases} = \rho_{r_2 kw}^{*}, & \text{if} \quad q_{r_2 kw}^{gr_1 u*} > 0, \\ \geq \rho_{r_2 kw}^{*}, & \text{if} \quad q_{r_2 kw}^{gr_1 u*} = 0; \end{cases}$$

and

$$\rho_{r_1 w}^{*} + \sum_{b=1}^{B} \mu_{bw}^{*} \alpha_{r_1 r_2 b} + \hat{c}_{r_2 kw}^{r_1}(Y_w^{2*}) \begin{cases} = \rho_{r_2 kw}^{*}, & \text{if} \quad y_{r_2 kw}^{r_1 *} > 0, \\ \geq \rho_{r_2 kw}^{*}, & \text{if} \quad y_{r_2 kw}^{r_1 *} = 0. \end{cases}$$

Definition: Electric Power Supply Chain Network Equilibrium

The equilibrium state of the electric power supply chain network with fuel supply markets is one where the fuel flows and electric power flows and prices satisfy the equilibrium conditions for the fuel markets, the optimality conditions for the power generators, the equilibrium conditions for the demand markets, and the equilibrium conditions for the ISO.

Theorem: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium

The equilibrium conditions governing the electric power supply chain network coincide with the solution of the variational inequality given by: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*) \in \mathcal{K}_1$ *satisfying*

$$\sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[\pi_{am}(Q^{1*}) + c^{am}_{gr_1uw}\right] \times [q^{am}_{gr_1uw} - q^{am*}_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[\frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}} + \eta^*_{gr_1uw}\right] \times [q_{gr_1uw} - q^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K}\left[\frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u*}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2*}_w)\right] \times [q^{gr_1u}_{r_2kw} - q^{gr_1u*}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\left[\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u*}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} + \sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b} - \rho^*_{r_2w}\right] \times [y^{gr_1u}_{r_2w} - y^{gr_1u*}_{r_2w}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[\frac{\partial c_{gr_1uw}(z^*_{gr_1uw})}{\partial z_{gr_1uw}} + \lambda^*_{gr_1uw} + \eta^*_{gr_1uw} - \varphi^*_{r_1w}\right] \times [z_{gr_1uw} - z^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K}\left[\rho^*_{r_1w} + \hat{c}^{r_1}_{r_2kw}(Y^{2*}_w) + \sum_{b=1}^{B}\mu^*_{bw}\alpha_{r_1r_2b}\right] \times [y^{r_1}_{r_2kw} - y^{r_1*}_{r_2kw}]$$

Theorem: Variational Inequality Formulation of the Electric Power Supply Chain Network Equilibrium (Cont'd)

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[Cap_{gr_1u} - q^*_{gr_1uw} - z^*_{gr_1uw}\right] \times [\eta_{gr_1uw} - \eta^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} \left[OP_{gr_1u} - z^*_{gr_1uw}\right] \times [\lambda_{gr_1uw} - \lambda^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{b=1}^{B}[TCap_b - \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}}\sum_{k=1}^{K} q^{gr_1u*}_{r_2kw} + \sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} y^{gr_1u*}_{r_2w} + \sum_{k=1}^{K} y^{r_1*}_{r_2kw}]\alpha_{r_1r_2b}] \times [\mu_{bw} - \mu^*_{bw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r=1}^{R}[\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} y^{gr_1u*}_{rw} - \sum_{r_2=1}^{R}\sum_{k=1}^{K} y^{r*}_{r_2kw}] \times [\rho_{rw} - \rho^*_{rw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}[\sum_{g=1}^{G}\sum_{u=1}^{N_{gr_1}} z^*_{gr_1uw} - OPR_{r_1}] \times [\varphi_{r_1w} - \varphi^*_{r_1w}] \geq 0,$$

$$\forall (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in \mathcal{K}_1, \qquad (1)$$

where $\mathcal{K}_1 \equiv \{(Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) | (Q^1, q, Q^2, Y^1, Y^2, Z, \eta, \lambda, \mu, \rho_3, \varphi) \in R_+^{AMNW+NRKW+NRW+4NW+R^2KW+BW+2RW}$ and the conservation of flow equations hold$\}$.

Theorem: Existence

If $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*)$ satisfies variational inequality (1) then $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*)$ is a solution to the variational inequality problem: determine $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*) \in \mathcal{K}_2$ satisfying

$$\sum_{w=1}^{W}\sum_{a=1}^{A}\sum_{m=1}^{M}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[\pi_{am}(Q^{1*}) + c^{am}_{gr_1uw}\right] \times [q^{am}_{gr_1uw} - q^{am*}_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\left[\frac{\partial f_{gr_1uw}(q^*_{gr_1uw})}{\partial q_{gr_1uw}}\right] \times [q_{gr_1uw} - q^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\sum_{k=1}^{K}\left[\frac{\partial c^{gr_1u}_{r_2kw}(q^{gr_1u*}_{r_2kw})}{\partial q^{gr_1u}_{r_2kw}} + \hat{c}^{gr_1u}_{r_2kw}(Q^{2*}_w)\right] \times [q^{gr_1u}_{r_2kw} - q^{gr_1u*}_{r_2kw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\sum_{r_2=1}^{R}\frac{\partial c^{gr_1u}_{r_2w}(y^{gr_1u*}_{r_2w})}{\partial y^{gr_1u}_{r_2w}} \times [y^{gr_1u}_{r_2w} - y^{gr_1u*}_{r_2w}]$$

$$+\sum_{w=1}^{W} L_w \sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}}\frac{\partial c_{gr_1uw}(z^*_{gr_1uw})}{\partial z_{gr_1uw}} \times [z_{gr_1uw} - z^*_{gr_1uw}]$$

$$+\sum_{w=1}^{W} L_w \sum_{r_1=1}^{R}\sum_{r_2=1}^{R}\sum_{k=1}^{K}\hat{c}^{r_1}_{r_2kw}(Y^{2*}_w) \times [y^{r_1}_{r_2kw} - y^{r_1*}_{r_2kw}] \geq 0, \quad \forall (Q^1, q, Q^2, Y^1, Y^2, Z) \in \mathcal{K}_2, \qquad (2)$$

Theorem: Existence (Cont'd)

where $\mathcal{K}_2 \equiv \{(Q^1, q, Q^2, Y^1, Y^2, Z) | (Q^1, q, Q^2, Y^1, Y^2, Z) \in R_+^{AMNW+NRKW+NRW+2NW+R^2KW}$ *and the conservation of flow equations and*

$$L_w \sum_{g=1}^{G} \sum_{r_1=1}^{R} \sum_{u=1}^{N_{gr_1}} y_{rw}^{gr_1 u} \geq L_w \sum_{r_2=1}^{R} \sum_{k=1}^{K} y_{r_2 kw}^{r}, \forall r; \forall w,$$

$$L_w \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} z_{gr_1 uw} \geq L_w OPR_{r_1 w}, \forall r_1; \forall w,$$

$$\text{and} \quad L_w \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} [\sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} \sum_{k=1}^{K} q_{r_2 kw}^{gr_1 u} + \sum_{g=1}^{G} \sum_{u=1}^{N_{gr_1}} y_{r_2 w}^{gr_1 u} + \sum_{k=1}^{K} y_{r_2 kw}^{r_1}] \alpha_{r_1 r_2 b} \leq L_w TCap_b, \quad \forall b; \forall w$$

are satisfied$\}$.

A solution to (2) is guaranteed to exist provided that $\mathcal{K}_2$ *is nonempty. Moreover, if* $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*)$ *is a solution to (2), there exist* $(\eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*) \in R_+^{2NW+BW+2RW}$ *with* $(Q^{1*}, q^*, Q^{2*}, Y^{1*}, Y^{2*}, Z^*, \eta^*, \lambda^*, \mu^*, \rho_3^*, \varphi^*)$ *being a solution to variational inequality (1).*

Theorem: Monotonicity

Suppose that all cost functions in the model are continuously differentiable and convex; all unit cost functions are monotonically increasing, and the inverse price functions at the fuel supply markets are monotonically increasing. Then the vector F that enters the variational inequality (1) is monotone, that is,

$$\left\langle (F(X') - F(X''))^T, X' - X'' \right\rangle \geq 0, \quad \forall X', X'' \in \mathcal{K}, X' \neq X''.$$

Modified Projection Method

Step 0: Initialization
Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and let α be a scalar such that $0 < \alpha \leq \frac{1}{\mathrm{L}}$, where L is the Lipschitz continuity constant.

Step 1: Computation
Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle \bar{X}^{\mathcal{T}} + \alpha F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 2: Adaptation
Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + \alpha F(\bar{X}^{\mathcal{T}}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}.$$

Step 3: Convergence Verification
If max $|X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} =: \mathcal{T} + 1$, and go to Step 1.

Modified Projection Method (Cont'd)

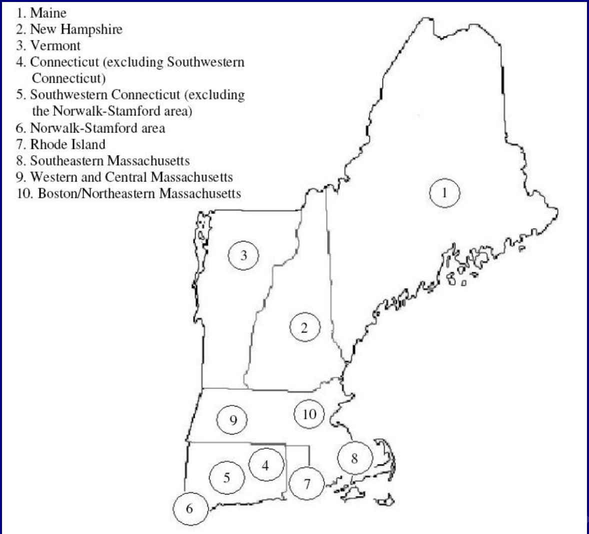
The method converges to a solution of the model provided that $F(X)$ is monotone and Lipschitz continuous, and a solution exists.

In Steps 1 and 2 of the modified projection method, due to the special structure of the underlying feasible set, the subproblems are completely separable and can be solved as W transportation network problems with the prices in each subproblem solvable in closed form.

Empirical Case Study and Examples

- New England electric power market and fuel markets
- 82 generators who own and operate 573 power plants
- 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal
- Ten regions (R=10): 1. Maine, 2. New Hampshire, 3. Vermont, 4. Connecticut(excluding Southwestern Connecticut), 5. Southwestern Connecticut(excluding Norwalk-Stamford area), 6. Norwalk-Stamford area, 7. Rhode Island, 8. Southeastern Massachusetts, 9. Western and Central Massachusetts, 10. Boston/Northeastern Massachusetts
- Hourly demand/price data of July 2006 ($24 \times 31 = 744$ scenarios)
- 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$).

The New England Electric Power Supply Chain Network with Fuel Supply Markets



Empirical Case Study and Examples

- Example 1: Simulation of the regional electric power prices
- Example 2: Sensitivity analysis for peak-hour electricity prices under natural gas and oil price variations
- Example 3: The impact of the oil price on the natural gas price through electric power markets
- Example 4: The impact of changes in the electricity demands for electricity on the electric power and fuel supply markets.

Example 1: Simulation of the Regional Electric Power Prices

Average Regional Demands for Each Demand Level (Mwh)

Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
1	1512	1425	1384	1292	1051	889
2	1981	1868	1678	1481	1193	1005
3	774	760	717	654	560	500
4	2524	2199	2125	1976	1706	1432
5	2029	1798	1636	1485	1257	1065
6	1067	931	838	740	605	509
7	1473	1305	1223	1112	952	801
8	2787	2478	2315	2090	1736	1397
9	2672	2457	2364	2262	2448	2186
10	4383	4020	3684	3260	2744	2384
Total	21201	19241	17963	16350	14252	12168

Example 1: Simulation of the Regional Electric Power Prices

Actual Regional Prices ($/Mwh)

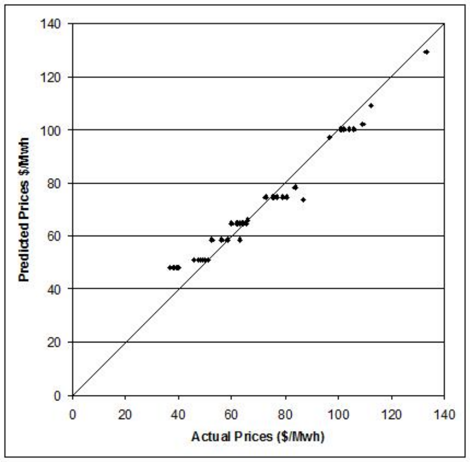
Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
ME	96.83	72.81	59.78	52.54	45.79	36.70
NH	102.16	77.17	63.07	56.31	48.20	38.35
VT	105.84	80.69	65.32	58.39	49.71	39.24
CT	133.17	112.25	86.85	65.97	50.92	39.97
RI	101.32	75.66	61.84	56.06	47.55	37.94
SE MA	101.07	75.78	62.09	56.27	47.54	38.05
WC MA	104.15	79.19	64.49	58.41	49.25	39.53
NE MA	109.29	83.96	63.93	63.02	48.11	38.22
Average	111.66	87.36	69.15	60.18	48.80	38.79

Example 1: Simulation of the Regional Electric Power Prices

Simulated Regional Prices ($/Mwh)

Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
ME	92.10	74.62	64.77	58.71	50.31	48.00
NH	100.28	74.62	64.77	58.71	50.31	48.00
VT	100.28	74.62	64.77	58.71	50.31	48.00
CT	131.80	109.09	70.80	64.77	50.31	48.00
RI	100.28	74.62	64.77	58.71	50.31	48.00
SE MA	100.28	74.62	64.77	58.71	50.31	48.00
WC MA	100.28	74.62	64.77	58.71	50.31	48.00
NE MA	102.21	78.43	64.82	58.71	50.31	48.00
Average	108.28	86.34	67.01	60.56	50.31	48.00
Average (*)	95.57	84.75	64.77	58.71	50.31	48.00
(*) is the simulated weighted average electricity price without the consideration of physical transmission constraints						

Actual Prices vs. Simulated Prices ($/Mwh)



Example 2: Peak Electric Power Prices under Fuel Price Variations

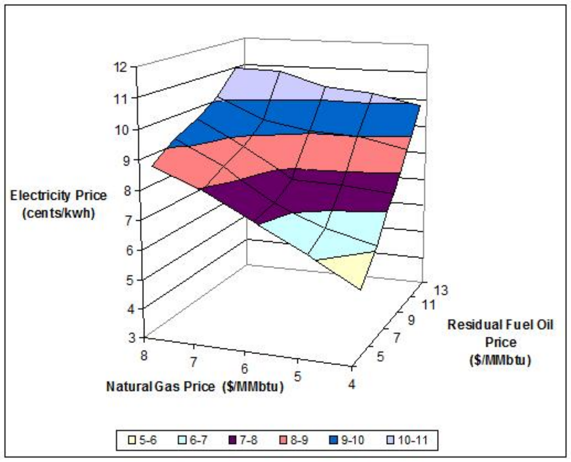
- Natural gas units and oil units generate 38% and 24% of electric power in New England, respectively.
- Generating units that burn gas or oil set electric power market price 85% of the time.

Example 2: Peak Electric Power Prices under Fuel Price Variations

Average Peak Electricity Prices under Fuel Price Variations

Electricity Price (cents/kwh)		Residual Fuel Oil Prices ($/MMBtu) 5.00	7.00	9.00	11.00	13.00
Natural Gas ($/MMBtu)	4.00	5.26	6.13	7.46	8.63	9.92
	5.00	6.15	6.56	7.65	9.01	10.31
	6.00	7.06	7.26	7.72	9.07	10.45
	7.00	7.94	8.37	8.71	9.41	10.91
	8.00	8.62	9.22	9.61	10.12	10.97

Example 2: Peak Electric Power Prices under Fuel Price Variations



Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

- Two cases: the high residual fuel oil price (7\$/MMBtu) and the low residual fuel oil price (4.4\$/MMBtu)
- We assumed that the natural gas price function (unit: \$/MMBtu) takes the form:

$$\pi_{GASm}(h) = 7 + 6\frac{\sum_{w=1}^{6}\sum_{m=1}^{6}\sum_{g=1}^{G}\sum_{r_1=1}^{R}\sum_{u=1}^{N_{gr_1}} q_{gr_1uw}^{GASm} - d_{GAS0}}{d_{GAS0} + \bar{d}_{GAS0}}.$$

Example 3: The Interactions Among Electric Power, Natural Gas and Oil Markets

The Price Changes of Natural Gas and Electric Power Under Residual Fuel Oil Price Variation

	Example 3.1		Example 3.2	
	High RFO	Low RFO	High RFO	Low RFO
RFO Price ($/MMBtu)	7.00	4.40	7.00	4.40
NG Demand (Billion MMBtu)	35.95	30.99	41.95	31.80
NG Price ($/MMBtu)	7.00	6.58	7.00	6.27
NG Price Percentage Change	-6.0%		-10.4%	
EP Ave. Price Blocks 1 and 2	8.28	5.94	7.08	5.86
EP Ave. Price Blocks 3 and 4	6.54	5.37	6.25	5.33
EP Ave. Price Blocks 5 and 6	4.99	4.55	4.96	4.44
NG=Natural Gas, RFO=Residual Fuel Oil, EP=Electric Power				

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

- When electricity demands increase (or decrease), the electric power prices will increase (or decrease) due to two main reasons:
    - Power plants with higher generating costs (e.g. heat rates) have to operate more (or less) frequently.
    - The demands for various fuels will also rise which may result in higher (or lower) fuel prices/costs.

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

- In August, 2006, the natural gas price soared by 14% because hot weather across the U.S. led to high electricity demand.
- In July 2007, the natural gas future price for September 2007 increased by 4.7% mainly because of the forecasted high electricity demands in Northeastern and Mid-western cities due to rising temperatures.

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

Prices Before the Demand Increase ($/Mwh)

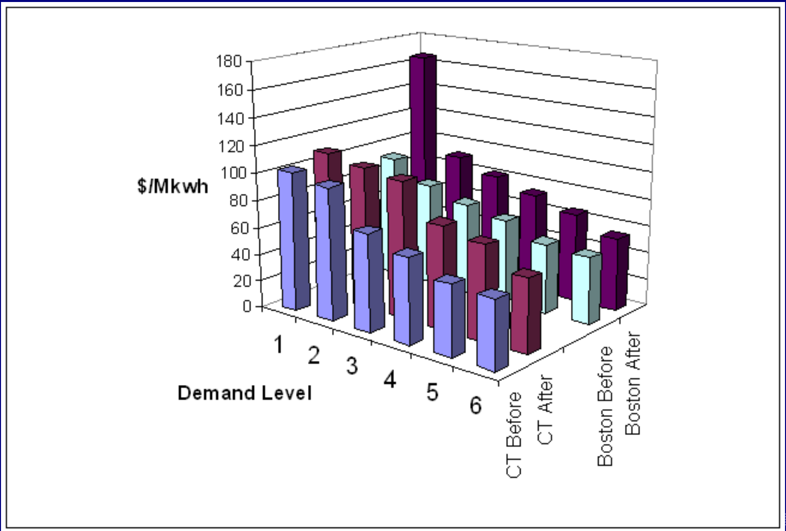
Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
ME	78.73	76.36	67.69	61.56	50.14	49.18
NH	84.82	76.36	67.69	61.56	50.14	49.18
VT	84.82	76.36	67.69	61.56	50.14	49.18
CT	101.81	97.45	71.27	62.22	51.46	49.18
RI	84.82	76.36	67.69	62.22	51.46	49.18
SE MA	84.82	76.36	67.69	62.22	51.46	49.18
WC MA	84.82	76.36	67.69	62.22	51.46	49.18
NE MA	91.30	76.36	67.69	62.22	51.46	49.18
Average	90.23	81.76	68.61	62.08	51.20	49.18
NG Demand	35.95 Billion MMBtu					
NG Price	7.00 $/MMBtu					

Example 4: The Impact of Electricity Demand Changes on the Electric Power and the Natural Gas Markets

Prices after the Demand Increase ($/Mwh)

Region	Block 1	Block 2	Block 3	Block 4	Block 5	Block 6
ME	78.73	83.45	81.55	73.33	65.14	53.46
NH	93.68	84.82	81.55	73.33	65.14	53.46
VT	93.68	84.82	81.55	73.33	65.14	53.46
CT	109.09	104.20	100.84	75.74	69.23	53.73
RI	93.68	84.82	81.55	73.33	65.14	53.73
SE MA	93.68	84.82	81.55	73.33	65.14	53.73
WC MA	93.68	84.82	81.55	73.33	65.14	53.73
NE MA	165.16	91.30	81.55	73.33	65.14	53.73
Average	111.48	91.04	86.49	73.95	66.16	53.68
NG Demand	43.62 Billion MMBtu					
NG Price	7.64 $/MMBtu					

Example 4: Electric Power Prices Before and After the Increase of Demands (Connecticut and Boston)



Conclusions

- We developed a new variational inequality model of electric power supply chain networks with fuel markets, which considered both economic transactions and physical transmission networks.
- We provided some qualitative properties of the model as well as a computational method.
- We then conducted a case study where our theoretical model was applied to the New England electric power network and fuel supply markets.
- We also conducted sensitivity analysis in order to investigate the electric power prices under fuel price variations.

Conclusions (Cont'd)

- We showed that not only the responsiveness of dual-fuel plants, but also the electric power market responsiveness, were crucial to the understanding and determination of the impact of the residual fuel oil price on the natural gas price.
- We applied our model to quantitatively demonstrate how changes in the demand for electricity influenced the electric power and fuel markets.
- The model and results presented in this paper are useful in determining and quantifying the interactions between electric power flows and prices and the various fuel supply markets.
- Such information is important to policy-makers who need to ensure system reliability as well as for the energy asset owners and investors who need to manage risk and evaluate their assets.

Thank You!

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