Influence of Beckmann, McGuire, and Winsten's Studies in the Economics of Transportation on Innovations in Modeling, Methodological Developments, and Applications

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The book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten was published in 1956 by Yale University Press and was a breakthrough in the rigorous modeling and analysis of transportation problems with a focus on congested highway systems as well as railroad systems.

Its impact has been seminal, far-reaching, and continues to this day.

- **In this paper, we focus on the influence of this book on innovations in modeling, methodological developments, and applications.**

- **In particular, this paper traces the impacts of the first part of the book, which is on highway transportation.**

- I first begin with a description of the context in which the writing of this book took place. The book was based on a Rand Corporation report of the same name, RM-1488, and dated May 12, 1955, with an introduction by Tjalling C. Koopmans, who twenty years after was awarded a Nobel Prize in Economics.
- Koopmans noted that the report consisted of exploratory studies with an intended audience of various professionals, including economists, traffic and railroad engineers, as well as operations researchers/management scientists, and mathematicians.

- **The report resulted from a research project conducted by the Cowles Commission for Research in Economics with funding provided by the Rand Corporation. Koopmans was the project leader. Beckmann, a mathematical economist, was especially interested at that time in linear programming and economic activity analysis. Winsten, a mathematician and economist, held a particular interest in applying probability concepts to industrial issues, whereas McGuire, an economist, provided a pragmatic and realistic check on the model development. In terms of the study of highway transportation, the main emphasis was on congestion.**

U. S. AIR FORCE
PROJECT RAND

RESEARCH MEMORANDUM

STUDIES IN THE ECONOMICS OF TRANSPORTATION *

Martin Beckmann
C. B. McGuire
Christopher B. Winsten

RM-1488

12 May 1955

Assigned to ______________________

57

The RAND Corporation
1700 MAIN ST. • SANTA MONICA • CALIFORNIA

* To be published by Yale University Press.

- **The topic of transportation had been addressed earlier in the context of optimal allocation of resources through linear programming by Hitchcock (1941) and Kantorovich (1942) (who later shared the Nobel Prize with Koopmans) as well as by Koopmans (1947) and Dantzig (1951). In such models, however, there was no congestion associated with transportation.**

- **The problem of users of a congested transportation network seeking to determine their travel paths from origins to their respective destinations appears as early as Pigou (1920), who considered a two-node, two-link (or path) network, and was further developed in Knight (1924). Both of these references are cited in the Beckmann, McGuire, and Winsten (1956) book.**

- **Fascinatingly, Koopmans in his introduction also acknowledged the work of Enke (1951) and Samuelson (1952) in terms of commodity transportation and the determination of interregional price differentials, a topic now known as spatial price equilibrium, and one which we return to later.**

- **In 1952, Wardrop had set forth two principles of transportation network utilization, which have come to be termed, respectively (cf. Dafermos and Sparrow (1969)), user-optimization and system-optimization.**
- **The first principle expresses that travelers select their routes of travel from origins to destinations independently and ultimately the journey times of all routes actually used between an origin/destination pair are equal and less than those which would be experienced by a single vehicle on any unused route. The user-optimized solution is also referred to as a *traffic network equilibrium* or a *traffic assignment*.**

- **The second principle, in contrast, reflects the situation in which there is a central controller who routes the traffic flows in an optimal manner from origins to the destinations so as to minimize the total cost in the network. That optimum is reached when the marginals of the total costs on used paths connecting an origin/destination pair are equal and minimal**.

- **Beckmann, McGuire, and Winsten (1956) were the first to provide a rigorous mathematical formulation of the conditions set forth by Wardrop's first principle that allowed for the ultimate solution of the traffic network equilibrium problem in the context of certain link cost functions which were increasing functions of the flows on the links.**

- **In particular, they demonstrated that the optimality conditions in the form of Kuhn-Tucker (1951) conditions of an appropriately constructed mathematical programming/optimization problem coincided with the statement that the travel costs on utilized routes/paths connecting each origin/destination pair of nodes in a transportation network have equal and minimal travel costs. Hence, no traveler, acting unilaterally will have any incentive to alter his path (assuming rational behavior) since his travel cost (travel time) is minimal.**

- **Interestingly, Charnes and Cooper (1958, 1961) in their papers had cited the work of Nash (1951), Wardrop (1952), and Prager (1954), with their (1958) paper also noting Duffin (1947), who provided a formulation of the equilibrium in electrical networks, but they did not cite Beckmann, McGuire, and Winsten (1956).**

- **Thus, a problem in which there are numerous decision-makers acting independently and as later also noted by Dafermos and Sparrow (1969) competing in the sense of Nash, could be reformulated (under appropriate assumptions) as a convex optimization problem with a single objective function subject to linear constraints and nonnegativity assumptions of the flow on the network.**
- **Prager (1954) had also recognized Wardrop's principles and in his paper emphasized that the traffic cost on a link may depend not only on the flow on that link but on other links in the network as well**.

- **Jorgensen (1963) in a report (actually his Master's thesis) did not cite Beckmann, McGuire, and Winsten (1956) but noted the work of Wardrop (1952) and Charnes and Cooper (1961) and developed an optimization reformulation of the traffic network equilibrium conditions in the case of fixed travel demands and links cost functions that were separable.**
- **Jorgensen (1963) also influenced the thesis of Dafermos (1968) upon which the paper of Dafermos and Sparrow (1969) is based.**

- **In this paper, I trace the impacts of the book. Such an assignment is challenging and daunting given the almost fifty years that have elapsed since its publication. Nevertheless, it is important to highlight and to emphasize further the impact of this monumental work, even if it is done through the prism of one's own experiences and knowledge of the literature, but accompanied by interactions with many leaders in the transportation science and broader scientific community whose work has been impacted by this volume.**

Innovations in Modeling and Methodological Developments

- **Algorithms and Computations for the Standard Models of BMW**
- **Toll Policies**
- **Extended Traffic Network Models Including Models of Urban Location**
- **Variational Inequality Formulations and Algorithms**
- **Multicriteria Decision-Making**
- **Stochastic Route Choice Modeling**
- **Dynamic Transportation Networks**
- **Sensitivity Analysis and Stability Analysis**

Algorithms and Computations for the Standard Models of BMW

- **The first major methodological innovation in a paper that cites Beckmann, McGuire, and Winsten (BMW) (1956) is the paper by Dafermos and Sparrow (1969) which not only coined the terms *user-optimization* and *system-optimization* to distinguish between Wardrop's first and second principles, respectively, and to help to clarify to underlying behavior of the travelers in these two contexts, but also developed algorithms that explicitly exploited the network structure of these two problems and established convergence results for the schemes.**

- **Moreover, that paper, provided not only equilibration algorithms for networks of any topology but also special-purpose ones in the case of special topologies for which the flows could be computed exactly and in closed form. Further, the paper discussed stability of the solution patterns, a topic whose importance was emphasized in BMW**.

- **Almond (1967) had constructed an algorithm for the determination of the user-optimized solution but in the case of very simple networks. Tomlin (1966), in turn, considered linear cost (congestion) functions and not nonlinear ones as had Dafermos and Sparrow (1969) and exploited that feature in the development of his algorithm. Almond (1967) cited BMW whereas Tomlin did not although he did refer to Jorgensen (1963).**
- **Leventhal, Nemhauser, and Trotter (1973) proposed a column generation procedure that could be embedded in the Dafermos and Sparrow general equilibration procedures to allow for path generation as needed (rather than apriori which could require large computer memory resources).**

- **The first innovations in algorithm development for traffic network equilibrium problems focused on the standard models of BMW where by standard is meant that the link cost functions were separable in that the cost on a link depended upon only the flow on that link.**
- **Effective schemes for such problems are important not only for such problems but also in the case of more general network models for which an optimization reformulation of the governing equilibrium conditions is not available and, hence, one must appeal to variational inequality formulations.**

- **Bruynooghe, Gilbert, and Sakarovitch (1969) considered the fixed demand model and also discussed two algorithms and cited the Beckmann, McGuire, and Winsten (1956) book. Netter (1971) further described the properties of system-optimized versus user-optimized solutions and referred to BMW. LeBlanc (1973); see also LeBlanc, Morlok, and Pierskalla (1975), proposed an algorithm based on the Frank-Wolfe (1956) convex programming scheme to solve the traffic assignment problem, and although he cited BMW, he did not cite Dafermos and Sparrow (1969).**

- **The use of concepts formalized in BMW to model traffic network equilibrium problems by defining the appropriate origin/destination pairs, links, paths, and associated link costs as well as travel demands was receiving increasing attention and recognition and has to-date been applied in settings distinct from transportation science.**

- **Indeed, it is quite remarkable how the fundamental work of BMW continues to be rediscovered, elaborated upon, and utilized in numerous applications.**

- **We further emphasize the importance of rigorous scientific methodologies for modeling, analysis, and solution of traffic network equilibrium problems, which are not only of theoretical interest, but also of great practical importance due to the growing congestion in developed countries as well as in developing countries.**
- **Bar-Gera (1999) has devised a convergent algorithm based on origin-based assignment which has been applied to solve networks of realistic size.**

Toll Policies

- **In 1971 Dafermos and Sparrow published a paper on optimal resource allocation and tolls, which would guarantee that once assigned, the user-optimized solution would coincide with the system-optimized solution so that individual travelers would behave in a manner that would also be optimal from a system or societal point of view. BMW had earlier discussed how efficiency toll rates could be determined, whereas Beckmann (1967a) described optimal tolls for highways, tunnels, and bridges.**

- **In both works, tolls were viewed as a means of bringing about the best utilization of the transportation network rather than as a means of construction financing per se.**
- **Dafermos and Sparrow (1971) proposed two types of toll policies, in link form and in path form, with the latter allowing for more flexibility from the planning perspective but resulting perhaps in subsidies unlike the link policy. That paper, as the paper of Dafermos and Sparrow (1969), was based on the thesis of Dafermos (1968), which, as we have noted earlier, cited BMW.**

- **Pigou in 1920 had proposed tolls which could be imposed by the government in such a way so that the altered user-optimized flow pattern would coincide with the total cost (system-optimized) optimized pattern.**
- **Another relevant early reference is that of Vickrey (1952), whose work in the pricing of transportation services later earned him a Nobel Prize, and who was cited in BMW. Walters (1961), subsequently, utilized the network model of BMW for toll determination.**

- **The topic of congestion pricing through tolls has been recently an active area of research and practice with tolls schemes being applied in various parts of the world, including, with some success, in London. For a recent approach and additional references, see Bergendorff, Hearn, and Ramana (1997).**

Extended Traffic Network Models Including Models of Urban Location

- **BMW focused on transportation networks in which the cost (also travel time) on a link, that is, road, depended solely upon the flow on that link. Under such an assumption (i.e., separable functions and necessarily *symmetric*) they could then prove their fundamental result.**

- **Dafermos in a series of papers in the early 70s, which cited BMW, developed *extended* traffic network models and also formulated tolls in the case of multiclass networks.**
- **Dafermos in her 1971 and 1972 papers demonstrated that an analogous reformulation of the traffic network equilibrium conditions as a convex optimization problem could be identified in the case of more general user link cost functions in which the cost on a link could depend on the flows on all links in the network, provided that a symmetry condition held either in the single-class user case or the multi-class user case, which allowed for different classes of travelers who perceive the travel cost on a link in an individual manner.**

- **She further demonstrated that one could transform that model into an extended, single-class one by constructing appropriate abstract copies of the multiclass network and by redefining the underlying functions and flows. In addition, extensions of the general equilibration algorithms contained in Dafermos and Sparrow (1969), along with convergence results, were obtained in Dafermos (1971, 1972). In 1973, Dafermos further generalized tolls to multiclass traffic networks, which are also now referred to as multimodal networks.**

- **Beckmann, McGuire, and Winsten (1956) clearly delineated that one should distinguish between short-run and long-run decision-making regarding transportation networks. In particular, they noted that if travelers have already made their origin and destination selections, then the decision becomes one where one must determine the optimal path to take between the two. However, in the long-run, travelers may wish to choose not only their routes but also perhaps their origins in the form of residences and/or destinations, say, in the form of places of employment.**

- **Motivated by such questions, Dafermos in 1976 demonstrated, through the use of abstract networks that one could capture such decision-making within a network equilibrium context. Again, the fundamental concepts devised and elaborated upon in BMW were now being applied to more complex decision-making which included not only route choice.**

- **In 1980, Boyce proposed a framework for constructing network equilibrium models of urban location which allowed for the incorporation of the trip distribution problem. In 1983, Boyce et al., motivated by the first author's work plus that of BMW and the contributions of Evans (1973, 1976) regarding the efficient and practical solution of network equilibrium problems, presented a unified approach (see also Boyce and Southworth (1979) and Erlander (1980)) to deriving models of urban location, destination, mode, and route choice.**

- **Moreover, selected parts of the modeling framework were implemented for the Chicago region. In Boyce et al. (1983), the calibration of the model parameters was described as was the estimation of the coefficients of the generalized link cost functions. See Boyce and Mattsson (1999) for an application of a network equilibrium model for residential location choice in relation to housing location and road tolls, along with additional citations.**

Variational Inequality Formulations and Algorithms

- **Smith (1979) provided an alternative formulation of traffic network equilibrium, which was identified by Dafermos (1980) to be a variational inequality problem. These fundamental papers, which cited BMW, enabled the modeling, analysis, and computation of solutions to traffic network equilibrium problems in which the symmetry assumption no longer held, which, simply stated, means that the cost on a link depends on the flow on another link in the same fashion that the cost on the other link depends on the former link's flow. In this case, important from the application standpoint, one could no longer reformulate the network equilibrium conditions as a solution to an optimization problem.**

- **The variational inequality framework would revolutionize the formulation, analysis, and computation of solutions to network equilibrium problems, in general, as well as other equilibrium problems.**

- **BMW specifically emphasized elastic demand traffic network problems and developed a model which allowed for the prediction not only of the traffic volumes on the links or roads of the network but also the travel demand associated with the origin/destination pairs. Hence, there may be times when travelers opt not to travel at all due to the cost associated with congestion.**
- **Dafermos in 1982, subsequently, recognizing the generality of the elastic demand traffic network model proposed a multiclass, asymmetric model and formulated and solved it as a variational inequality problem**.

Multicriteria Decision-Making

- **The recognition that different criteria in addition to time and cost might be applicable in transportation route choice selection, notably, that of *risk* was explicit in the book of Beckmann, McGuire, and Winsten (1956). This is especially timely given the new world scenario and further underscores the brilliance of this book and the creativity and longevity of the authors' ideas and contributions.**

- **Indeed, although Schneider (1968) and Quandt (1967) proposed multicriteria traffic network equilibrium models, it was actually Dial (1979) who further developed such ideas and Dafermos (1981) who introduced congestion effects into such a model and formulated it as a variational inequality problem (in fact, an infinite-dimensional one).**

Stochastic Route Choice Modeling

- **The first stochastic route choice model was proposed by Dial (1971) who developed a logit model that was flow-independent. Daganzo and Sheffi (1977) constructed a stochastic user equilibrium model in which at the equilibrium state, no traveler can improve upon his *perceived* travel time by unilaterally changing routes. Additional background on such models, can be found in the book by Sheffi (1985). See also the review articles by Boyce, LeBlanc, and Chon (1988) and Florian and Hearn (1995), which also discuss deterministic models.**

Dynamic Transportation Networks

- **Although Beckmann, McGuire, and Winsten (1956) did not explicitly formulate dynamic traffic network models, the recognition of the importance of such models was explicit in the book.**
- **Yagar (1971), Hurdle (1974), and Merchant and Nemhauser (1978a, b) were some of the first contributors to the development of dynamic models with explicit flows, and the work of Merchant and Nemhauser (1978a, b) is often credited with being the first to consider dynamic route choices over general networks.**

- **In particular, they studied dynamic system-optimized networks in the case of single destination networks and athough they did not cite BMW, they did reference Dafermos and Sparrow (1969).**
- **Mahmassani and Herman (1984) developed a dynamic user equilibrium departure time and route choice model.**
- **Carey (1987), in turn, did reference Beckmann, McGuire, and Winsten (1956) and provided a convex programming formulation of a dynamic system-optimized traffic network which could handle multiple destinations and multiple commodities.**

- **Today, variational inequality theory has since become the theoretical basis for the analysis and computation of Wardrop equilibria in a within-day static traffic network.**
- **Indeed, motivated by realistic concerns, within-day dynamic traffic assignment problem is receiving increasing attention (cf. Janson (1991), Smith (1993), Friesz et al. (1993), Ran and Boyce (1994), Wu (1994), and Wu et al. (1998), among others). Underlying a dynamic user equilibrium is a ``doubly" dynamic system which is comprised of a day-to-day adjustment process and a within-day realization process.**

- **The day-to-day adjustment process addresses the users' behavior in acquiring information and in adjusting their departure time and route choices (see, e. g., Smith (1984), Mahmassani (1990), Friesz, et al. (1994), Zhang and Nagurney (1996), Nagurney and Zhang (1997), Zhang and Nagurney (1997)). The within-day realization process addresses the real time dynamic traffic flow as the realization of the users' route choices on the particular day, which, in turn, results in updated information feedback to the day-to-day process.**

- **A dynamic loading operation (Wu et al. (1998)) is involved in this realization process that loads the dynamic path departure rates into dynamic link volumes which determine the dynamic link travel times as the feedback to the travelers.**

- **For some additional insights, see Zhang, Nagurney, and Wu (2001).**

- **Dupuis and Nagurney (1993), motivated in great part by the need to introduce dynamics into the formal modeling and analysis of network systems, including transportation networks, that had been studied primarily at an equilibrium state, using, for example, variational inequality theory, developed the basic theory of existence and uniqueness as well as computational procedures for what are now termed *projected dynamical systems* (cf. also Zhang and Nagurney (1995) and Nagurney and Zhang (1996)).**

Sensitivity Analysis and Stability Analysis

- **The importance of stability analysis was recognized in Beckmann, McGuire, and Winsten (1956). Dafermos and Sparrow (1969), subsequently, obtained stability analysis results in the context of user-optimized models in the static setting.**

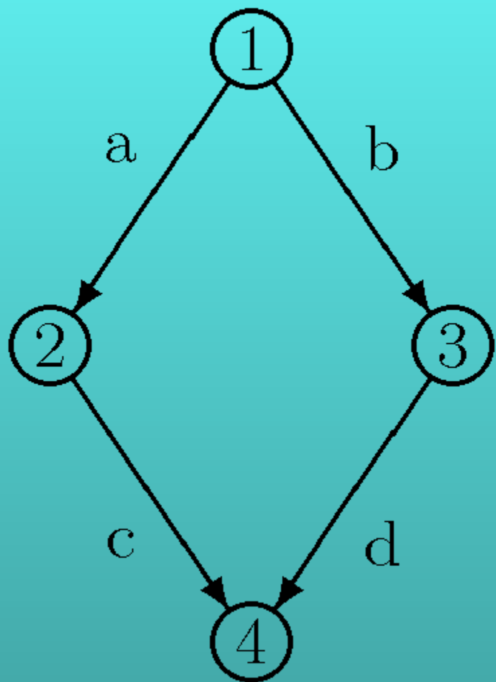
- **More recently, Nagurney and Zhang (1996), motivated by the connection between finite-dimensional variational inequality problems and dynamical systems as defined by Dupuis and Nagurney (1993) (see also Zhang and Nagurney (1995)), and as discussed above, obtained local and global stability analysis results for dynamic traffic network problems modeled as projected dynamical systems. Stability analysis using Lyapunov functions was addressed by Smith (1979, 1984) in some of his major works.**

- **Interestingly, Braess (1968), whose well-known paradox motivated much of the subsequent research in sensitivity analysis and networks, cited neither Wardrop (1952) nor Beckmann, McGuire, and Winsten (1956). That paper was followed by the contributions of Murchland (1970), who elaborated upon the Braess paradox and reflected upon it in the context of BMW and Beckmann (1967b).**

- **Fisk (1979) also cited BMW and identified additional paradoxical phenomena in traffic networks. Stewart (1980) and Steinberg and Zangwill (1983) further spurred the investigation of sensitivity analysis in network equilibrium problems.**
- **The thesis of Nagurney (1983) (see also Dafermos and Nagurney (1984 a, b, c)) addressed such issues and computational ones for general network equilibrium problems in a variational inequality framework. Dafermos and Nagurney (1984d) obtained stability and sensitivity analysis results for a general network equilibrium travel choice model with elastic demands using the variational inequality formulation derived therein and noted BMW.**

- **Today, paradoxes on networks, due to alternative behaviors of decision-makers, are garnering increasing attention in other scientific communities, including that of computer science.**

Braess Paradox



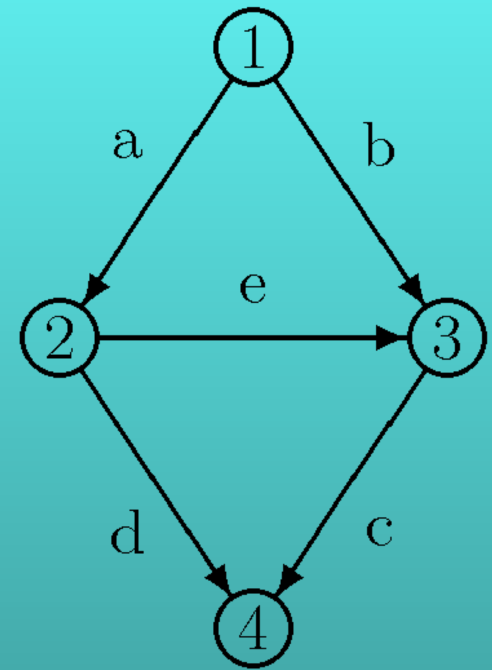
$c_a(f_a)=10f_a$

$c_b(f_b)=f_b+50$

$c_c(f_c)=f_c+50$

$c_d(f_d)=10f_d$

$c_c(f_e)=f_e+10$



O/D w=(1,4) travel demand is $d_w=6$.
Paths $p_1=(a,c)$ $p_2=(b,d)$.
The user-optimized solution in path flows: $x_{p1}{}^* = x_{p2}{}^* =3$
The induced link flow pattern:
The link travel costs:
$c_a=30$ $c_b=53$ $c_c =53$ $c_d=30$,
and the user path travel costs:
$C_{p1}=c_a+c_c=83$ $C_{p2}=c_b+c_d=83$.

Consider the addition of a new road/link e to the network
A new path $p_3=(a,e,c)$ is available.
The new user-optimized solution in path flows: $x_{p1}{}^* = x_{p2}{}^* = x_{p3}{}^* = 2$
The induced the link flow pattern:
$f_a{}^*=4$ $f_b{}^*=2$ $f_c{}^*=2$ $f_d{}^* =4$ $f_e{}^*=2$
and associated user path travel costs:
$C_{p1} = C_{p2} = C_{p3} = 92$.

Network Equilibrium Applications

- **I now highlight the many applications whose further development has benefited from the book by Beckmann, McGuire, and Winsten (1956).**

- **Beckmann (1967b), in his survey article, noted that there were analogues of the elastic demand network equilibrium model for problems other than road traffic and included examples to the distribution of electric current, steam, water, and natural gas distribution, as well as to the routing of messages in a communications network.**

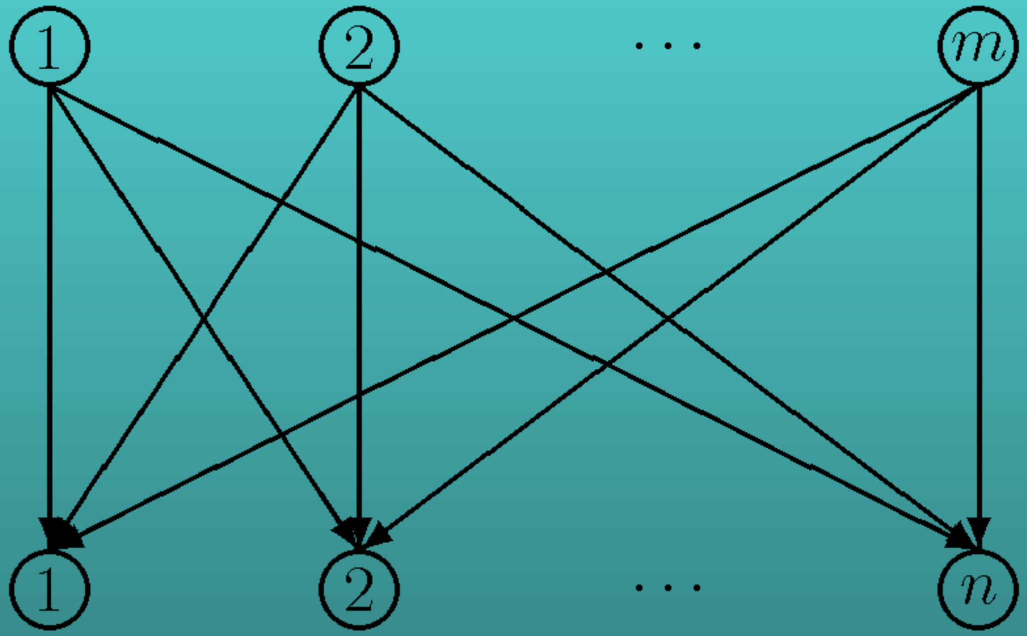
Applications

- **Spatial Price Equilibrium Networks**
- **General Economic Equilibrium**
- **Oligopolistic Market Equilibrium and Game Theory**
- **Supernetworks: Applications to Telecommuting Decision-Making and Teleshopping Decision-Making**
- **Supply Chain Networks**
- **Knowledge Networks**
- **Computer Scientists Discover Beckmann, McGuire, and Winsten**

Spatial Price Equilibrium Networks

- **Koopmans, in his introduction, in discussing the railroad transportation contributions in the BMW book, noted the work of Enke (1951) and Samuelson (1952) in the development of frameworks (the former using analogues to electronic circuits and the latter to a linear programming problem) for the determination of interregional commodity flows and prices in the case of separated markets.**
- **Takayama and Judge (1964) in their first major paper on spatial equilibrium demonstrated how, in the case of linear regional supply and demand functions and fixed interregional transportation costs, the governing spatial price equilibrium conditions could be reformulated as the Kuhn-Tucker conditions of a quadratic programming problem. In the paper, the authors thank first Martin Beckmann for helpful comments.**

The Structure of Classical Spatial Price Networks



- **Florian and Los (1982) provided a synthesis of the Samuelson (1952) model and the BMW network equilibrium model with elastic/variable demand to construct a spatial price equilibrium model on a general network. They also considered multicommodity models and demonstrated that the governing equilibrium conditions satisfy a variational inequality problem akin to those arising in traffic network equilibrium models.**

- **Others had also been developing and extending the basic spatial price equilibrium models of Samuelson (1952) and Takayama and Judge (1964, 1971) (for a list of references, see Nagurney (1993)).**

- **It was researchers in transportation science that truly exploited the connections between the two subjects which had actually been identified as early as the seminal book.**

- **Nagurney and Dafermos (1985) established an isomorphism between spatial price and traffic network equilibrium problems which was further elaborated upon by Dafermos (1986) in the context of multicommodity/multiclass networks.**

- **Friesz et al. (1983, 1984), citing BMW, provided additional contributions to the modeling, analysis, and solution of spatial price network equilibrium problems and forged the topic of freight network equilibrium.**

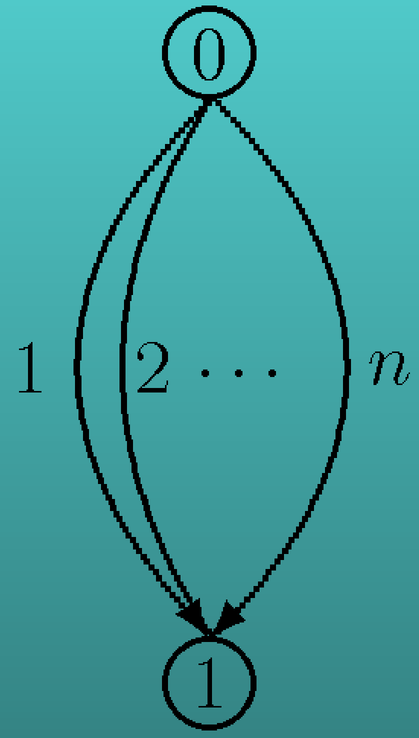
- **Nagurney (1987) demonstrated the efficient solution of spatial price equilibrium problems whereas Nagurney, Nicholson, and Bishop (1996) discussed the solution of large-scale such problem in the case of ad valorem tariffs. Finally, utilizing the theory of projected dynamical systems,**

General Economic Equilibrium

- **Spatial price equilibrium models, in contrast to general economic equilibrium models, are necessarily partial equilibrium models. The network structure of spatial price equilibrium problems considered today often corresponds to the physical transportation network.**

- **The general economic equilibrium problem due to Walras (1874) has also been extensively studied (see, e.g., Border (1985)) both from qualitative as well as quantitative perspectives (cf. Dafermos (1990) and the references therein). The Walrasian price equilibrium problem can also be cast into a network equilibrium form as shown in Zhao and Nagurney (1993), who recognized the work of BMW (see also Nagurney (1993)).**

Network Structure of Walrasian Price Equilibrium



Oligopolistic Market Equilibrium and Game Theory

- **Game theory, although not explicitly recognized in the sense of Nash (1951) (see also Nash (1950)) in the work of BMW, but noted in the Dafermos and Sparrow (1969) paper and cited by Charnes and Cooper (1958, 1961), has had an enormous impact not only on economics but lately also in computer science.**

- **Such problems date to Cournot (1838) and Nash equilibria in the context of oligopoly problems have been shown to satisfy variational inequalities by Gabay and Moulin (1982) and solved thus by Harker (1984, 1986) and by Nagurney (1988) (see also Murphy, Sherali, and Soyster (1982)).**

- **Nagurney (1993) demonstrated that the classical aspatial Cournot oligopoly market equilibrium problem could also be cast into a network equilibrium framework on an abstract network (of the same structure as that underlying the Walrasian price equilibrium problem in Figure 4) but with elastic demand and cited Beckmann, McGuire, and Winsten (1956). In the network setting, the links correspond to the firms and the flows on the links are the production outputs.**

- **Spence (1976) had noted that in the case of a linear demand functions and quadratic production cost function for each firm in the oligopoly, the equilibrium production outputs could be determined as the solution of a convex optimization problem**

- **Dafermos and Nagurney (1987) established the connection between spatial oligopolies operating in a Nash-Cournot sense and spatial price equilibrium problems.**

- **Devarajan (1981), motivated by the Dafermos and Sparrow (1969) paper, established that a continuous flow, user-optimized network is a pure-strategy Nash equilibrium in a game with a continuum of pure strategies.**

- **Haurie and Marcotte (1985) further tightened the connection between Nash-Cournot equilibria and Wardrop equilibria**.

Supernetworks: Applications to Telecommuting Decision-Making and Teleshopping Decision-Making

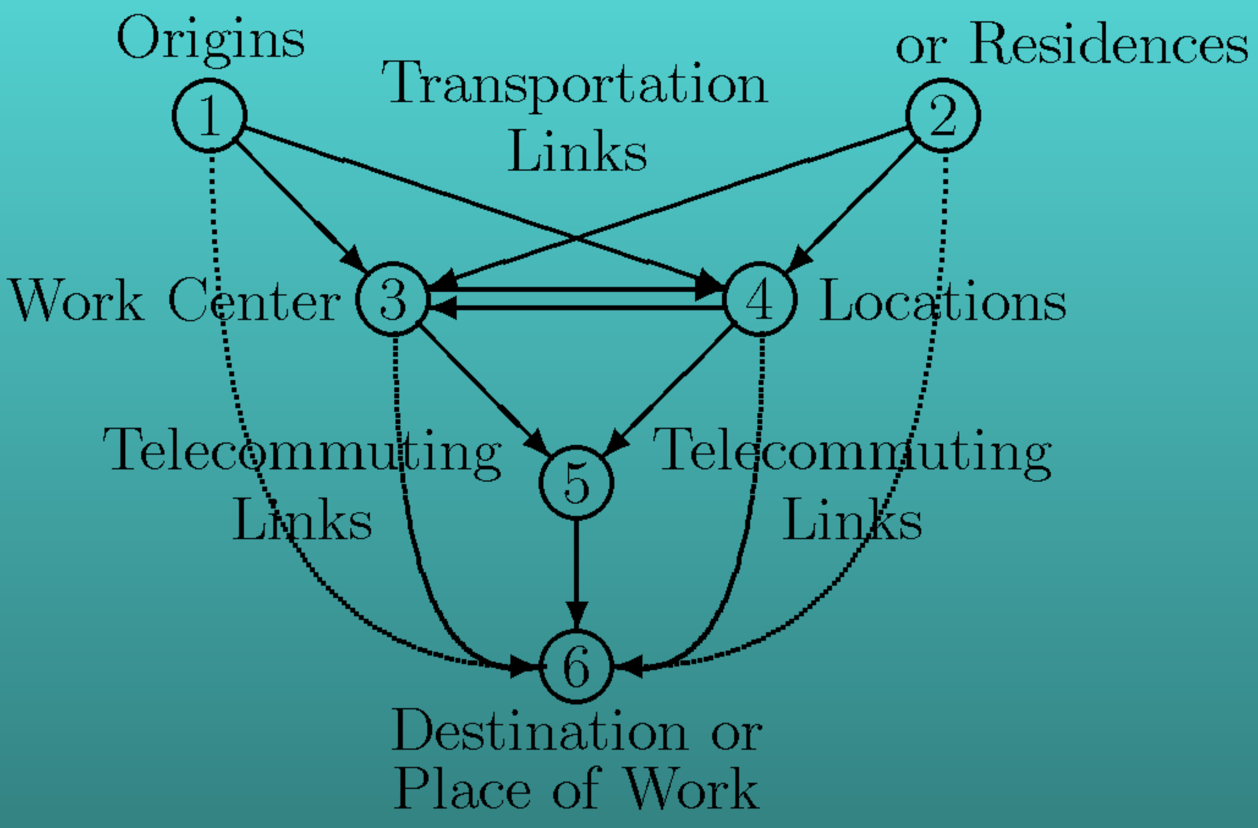
- **The growing impact of the Information Age, coupled with similarities between traffic networks and communications networks in terms of the relevance of such concepts as system-optimization and user-optimization, along with issues of centralized versus decentralized control, have provided a setting in which the relationships between decision-making on such networks and associated trade-offs could be explored (see Nagurney and Dong (2002a), Nagurney, Dong, and Mokhtarian (2001, 2002)).**

Telecommuting vs. Commuting Decision-Making

- **The decision-makers in the context of the telecommuting versus commuting decision-making application are travelers, who seek to determine their *optimal* routes of travel from their origins, which are residences, to their destinations, which are their places of work.**

- **The supernetwork representing the problem under study can be as general as necessary and a path may consist of a set of links corresponding to physical and virtual transportation choices such as would occur if a worker were to commute to a work center from which he could then telecommute.**

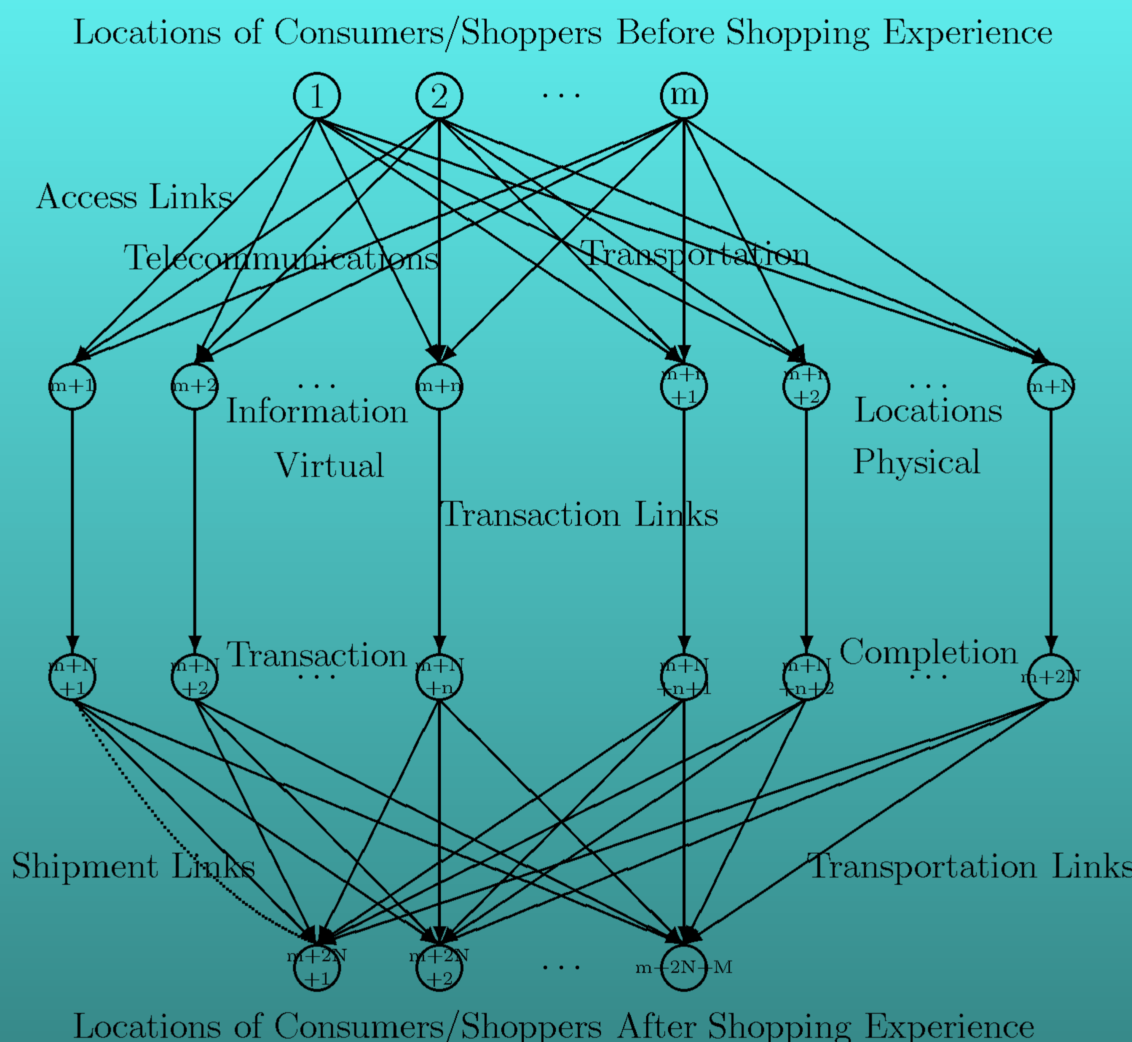
A Supernetwork Conceptualization of Commuting vs Telecommuting



Teleshopping vs Shopping Decision-Making

- **Now a multicriteria network equilibrium model for teleshopping decision-making is described.**

- **For further details, including numerical examples, see Nagurney and Dong (2002a) and the papers by Nagurney, Dong, and Mokhtarian (2001, 2002).**


Locations of Consumers/Shoppers Before Shopping Experience
1
2
m
Access Links
Telecommunications
Transportation
m+1
m+2
m+n
m+n +1
m+n +2
m+N
Information
Virtual
Locations
Physical
Transaction Links
m+N +1
m+N +2
m+N +n
m+N +n+1
m+N +n+2
m+2N
Transaction
Completion
Shipment Links
Transportation Links
m+2N +1
m+2N +2
m+2N+M
Locations of Consumers/Shoppers After Shopping Experience

Supply Chain Networks

- **Beckmann, McGuire, and Winsten (1956) explicitly recognized the generality of networks as a means of conceptualizing even decision-making of a firm with paths corresponding to production processes and the links corresponding to transformations as the material moved down the path from the origin to the destination. The paths then abstracted the choices or production possibilities available to a firm.**

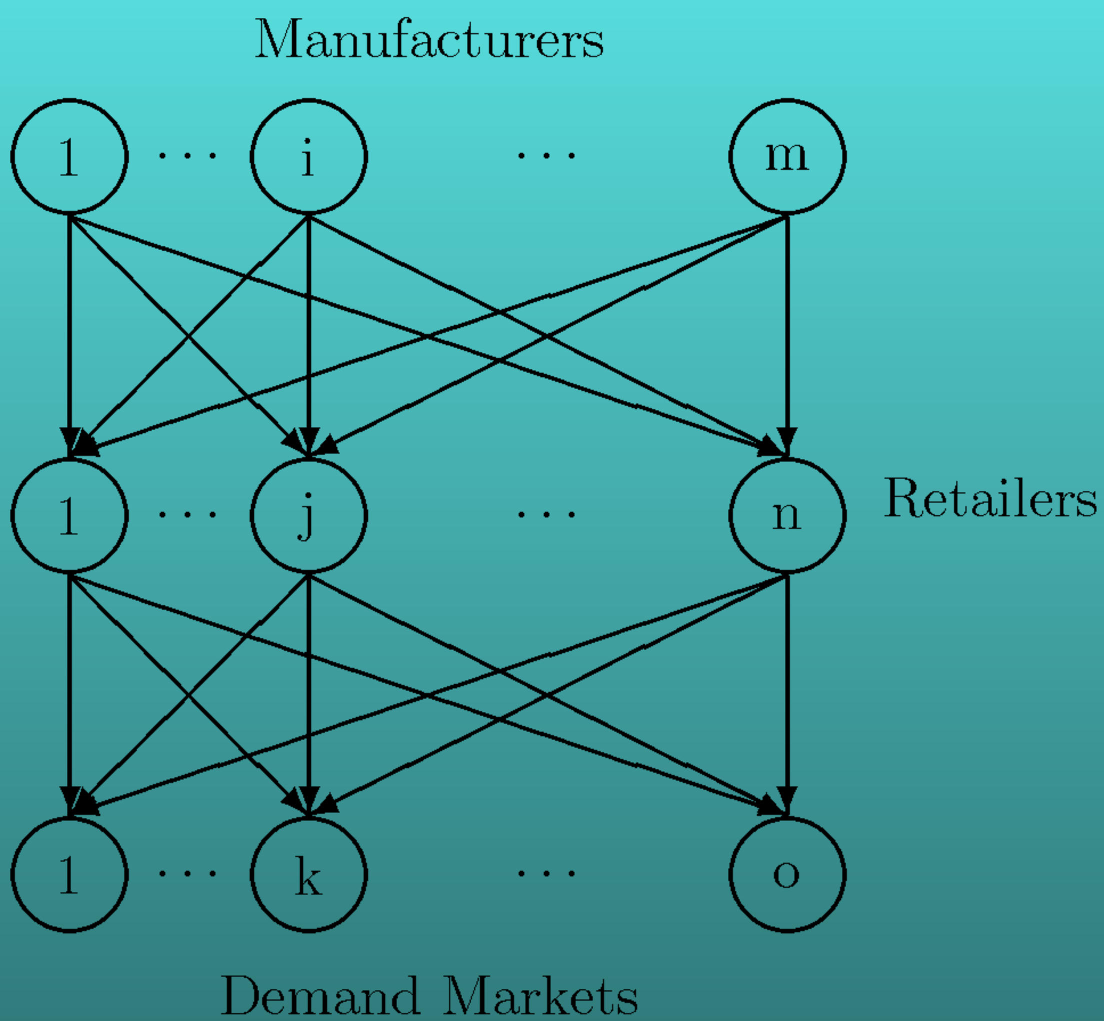
- **Zhang, Dong, and Nagurney (2003) have recently generalized Wardrop's second principle to consider not only paths but *chains* in the network to identify the *winning* supply chains.**

- **In that application context, paths correspond to production processes and links can be either operation or interface links. Their framework allows for the modeling of competition between supply chains which may entail several firms (producing, transporting, retailing, etc.).**

- **The first work on utilizing network equilibrium concepts in the context of supply chain applications is due to Nagurney, Dong, and Zhang (2001).**

- **The decision-makers, now located at the nodes of the network, are faced with their individual objective functions, which can include profit-maximization, and one seeks to determine not only the optimal/equilibrium flows between tiers of nodes but also the prices of the product at the various tiers. The model therein was subsequently generalized to include electronic commerce by Nagurney et al. (2002).**

A Supply Chain Network



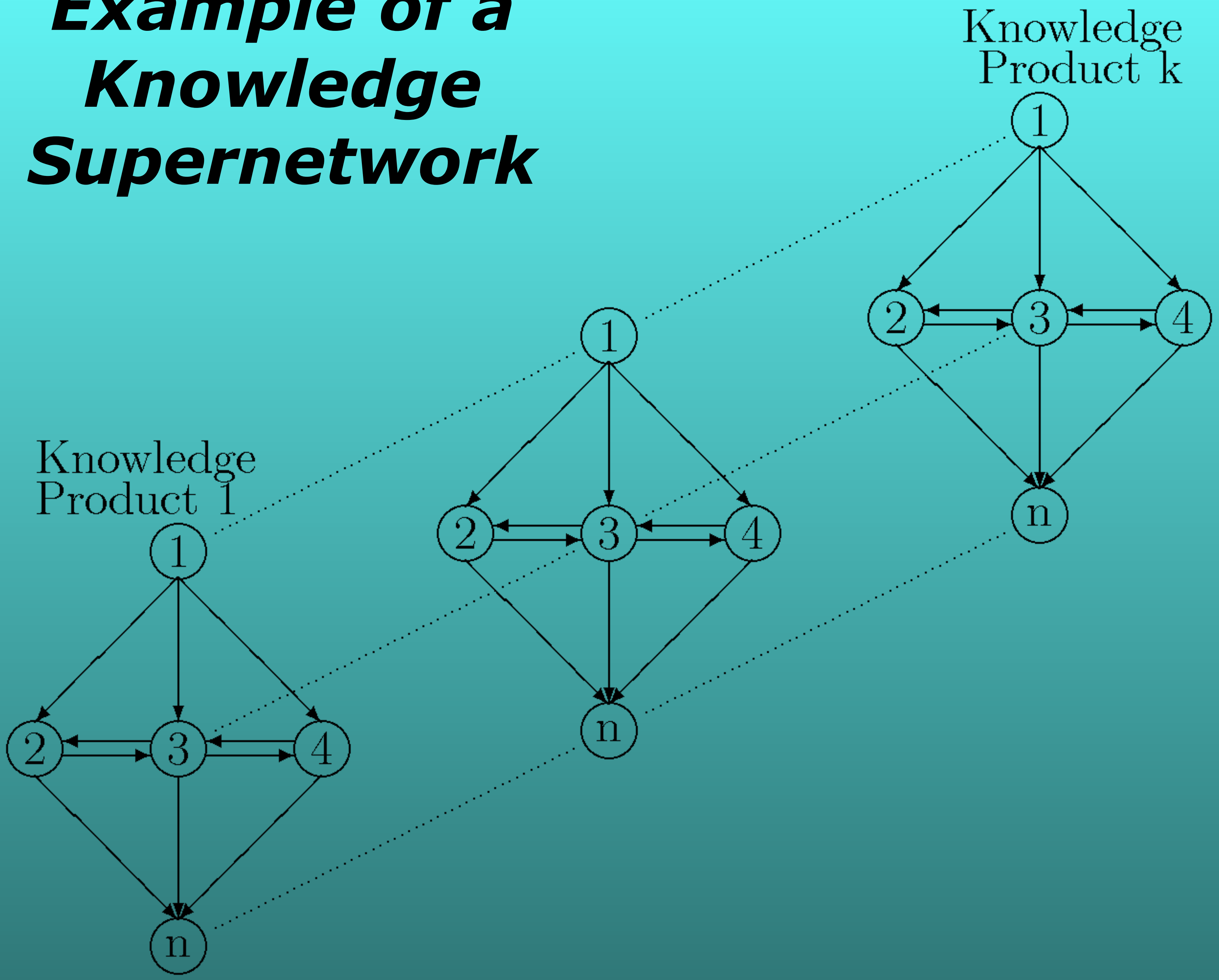
Knowledge Networks

- **The concept of a network equilibrium first formulated rigorously by Beckmann, McGuire, and Winsten (1956) is much broader than its original application context -- that of transportation networks. Its generality was apparent in the book since the authors themselves discussed other application settings, including the application of the concepts to a firm and its production possibilities.**

- **There has been much research conducted in the modeling of knowledge networks from an economic perspective and, notably, by researchers in transportation (cf. Karlqvist and Lundqvist (1972), Batten, Kobayashi, and Andersson (1989), Kobayashi (1995), Nagurney (1999), and the references therein) and even Beckmann (1993, 1994) and the volume edited by Beckmann et al. (1998).**
- **Beckmann (1994) noted BMW but in the sense that the topic of transportation networks had been the study of operations researchers, applied mathematicians, and economic theorists while that of knowledge networks had not.**

- **Nagurney and Dong (2003) proposed a framework for the modeling and analysis of knowledge intensive organizations including news organizations, intelligence agencies, and/or global financial institutions. Their perspective used the supernetwork concept of Nagurney and Dong (2002a) and the network equilibrium concept of Beckmann, McGuire, and Winsten (1956) to identify the knowledge products, the O/D pairs, the paths and their meanings, along with the links and flows in a variety of knowledge organization contexts.**
- **The need for research on such topics as postulated by Beckmann (1994) was now becoming a reality.**

Example of a Knowledge Supernetwork



- **This application setting further demonstrates the power of the concepts introduced in Beckmann, McGuire, and Winsten .**

- **In Nagurney and Dong (2003), we discuss the types of factors of production associated with the links are useful in knowledge production**.

Computer Scientists Discover Beckmann, McGuire, and Winsten

- **Beckmann (1967b) noted the relevance of network equilibrium concepts to communication networks.**
- **Bertsekas and Gallager (1987) realized the similarities between communication and transportation networks as well and were familiar with the algorithms of Dafermos and Sparrow (1969).**

- **It was, however, the Braess paradox which, subsequently, provided one of the main linkages between transportation science and computer science.**

- **Korilis, Lazar, and Orda (1999), in turn, developed methods to show how resources could be added efficiently to a noncooperative network, including the Internet, so that the Braess paradox would not occur and cited the work of Dafermos and Nagurney (1984a).**

- **Roughgarden (2002a), in his thesis, further elaborated upon the Braess paradox and focused on the quantification of the worst possible loss in network performance arising from noncooperative behavior.**

- **He recognized the importance of the work of Koutsoupias and Papadimitrou (1999), who are computer scientists, and who proposed the idea of bounding of the inefficiency of Nash equilibria, and that of Beckmann, McGuire, and Winsten (1956) and Dafermos and Sparrow (1969).**

- **Hence, almost 50 years after its publication, Beckmann, McGuire, and Winsten (1956) is finding applications in disciplines that did not even exist when the book was published!**

- **I expect that there will be continuing cross-fertilization between many fields in which networks play a prominent role, with BMW serving as one of the fundamental references.**

Personal Reflections and Comments

- **I was privileged to have had Martin Beckmann on my doctoral dissertation committee at Brown University with the chair of the committee being Stella Dafermos, who passed away in 1990.**

- **Although I could not locate a copy of BMW for purchase, Stella had given me copies of parts of it for use in my research and it became a reference that has served me well and that I have carried with me on many travels and while living abroad and doing research.**

- **Amazingly, Brown had been home to such luminaries in transportation as William Prager, who in 1954 published a paper, which discussed the importance of extended type of traffic network models in which the cost on a link could depend not only on its own flow, to Gordon Newell, for a period of time, to Beckmann, as well as to Dafermos**

- **I had met Prager who presented a seminar as part of freshman week activities at Brown but did not have a course with Stella Dafermos until becoming a graduate student. Several of my friends, including my college room-mate, did take courses in operations research and transportation from Dafermos so I would hear often about her as an individual (the only female faculty member in either Applied Mathematics or Engineering at that time).**

- **Stella had been introduced to operations research by her thesis advisor at John Hopkins University, F. Tom Sparrow, who, subsequently, moved to Purdue University.**

- **She also benefited greatly from support provided by Alan Goldman who, at that time, was with the National Bureau of Standards and from technical assistance from George Nemhauser, who is now at Georgia Tech.**

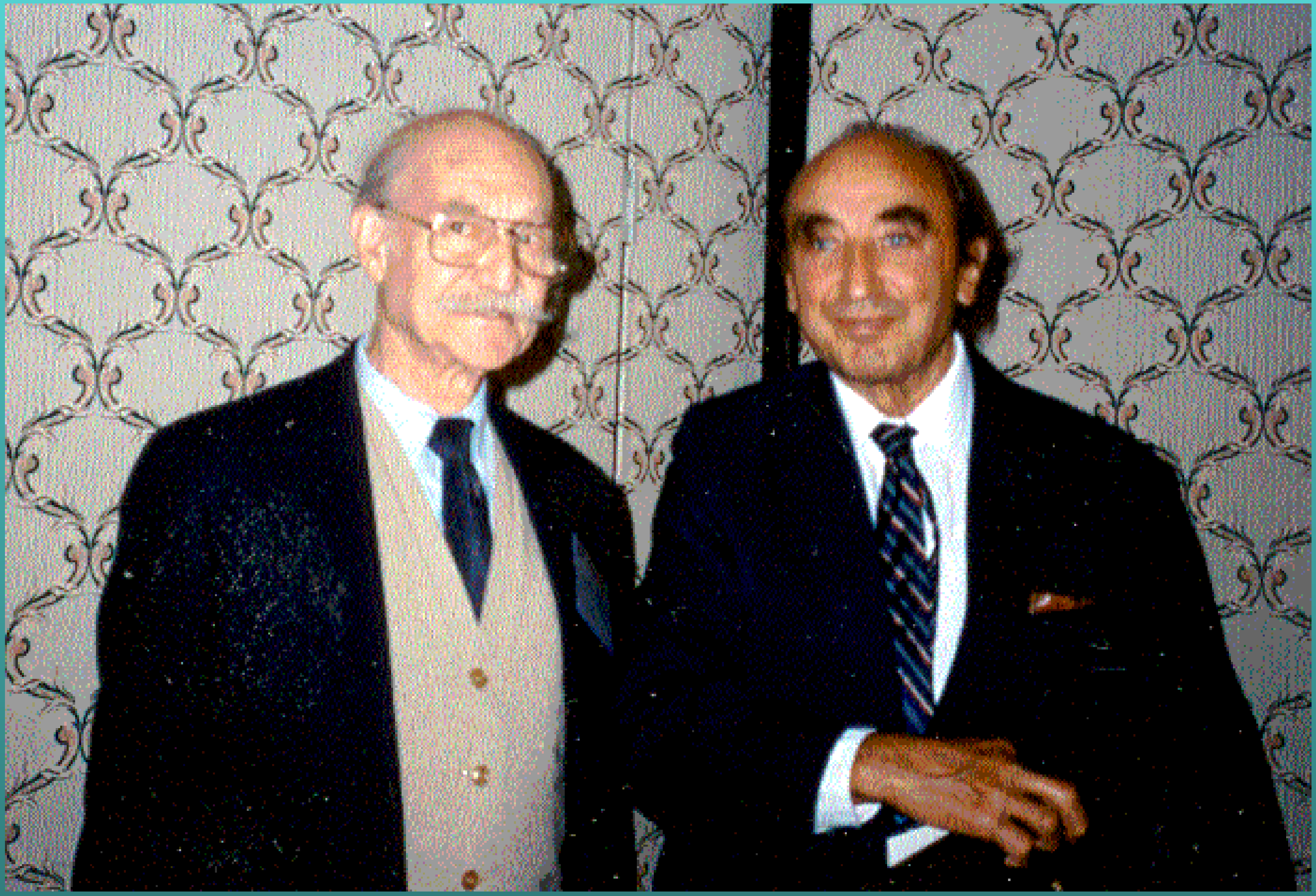
- **Upon graduation from Hopkins and prior to following her husband to Brown, she was at Cornell.**

- **The intellectual journey that Beckmann and Dafermos started me on and influenced numerous others has been fascinating and never dull. It has taken me to many countries, including Canada, Sweden, Russia, Japan, and Australia, and the intellectual inquiries and excitement continue to be fueled by interactions with students, collaborators, and many international colleagues.**

- **Through the Robert Herman Lifetime Achievement Award sponsored by the Transportation and Logistics Section of INFORMS (and named after its first recipient, Robert Herman), the achievements and sustained contributions of innovators in transportation science have been recognized.**

- **Past winners: Robert Herman, Martin Beckmann, Michael Florian, Denos Gazis, Amedeo Odoni, and, most recently, David E. Boyce.**

Martin Beckmann with Robert Herman in 1994 upon his receipt of the Lifetime Achievement Award in Transportation



Beckmann being Congratulated by Michael Florian

Regional Science Conference Mallacoota Victoria Australia - 1992

Mallacoota, December 1992

On the Beach
Mallacoota, Australia - 1992

Anna Nagurney with her PhD Committee, Professors Majda, Beckmann, and Dafermos at the Post-Defense Party at Brown.

Stella Dafermos and Anna Nagurney Athens Greece - 1987

Thank you!

A copy of the paper accompanying this talk can be found at:

http://supernet.som.umass.edu/articles/beckmann.pdf