Supernetworks: Decision-Making in the 21st Century

Anna Nagurney
John F. Smith Memorial Professor
and
Director
The Virtual Center for Supernetworks
University of Massachusetts - Amherst

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Eugene M. Isenberg School of Management

The Virtual Center for Supernetworks
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We are in a New Era of Decision-Making Characterized by:

- complex interactions among decision-makers in organizations;
- alternative and at times conflicting criteria used in decision-making;
- global reach of many decisions;
- high impact of many decisions;
- increasing risk and uncertainty, and
- the importance of dynamics and realizing a fast and sound response to evolving events.

The complexity of today's decision-making environments in organizations requires the development and harnessing of *appropriate and rigorous scientific and engineering tools* which must be based on *information technology* since only such technology provides one with the speed and accuracy needed to model complex interactions and to optimize accordingly.

The New Era is Network-Based with the Internet providing critical infrastructure along with transportation/logistical networks as well as other telecommunication networks and energy networks.

No longer are networks independent of one another but critically linked with major questions arising regarding decision-making and appropriate management tools.

Indeed, the events of 9/11 coupled with the recent computer worm and viruses along with the biggest blackout in US history demonstrate irrevocably that *we must as a nation harness the best and most powerful methodologies for the modeling, analysis, and solution of complex decision-making problems.*

We at the *Isenberg School of Management* have established the *Virtual Center for Supernetworks*, which along with the Supernetworks Laboratory for Computation and Visualization, serves as a resource for researchers, educators, and practitioners.

The center emphasizes the importance of network systems, their modeling, and analysis, and simultaneously expands upon scientific network tools for decision-making.

The center team is multidisciplinary and multicultural and at present consists of doctoral students from three different countries.

The center supports *undergraduates in research* since they are our future and provide new and fresh perspectives.

Center associates from different academic institutions and industry work closely with the center director and student associates.

The Supernetwork Team 2004-2005















Background

*Throughout history, **networks** have served as the foundation for connecting humans to one another and their activities.*

- Roads were laid, bridges built, and waterways crossed so that humans, be they on foot, on animal, or vehicle could traverse physical distance through ***transportation***. The airways were conquered through flight.
- ***Communications*** were conducted using the available means of the period, from smoke signals, drum beats, and pigeons, to the telegraph, telephone, and computer networks of today.

Importance of Networks to the Economy and the Nation

- US consumers, businesses, and governments spend about $950 billion on transportation annually (US DOT).

- Corporate buyers spend about $517.6 billion on telecommunications annually (Purchasing).

- Energy expenditures in the United States are about $515.8 billion a year (US Dept. of Commerce).

Information technology has transformed the ways in which individuals work, travel, and conduct their daily activities, with profound implications for existing and future networks.

The ***decision-making process*** itself has been altered due to the addition of alternatives and options which were not possible or even feasible.

The ***boundaries*** for decision-making have been redrawn as individuals can now work from home or purchase products from work.

We live in an era in which the freedom to choose is weighted by the immensity of choices and possibilities:

- Where should one live?
- Where should one work? And when?
- How should one travel? Or communicate? And with whom?
- Where should one shop? And how?

Not only has individual *decision-making* been transformed in this new era but *organizations* have as well.

How do we capture in a scientific manner *cooperation vs. competition* and the ramifications, *centralized control vs. decentralized control*, and *different criteria* in decision-making?

Moreover, what are the results on the flows on the networks be they in the form of vehicles, messages, products, and/or services, as well as financial?

Who wins and who loses?

Classical Networks

Network System	Nodes	Links	Flows
Transportation Urban Air Rail	 Intersections, Homes, Places of Work Airports Railyards	 Roads Airline Routes Railroad Track	 Autos Planes Trains
Manufacturing and Logistics	Distribution Points, Processing Points	Routes Assembly Line	Parts, Products
Communication	Computers Satellites Phone Exchanges	Cables Radio Cables, Microwaves	Messages Messages Voice, Video
Energy	Pumping Stations Plants	Pipelines Pipelines	Water Gas, Oil

Transportation science has historically been the discipline that has pushed the frontiers in terms of methodological developments for such problems (which are often large-scale) beginning with the work of Beckmann, McGuire, and Winsten (1956).

Dafermos (1980) later showed that the traffic network equilibrium conditions as formulated by Smith (1979) were a finite-dimensional variational inequality and then utilized the theory to establish both existence and uniqueness results of the equilibrium traffic flow pattern as well as to propose an algorithm with convergence results.

Finite-dimensional variational inequality theory has been applied to-date to the wide range of equilibrium problems noted above, as well as to game theoretic problems, such as oligopolistic market equilibrium problems, and to general economic equilibrium problems (see, e.g., Nagurney (1993) and the references therein).

Reality of Today's Networks

Characteristics of Today's Networks:

- *large-scale nature* and complexity of network topology;
- *congestion*;
- alternative behavior of users of the network, which may lead to *paradoxical phenomena*;
- the *interactions among networks* themselves such as in transportation versus telecommunications networks;
- *policies* surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.

Large-Scale Nature and Complexity

- Chicago's Regional Transportation Network has 12,982 nodes, 39,018 links, and 2,297,945 origin/destination pairs.

- AT&T's domestic network has 100,000 origin/destination pairs. In the detail graph applications in which nodes are phone numbers and edges are calls, there are 300 million nodes and 4 billion edges.

Congestion

- In the case of *transportation networks* in the United States alone, congestion results in $100 billion in lost productivity, whereas the figure in Europe is estimated to be $150 billion.

- In terms of the *Internet*, the FCC reports that the volume of traffic is doubling every 100 days, which is remarkable given that telephone traffic has typically increased only by about 5 percent a year.

System-Optimization versus User-Optimization

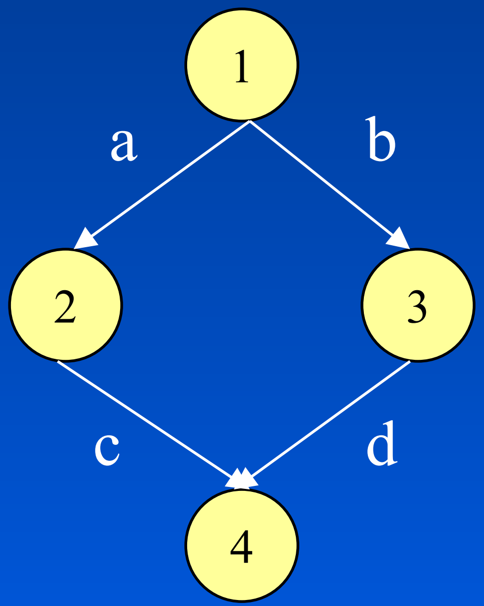
In transportation networks, travelers select their routes of travel from an origin to a destination so as to minimize their own travel cost or travel time, which although *optimal* from an individual's perspective (*user-optimization*) may not be optimal from a societal one (*system-optimization*) where one has control over the flows on the network.

The Braess' Paradox

Assume a network with a single O/D pair (1,4) . There are 2 paths available to travelers: p_1=(a,c) and p_2=(b,d).

For a travel demand of 6, the equilibrium path flows are $x_{p_1}{}^* = x_{p_2}{}^* = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83$.



$c_a(f_a)=10\ f_a \quad c_b(f_b) = f_b+50$

$c_c(f_c) = f_c+50 \quad c_d(f_d) = 10f_d$

Adding a Link
Increased Travel Cost for All!

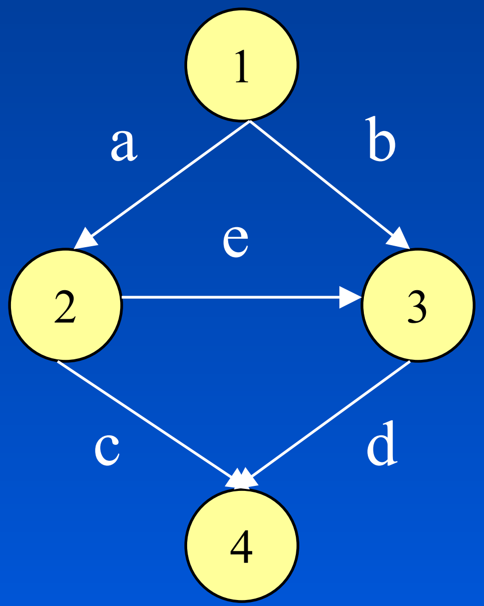
Adding a new link creates a new path p_3=(a,e,d).

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, C_{p_3}=70.

The new equilibrium flow pattern network is $x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.

The equilibrium path travel costs:

$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

This phenomenon is also relevant to telecommunications networks and, in particular, to the Internet which is another example of a noncooperative network.

Recently, we have discovered paradoxes in networks with zero emission links such as telecommmunication networks:

- The addition of a zero emission link may result in an increase in total emissions with no change in demand!

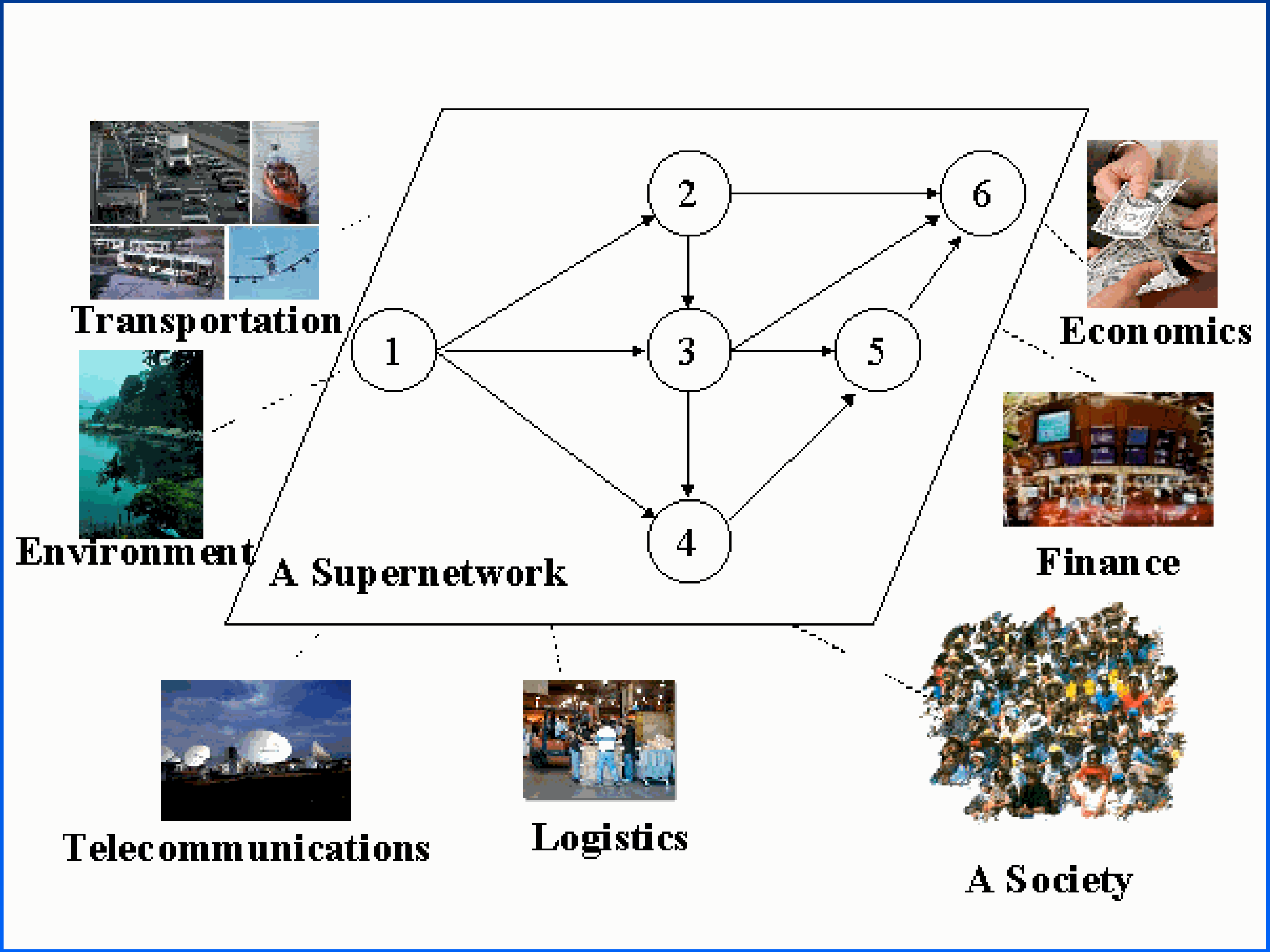
- A decrease in demand on a network with a zero emission link may result in an increase in total emissions!

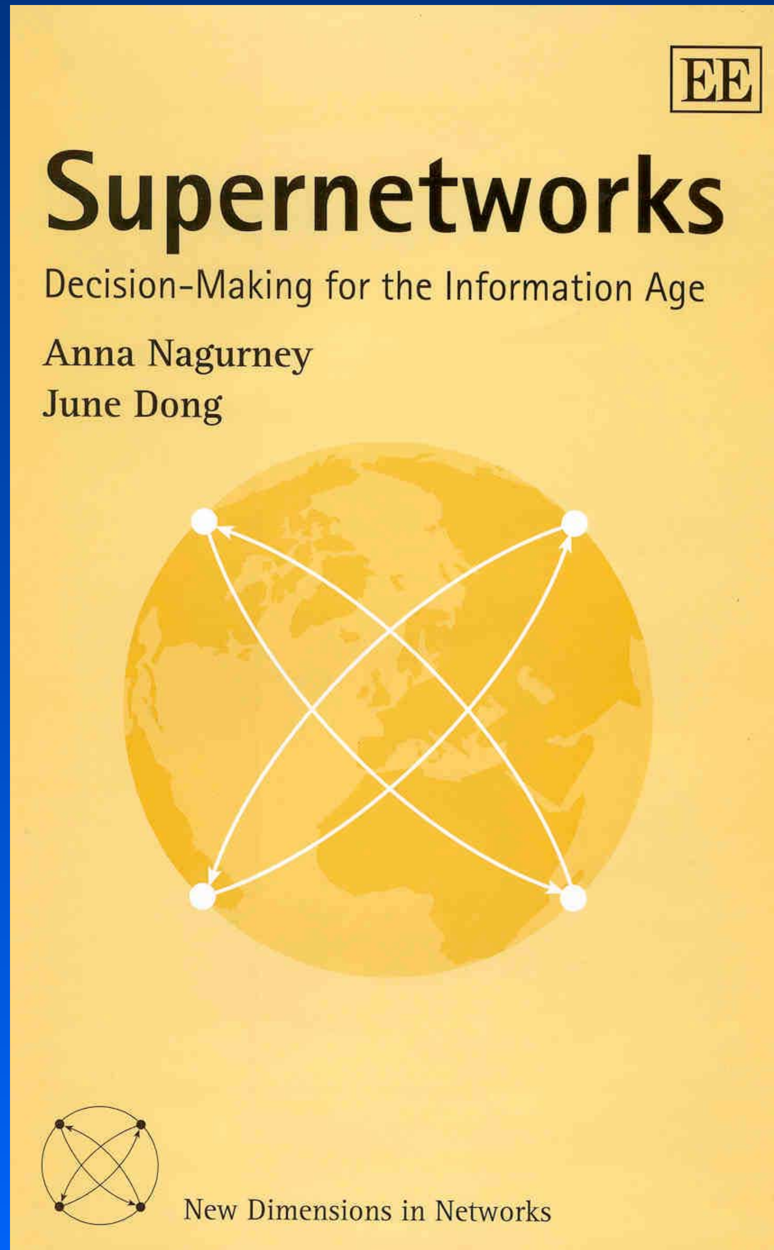
One must *incorporate the network topology*, the relevant cost and demand structure, as well as the behavior of the users of the network(s) into any network-based policy!

These paradoxes further illustrate the interconnectivity among distinct network systems and that they cannot be studied simply in isolation!!!

A New Paradigm

Supernetworks: A New Paradigm

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Supernetworks
Decision-Making for the Information Age
Anna Nagurney
June Dong
New Dimensions in Networks

Supernetworks

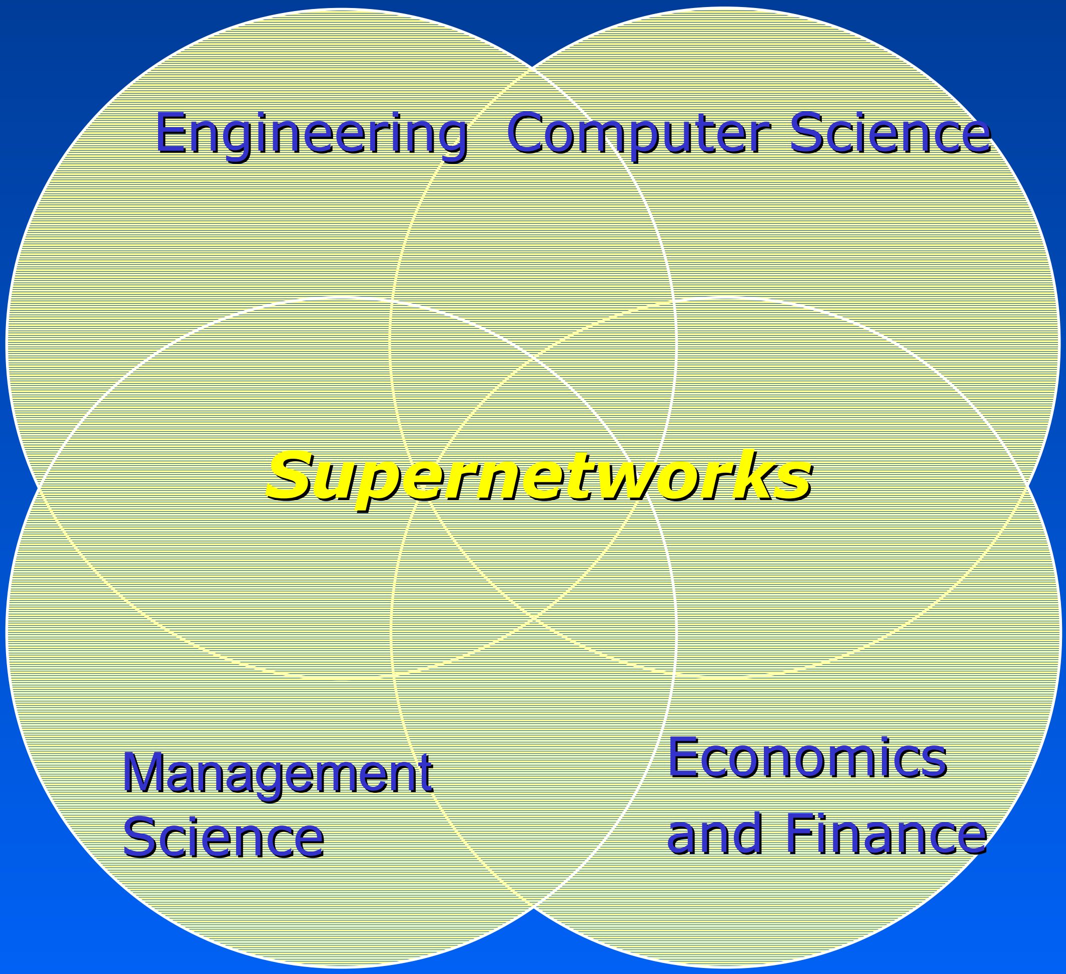
- Supernetworks may be comprised of such networks as transportation, telecommunication, logistical, and/or financial networks.
- They may be *multilevel* as when they formalize the study of supply chain networks or *multitiered* as in the case of financial networks with intermediation.
- Decision-makers may be faced with multiple criteria; thus, the study of supernetworks also includes the study of *multicriteria decision-making*.

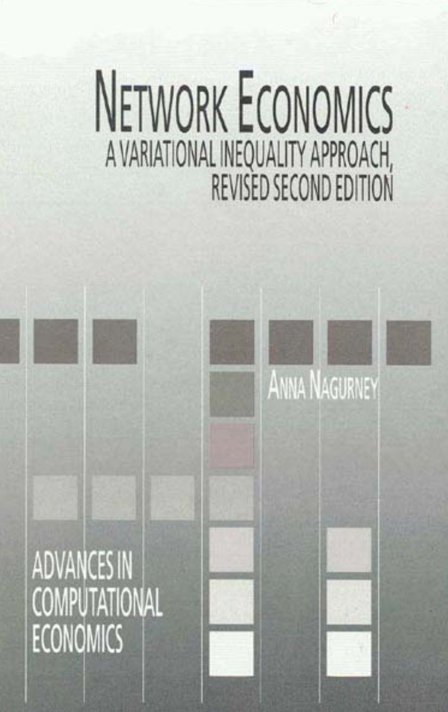
New Tools

The tools that we have been using in our supernetworks research include:

- network theory
- optimization theory
- game theory
- variational inequality theory and
- projected dynamical systems theory (which we have been instrumental in developing)
- network visualization tools.

A Multidisciplinary Paradigm

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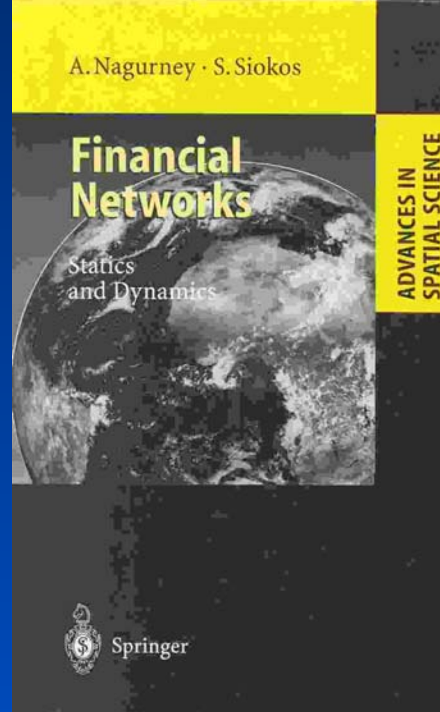
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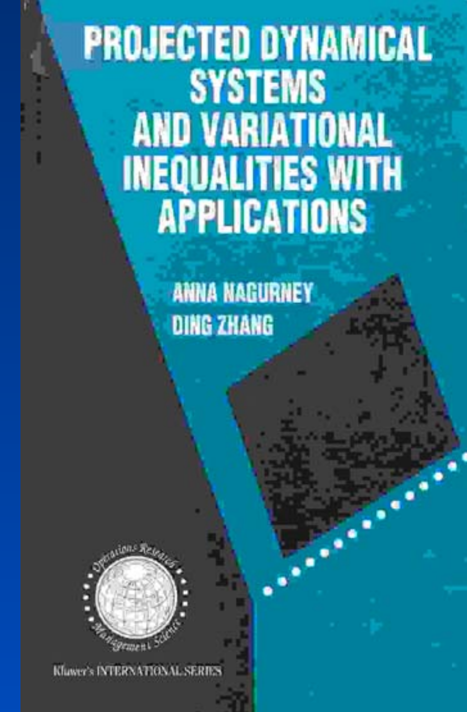
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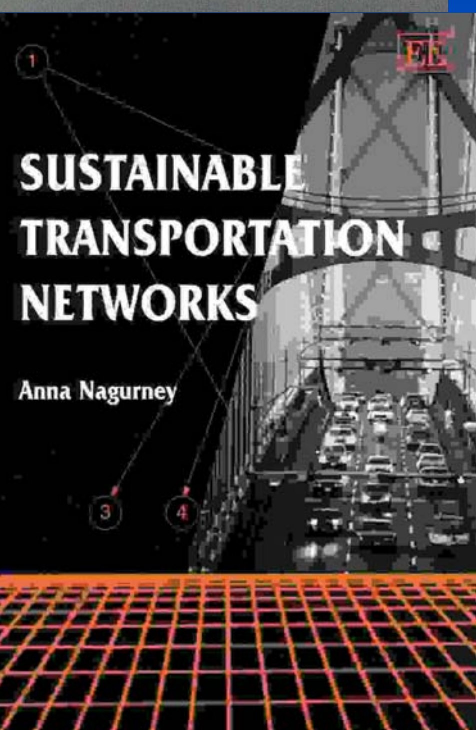
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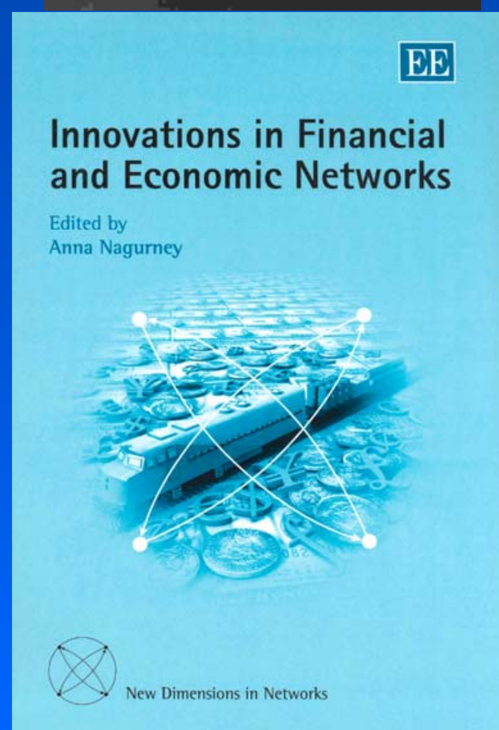
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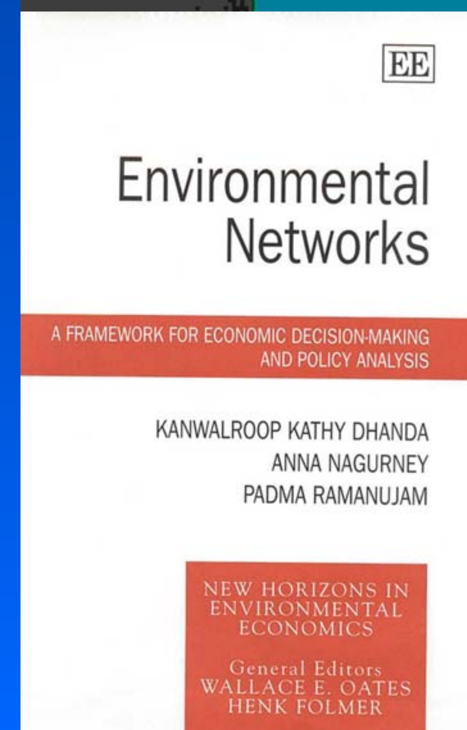
New Dimensions in Networks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Environmental
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A FRAMEWORK FOR ECONOMIC DECISION-MAKING
AND POLICY ANALYSIS
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We are interested not only in addressing topological issues in terms of connectivity but in predicting the various flows on the networks whether physical or abstract subject to human decision-making under the associated constraints, be they budget, time, security, risk, and/or cost-related.

Some Successes

- We were the first the lay down the theoretical foundations for dynamical systems with constraints (called projected dynamical systems) which allows for the qualitative analysis of such systems including stability analysis along with discrete-time algorithms.

- The applications that we have studied range from dynamic transportation networks to global supply chains and international financial networks with risk management.

- We were the first to quantify and model decision-making on *multitiered networks* as well as *multilevel networks* (along with the dynamics).

- We have made fundamental extensions to *multicriteria decision-making* on networks with *mutiple decision-makers*.

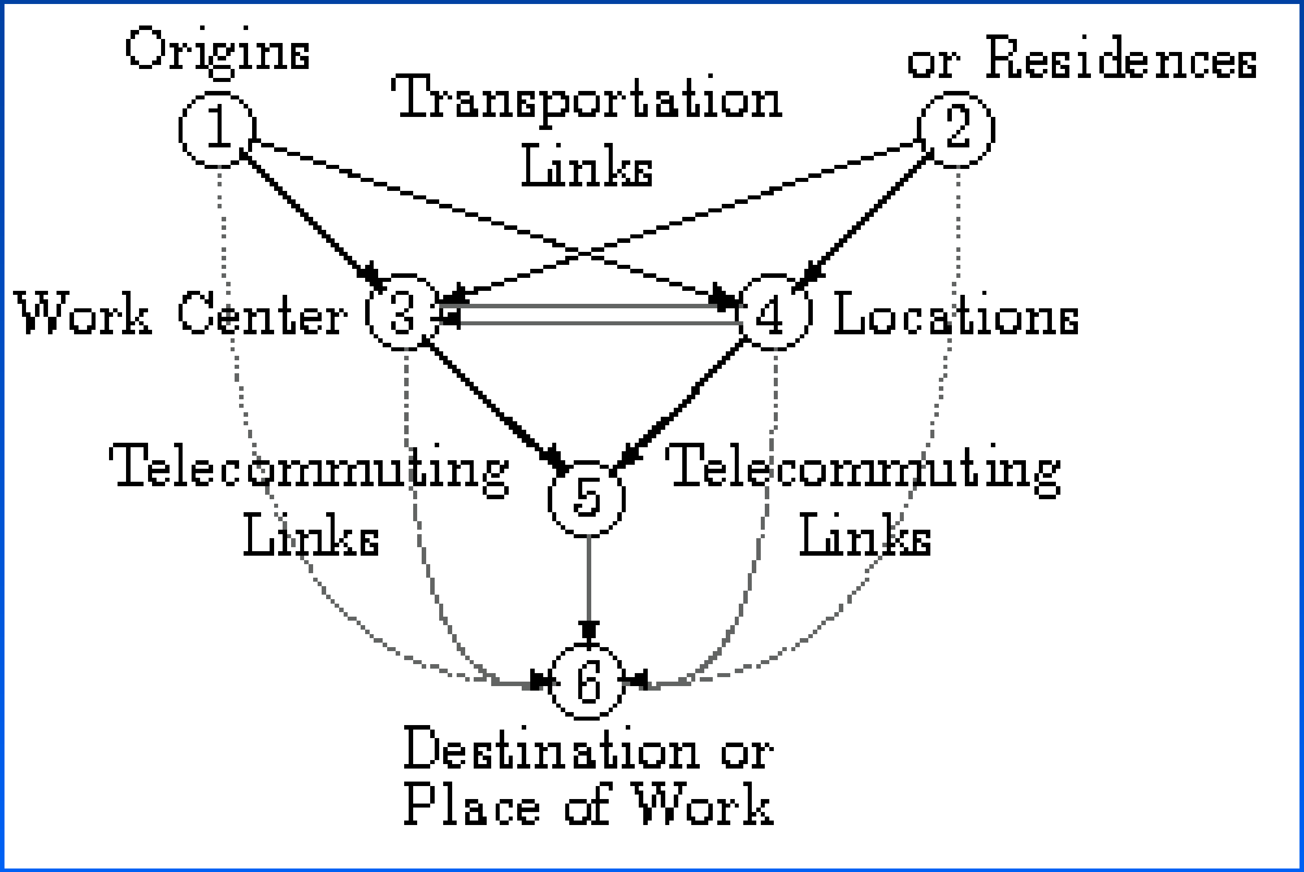
- We have demonstrated through several distinct network systems how *risk and stochastic components* could be directly incorporated into a variational inequality framework.

Novel Applications

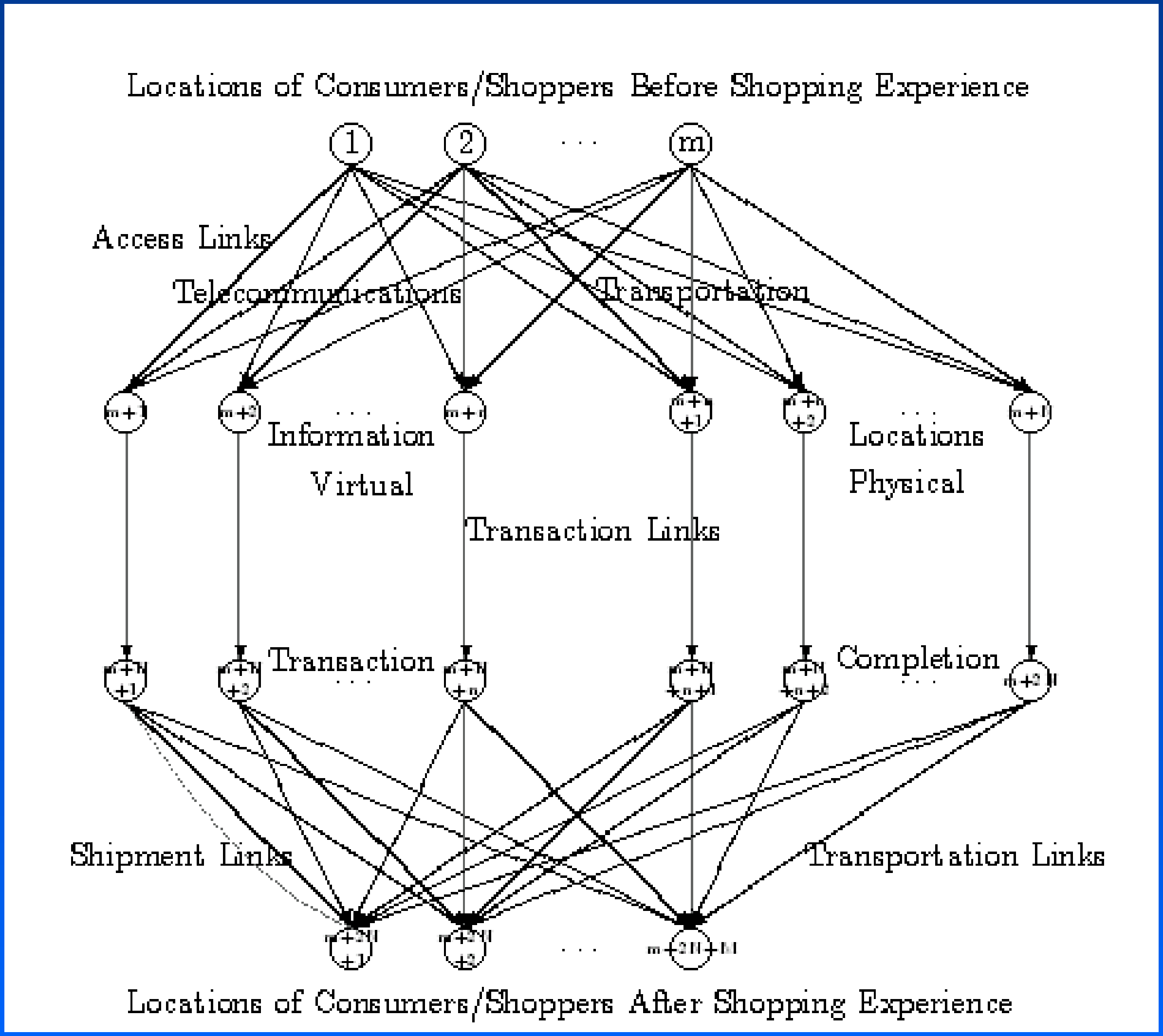
Applications of Supernetworks

- Telecommuting/Commuting Decision-Making
- Teleshopping/Shopping Decision-Making
- Supply Chain Networks with Electronic Commerce
- Financial Networks with Electronic Transactions
- Reverse Supply Chains with E-Cycling
- Knowledge Networks
- Energy Networks/Power Grids

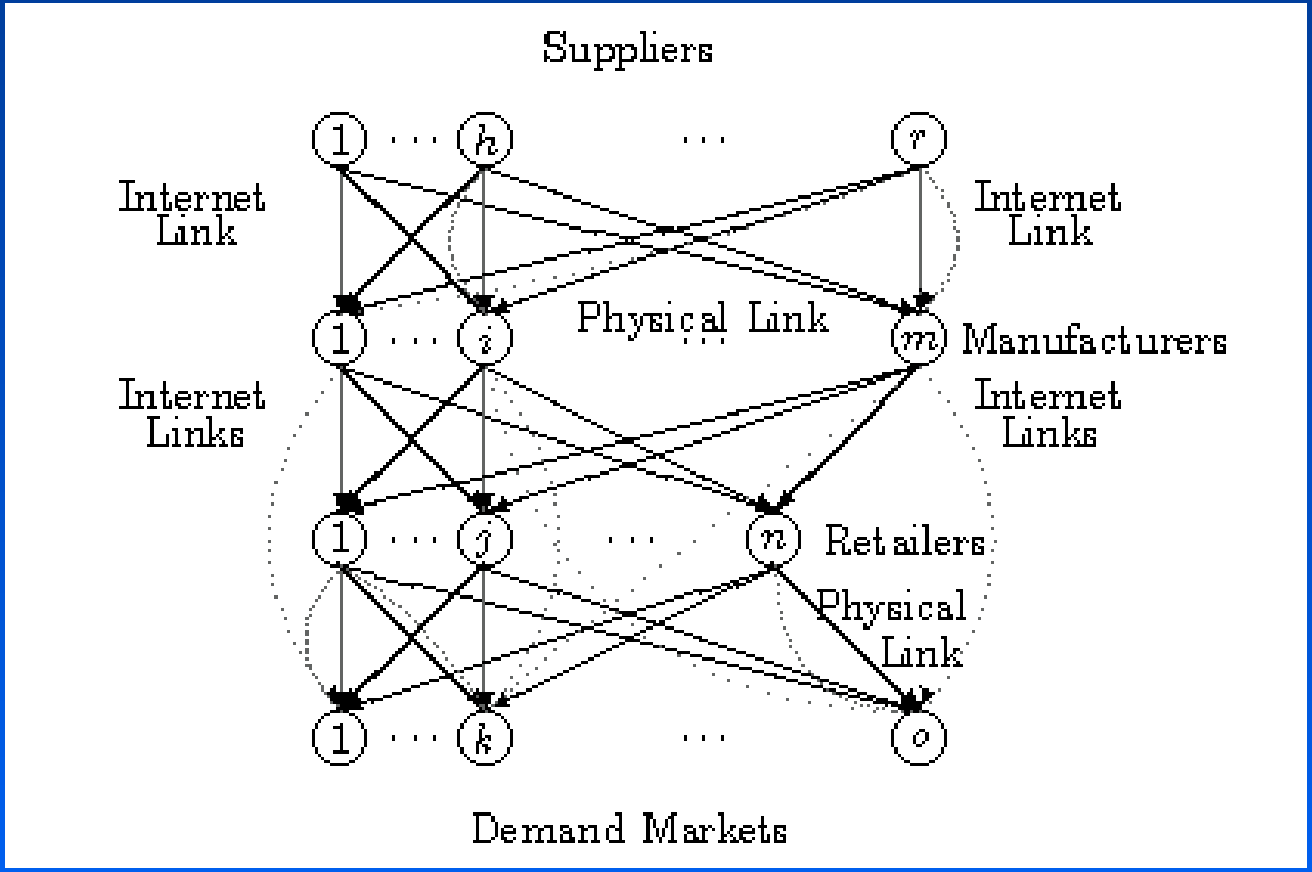
A Supernetwork Conceptualization of Commuting versus Telecommuting



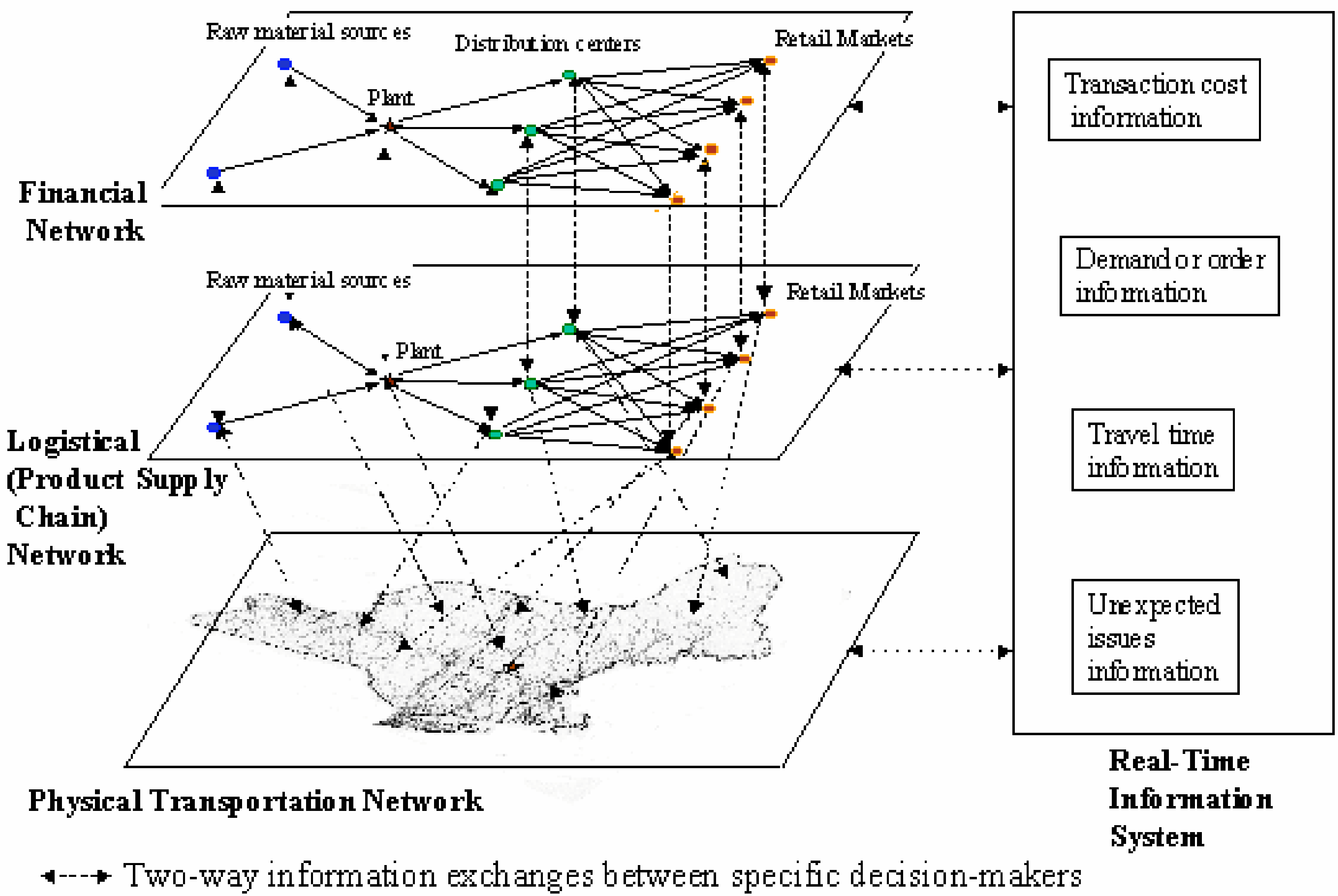
A Supernetwork Framework for Teleshopping versus Shopping



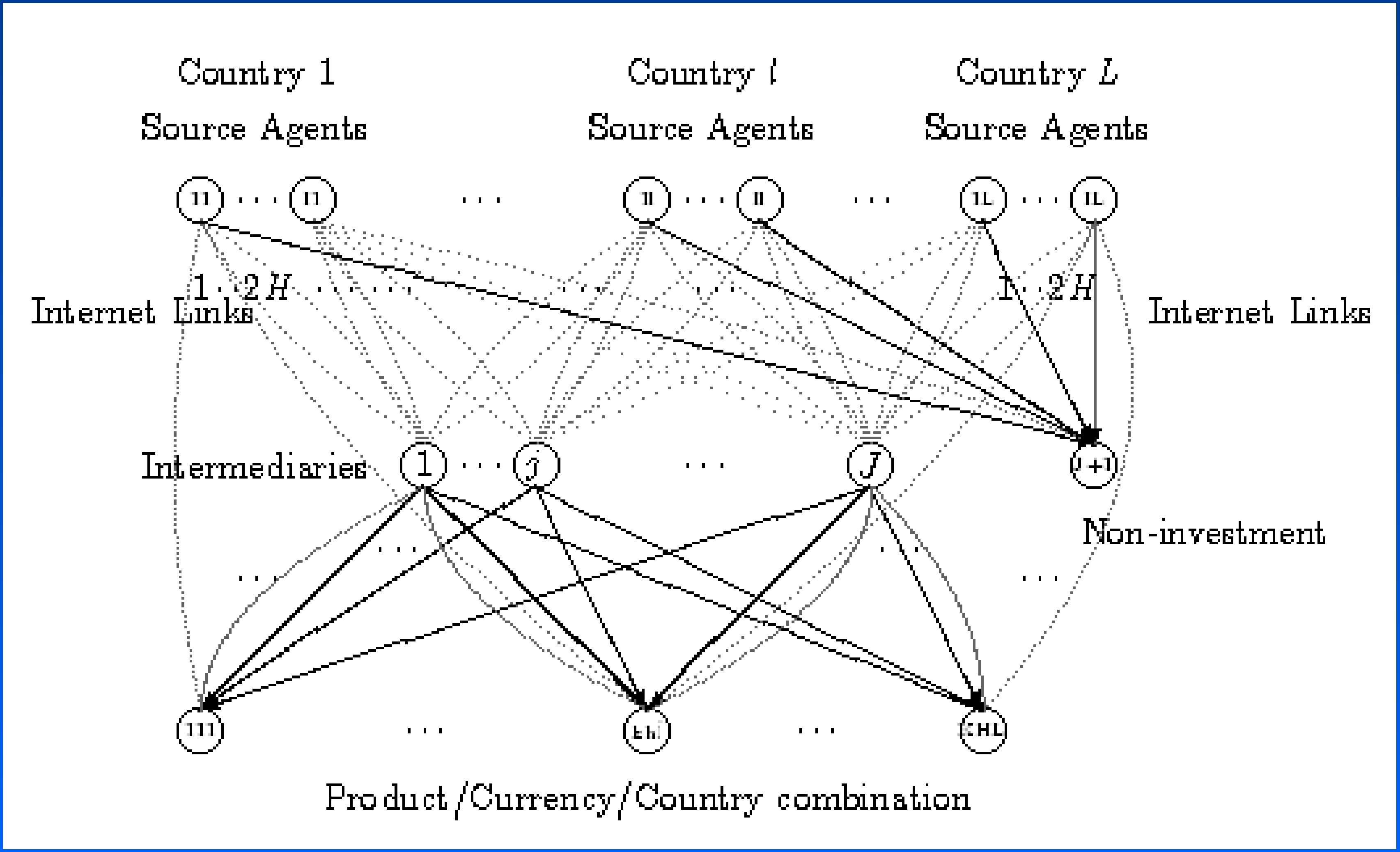
The Supernetwork Structure of a Supply Chain Network



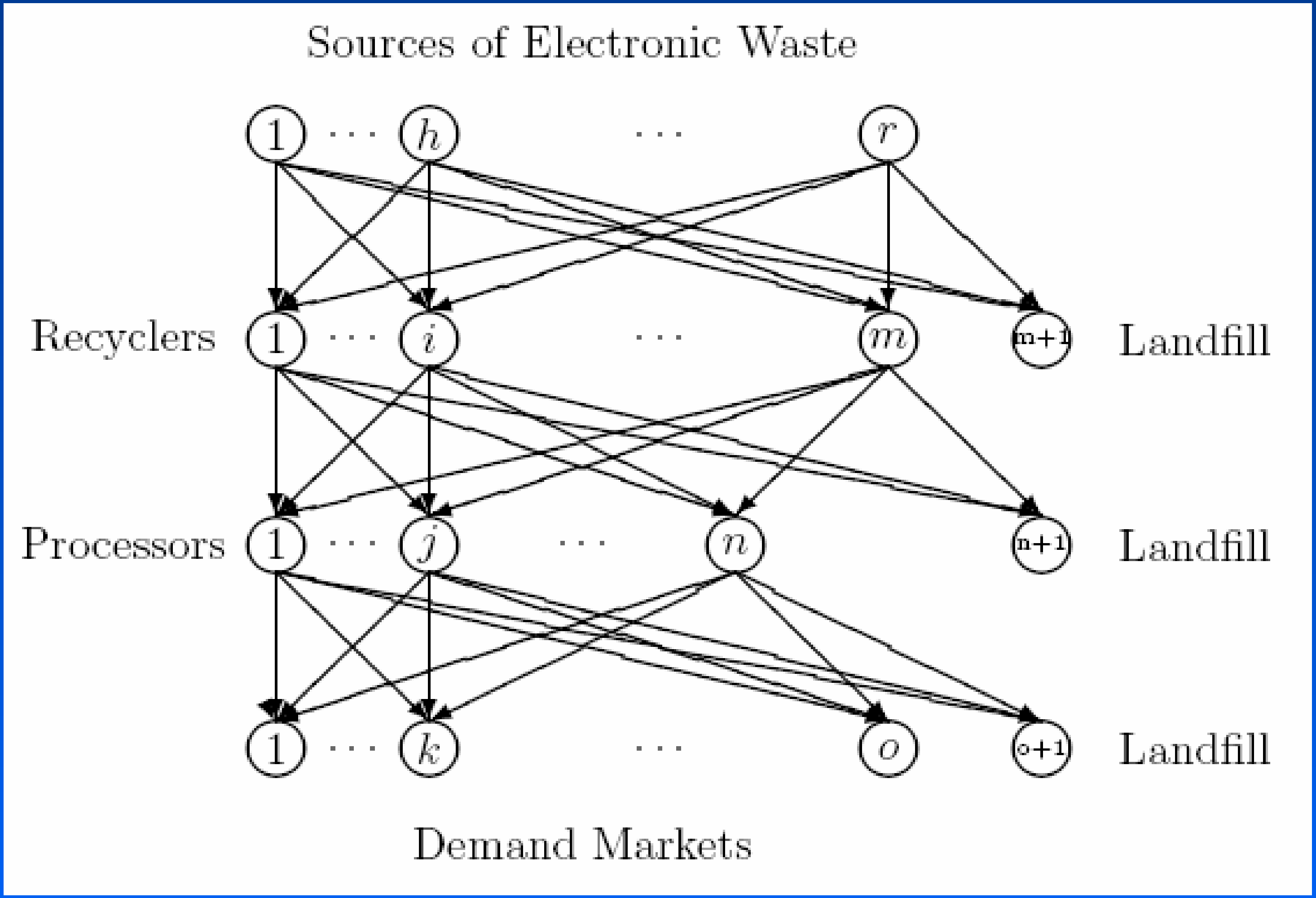
Supply Chain -Transportation Supernetwork Representation



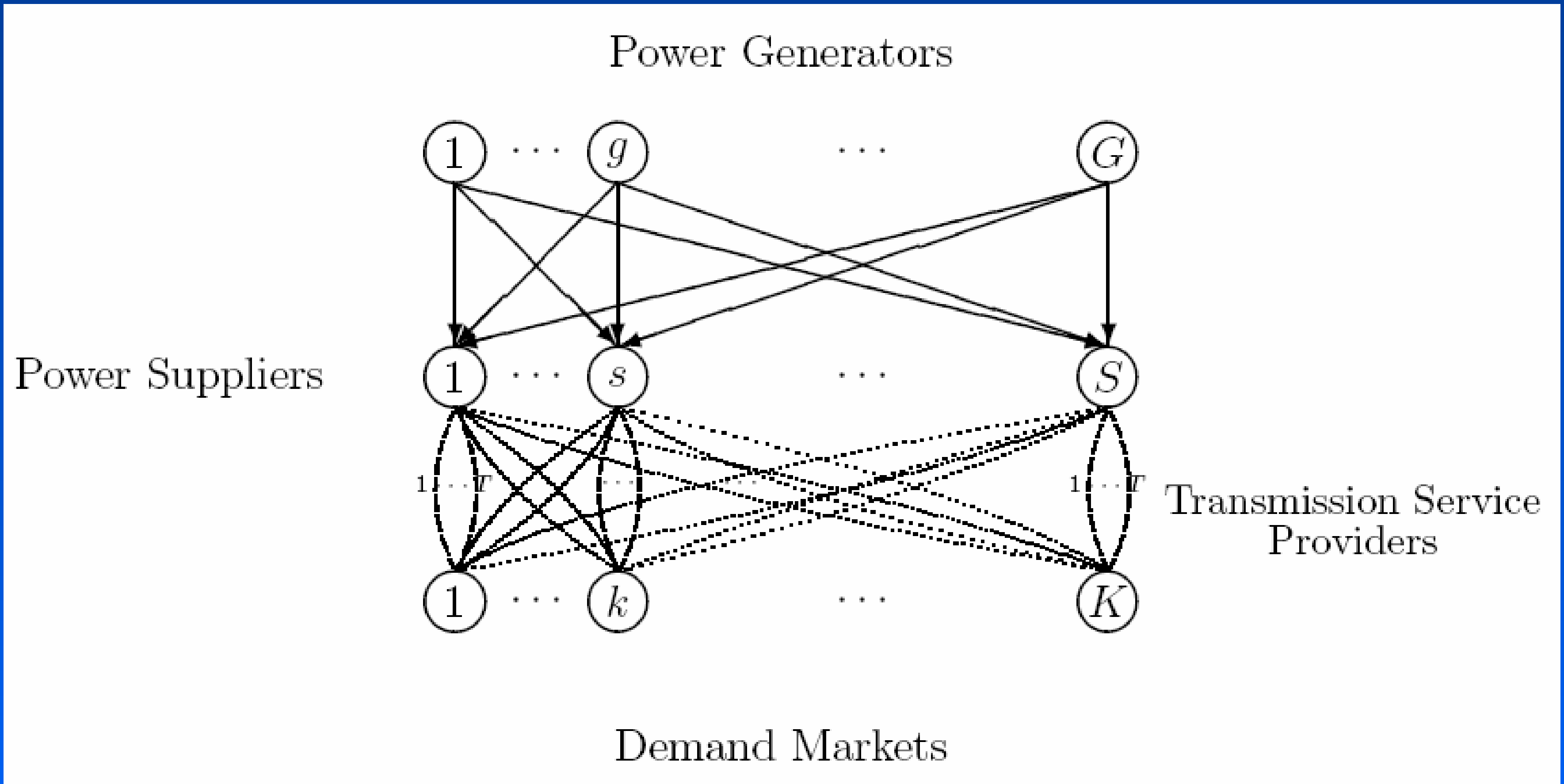
International Financial Networks with Electronic Transactions



The 4-Tiered E-Cycling Network



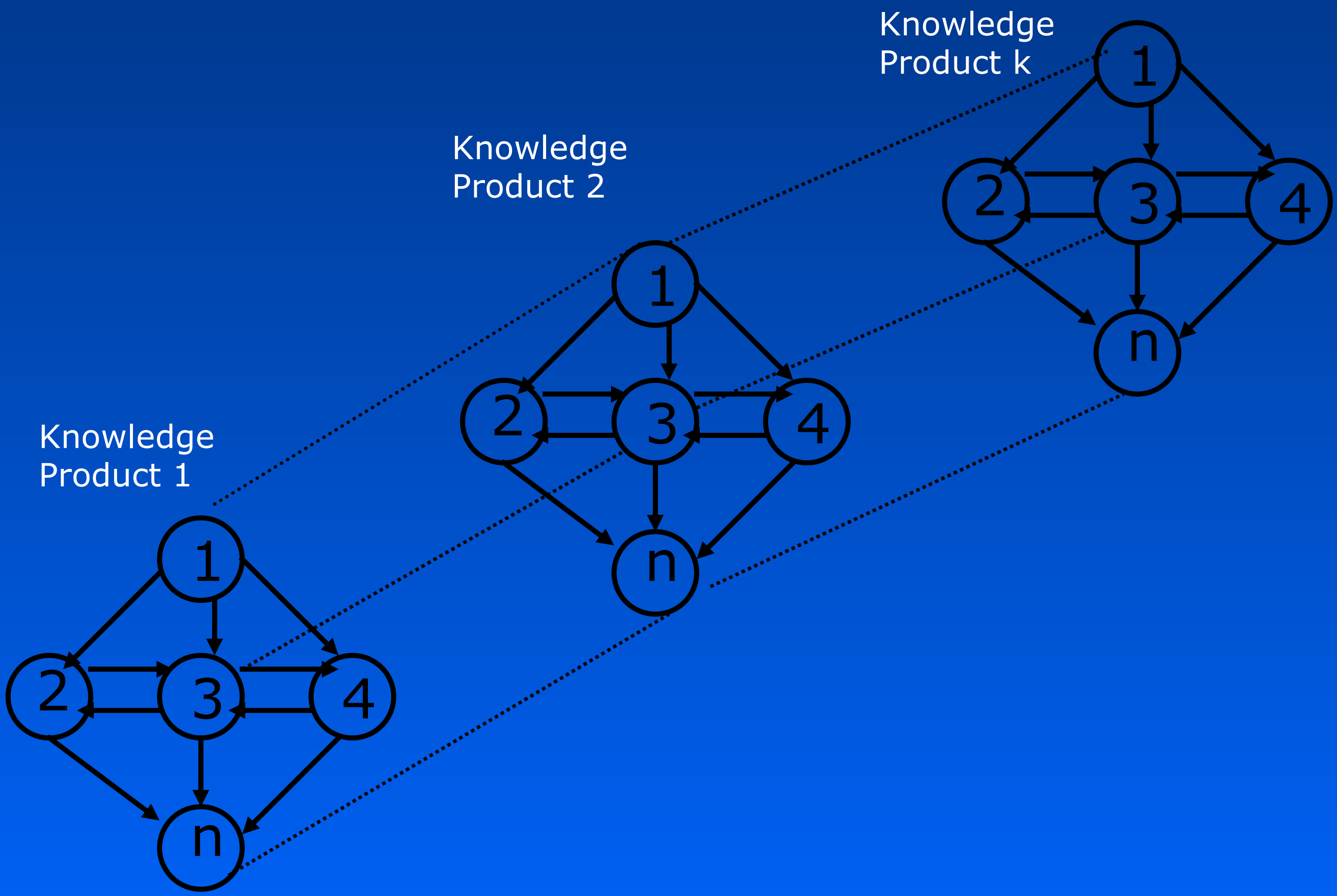
The Electric Power Supply Chain Network



New Directions

The ***knowledge supernetwork theory*** that we are developing is *multidimensional* in scope and *conceptualizes* and *abstracts* complex dynamic business processes and their outcomes as *multitiered* ***and*** *multilevel networks on which multicriteria decision-makers interact*, both competitively and cooperatively, and the effects of their decisions may affect a variety of flows .

A Knowledge Supernetwork



Supernetworks Integrating Social Networks

The models explicitly consider the role that relationship levels play in other network systems and include multicriteria decision-making with individual weights for the criteria such as:

– maximization of profit
– minimization of risk
– maximization of relationship value.

Supernetworks Integrating Social Networks

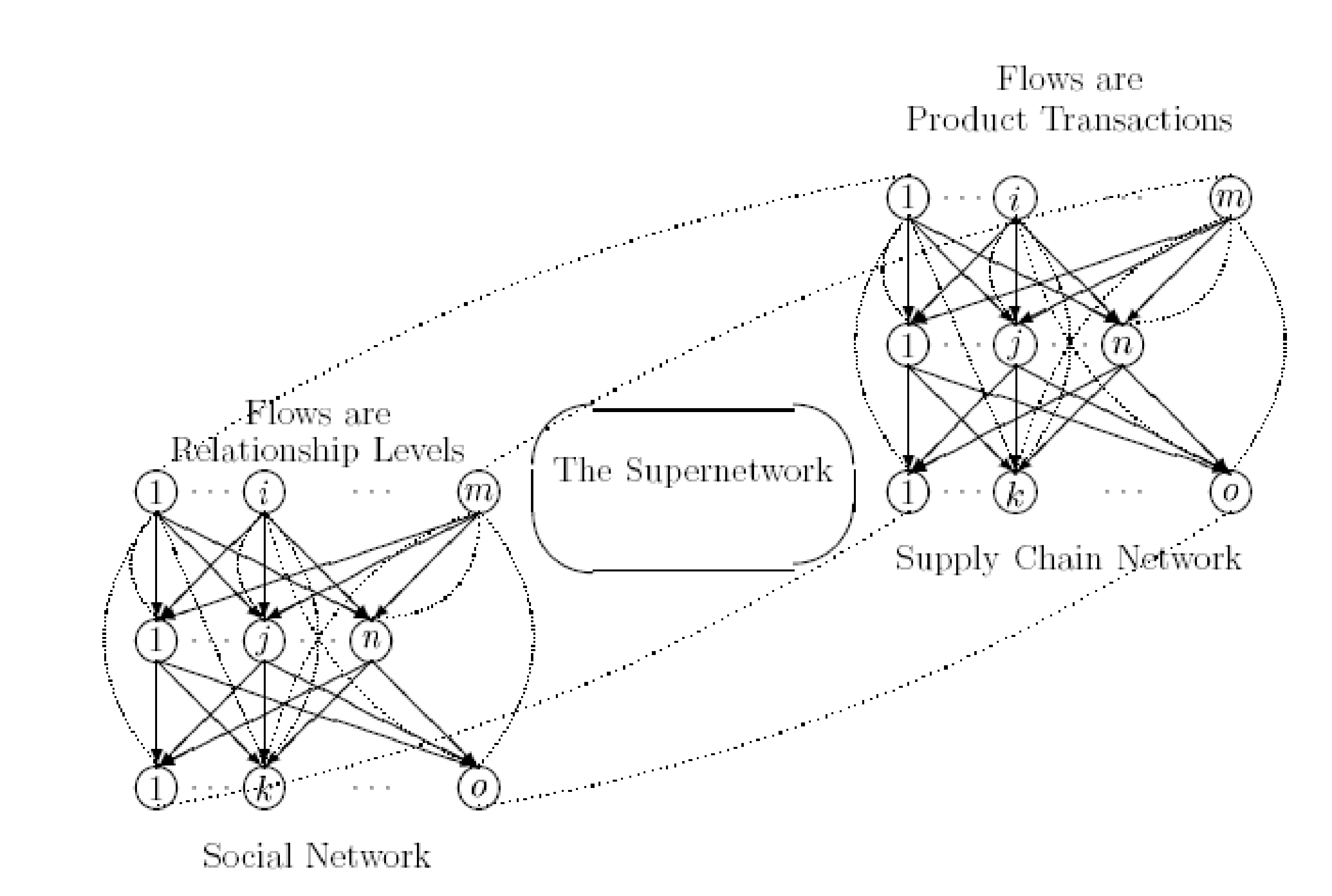
Decision-makers in the network can decide about the relationship levels [0,1] that they want to establish.

Establishing relationship levels incurs some costs.

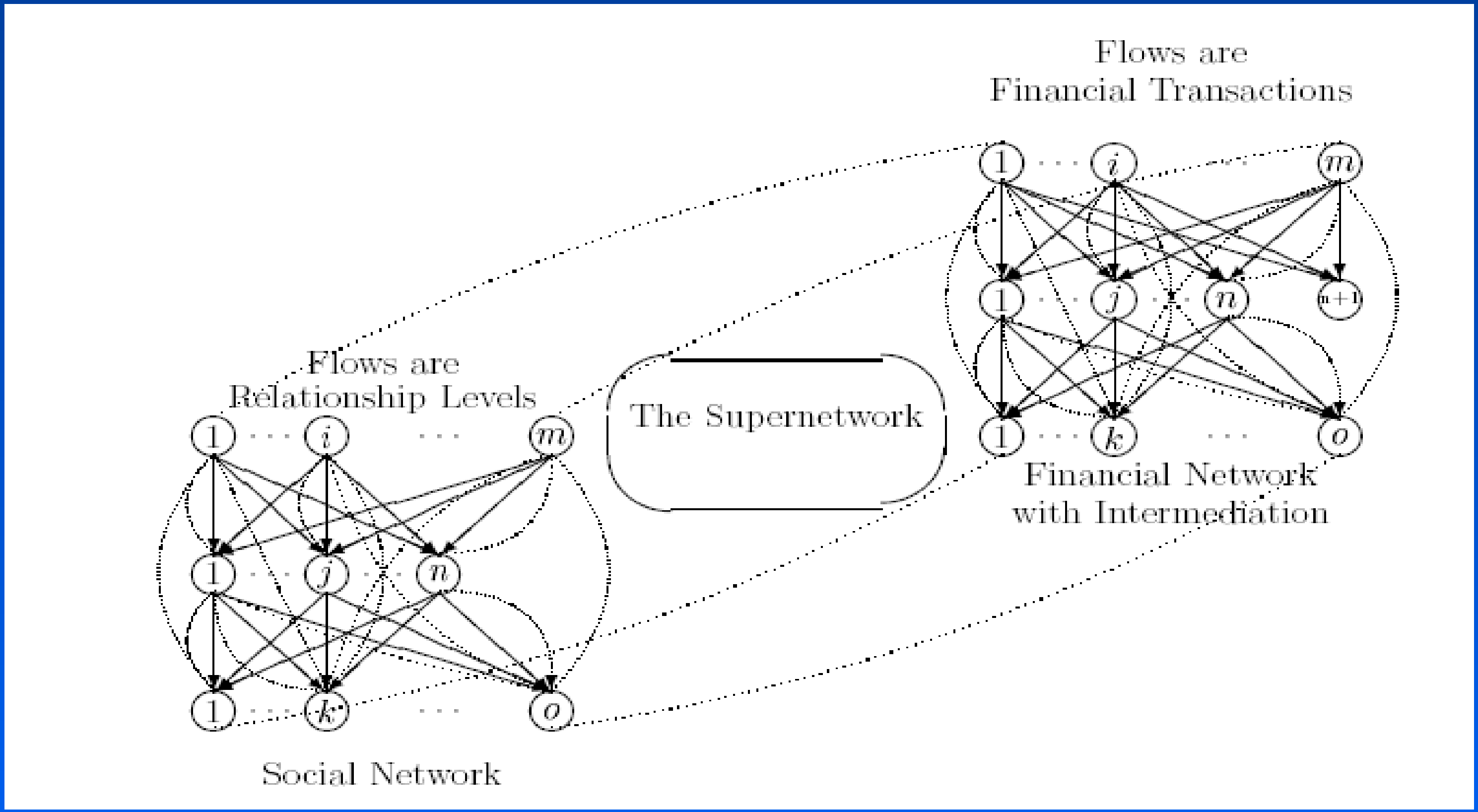
Higher relationship levels

- Reduce transaction costs
- Reduce risk
- Have some additional value (relationship value).

Supernetwork Structure: Integrated Supply Chain/Social Network System



Supernetwork Structure: Integrated Financial/Social Network System



Supernetworks Integrating Social Networks

Dynamic evolution of:

- Product transactions/financial flows and associated prices on the supply chain network/financial network with intermediation

- Relationship levels on the social network

Computational Procedure

We use the Euler Method to solve the Variational Inequality (VI) problems and to track the dynamic trajectories associated with the projected dynamical systems. The VI is in standard form:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \to 0$, as $T \to \infty$

and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

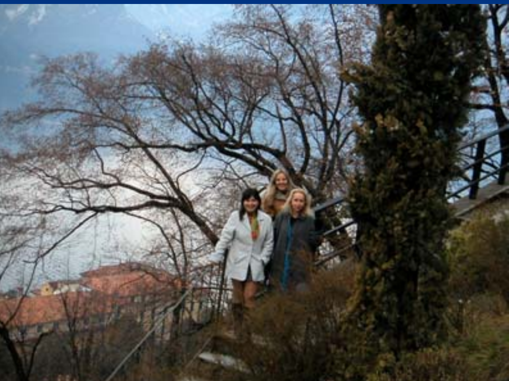
$$y = P_{\mathcal{K}} X = \arg\min_{z \in \mathcal{K}} \|X - z\|.$$

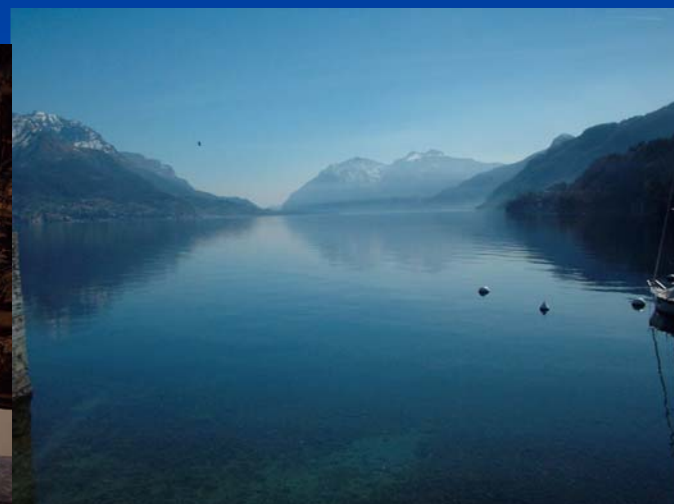
Step 2: Convergence Verification

If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1.

New Challenges, Results, and Opportunities

Bellagio Research Team Residency March 2004



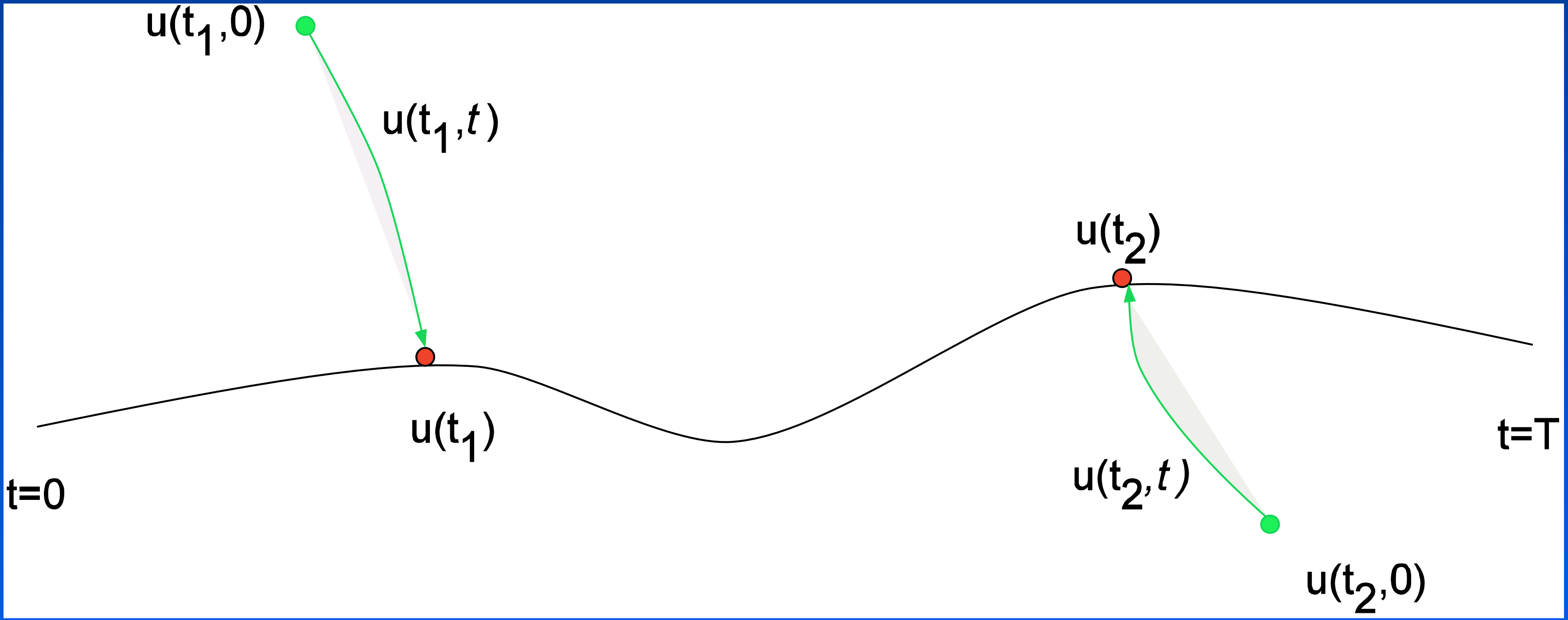



- We are working with Professors Cojocaru and Daniele on infinite-dimensional projected dynamical systems and evolutionary variational inequalities and their relationships and unification.

- This allows us to model dynamic networks over different time scales.

- In addition, there are ties with evolutionary game theory.

A Pictorial of the Double-Layered Dynamics



HOW THESE CONCEPTS CAN BE APPLIED TO TRAFFIC NETWORKS

To use these concepts/results we apply the following steps:

- we discretize the evolution time interval of the EVI;

- we obtain a finite collections of PDS's, defined on distinct closed, convex set K_t;

- we compute the equilibria of each PDS, i.e., we find the equilibria at the discrete chosen moments $t \in [0, T]$;

- we interpolate the sequence of equilibria and obtain an approximation of the curve(s) of equilibria.

A DYNAMIC NETWORK EXAMPLE

Consider a network consisting of a single origin/destination pair of nodes and two paths connecting these nodes of a single link each. The feasible set is given before where $u(t)$ denotes the vector of path flows at t. The cost functions on the paths are defined as: $2u_1(t) - 1.5$ for the first path and $u_2(t) - 1$ for the second path. We consider a vector field F given by

$$F : K \to L^2([0,1], R^2),$$

$$(F_1(u(t)), F_2(u(t))) = (2u_1(t) - 1.5, u_2(t) - 1).$$

The theory of EVI states that the system has a unique equilibrium, since F is strictly monotone, for any arbitrarily fixed point $t \in [0,2]$. One can easily see that

$$\langle F(u_1, u_2) - F(v_1, v_2), (u_1 - v_1, u_2 - v_2)\rangle = 2(u_1 - v_1)^2 + (u_2 - v_2)^2 > 0,$$

for any

$$u \neq v \in L^2([0,2], R^2).$$

With the help of PDS theory, we can compute an approximate curve of equilibria, by choosing $t_0 \in \left\{\frac{k}{4} | k \in \{0, \ldots, 8\}\right\}$. Therefore, we obtain a sequence of PDS defined by the vector field

$$-F(u_1(t_0), u_2(t_0)) = (-2u_1(t_0) + 1.5, -u_2(t_0) + 1)$$

on nonempty, closed, convex, 1-dimensional subsets

$$K_{t_0} := \left\{ \left\{ [0, t_0] \times \left[0, \frac{3}{2}t_0\right] \right\} \cap \{x + y = t_0\} \right\}.$$

For each we can compute the unique equilibrium of the system at t_0, i.e., the point

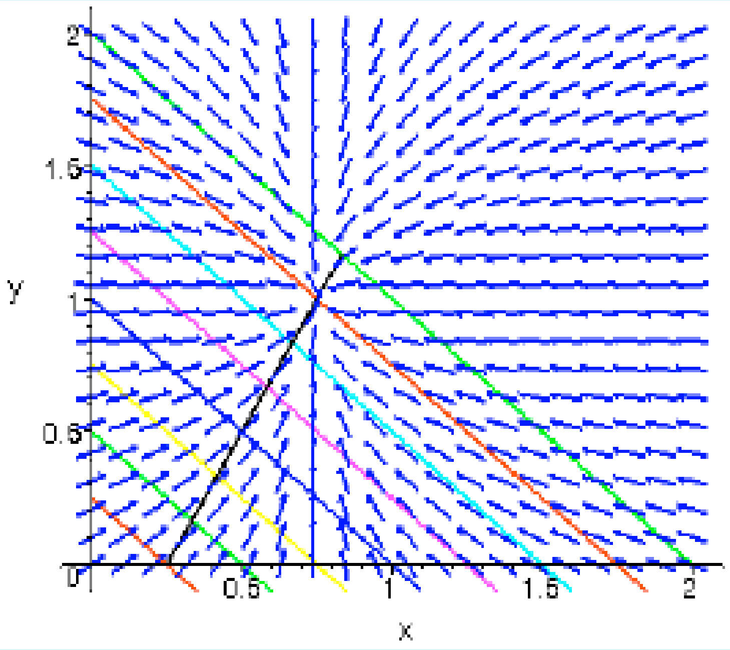
$$(u_1(t_0), u_2(t_0)) \in R^2$$

such that $-F(u_1(t_0), u_2(t_0)) \in N_{K_{t_0}}(u_1(t_0), u_2(t_0))$.

Using a simple MAPLE computation, we obtain that the equilibria are the points:

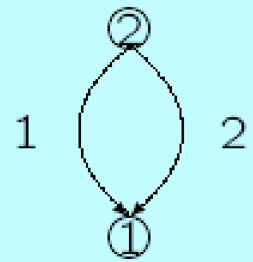
$$\left\{(0,0), \left(\frac{1}{4},0\right), \left(\frac{1}{3},\frac{1}{6}\right), \left(\frac{5}{12},\frac{1}{3}\right), \left(\frac{1}{2},\frac{1}{2}\right), \left(\frac{7}{12},\frac{2}{3}\right), \left(\frac{2}{3},\frac{5}{6}\right), \left(\frac{3}{4},1\right), \left(\frac{5}{6},\frac{7}{6}\right)\right\}.$$

Interpolating these points we obtain the approximate curve of network equilibria as displayed in the Figure.



ANOTHER DYNAMIC TRAFFIC NETWORK EXAMPLE

We again consider a transportation network consisting of a single origin/destination pair of nodes and two paths connecting these nodes of a single link each.



Network Structure of the Numerical Examples

The feasible set K is as before, where we take $p := 2$. We also have that $q := 2$, $j := 1$, $T := 2$, $\rho(t) := t$, and $\xi_{ji} := 1$ for $i \in \{1, 2\}$:

$$K = \bigcup_{t \in [0,2]} \Big\{ u \in L^2([0,2],^2) |$$

$$(0,0) \leq (u_1(t), u_2(t)) \leq \left(t, \frac{3}{2}t\right) \text{ a.e. in } [0,2];$$

$$\sum_{i=1}^{2} u_i(t) = t \text{ a.e. in } [0,2] \Big\}.$$

In this application $u(t)$ denotes the vector of path flows at t. The cost functions on the paths are defined as: $u_1(t) + 1$ for the first path and $u_2(t) + 2$ for the second path. We consider a vector field F defined by

$$F : L^2([0,2],^2) \to L^2([0,2],^2);$$

$$(F_1(u(t), F_2(u(t))) = (u_1(t) + 1, u_2(t) + 2).$$

The theory of EVI (as described above) states that the system has a unique equilibrium, since F is strictly monotone, for any arbitrarily fixed point $t \in [0,2]$. Indeed, one can easily see that $\langle F(u_1,u_2) - F(v_1,v_2), (u_1 - v_1, u_2 - v_2)\rangle = (u_1 - v_1)^2 + (u_2 - v_2)^2 > 0$, for any $u \neq v \in L^2([0,2],^2)$. With the help of PDS theory, we can compute an approximate curve of equilibria, by selecting $t_0 \in \left\{\frac{k}{4} | k \in \{0,\ldots,8\}\right\}$. Hence, we obtain a sequence of PDS defined by the vector field $-F(u_1(t_0), u_2(t_0)) = (-u_1(t_0) + 1, -u_2(t_0) + 2)$ on nonempty, closed, convex, 1–dimensional subsets:

$$_{t_0} := \left\{ \left\{ [0, t_0] \times \left[0, \frac{3}{2} t_0\right] \right\} \cap \{x + y = t_0\} \right\}.$$

For each, we can compute the unique equilibrium of the system at the point t_0, that is, the point:

$$(u_1(t_0), u_2(t_0)) \in R^2 \text{ such that}$$

$$-F(u_1(t_0), u_2(t_0)) \in N_{K_{t_0}}(u_1(t_0), u_2(t_0)).$$

Proceeding in this manner, we obtain the equilibria consisting of the points:

$$\left\{(0,0), \left(\frac{1}{4}, 0\right), \left(\frac{1}{2}, 0\right), \left(\frac{3}{4}, 0\right), (1,0), \left(\frac{9}{8}, \frac{1}{8}\right), \left(\frac{5}{4}, \frac{1}{4}\right), \left(\frac{11}{8}, \frac{3}{8}\right), \left(\frac{3}{2}, \frac{1}{2}\right)\right\}.$$

The interpolation of these points yields the curve of equilibria.

We note that due to the simplicity of the network topology and the linearity (and separability of the cost functions in this example) we can also obtain explicit formulae for the path flows over time as given below:

$$\begin{cases} u_1(t) = t, \\ u_2(t) = 0 \end{cases} \quad \text{if} \quad 0 \leq t \leq 1$$

and

$$\begin{cases} u_1(t) = \frac{t+1}{2}, \\ u_2(t) = \frac{t-1}{2}. \end{cases} \quad \text{if} \quad 1 \leq t \leq 2.$$

Summary and Conclusions

We have described the *realities* surrounding networks today and the challenges and *complexities* posed for their analysis and study.

We have argued for *new paradigms* to capture decision-making in the Information Age.

We have focused on the concept of *supernetworks* and have discussed a variety of applications.

There has never been a more exciting time to be doing research in operations research / management science.

It is this discipline that is instrumental in building bridges with economics, finance, computer science as well as different fields of engineering.

Operations researchers / management scientists have the technical expertise as well as the application-based practical know-how to truly make a difference in this world.

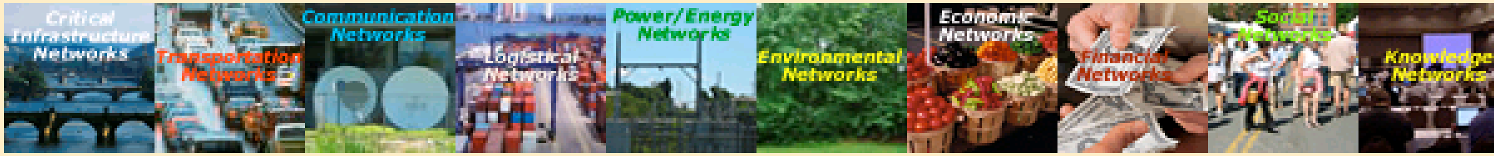
Thank you!

For more information, see
http://supernet.som.umass.edu

Eugene M. Isenberg School of Management

The Virtual Center for Supernetworks

The Virtual Center for Supernetworks



Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

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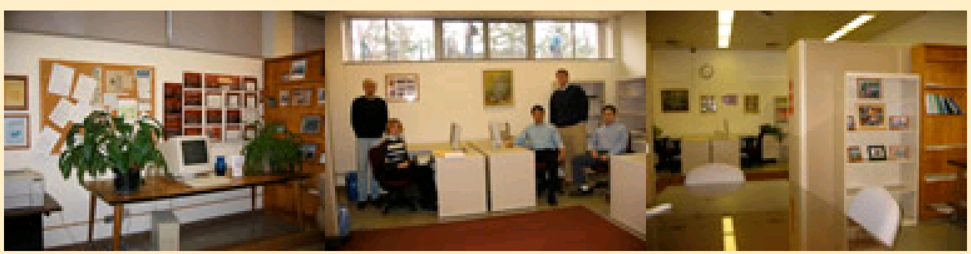
The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.



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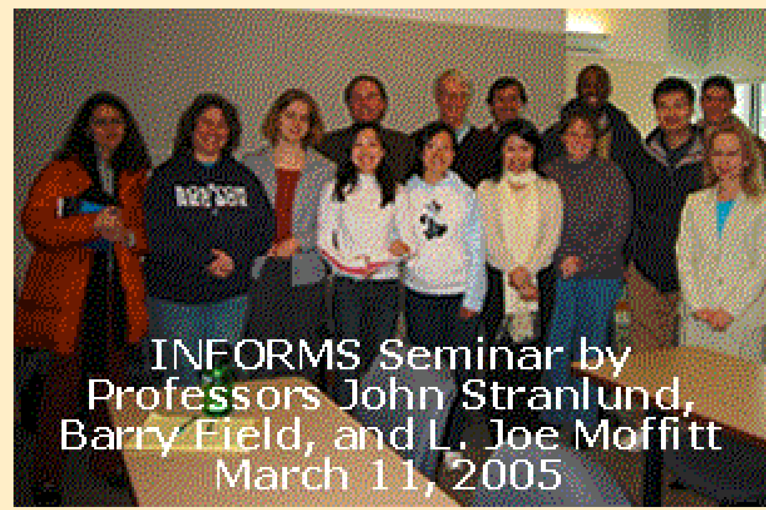

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