A Bi-Criteria Measure to Assess Supply Chain Network Performance for Critical Needs Under Capacity and Demand Disruptions

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Outline

- Background and Research Motivation
- The Supply Chain Network Model for Critical Needs Under Disruptions
- Performance Measurement of Supply Chain Networks for Critical Needs
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Background and Research Motivation

- From January to October 2005 alone, an estimated 97,490 people were killed in disasters globally; 88,117 of them lost their lives because of natural disasters (Braine 2006).
- Some deadliest examples of disasters that have been witnessed in the past few years:
  - September 11 attacks in 2001;
  - The tsunami in South Asia in 2004;
  - Hurricane Katrina in 2005;
  - Cyclone Nargis in 2008; and
  - The earthquakes in Sichuan, China in 2008.
  - The earthquakes in Japan in 2011.

Delivering the humanitarian relief supplies to the victims was a major challenge.

Disruptions to Critical Needs Supply Chains Capacities & Uncertainties in Demands

- Chiron Corporation experienced contamination in its production process – flu vaccine supplies in the US cut by 50% (Fink (2004)).
- Winter storm in China in 2008 destroyed crop supplies – sharp food price inflation.
- Overestimation of the demand for certain products resulted in a surplus of supplies with around $81 million of MREs being destroyed by FEMA (Stamm and Villareal (2009)).
- Of the approximately 1 million individuals evacuated after Katrina, about 100,000 suffered from diabetes, which requires daily medical supplies and caught the logistics chain completely off-guard (Cefalu et al. (2006)).
- "...thousands of lives could have been saved in the tsunami and other recent disasters if simple, cost effective measures like evacuation training and storage of food and medical supplies had been put in place to protect vulnerable communities."

Goals of Critical Needs Supply Chains

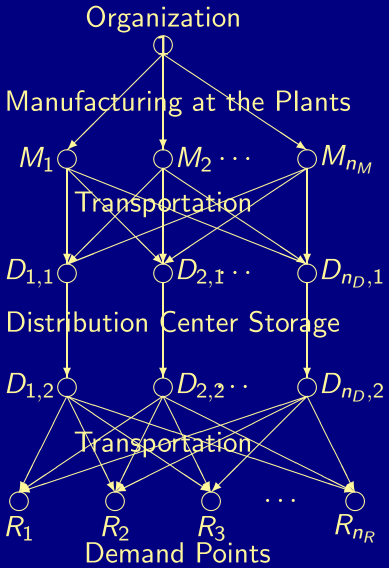
- The goals for humanitarian relief chains, for example, include cost reduction, capital reduction, and service improvement (cf. Beamon and Balcik (2008) and Altay and Green (2006)).

- "A successful humanitarian operation mitigates the urgent needs of a population with a sustainable reduction of their vulnerability in the shortest amount of time and with the least amount resources." (Tomasini and van Wassenhove (2004)).

Our paper considers the following factors:

- Supply chain capacities may be affected by disruptions;
- Demands may be affected by disruptions;
- Disruption scenarios are categorized into two types; and
- Organizations (NGOs, government, etc.) responsible for ensuring that the demand for the essential product be met is considering the possible supply chain activities, associated with the product, which are represented by network.

Topology of Supply Chain Network for Critical Needs



The Supply Chain Network Model for Critical Needs Under Disruptions: Case I: Demands Can be Satisfied Under Disruptions

We are referring, in this case, to the disruption scenario set Ξ^1.

$$v_k^{\xi_i^1} \equiv \sum_{p \in P_{w_k}} x_p^{\xi_i^1}, \quad k = 1, \ldots, n_R, \forall \xi_i^1 \in \Xi^1; \ i = 1, \ldots, \omega^1, \qquad (1)$$

where $v_k^{\xi_i^1}$ is the demand at demand point k under disruption scenario ξ_i^1; $k = 1, \ldots, n_R$ and $i = 1, \ldots, \omega^1$.

Let $f_a^{\xi_i^1}$ denote the flow of the product on link a under disruption scenario ξ_i^1. Hence, we must have the following conservation of flow equations satisfied:

$$f_a^{\xi_i^1} = \sum_{p \in P} x_p^{\xi_i^1} \delta_{ap}, \quad \forall a \in L, \forall \xi_i^1 \in \Xi^1; \ i = 1, \ldots, \omega^1 \qquad (2)$$

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link.

The total cost on a link, be it a manufacturing/production link, a transportation link, or a storage link is assumed to be a function of the flow of the product on the link We have that

$$\hat{c}_a = \hat{c}_a(f_a^{\xi_i^1}), \quad \forall a \in L, \forall \xi_i^1 \in \Xi^1; \; i = 1, \ldots, \omega^1. \tag{3}$$

We further assume that the total cost on each link is convex and continuously differentiable. We denote the nonnegative capacity on a link a under disruption scenario ξ_i^1 by $u_a^{\xi_i^1}$, $\forall a \in L$, $\forall \xi_i^1 \in \Xi^1$, with $i = 1, \ldots, \omega^1$.

The supply chain network optimization problem for critical needs faced by the organization can be expressed as follows: Under the disruption scenario ξ_i^1, the organization must solve the following problem:

$$\text{Minimize} \quad TC^{\xi_i^1} = \sum_{a \in L} \hat{c}_a(f_a^{\xi_i^1}) \tag{4}$$

subject to: constraints (1), (2) and

$$x_p^{\xi_i^1} \geq 0, \quad \forall p \in P, \forall \xi_i^1 \in \Xi^1; \ i = 1, \ldots, \omega^1, \tag{5}$$

$$f_a^{\xi_i^1} \leq u_a^{\xi_i^1}, \quad \forall a \in L. \tag{6}$$

Denote $K^{\xi_i^1}$ as the feasible set such that

$$K^{\xi_i^1} \equiv \{(x^{\xi_i^1}, \lambda^{\xi_i^1}) | x^{\xi_i^1} \text{ satisfies (1)}, \; x^{\xi_i^1} \in R_+^{n_P} \text{ and } \lambda^{\xi_i^1} \in R_+^{n_L}\}.$$

Theorem 1

The optimization problem (4), subject to constraints (1), (2) (5) and (6), is equivalent to the variational inequality problem: determine the vector of optimal path flows and the vector of optimal Lagrange multipliers $(x^{\xi_i^1 *}, \lambda^{\xi_i^1 *}) \in K^{\xi_i^1}$, such that:

$$\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[\frac{\partial \hat{C}_p(x^{\xi_i^1 *})}{\partial x_p} + \sum_{a \in L} \lambda_a^{\xi_i^1 *} \delta_{ap} \right] \times [x_p^{\xi_i^1} - x_p^{\xi_i^1 *}] + \sum_{a \in L} [u_a^{\xi_i^1} - \sum_{p \in P} x_p^{\xi_i^1 *} \delta_{ap}]$$

$$\times [\lambda_a^{\xi_i^1} - \lambda_a^{\xi_i^1 *}] \geq 0, \quad \forall (x^{\xi_i^1}, \lambda^{\xi_i^1}) \in K^{\xi_i^1}, \qquad (7)$$

where $\frac{\partial \hat{C}_p(x^{\xi_i^1})}{\partial x_p} \equiv \sum_{a \in L} \frac{\partial \hat{c}_a(f_a^{\xi_i^1})}{\partial f_a} \delta_{ap}$ for paths $p \in P_{w_k}$; $k = 1, \ldots, n_R$.

The Supply Chain Network Model for Critical Needs Under Disruptions: Case II: Demands Cannot be Satisfied Under Disruptions

The disruption scenario set in this case is Ξ^2; that is to say, the optimization problem (4) is not feasible anymore. A max-flow algorithm can be used to decide how much demand can be satisfied.

Performance Measurement of Supply Chain Networks for Critical Needs

- Performance Measure I: Demands Can be Satisfied:

  For disruption scenario ξ_i^1, the corresponding network performance measure is:

$$\mathcal{E}_1^{\xi_i^1}(G, \hat{c}, v^{\xi_i^1}) = \frac{TC^{\xi_i^1} - TC^0}{TC^0} \quad (8)$$

  where TC^0 is the minimum total cost obtained as the solution to the cost minimization problem (4).

- Performance Measure II: Demands Cannot be Satisfied:

  For disruption scenario ξ_i^2, the corresponding network performance measure is:

$$\mathcal{E}_2^{\xi_i^2}(G, \hat{c}, v^{\xi_i^2}) = \frac{TD^{\xi_i^2} - TSD^{\xi_i^2}}{TD^{\xi_i^2}}, \quad (9)$$

  where $TSD^{\xi_i^2}$ is the total satisfied demand and $TD^{\xi_i^2}$ is the total (actual) demand under disruption scenario ξ_i^2.

Definition: Bi-Criteria Performance Measurement of a Supply Chain Network for Critical Needs

The performance measurement, $\mathcal{E}$, of a supply chain network for critical needs under disruption scenario sets Ξ^1 and Ξ^2 and with associated probabilities $p_{\xi_1^1}, p_{\xi_2^1}, \ldots, p_{\xi_{\omega^1}^1}$ and $p_{\xi_1^2}, p_{\xi_2^2}, \ldots, p_{\xi_{\omega^2}^2}$, respectively, is defined as:

$$\mathcal{E} = \epsilon \times (\sum_{i=1}^{\omega^1} \mathcal{E}_1^{\xi_i^1} p_{\xi_i^1}) + (1-\epsilon) \times (\sum_{i=1}^{\omega^2} \mathcal{E}_2^{\xi_i^2} p_{\xi_i^2}) \tag{10}$$

where ϵ is the weight associated with the network performance when demands can be satisfied, which has a value between 0 and 1. The higher ϵ is, the more emphasis is put on the cost efficiency.

Modified Projection Method (cf. Korpelevich (1977) and Nagurney (1993))

Step 0: Initialization
Set $X^{\xi_i^1 0} \in K^{\xi_i^1}$. Let $\mathcal{T} = 1$ and set α such that $0 < \alpha \leq \frac{1}{L}$ where L is the Lipschitz constant for the problem.

Step 1: Computation
Compute $\bar{X}^{\xi_i^1 \mathcal{T}}$ by solving the variational inequality subproblem:

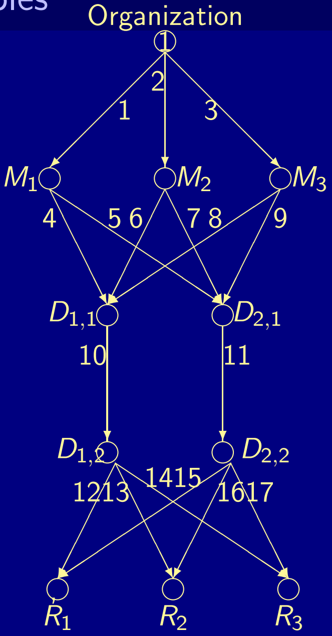
$$< (\bar{X}^{\xi_i^1 \mathcal{T}} + \alpha F(X^{\xi_i^1 \mathcal{T}-1}) - X^{\xi_i^1 \mathcal{T}-1})^T, X^{\xi_i^1} - \bar{X}^{\xi_i^1 \mathcal{T}} >, \ \forall X^{\xi_i^1} \in K^{\xi_i^1}. \quad (11)$$

Step 2: Adaptation
Compute $X^{\xi_i^1 \mathcal{T}}$ by solving the variational inequality subproblem:

$$< (X^{\xi_i^1 \mathcal{T}} + \alpha F(\bar{X}^{\xi_i^1 \mathcal{T}-1}) - X^{\xi_i^1 \mathcal{T}-1})^T, X^{\xi_i^1} - X^{\xi_i^1 \mathcal{T}} >, \ \forall X^{\xi_i^1} \in K^{\xi_i^1}. \quad (12)$$

Step 3: Convergence Verification
If $\max |X_l^{\xi_i^1 \mathcal{T}} - X_l^{\xi_i^1 \mathcal{T}-1}| \leq e$, for all l, with $e > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} = \mathcal{T} + 1$, and return to Step 1.

Numerical Examples



Total Cost Functions, Capacities, and Solution for the Baseline Numerical Example Under No Disruptions

Link a	$\hat{c}_a(f_a)$	u_a^0	f_a^{0*}	λ_a^{0*}
1	$f_1^2 + 2f_1$	10.00	3.12	0.00
2	$.5f_2^2 + f_2$	10.00	6.88	0.00
3	$.5f_3^2 + f_3$	5.00	5.00	0.93
4	$1.5f_4^2 + 2f_4$	6.00	1.79	0.00
5	$f_5^2 + 3f_5$	4.00	1.33	0.00
6	$f_6^2 + 2f_6$	4.00	2.88	0.00
7	$.5f_7^2 + 2f_7$	4.00	4.00	0.05
8	$.5f_8^2 + 2f_8$	4.00	4.00	2.70
9	$f_9^2 + 5f_9$	4.00	1.00	0.00
10	$.5f_{10}^2 + 2f_{10}$	16.00	8.67	0.00
11	$f_{11}^2 + f_{11}$	10.00	6.33	0.00
12	$.5f_{12}^2 + 2f_{12}$	2.00	3.76	0.00
13	$.5f_{13}^2 + 5f_{13}$	4.00	2.14	0.00
14	f_{14}^2	4.00	2.76	0.10
15	$f_{15}^2 + 2f_{15}$	2.00	1.24	0.00
16	$.5f_{16}^2 + 3f_{16}$	4.00	2.86	0.00
17	$.5f_{17}^2 + 2f_{17}$	4.00	2.24	0.00

There Are Three Disruption Scenarios – Example Set I

1. The capacities on the manufacturing links 1 and 2 are disrupted by 50% and the demands remain unchanged. (Disruption type 1)
2. The capacities on the storage links 10 and 11 are disrupted by 20% and the demands at the demand points 1 and 2 are increased by 20%. (Disruption type 1)
3. The capacities on links 12 and 15 are decreased by 50% and the demand at demand point 1 is increased by 100%. The probabilities associated with these three scenarios are: 0.4, 0.3, 0.2, respectively, and the probability of no disruption is 0.1.(Disruption type 2)

For 1 and 2, we have that $TC^{\xi_1^1} = 299.02$ and $TC^{\xi_2^1} = 361.41$.and therefore, $\mathcal{E}_1^{\xi_1^1} = \frac{TC^{\xi_1^1} - TC^0}{TC^0} = 0.0296$ and $\mathcal{E}_1^{\xi_2^1} = \frac{TC^{\xi_2^1} - TC^0}{TC^0} = 0.2444$.

For 3, we have that $\mathcal{E}^{\xi_1^2} = \frac{TD^{\xi_1^2} - TSD^{\xi_1^2}}{TD^{\xi_1^2}} = 0.7000$.

We let $\epsilon = 0.2$ to reflect the importance of being able to satisfy demands and we compute the bi-criteria supply chain performance measure as:

$$\mathcal{E} = 0.2 \times (0.4 \times \mathcal{E}_1^{\xi_1^1} + 0.3 \times \mathcal{E}_1^{\xi_2^1}) + 0.8 \times (0.2 \times \mathcal{E}_2^{\xi_1^2}) = 0.1290.$$

Example Set II

Everything is the same as in Example Set 1 above except that under the third scenario above, the disruptions have decreased the demand by 20% at demand point 1 but have increased the demand by 20% at demand point 2.

It is reasonable to assume that the critical needs demands may move from one point to another under disruptions and it is important for a supply chain to be able to meet the demands in such scenarios. Indeed, those affected may need to be evacuated to other locations, thereby, altering the associated demands. Under this scenario, we know that all the demands can be satisfied and we have that $TC^{\xi_3^1} = 295.00$, which means that $\mathcal{E}^{\xi_3^1} = \frac{TC^{\xi_3^1} - TC^0}{TC^0} = 0.0157$. Hence, given the same weight ϵ as in the First Case, the bi-criteria supply chain performance measure is now:

$$\mathcal{E} = 0.2 \times (0.4 \times \mathcal{E}_1^{\xi_1^1} + 0.3 \times \mathcal{E}_1^{\xi_2^1} + .2 \times \mathcal{E}_1^{\xi_3^1}) + 0.8 \times (0.2 \times \mathcal{E}_2^{\xi_1^2}) = 0.0177.$$

Conclusions

- We developed a supply chain network model for critical needs, which captures disruptions in capacities associated with the various supply chain activities of production, transportation, and storage, as well as those associated with the demands for the product at the various demand points.
- We showed that the governing optimality conditions can be formulated as a variational inequality problem with nice features for numerical solution.
- We proposed two distinct supply chain network performance measures for critical needs products. We then constructed a bi-criteria supply chain network performance measure and used it for the evaluation of distinct supply chain networks. The bi-criteria measure allows for the comparison of the robustness of different supply chain networks under a spectrum of real-world scenarios.
- We illustrated the new concepts with numerical examples in which the supply chains were subject to a spectrum of disruptions involving capacity reductions as well as demand changes.

Thank You!