The Evolution and Integration of Social and Financial Networks with Applications

Anna Nagurney

John F. Smith Memorial Professor

University of Massachusetts - Amherst

Cambridge Colloquium on Complexity and Social Networks

Kennedy School of Government

December 12, 2005

RADCLIFFE INSTITUTE FOR ADVANCED STUDY

HARVARD UNIVERSITY

Eugene M. Isenberg School of Management

The Virtual Center for Supernetworks

http://supernet.som.umass.edu

Funding for research provided by:

National Science Foundation

AT&T Foundation

John F. Smith Memorial Fund - University of Massachusetts at Amherst

THE ROCKEFELLER FOUNDATION

RADCLIFFE INSTITUTE FOR ADVANCED STUDY
HARVARD UNIVERSITY

Outline of Presentation:

- **Background**
- **Scientific Study of Networks**
- **Interdisciplinary Impact of Networks**
- **Characteristics of Networks Today**
- **The Braess Paradox and a Discovery**
- **Supernetworks - A New Paradigm with Some Applications**
- **Supernetworks Integrating Social and Financial Networks**
- **Social Networks and Supply Chains**

We are in a New Era of Decision-Making Characterized by:

- ***complex interactions*** **among decision-makers in organizations;**
- **alternative and at times** ***conflicting criteria*** **used in decision-making;**
- ***global reach*** **of many decisions;**
- ***high impact*** **of many decisions;**
- **increasing** ***risk and uncertainty*****, and**
- **the** ***importance of dynamics*** **and realizing a fast and sound response to evolving events.**

Network-Based New Era

- **Physical networks**
  - **Internet**
  - **Transportation/logistical networks**
  - **Energy/Power networks…**

- **Abstract networks**
  - **Social networks**
  - **Knowledge networks**

No longer are networks independent of one another but critically linked with major questions arising regarding decision-making and appropriate management tools.

Moreover, interactions between decision-makers and individuals can be modeled as networks and decision-making processes as well!

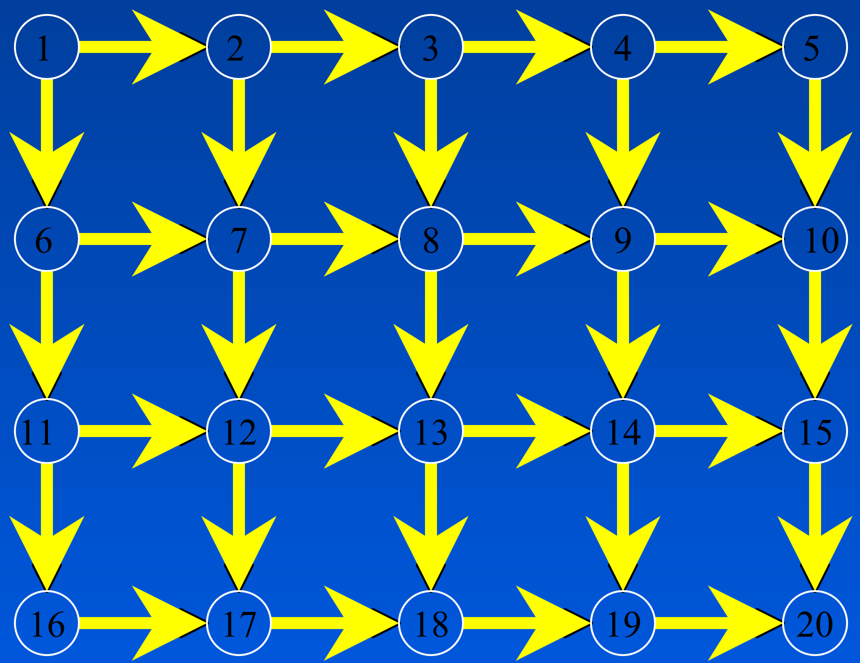
The scientific study of networks involves:

- how to **model** such applications as **mathematical entities**,
- how to **study the models** qualitatively,
- how to design **algorithms** to solve the resulting models.

The basic components of networks are:

- **Nodes**
- **Links or arcs**
- **Flows**

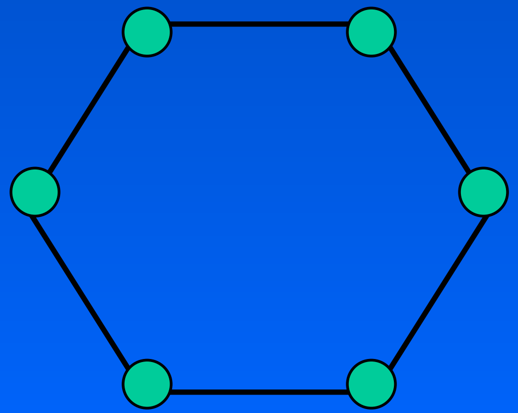
Nodes Links Flows



Brief History of the Science of Networks

1736 - Euler credited with the earliest paper on graph theory - Konigsberg bridges problem.

1758 - Quesnay in his *Tableau Economique* introduced an abstract network in the form of a graph to depict the circular flow of financial funds in an economy.



1781 - Monge, who had worked under Napoleon Bonaparte in providing infrastructure support for his army, publishes what is probably the first paper on transportation in minimizing the cost associated with backfilling n places from m other places with surplus brash with cost being proportional to distance.

1838 - Cournot not only states that a competitive price is determined by the intersection of supply and demand curves but does it in the context of spatially separate markets in which transportation costs are included.

1841 - Kohl considered a two node, two route transportation network problem.

1845 - Kirchhoff wrote *Laws of Closed Electric Circuits.*

1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

1936 - Konig published the first book on graph theory.

1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

1952 - Copeland in his book asked "Does money flow like water or electricity?"

1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

1969 - Dafermos and Sparrow coined the terms *user-optimization* and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

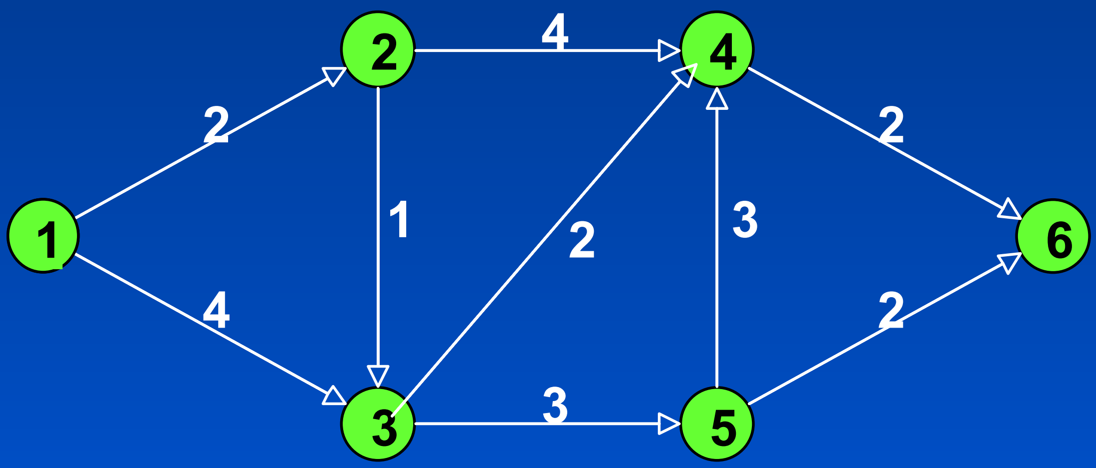
In a basic network problem domain:

one wishes to move the flow from one node to another in a way that is as efficient as possible.

Classic Examples of Network Problems Are:

- **The Shortest Path Problem**
- **The Maximum Flow Problem**
- **The Minimum Cost Flow Problem.**

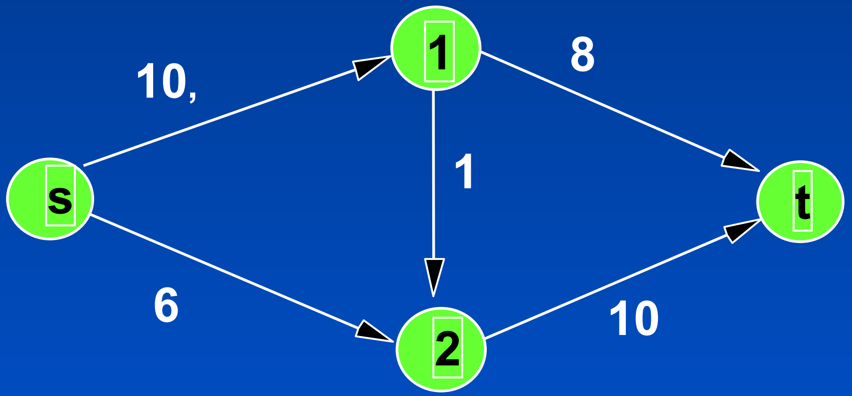
The Shortest Path Problem



Consider a network with Origin Node 1 and a Destination Node 6.

What is the shortest path from 1 to 6?

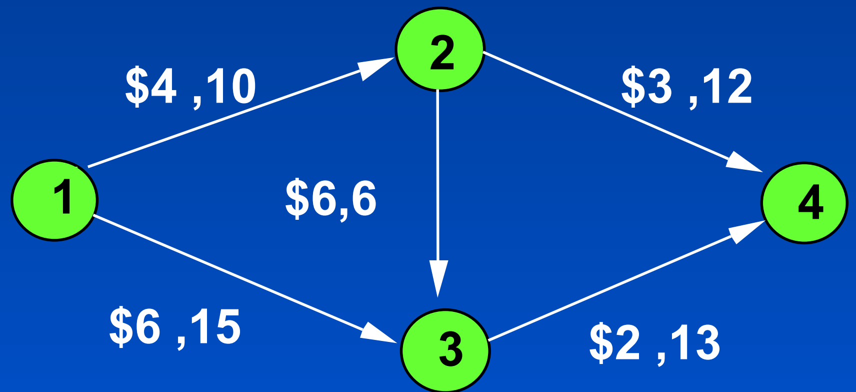
The Maximum Flow Problem



Each link has a maximum capacity.

How does one Maximize the flow from s to t, subject to the link capacities?

The Minimum Cost Flow Problem



Each link has a linear cost and a maximum capacity.

How does one Minimize Cost for a given flow from 1 to 4?

Network problems arise in other *surprising and fascinating* ways for problems, which at first glance and on the surface, may not appear to involve networks at all.

***The study of networks is not limited to only physical networks* but also to *abstract* networks in which nodes do not coincide to locations in space.**

The advantages of a scientific network formalism:

- **many present-day problems are *concerned with flows* (material, human, capital, informational, etc.) over space and time and, hence, ideally suited as an application domain for network theory;**

- **provides a *graphical or visual depiction* of different problems;**

- **helps to *identify similarities and differences* in distinct problems through their underlying network structure;**

- **enables the application of *efficient network algorithms*;**

- **allows for the study of disparate problems through a *unifying methodology.***

One of the primary purposes of scholarly and scientific investigation is to ***structure*** the world around us and to ***discover patterns*** that cut across boundaries and, hence, help to ***unify diverse applications***.

Network theory provides us with a ***powerful methodology*** to establish connections with different disciplines and to ***break down boundaries***.

Interdisciplinary Impact of Networks

Mathematics

Networks

Economics

- Interregional Trade
- General Equilibrium
- Industrial Organization
- Portfolio Optimization
- Flow of Funds Accounting

Engineering

- Energy
- Manufacturing
- Telecommunications
- Transportation

Biology

- DNA Sequencing
- Targeted Cancer Therapy

Sociology

- Social Networks
- Organizational Theory

Computer Science

- Routing Algorithms

Characteristics of Networks Today

- ***large-scale nature*** **and complexity of network topology;**
- ***congestion*;**
- **alternative behavior of users of the network, which may lead to *paradoxical phenomena*;**
- **the *interactions among networks* themselves such as in transportation versus telecommunications networks;**
- ***policies*** **surrounding networks today may have a *major impact* not only economically but also *socially, politically, and security-wise*.**

- **alternative behaviors of the users of the network**

  - **system-optimized versus**

  - **user-optimized (network equilibrium),**

  which may lead to

  paradoxical phenomena.

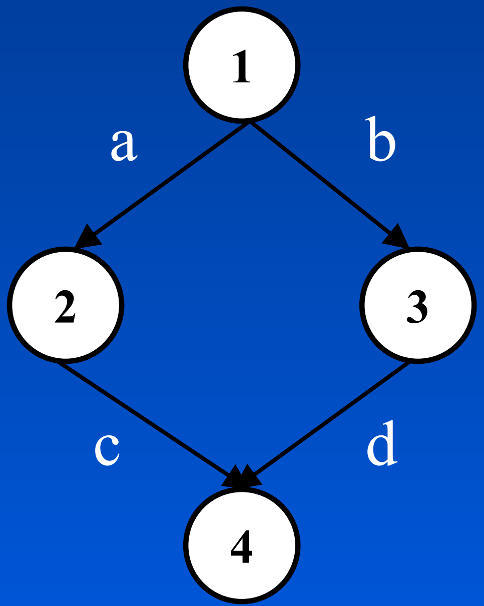
The Braess' Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1=(a,c)$ and $p_2=(b,d)$.

For a travel demand of 6, the equilibrium path flows are $x_{p_1}^* = x_{p_2}^* = 3$ and

The equilibrium path travel cost is

$C_{p_1} = C_{p_2} = 83$.



$c_a(f_a)=10\,f_a \quad c_b(f_b) = f_b+50$

$c_c(f_c) = f_c+50 \quad c_d(f_d) = 10\,f_d$

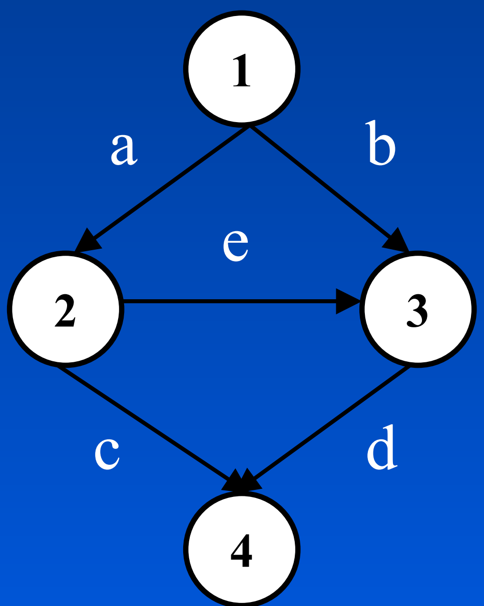
Adding a Link Increased Travel Cost for All!

Adding a new link creates a new path p_3=(a,e,d).

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, C_{p_3}=70.

The new equilibrium flow pattern network is

$x_{p_1}{}^* = x_{p_2}{}^* = x_{p_3}{}^* = 2.$



$c_e(f_e) = f_e + 10$

The equilibrium path travel costs: $C_{p_1} = C_{p_2} = C_{p_3} = 92.$

This phenomenon is relevant to telecommunications networks and the Internet which is another example of a

noncooperative network.

The Price of Anarchy!!!

The 1968 Braess article has been translated from German to English and appears as

On a Paradox of Traffic Planning

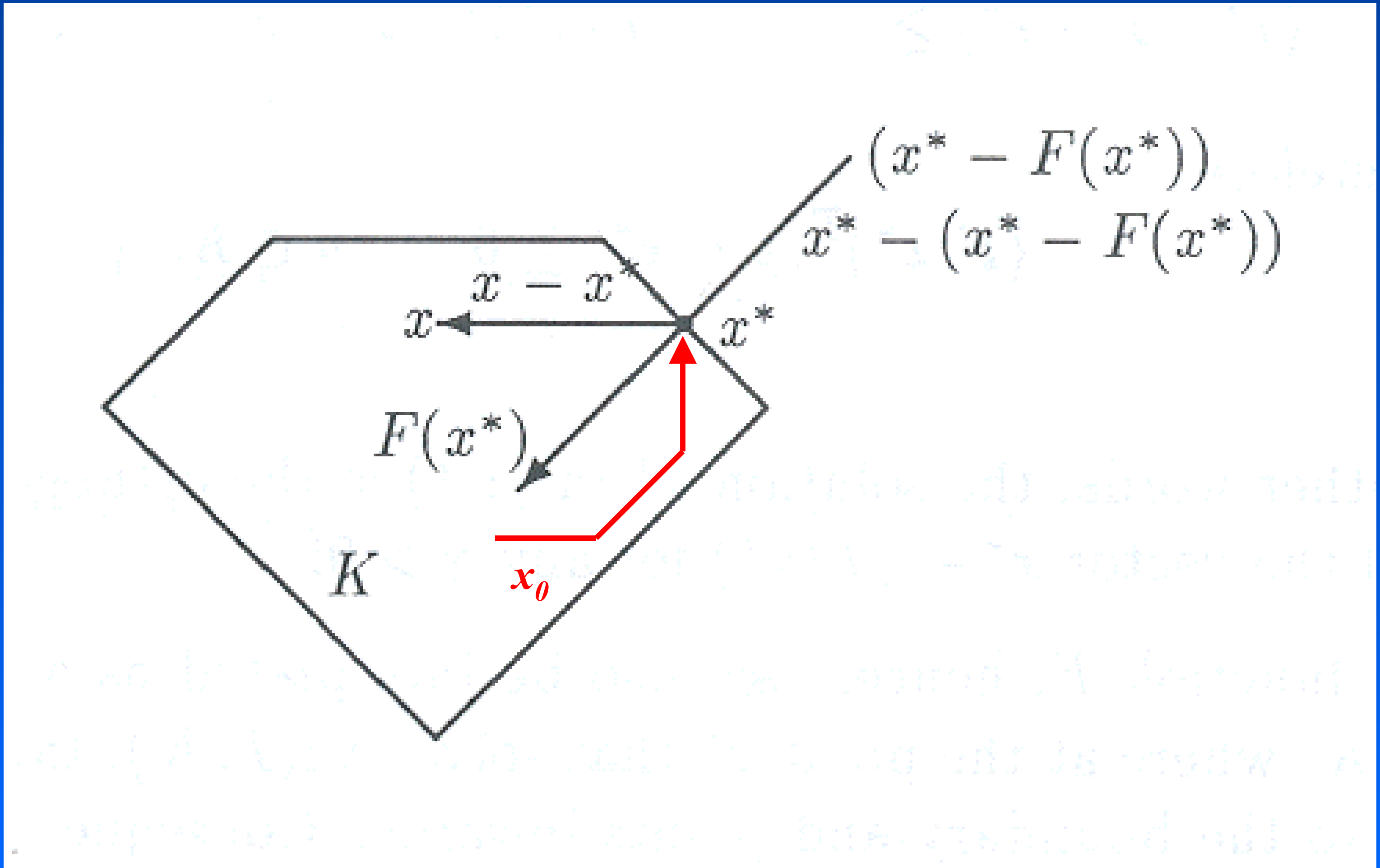
by Braess, Nagurney, Wakolbinger

in the November 2005 issue of *Transportation Science.*

The tools that we are using in our dynamic network research include:

- network theory
- optimization theory
- game theory
- variational inequality theory including evolutionary
- projected dynamical systems theory
- double layered dynamics theory
- network visualization tools.

A Geometric Interpretation of a Variational Inequality and a Projected Dynamical System



EQUILIBRIA of PDS and VARIATIONAL INEQUALITIES

An important feature of any PDS is that it is intimately related to a variational inequality problem (VI).

Theorem

The equilibria of a PDS:

$$\frac{\partial}{\partial t}(x(t)) = \Pi_K(x(t), -F(x(t)))$$

$$= \lim_{\delta \to 0} \frac{P_K(x(t) - \delta F(x(t))) - x(t)}{\delta}, \quad x(0) = x_0,$$

that is, $x^* \in K$ *such that*

$$\Pi_K(x^*, -F(x^*)) = 0$$

are solutions to the $\mathrm{VI}(F, K)$: *find* $x^* \in K$ *such that*

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall x \in K,$$

and vice-versa, where $\langle \cdot, \cdot \rangle$ *denotes the inner product on* X, *where* X *is a Hilbert space.*

A Discovery through the Investigation of Network Dynamics in the Form of Increasing Travel Demand

What happens if the demand is varied in the Braess Network?

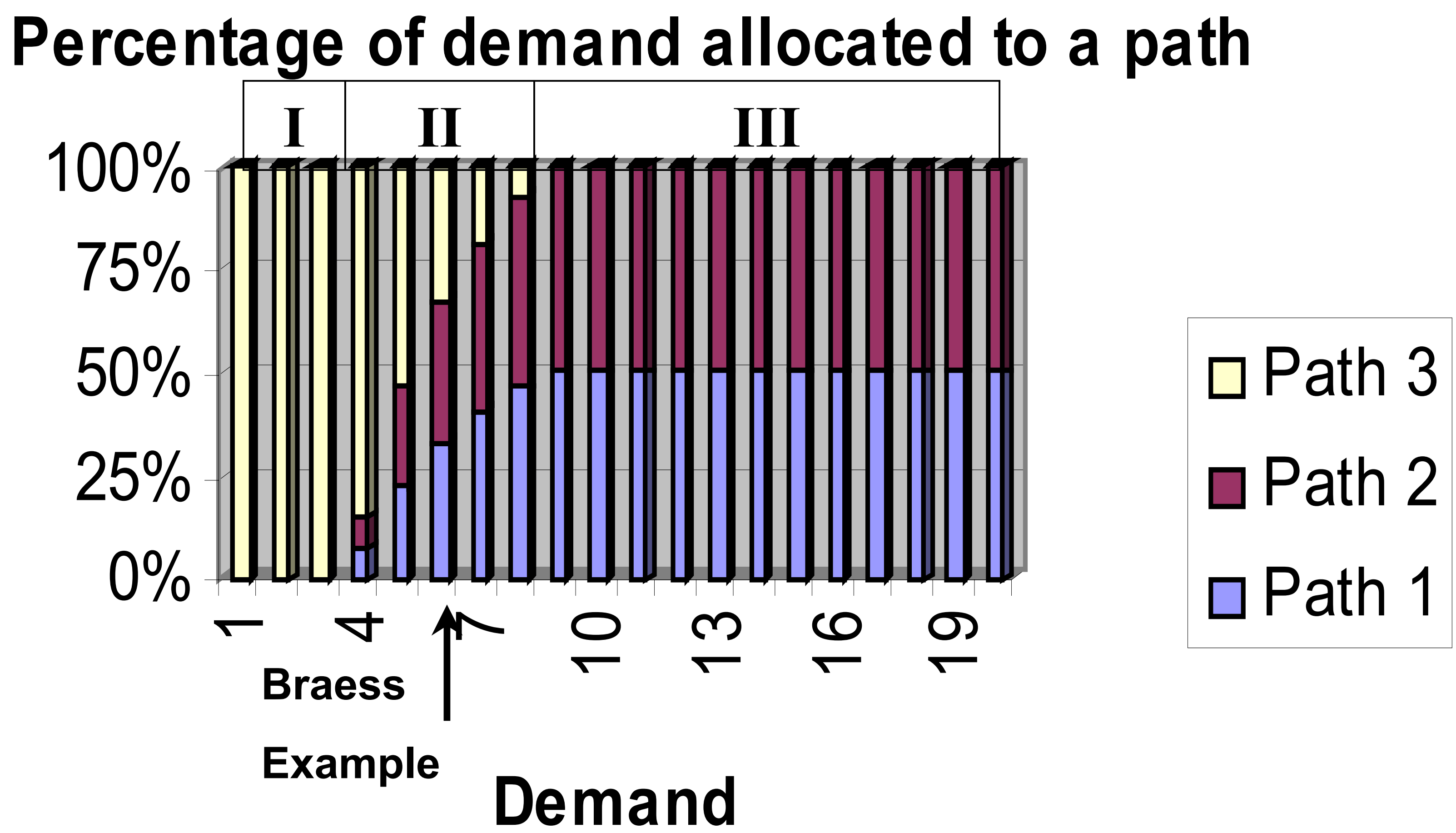
Behavior and Induced Flows Matter!!!

The answer lies in the solution of an *Evolutionary (Time-Dependent) Variational Inequality.*

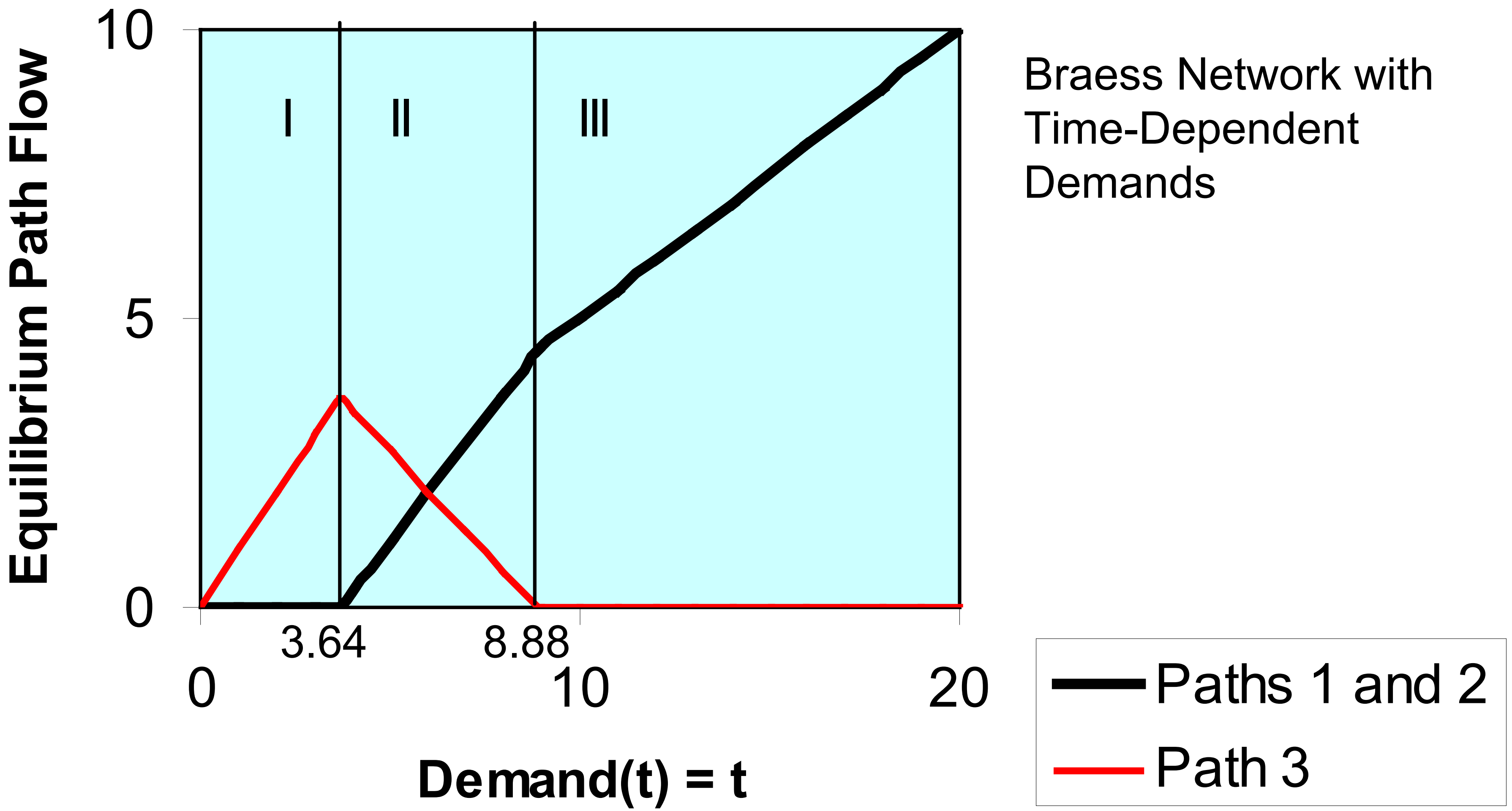
Find $x^* \in K$, such that

$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle \, dt \geq 0 \qquad \forall x \in K$$

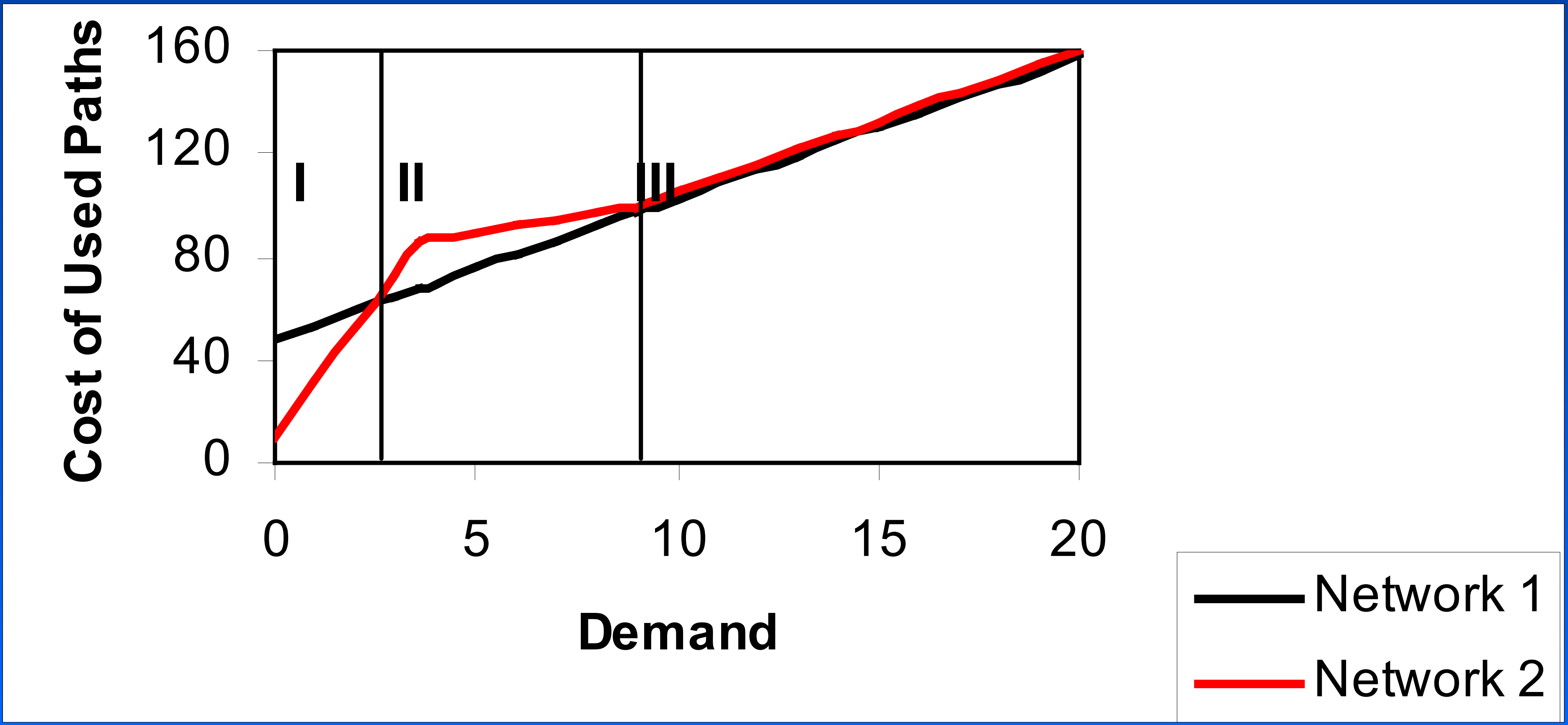
What happens if the demand changes?



The Solution of an Evolutionary (Time Dependent) Variational Inequality



In Regime II, the Addition of a New Road Makes Everyone Worse Off!



The new road is NEVER used after a certain demand is reached even if the travel demand approaches infinity.

Hence, in general, except for a limited range of travel demand, building the new road is a complete waste!

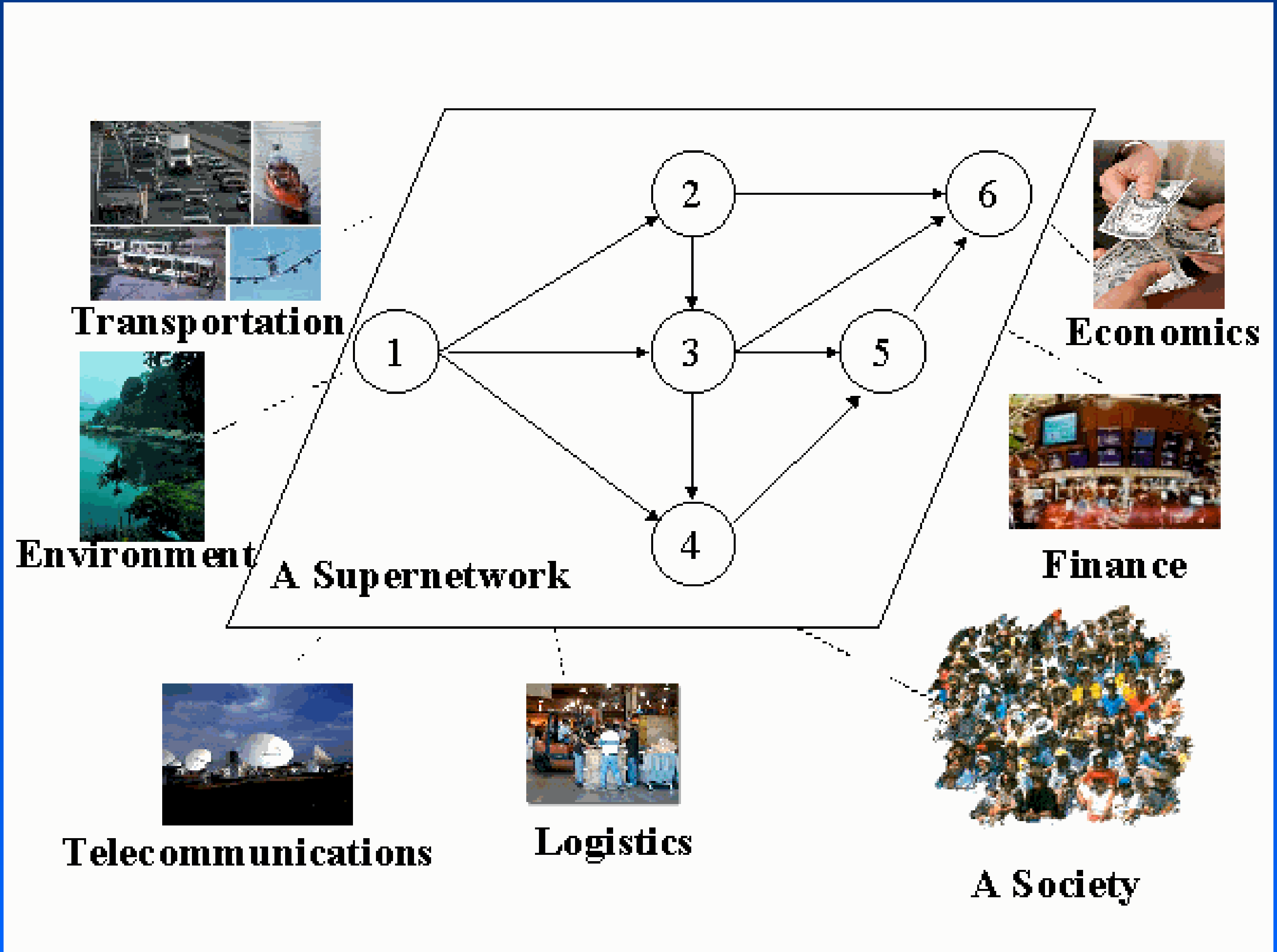
Supernetworks: A New Paradigm

Supernetworks may be comprised of such networks as *transportation, telecommunication, logistical, and/or financial networks.*

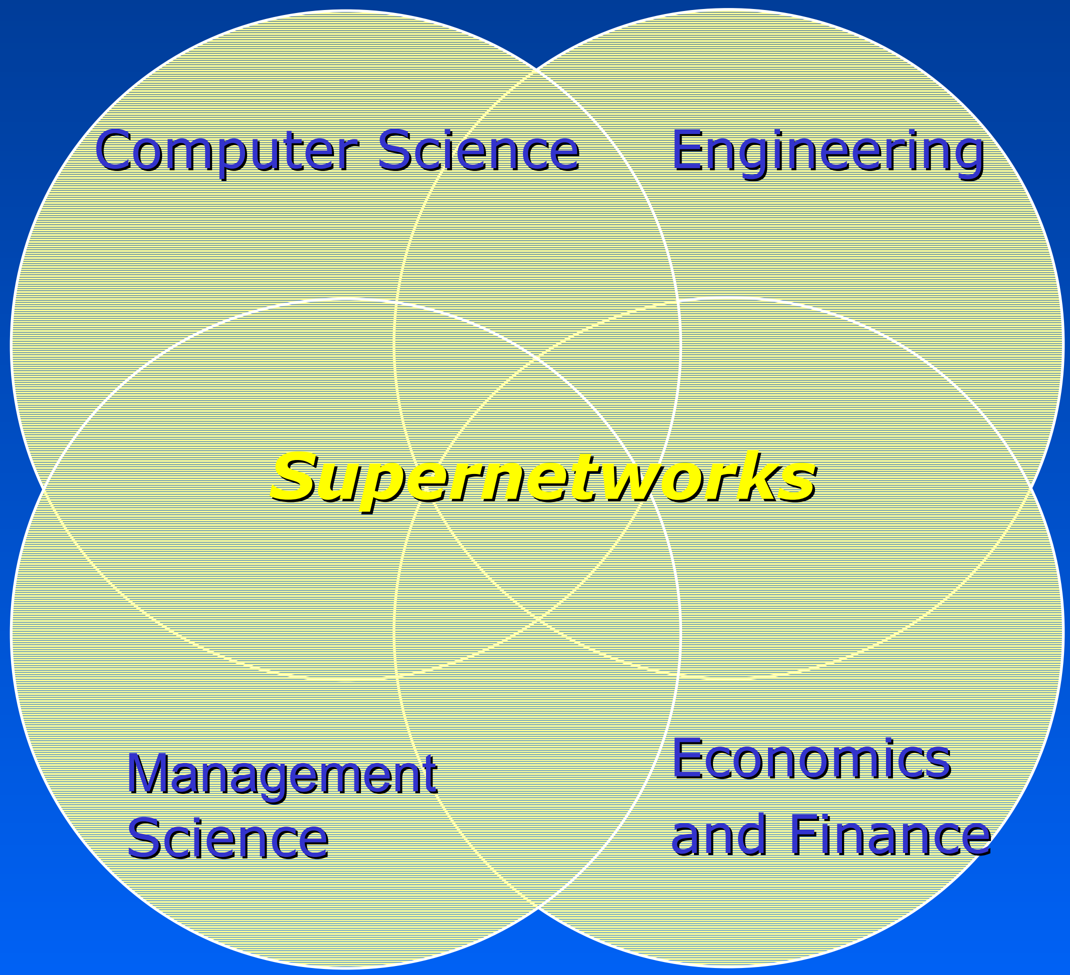
They may be *multilevel* as when they formalize the study of supply chain networks or *multitiered* as in the case of financial networks with intermediation.

Decision-makers may be faced with *multiple criteria;* thus, the study of supernetworks also includes the study of multicriteria decision-making.

Supernetworks: A New Paradigm



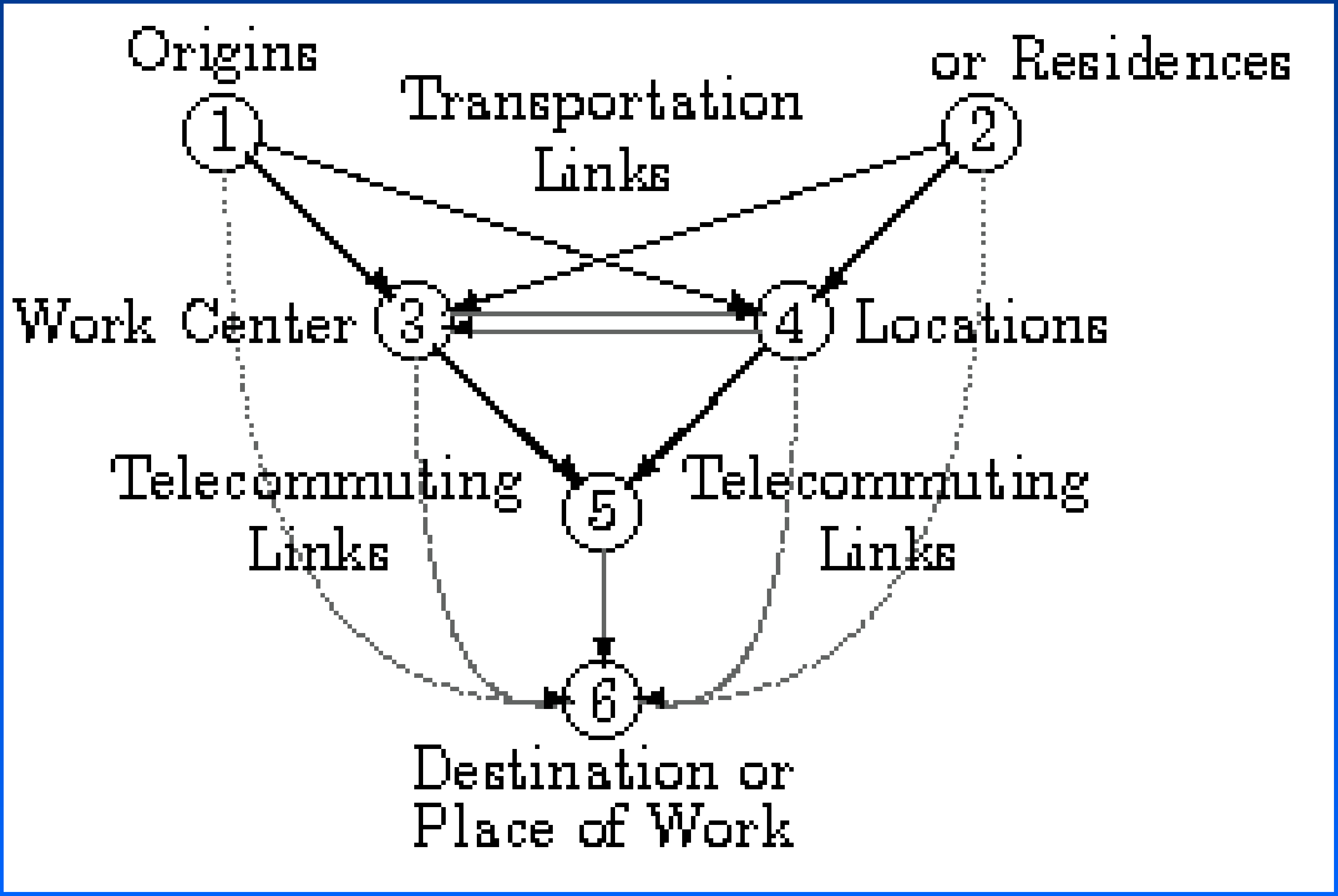
A Multidisciplinary Approach



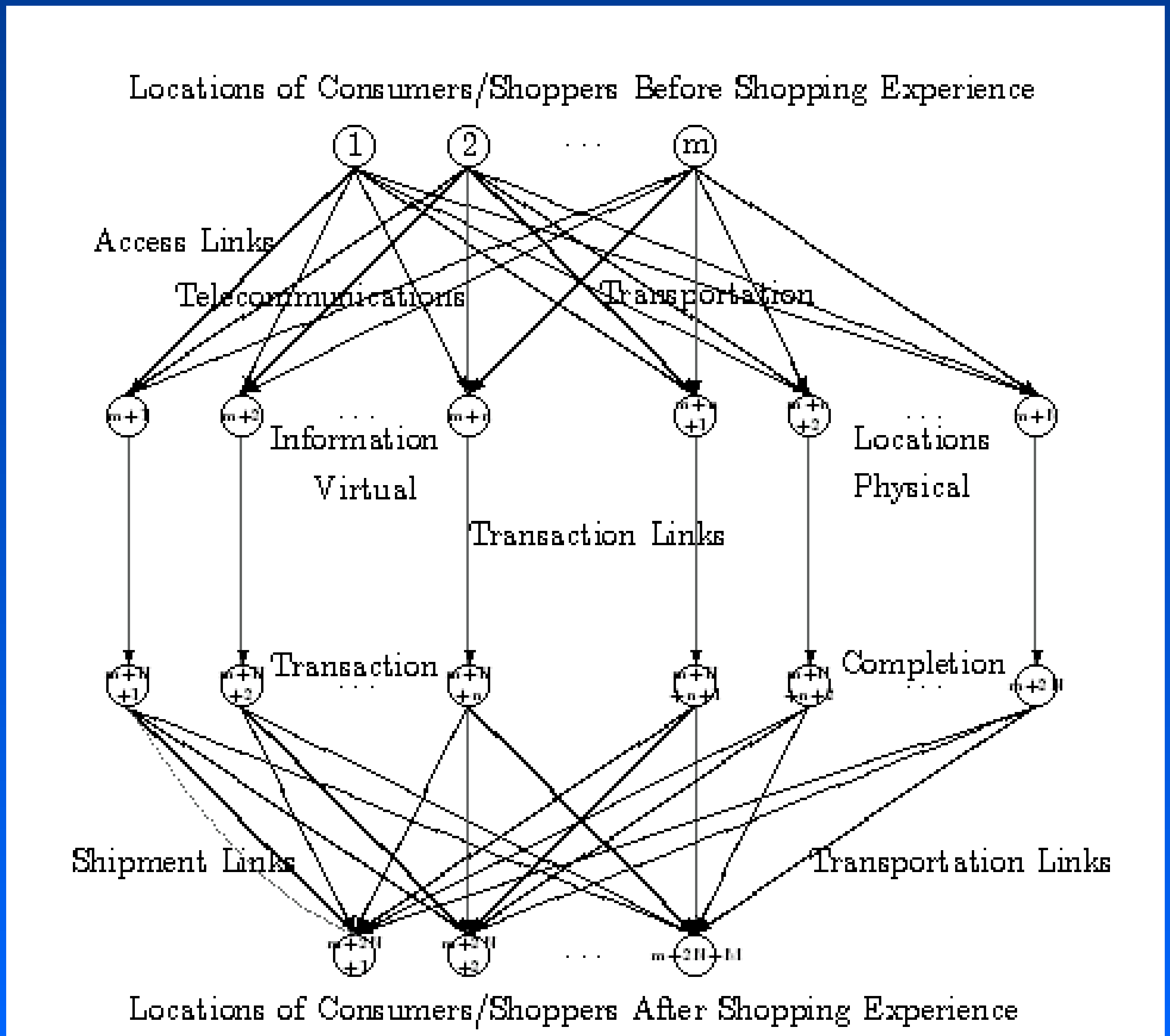
Applications of Supernetworks

- **Telecommuting/Commuting Decision-Making**
- **Teleshopping/Shopping Decision-Making**
- **Supply Chain Networks with Electronic Commerce**
- **Reverse Supply Chains with E-Cycling**
- **Knowledge Networks**
- **Energy Networks/Power Grids**
- **Financial Networks with Electronic Transactions**

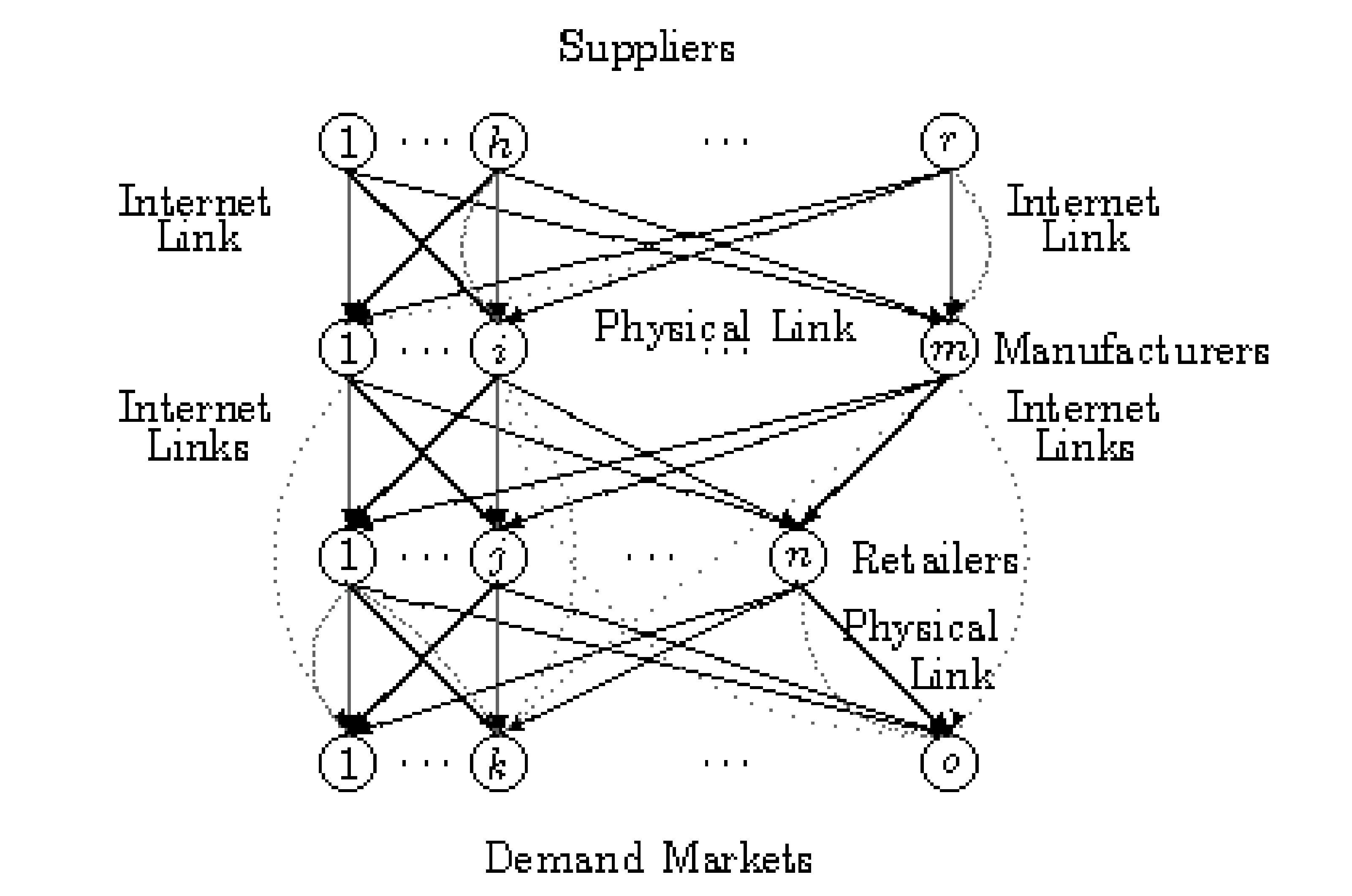
A Supernetwork Conceptualization of Commuting versus Telecommuting



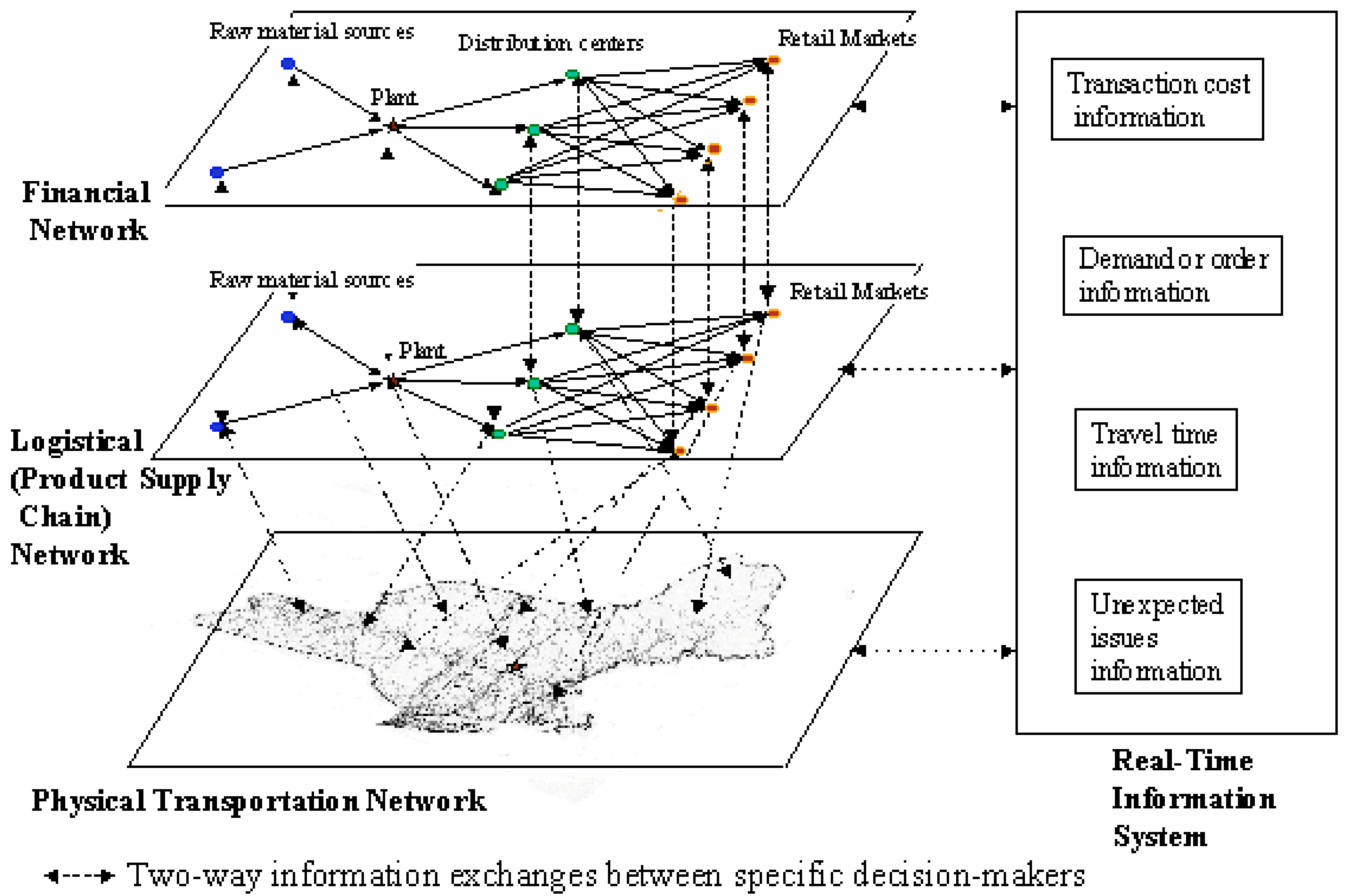
A Supernetwork Framework for Teleshopping versus Shopping



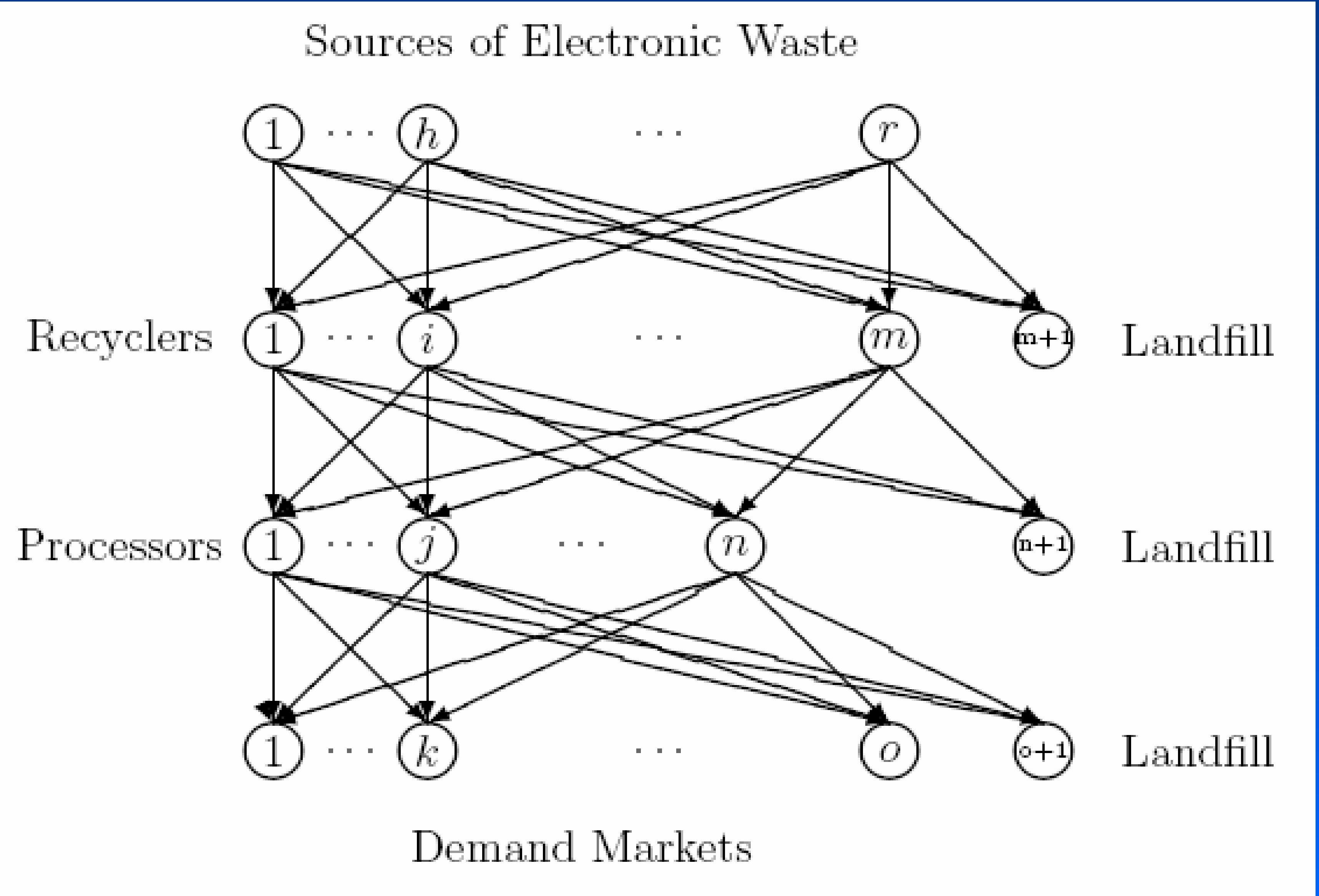
The Supernetwork Structure of a Supply Chain Network



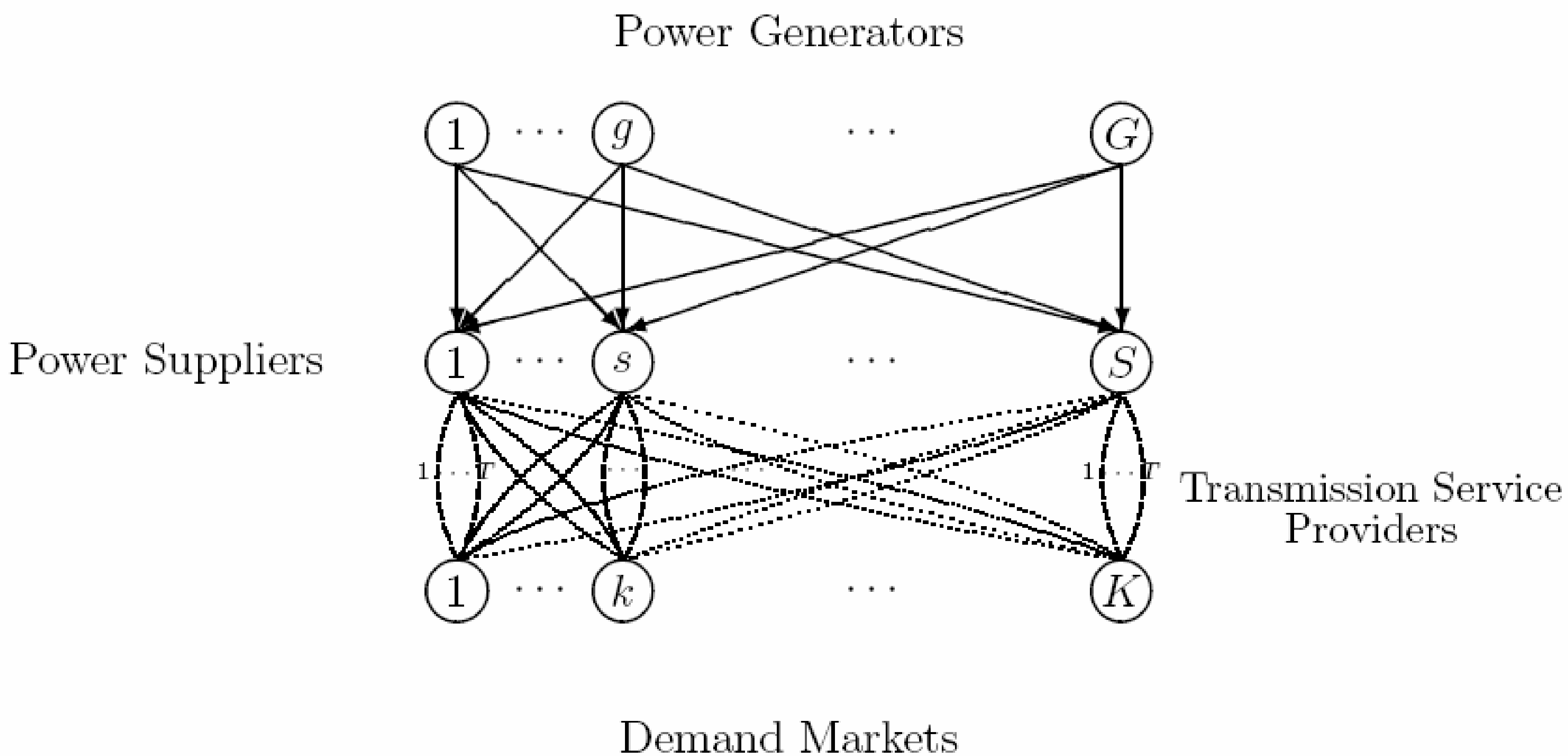
Supply Chain -Transportation Supernetwork Representation



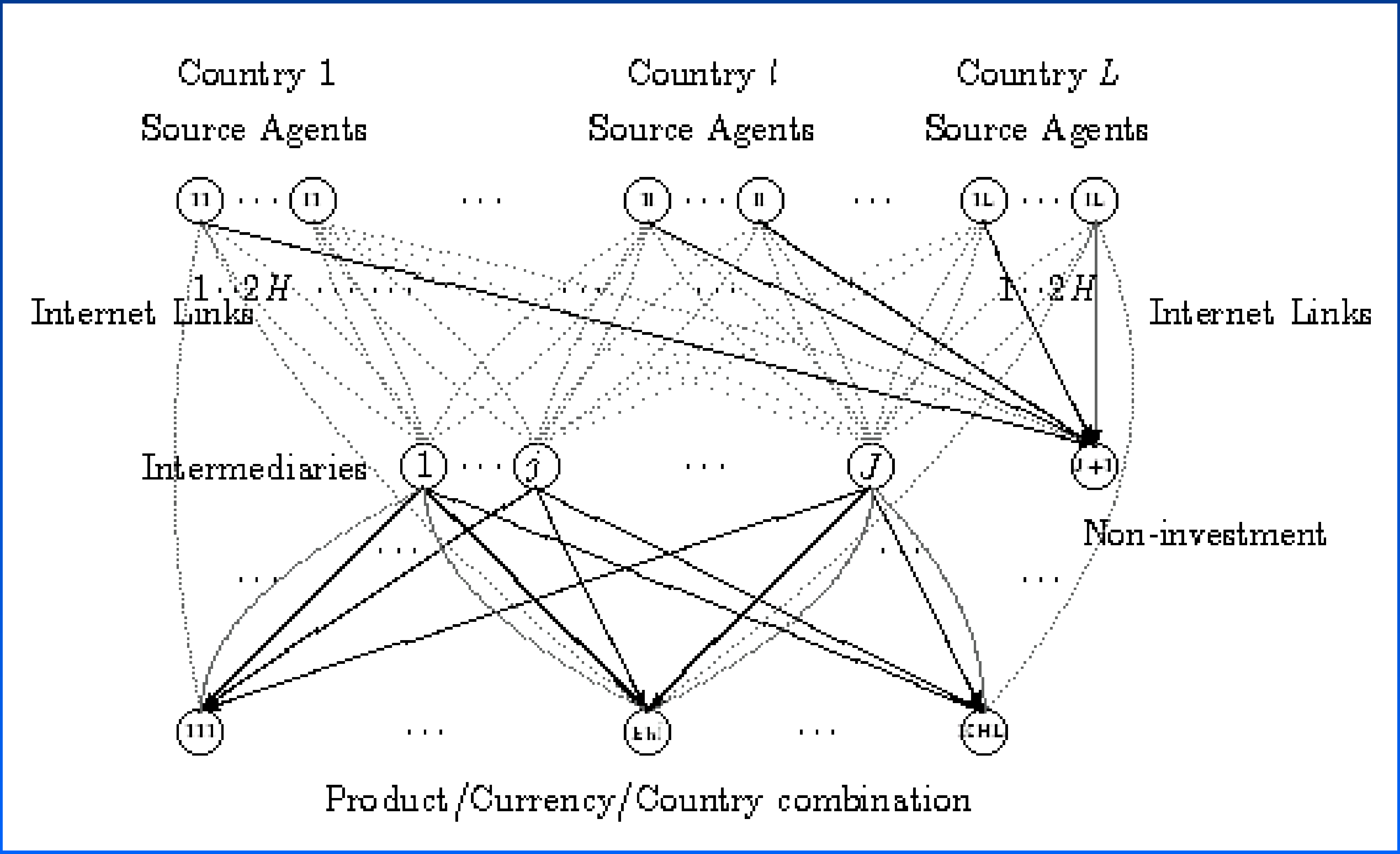
The 4-Tiered E-Cycling Network



The Electric Power Supply Chain Network

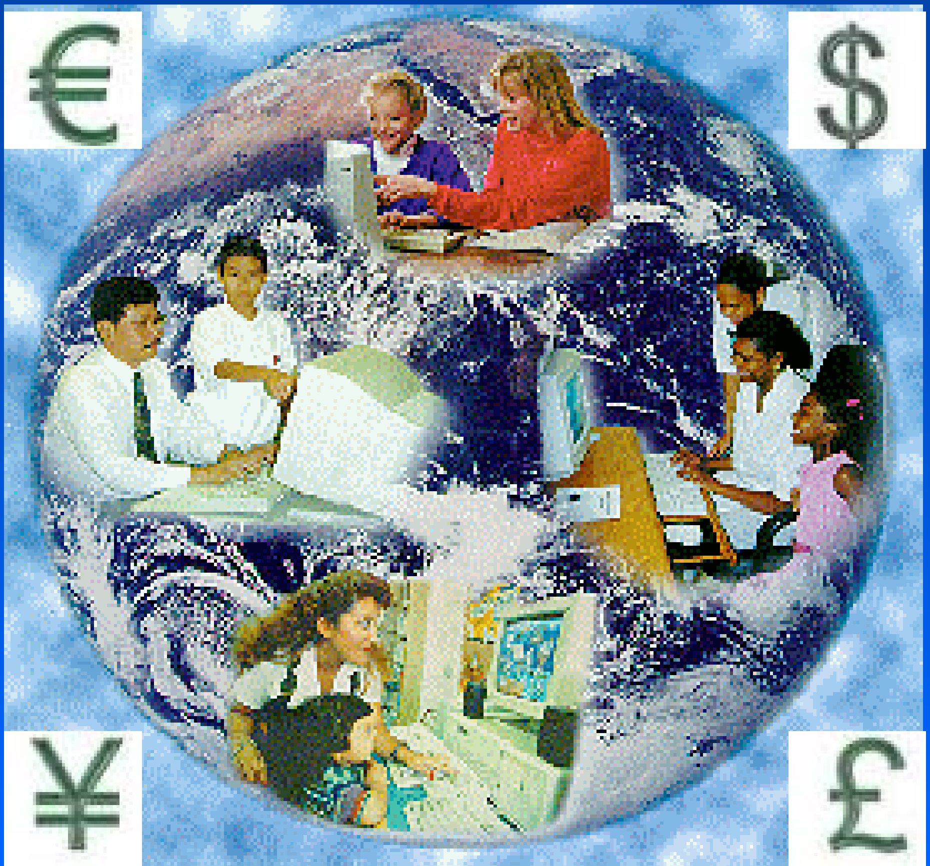


International Financial Networks with Electronic Transactions



Research Motivation

Can Social Networks and Financial Networks be Unified?



Especially given the importance of electronic financial transactions:

- In 2001 15 million Americans paid their bills online with up to 46 million expected by 2005.
- $160 billion in mortgages were taken out online in the US (cf. Mullaney and Little (2002)).

Strong importance of personal relationships in financial transactions.

Definition of a Social Network

A social network is a set of actors that may have relationships with one another. Networks can have few or many actors (nodes), and one or more kinds of relations (edges) between pairs of actors **(Hannemann (2001)).**

Roles of Social Networks in Economic Transactions

Examples from Sociology:

Granovetter (1985)
Uzzi (1996)

Examples from Economics:

Williamson (1983)
Joskow (1988)
Crawford (1990)
Vickers and Waterson (1991)
Muthoo (1998)

Roles of Social Networks in Economic Transactions

Examples from Marketing:

Ganesan (1994)

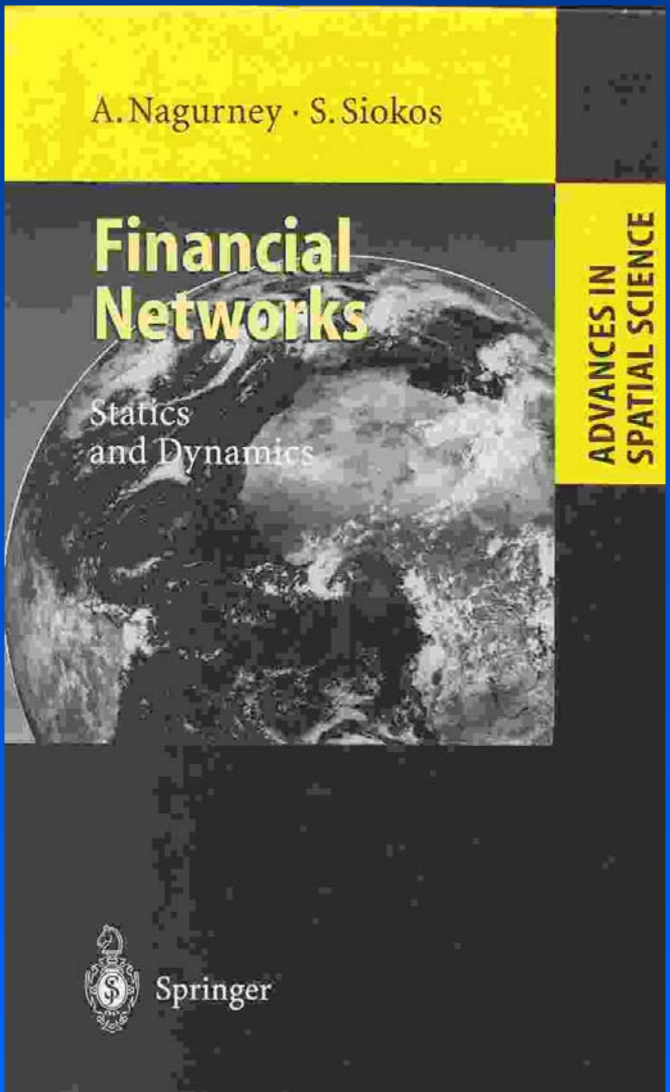
Bagozzi (1995)

Importance of Relationships in Financial Transactions

- **Examples in the context of micro-financing**
  - **Ghatak (2002), Anthony (1997)**

- **Examples in the context of lending**
  - **Sharpe (1990), Petersen and Rajan (1994, 1995), Berger and Udell (1995), Uzzi (1997, 1999), DiMaggio and Louch (1998), Arrow (1998), Wilner (2000), Burt (2000), Boot and Thakor (2000)**

Some of the Related Financial Network Literature



Related Literature

- Nagurney, A. and Ke, K. (2001), "Financial Networks with Intermediation," *Quantitative Finance* 1, 441-451.
- Nagurney, A. and Ke, K. (2003), "Financial Networks with Electronic Transactions: Modeling, Analysis, and Computations," *Quantitative Finance* 3, 71-87.
- Nagurney, A. and Cruz, J. (2003), "International Financial Networks with Electronic Transactions," in *Innovations in Financial and Economic Networks*, Edward Elgar Publishers, Cheltenham, England.

More Related Literature

– **Nagurney, A. and Cruz, J. (2004), "Dynamics of International Financial Networks with Risk Management,"** ***Quantitative Finance*** **4, 276-291.**
– **Nagurney, A., Wakolbinger, T., and L. Zhao, "The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels," to appear in** ***Computational Economics*** **(2006)**

Supernetwork Integrating Social Networks with Financial Networks

- Models the interaction of financial and social networks
- Captures interactions among individual sectors
- Includes electronic transactions
- Allows for non-investment
- Incorporates transaction costs and risk
- Shows the dynamic evolution of
  - Financial flows and associated prices on the financial network with intermediation
  - Relationship levels on the social network

Model Assumptions

- **3 tiers of decision-makers: source agents, intermediaries and demand markets**
- **Source agents can transact either physically or electronically with the intermediaries**
- **Source agents can transact directly with the demand markets via internet links**
- **Intermediaries can transact either physically or electronically with the demand markets**

Multicriteria Decision-Makers

Source agents and intermediaries:

- **Maximize net revenue**
- **Minimize risk**
- **Maximize relationship value**
- **Individual weights assigned to the different criteria.**

Role of Relationships

- Decision-makers in the network can decide about the relationship levels [0,1] that they want to establish.

- Establishing relationship levels incurs some costs.

- Higher relationship levels
  - Reduce transaction costs
  - Reduce risk
  - Have some additional value.

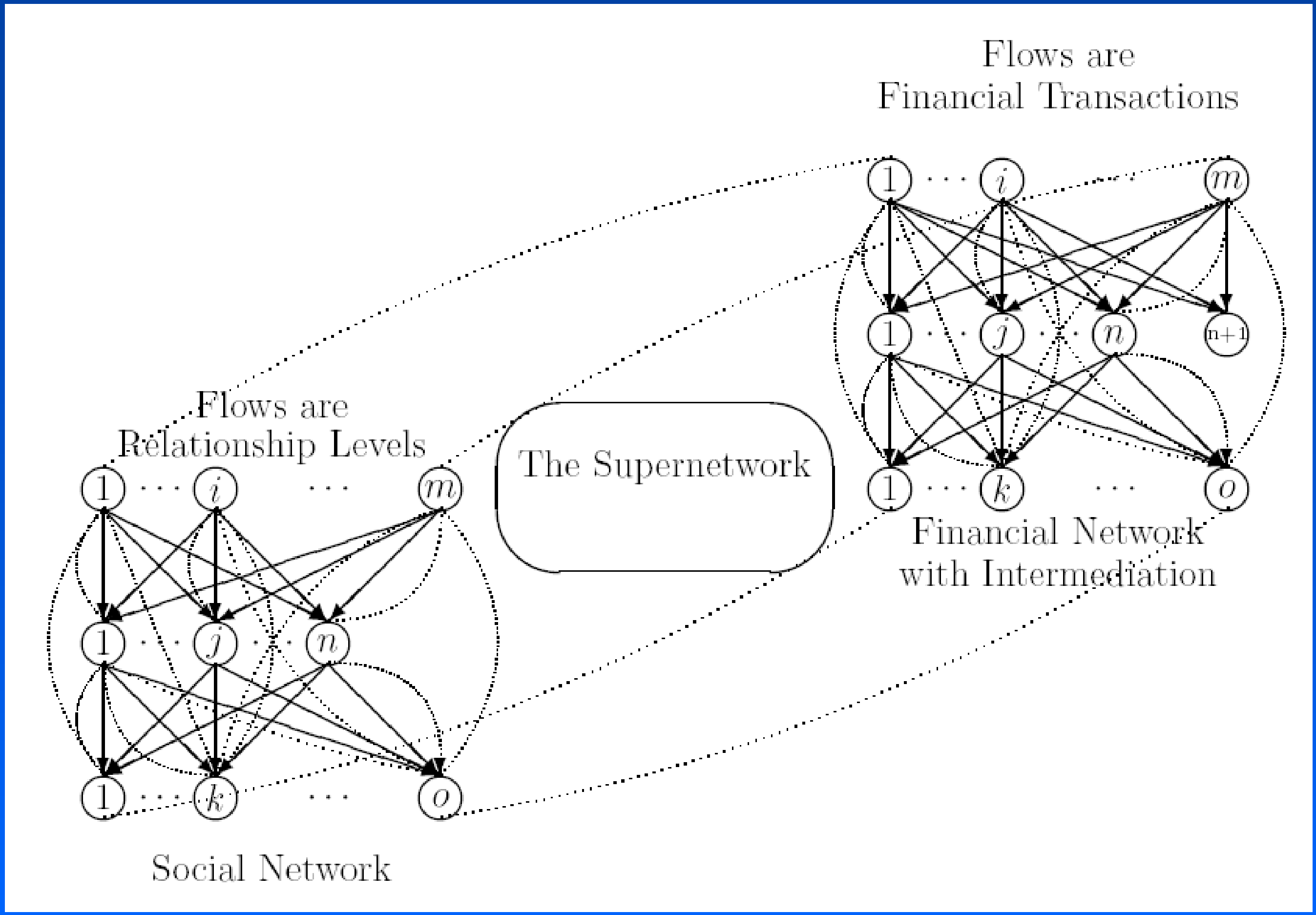
Novelty of Our Research

Supernetworks show the dynamic co-evolution of financial (financial product, price and even informational) flows and the social network structure.

Financial flows and social network structure are interrelated.

Network of relationships has a measurable economic value.

Supernetwork Structure: Integrated Financial/ Social Network System



A Source Agent's Multicriteria Decision-Making Problem

$$\text{Maximize } U^i = \sum_{j=1}^{n}\sum_{l=1}^{2}\rho^*_{1ijl}q_{ijl} + \sum_{k=1}^{o}\rho^*_{1ik}q_{ik} - \sum_{j=1}^{n}\sum_{l=1}^{2}c_{ijl}(q_{ijl},h_{ijl}) - \sum_{k=1}^{o}c_{ik}(q_{ik},h_{ik})$$

$$-\sum_{j=1}^{n}\sum_{l=1}^{2}b_{ijl}(h_{ijl}) - \sum_{k=1}^{o}b_{ik}(h_{ik}) - \alpha_i(\sum_{j=1}^{n}\sum_{l=1}^{2}r_{ijl}(q_{ijl},h_{ijl}) + \sum_{k=1}^{o}r_{ik}(q_{ik},h_{ik}))$$

$$+\beta_i(\sum_{j=1}^{n}\sum_{l=1}^{2}v_{ijl}(h_{ijl}) + \sum_{k=1}^{o}v_{ik}(h_{ik}))$$

subject to:

$$q_{ijl} \geq 0, \quad \forall j,l, \quad q_{ik} \geq 0, \quad \forall k,$$

$$0 \leq h_{ijl} \leq 1, \quad \forall j,l, \quad 0 \leq h_{ik} \leq 1, \quad \forall k,$$

Optimality Condition of Source Agents

determine $(Q^{1*}, Q^{2*}, h^{1*}, h^{2*}) \in \mathcal{K}_1$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} - \rho^*_{1ijl}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik}, h^*_{ik})}{\partial q_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik}, h^*_{ik})}{\partial q_{ik}} - \rho^*_{1ik}\right] \times [q_{ik} - q^*_{ik}]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial b_{ijl}(h^*_{ijl})}{\partial h_{ijl}} - \beta_i \frac{\partial v_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \alpha_i \frac{\partial r_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}}\right] \times \left[h_{ijl} - h^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\frac{\partial c_{ik}(q^*_{ik}, h^*_{ik})}{\partial h_{ik}} + \frac{\partial b_{ik}(h^*_{ik})}{\partial h_{ik}} - \beta_i \frac{\partial v_{ik}(h^*_{ik})}{\partial h_{ik}} + \alpha_i \frac{\partial r_{ik}(q^*_{ik}, h^*_{ik})}{\partial h_{ik}}\right] \times [h_{ik} - h^*_{ik}] \geq 0,$$

$$\forall (Q^1, Q^2, h^1, h^2) \in \mathcal{K}_1, \qquad (21)$$

where

$$\mathcal{K}_1 \equiv \left[(Q^1, Q^2, h^1, h^2) \mid q_{ijl} \geq 0,\ q_{ik} \geq 0,\ 0 \leq h_{ijl} \leq 1,\ 0 \leq h_{ik} \leq 1,\ \forall i, j, l, k, \text{and (1) holds}\right].$$

A Financial Intermediary's Multicriteria Decision-Making Problem

$$\text{Maximize} \quad U^j = \sum_{k=1}^{o}\sum_{l=1}^{2} \rho^*_{2jkl} q_{jkl} - c_j(Q^1) - \sum_{i=1}^{m}\sum_{l=1}^{2} \hat{c}_{ijl}(q_{ijl}, h_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} c_{jkl}(q_{jkl}, h_{jkl})$$

$$-\sum_{i=1}^{m}\sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} - \sum_{i=1}^{m}\sum_{l=1}^{2} \hat{b}_{ijl}(h_{ijl}) - \sum_{k=1}^{o}\sum_{l=1}^{2} b_{jkl}(h_{jkl}) - \delta_j(\sum_{i=1}^{m}\sum_{l=1}^{2} \hat{r}_{ijl}(q_{ijl}, h_{ijl}) + \sum_{k=1}^{o}\sum_{l=1}^{2} r_{jkl}(q_{jkl}, h_{jkl}))$$

$$+\gamma_j(\sum_{i=1}^{m}\sum_{l=1}^{2} \hat{v}_{ijl}(h_{ijl}) + \sum_{k=1}^{o}\sum_{l=1}^{2} v_{jkl}(h_{jkl})) \tag{41}$$

subject to:

$$\sum_{k=1}^{o}\sum_{l=1}^{2} q_{jkl} \leq \sum_{i=1}^{m}\sum_{l=1}^{2} q_{ijl} \tag{42}$$

$$q_{ijl} \geq 0, \quad \forall i, l, \quad q_{jkl} \geq 0, \quad \forall k, l, \tag{43}$$

$$0 \leq h_{ijl} \leq 1, \quad \forall i, l, \quad 0 \leq h_{jkl} \leq 1, \quad \forall k, l. \tag{44}$$

Optimality Conditions of Intermediaries

determine $(Q^{1*}, Q^{3*}, h^{1*}, h^{3*}, \epsilon^*) \in \mathcal{K}_2$, such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} + \rho^*_{1ijl} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial q_{ijl}} - \epsilon^*_j\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial q_{jkl}} - \rho^*_{2jkl} + \epsilon^*_j + \delta_j \frac{\partial r_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial q_{jkl}}\right] \times \left[q_{jkl} - q^*_{jkl}\right]$$

$$+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[\frac{\partial \hat{c}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} - \gamma_j \frac{\partial \hat{v}_{ijl}(h^*_{ijl})}{\partial h_{ijl}} + \delta_j \frac{\partial \hat{r}_{ijl}(q^*_{ijl}, h^*_{ijl})}{\partial h_{ijl}} + \frac{\partial \hat{b}_{ijl}(h^*_{ijl})}{\partial h_{ijl}}\right] \times \left[h_{ijl} - h^*_{ijl}\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\frac{\partial c_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial h_{jkl}} - \gamma_j \frac{\partial v_{jkl}(h^*_{jkl})}{\partial h_{jkl}} + \delta_j \frac{\partial r_{jkl}(q^*_{jkl}, h^*_{jkl})}{\partial h_{jkl}} + \frac{\partial b_{jkl}(h^*_{jkl})}{\partial h_{jkl}}\right] \times \left[h_{jkl} - h^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{o}\sum_{l=1}^{2} q^*_{jkl}\right] \times \left[\epsilon_j - \epsilon^*_j\right] \geq 0, \quad \forall (Q^1, Q^3, h^1, h^3, \epsilon) \in \mathcal{K}_2, \qquad (45)$$

where

$$\mathcal{K}_2 \equiv \left[(Q^1, Q^3, h^1, h^3, \epsilon) \mid q_{ijl} \geq 0, q_{jkl} \geq 0, 0 \leq h_{ijl} \leq 1, 0 \leq h_{jkl} \leq 1, \epsilon_j \geq 0, \forall i, j, l, k\right],$$

Equilibrium Conditions for the Demand Markets

for all intermediaries: j; $j = 1, ..., n$ and all modes l; $l = 1, 2$:

$$\rho^*_{2jkl} + \hat{c}_{jkl}(Q^{2*}, Q^{3*}, h^{2*}, h^{3*}) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^*_{jkl} > 0 \\ \geq \rho^*_{3k}, & \text{if} \quad q^*_{jkl} = 0, \end{cases}$$

and for all source agents i; $i = 1, ..., m$:

$$\rho^*_{1ik} + \hat{c}_{ik}(Q^{2*}, Q^{3*}, h^{2*}, h^{3*}) \begin{cases} = \rho^*_{3k}, & \text{if} \quad q^*_{ik} > 0 \\ \geq \rho^*_{3k}, & \text{if} \quad q^*_{ik} = 0. \end{cases}$$

In addition, we must have that

$$d_k(\rho^*_3) \begin{cases} = \sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik}, & \text{if} \quad \rho^*_{3k} > 0 \\ \leq \sum_{j=1}^{n} \sum_{l=1}^{2} q^*_{jkl} + \sum_{i=1}^{m} q^*_{ik}, & \text{if} \quad \rho^*_{3k} = 0. \end{cases}$$

VI Formulation of the Equilibrium Conditions for the Demand Markets

determine $(Q^{2^*}, Q^{3^*}, \rho_3^*) \in R^{2no+mo+n}$, such that

$$\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\rho_{2jkl}^* + \hat{c}_{jkl}(Q^{2^*}, Q^{3^*}, h^{2*}, h^{3*}) - \rho_{3k}^*\right] \times \left[q_{jkl} - q_{jkl}^*\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[\rho_{1ik}^* + \hat{c}_{ik}(Q^{2^*}, Q^{3^*}, h^{2*}, h^{3*}) - \rho_{3k}^*\right] \times \left[q_{ik} - q_{ik}^*\right]$$

$$+\sum_{k=1}^{o}\left[\sum_{l=1}^{2}\sum_{j=1}^{n} q_{jkl}^* + \sum_{i=1}^{m} q_{ik}^* - d_k(\rho_3^*)\right] \times \left[\rho_{3k} - \rho_{3k}^*\right] \geq 0, \quad \forall (Q^2, Q^3, \rho_3) \in R_+^{mo+2no+n}.$$

The Equilibrium State

Definition 1: The equilibrium state of the supernetwork integrating the financial network with the social network is one where the financial flows and relationship levels between the tiers of the network coincide and the financial flows, relationship levels, and prices satisfy the sum of the two sets of optimality conditions and the demand market equilibrium conditions.

The equilibrium state is equivalent to a VI of the form:

$$\text{determine } X^* \in \mathcal{K} \text{ satisfying } \langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

Projected Dynamical System

- The dynamic models can be rewritten as a projected dynamical system defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{80}$$

where $\Pi_{\mathcal{K}}$ denotes the projection of $-F(X)$ onto $\mathcal{K}$ at X and X_0 is equal to the point corresponding to the initial financial product transactions, relationship levels, shadow prices, and the demand market prices.

- The set of stationary points of the projected dynamical system coincides with the set of solutions of the variational inequality problem.

The Disequilibrium Dynamics

The trajectory of the PDS describes the dynamic evolution of:

- the financial product transactions on the financial network
- the relationship levels on the social network
- the demand market prices
- the Lagrange multipliers or shadow prices associated with the intermediaries.

The projection operation guarantees that the constraints underlying the supernetwork system are not violated.

Dynamics of Demand Market Prices

The demand market prices evolve according to the difference between the demand at the market (as a function of the prices at the demand markets at that time) and the amount of the financial product transactions.

The projection operator guarantees that the prices do not take on negative values.

Dynamics of Shadow Prices

The Lagrange multipliers/shadow prices associated with the intermediaries evolve according to the difference between the sum of the financial product transacted with the demand markets and that obtained from the source agents.

The projection operator guarantees that these prices do not become negative.

Dynamics of Relationship Levels

The relationship levels evolve on the social network links of the supernetwork according to the difference between the corresponding weighted relationship value, the sum of the various marginal costs and weighted marginal risks.

The relationship levels are guaranteed to remain within the range zero to one.

Dynamics of Financial Product Transactions

The financial product transactions evolve on the financial network links according to the difference between the characteristic price and various marginal and unit costs plus the weighted marginal risks.

These flows are guaranteed to not assume negative values due to the projection operation.

Qualitative Properties

We have established:

- **Existence of a solution to the VI**
- **Uniqueness of a solution to the VI**
- **Conditions for the existence of a unique trajectory to the projected dynamical system**
- **Convergence of the Euler method.**

Computational Procedure: The Euler Method

Step 0: Initialization

Set $X^0 \in \mathcal{K}$ and set $T = 0$. T is an iteration counter which may also be interpreted as a time period.

Step 1: Computation

Compute X^{T+1} by solving the variational inequality problem:

$$X^{T+1} = P_{\mathcal{K}}(X^T - a_T F(X^T)),$$

where $\{a_T\}$ is a sequence of positive scalars satisfying: $\sum_{T=0}^{\infty} a_T = \infty$, $a_T \to 0$, as $T \to \infty$

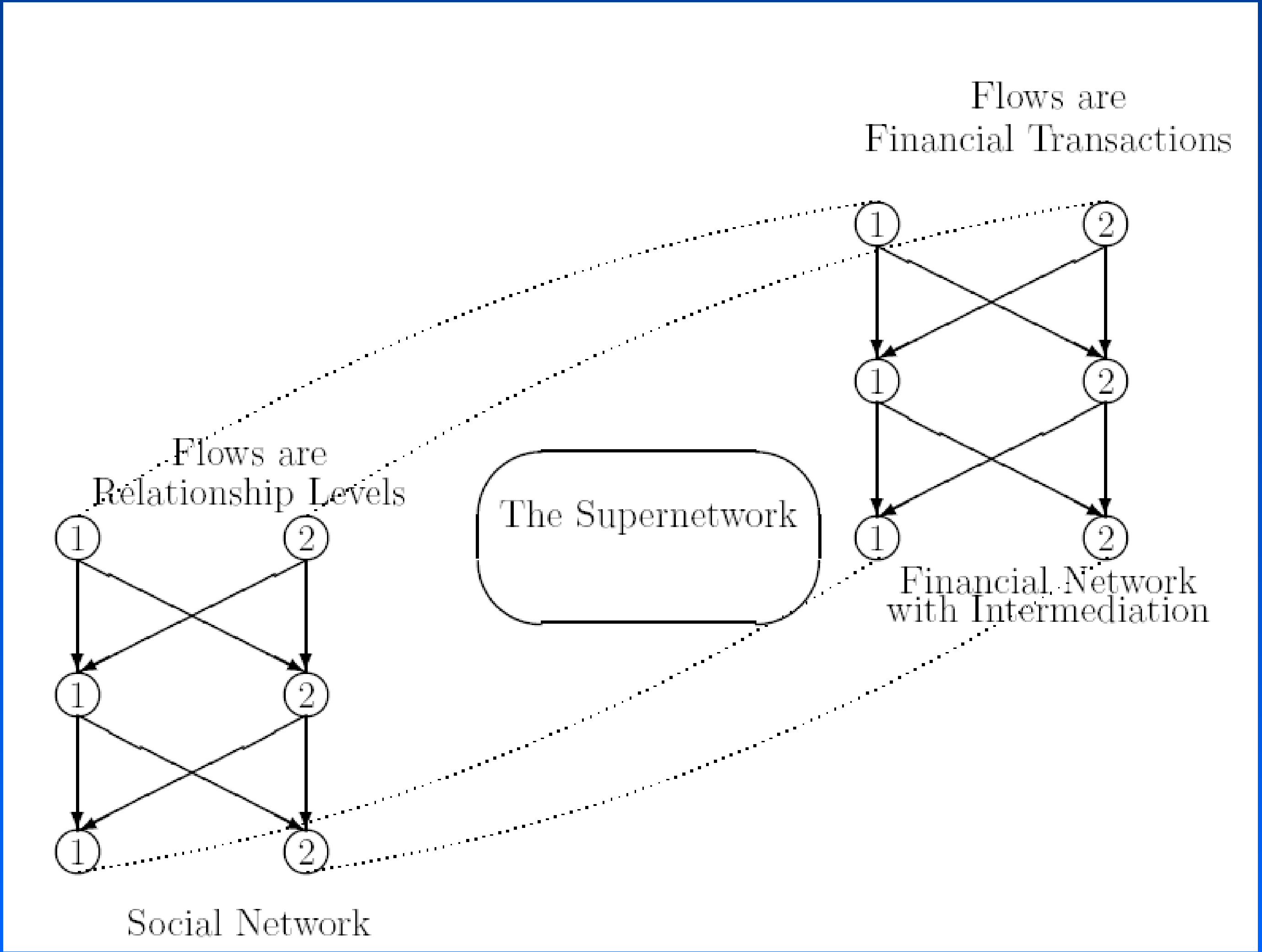
and $P_{\mathcal{K}}$ is the projection of X on the set $\mathcal{K}$ defined as:

$$y = P_{\mathcal{K}}X = \arg\,\min_{z \in \mathcal{K}} \|X - z\|.$$

Step 2: Convergence Verification

If $\|X^{T+1} - X^T\| \leq \epsilon$, for some $\epsilon > 0$, a prespecified tolerance, then stop; else, set $T = T + 1$, and go to Step 1,

Supernetwork Structure of the Numerical Examples



Financial Network Numerical Examples

- 2 source agents, 2 intermediaries, 2 demand markets
- No electronic transactions
- Transactions only between source agents and intermediaries and between intermediaries and demand markets
- Financial holdings of each source agent are 20
- Variance-covariance matrices are equal to identity matrices

Financial Network Numerical Examples

Transaction cost functions of source agents

$$c_{ij1}(q_{ij1}, h_{ij1}) = .5q_{ijl}^2 + 3.5q_{ijl} - h_{ij1}, \quad \text{for } i = 1,2; j = 1,2.$$

Handling cost functions of intermediaries

$$c_j(Q^1) = .5(\sum_{i=1}^{2} q_{ij1})^2, \quad \text{for } j = 1,2.$$

Transaction cost functions of intermediaries

$$\hat{c}_{ij1}(q_{ij1}, h_{ijl}) = 1.5q_{ijl}^2 + 3q_{ijl}, \quad \text{for } i = 1,2; j = 1,2.$$

Financial Network Numerical Examples

- **Demand functions**

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

- **Transaction cost functions for demand markets**

$$\hat{c}_{jk1}(Q^2, Q^2, h^2, h^3) = q_{jk1} - h_{jk1} + 5, \quad \text{for } j = 1, 2; k = 1, 2.$$

- **Relationship value functions**

$$v_{ij1}(h_{ij1}) = h_{ij1}, \quad \forall i, j; \quad v_{jk1}(h_{jk1}) = h_{jk1}, \quad \forall j, k,$$

- **Relationship cost functions**

$$b_{ijl}(h_{ijl}) = 2h_{ijl} + 1, \quad \forall i, j, l = 1; \quad b_{jkl}(h_{jk1}) = h_{jk1} + 1, \quad \forall j, k.$$

Differences among Financial Network Examples

- Example 1
  - The weight for risk and relationship value is equal to 1.
- Example 2
  - The weight for relationship value for the two source agents increased from 1 to 10.
- Example 3
  - Like Example 2 but demand function changed to $d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1100.$

Financial Network Example 1 Discussion

We set the weights associated with the risk functions and the relationship values to 1.

The financial flow on each link was equal to 1. There was slack associated with the source agent's financial transactions and, in fact, 18 units of financial flows were not allocated to any financial intermediary from each source agent.

The equilibrium relationship levels were all equal to 0.

Financial Network Example 2 Discussion

Now the relationship levels associated with the source agents' transactions increased from their values of 0 in Example 1 to new equilibrium levels of 1 but the financial flows stayed the same.

Hence, whereas before there were no relationships and, in effect, the social network component of the supernetwork could be entirely eliminated, the relationship levels between the source agents and the financial intermediaries were at their highest possible levels.

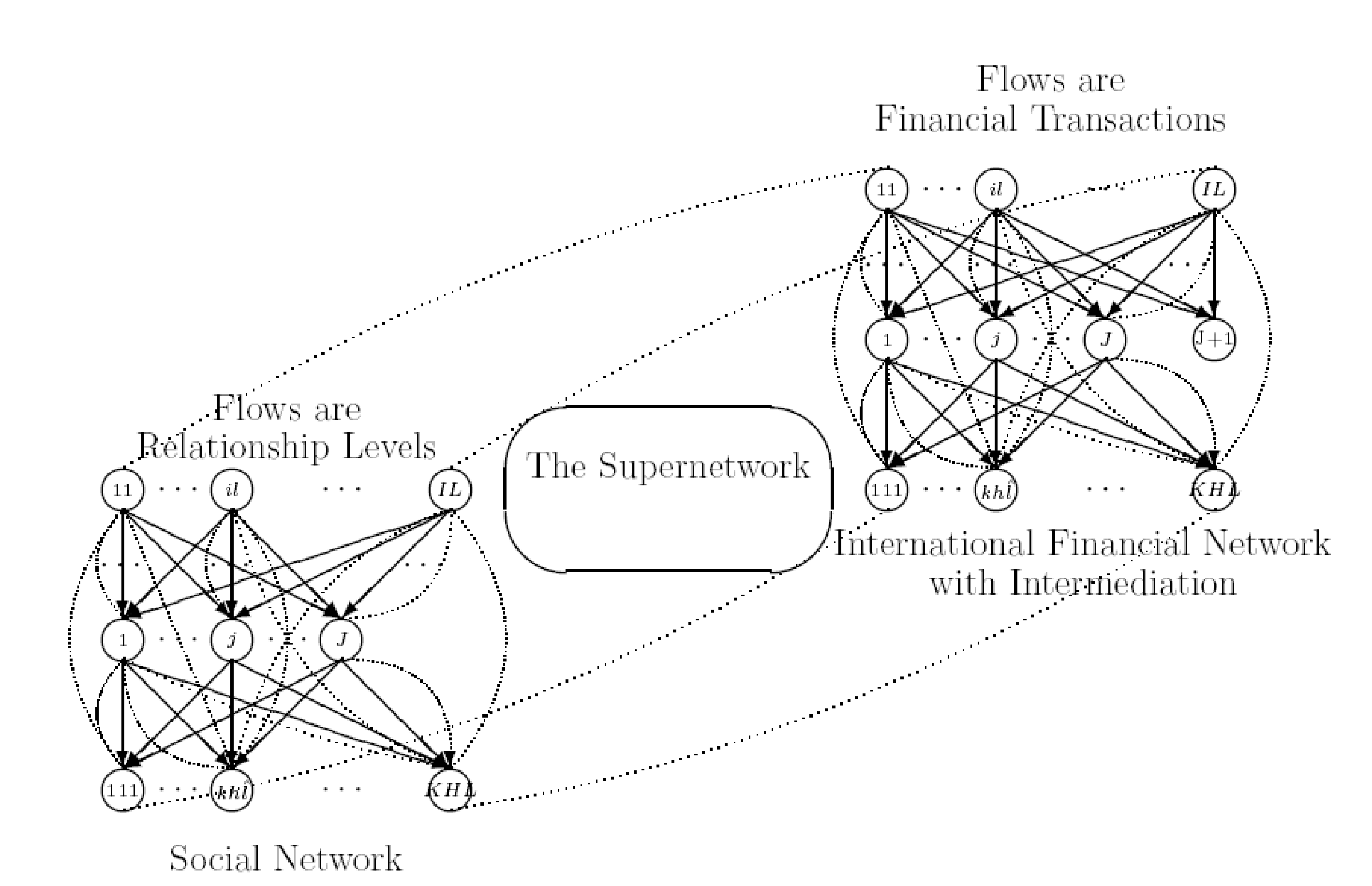
Financial Network Example 3 Discussion

The relationship levels remained as in Example 2.

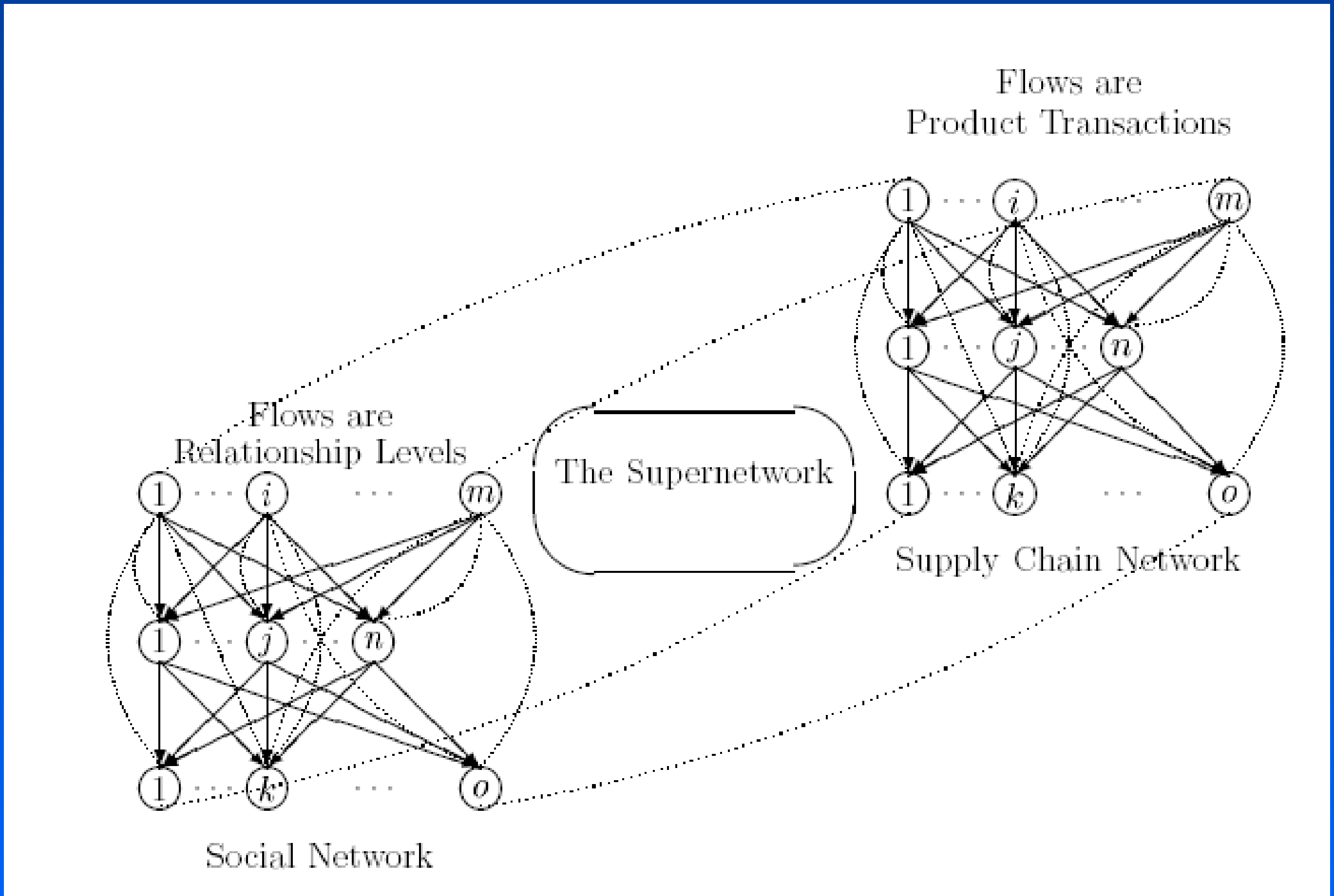
It is worth noting that in this, as in the preceding examples, the budget constraint did not hold tightly for each source agent, that is, not all the financial holdings were allocated.

The top tier financial flows increased as did the flows to the first demand market; the others decreased.

International Financial/ Social Network System



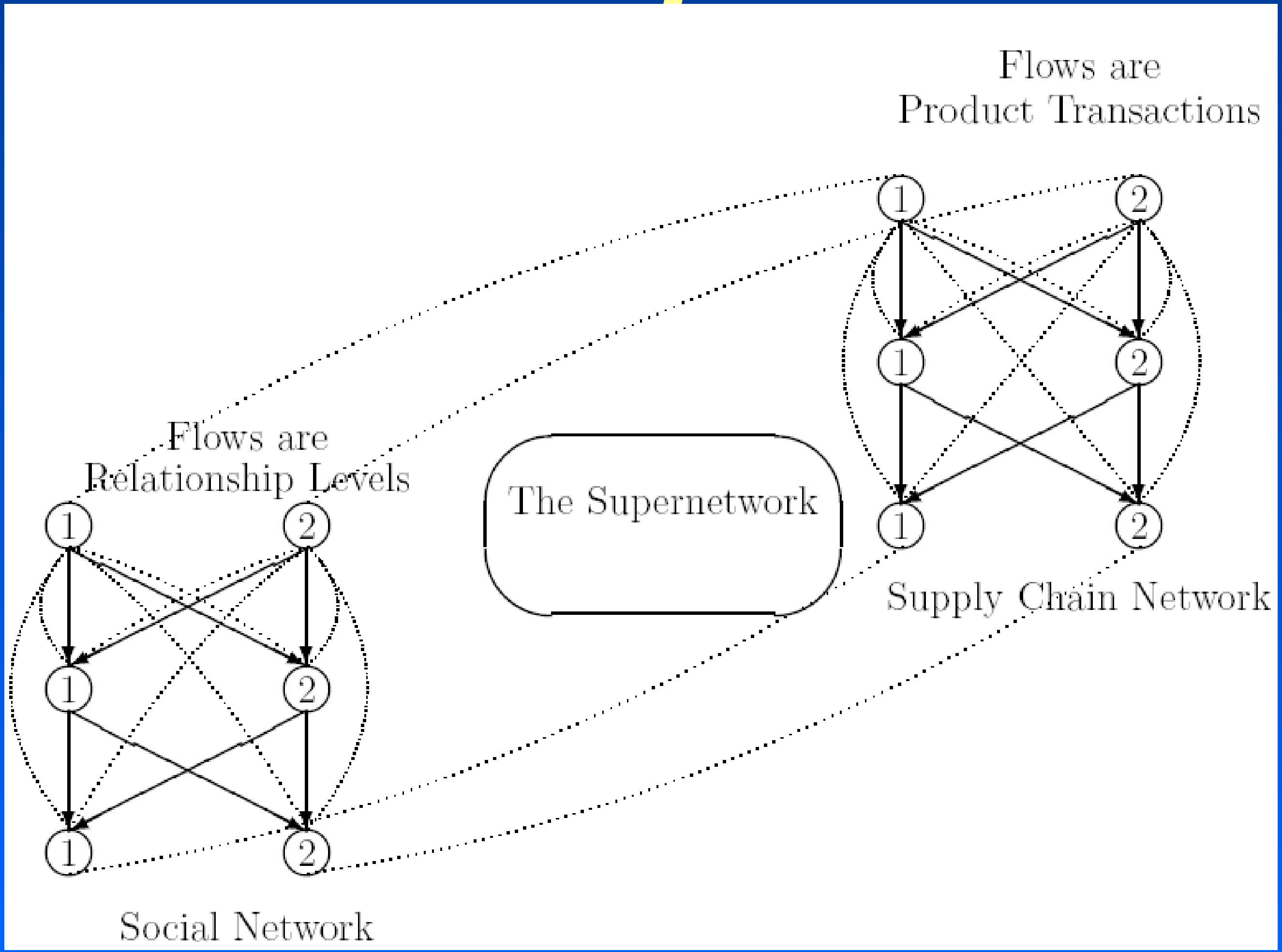
Supply Chain/Social Network System



Characteristics of the Supply Chain Numerical Examples

- 2 manufacturers
- 2 retailers
- 2 demand markets
- Physical and electronic transactions between manufacturers and retailers
- Electronic transactions between manufacturers and demand markets
- Physical transactions between retailers and demand markets

Network Structure of the Supply Chain Numerical Examples



Supply Chain Examples: 1-3 Manufacturer Information

2 manufacturers

Production cost functions

$$f_1(q) = 2.5q_1^2 + q_1 q_2 + 2q_1, \quad f_2(q) = 2.5q_2^2 + q_1 q_2 + 2q_2.$$

Transaction cost functions

$$c_{ij1}(q_{ij1}, h_{ij1}) = .5q_{ij1}^2 + 3.5q_{ij1} - h_{ij1}, \quad \forall i, j,$$

$$c_{ij2}(q_{ij2}, h_{ij2}) = 1.5q_{ij2}^2 + 3q_{ij2} - .5h_{ij2}, \quad \forall i, j.$$

$$c_{ik}(q_{ik}, h_{ik}) = q_{ik}^2 + 2q_{ik} - 2h_{ik}, \quad \forall i, k.$$

Supply Chain Examples 1-3: Retailer Information

2 retailers

Handling cost functions

$$c_1(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i1})^2, \quad c_2(Q^1) = .5(\sum_{i=1}^{2}\sum_{l=1}^{2} q_{i2})^2.$$

Transaction cost functions

$$\hat{c}_{ijl}(q_{ijl}, h_{ijl}) = 1.5q_{ijl}^2 + 3q_{ijl}, \quad \forall i, j, l.$$

Supply Chain Examples: 1-3 Demand Market Information

2 demand markets

Demand functions

$$d_1(\rho_3) = -2\rho_{31} - 1.5\rho_{32} + 1000, \quad d_2(\rho_3) = -2\rho_{32} - 1.5\rho_{31} + 1000,$$

Transaction cost functions

$$\hat{c}_{jk}(q_{jk}, h_{jk}) = q_{jk} - h_{jk} + 5, \quad \forall j, k,$$

$$\hat{c}_{ik}(q_{ik}, h_{ik}) = q_{ik} + 1, \quad \forall i, k.$$

Supply Chain Examples: 1-3 Relationship Functions

Relationship value functions

$$v_{ijl}(h_{ijl}) = h_{ijl}, \quad \forall i, j, l; \quad v_{ik}(h_{ik}) = h_{ik}, \quad \forall i, k; \quad v_{jk}(h_{jk}) = h_{jk}, \quad \forall j, k.$$

Relationship cost functions

$$b_{ijl}(h_{ijl}) = 2h_{ijl} + 1, \quad \forall i, j, l;$$

$$b_{ik}(h_{ik}) = h_{ik} + 1, \quad \forall i, k;$$

$$b_{jk}(h_{jk}) = h_{jk} + 1, \quad \forall j, k.$$

Differences among the Supply Chain Examples

Example 1

- **All weights for relationship values are equal to 1.**

Example 2

- **The weights for relationship values for the two manufacturers increased from 1 to 10.**

Example 3

- **The weights for relationship values for the two manufacturers increased from 10 to 20.**

Supply Chain Example 1 Discussion

The relationship levels were all equal to 0 except for the relationship levels between manufacturers and the retailers transacting via the Internet, whose relationship levels were the strongest, i.e., equal to 1.

Hence, the supernetwork in equilibrium consists of the supply chain network and the links on the social network joining the manufacturers with the retailers through the Internet.

In Example 2 we increased the weight associated with the relationship values associated with the manufacturers.

Supply Chain Example 2 Discussion

With the increase in weights associated with the manufacturers' relationship levels, the relationship levels between manufacturers and the retailers for both modes of transaction were at the highest levels, that is, all were equal to 1.

In addition, the relationship levels between retailers and the demand markets increased. This may be due to the fact that since the product transactions increased it made sense for the retailers to increase their relationship levels since, in view, of the transaction cost functions (which are decreasing in the relationship levels), these costs would be reduced.

All the product transactions increased (relative to those obtained in Example 1), except for the transactions associated with B2C commerce.

Hence, the social network component (in equilibrium) in Supply Chain Example 2 is much denser than that in Example 1.

We now have positive equilibrium relationship levels not only on the Internet links between manufacturers and retailers but also on the physical links between manufacturers and retailers, as well as on the links on the social network representing retailers transacting with the demand markets.

Supply Chain Example 3 Discussion

- **Since the weights associated with the relationships at the manufacturers further relative to the weights in Example 2, the relationship levels that were already at level 1 could not increase more (since they are already at their upper bounds) even with an increase in weight.**
- **The network topology of the supernetwork in equilibrium for this example was that obtained for Example 2.**

Types of Simulations that can be Performed

We can simulate:

- **Changes in production, transaction, handling, and relationship production cost functions**
- **Changes in demand and risk functions**
- **Changes in weights for relationship value and risk**
- **Addition and removal of actors**
- **Addition and removal of multiple transaction modes.**

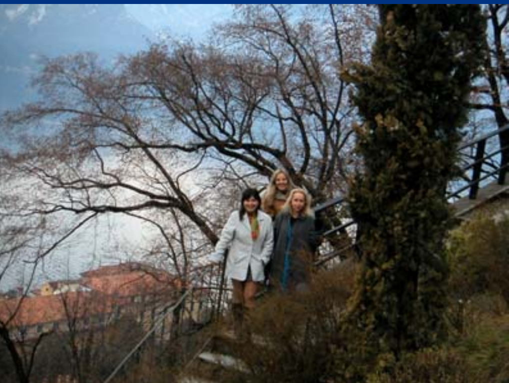
Summary

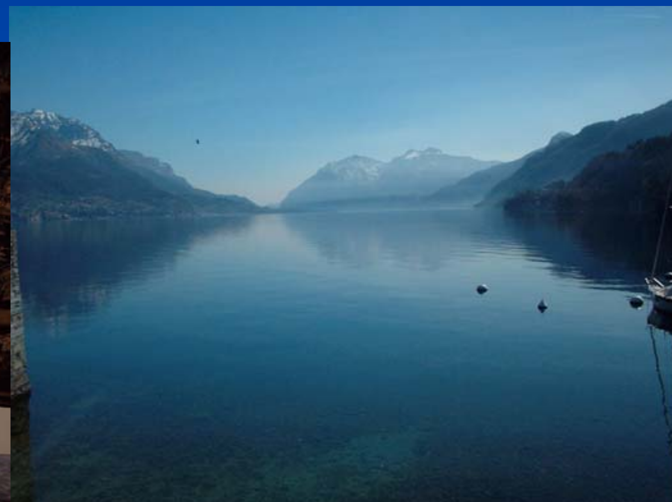
We modeled the behavior of the decision-makers, their interactions, and the dynamic evolution of the associated variables.

We studied the problems qualitatively as well as computationally.

We developed algorithms, implemented them, and established conditions for convergence.

Bellagio Research Team Residency March 2004





Present and Future Work

We are working on infinite-dimensional projected dynamical systems and evolutionary variational inequalities and their relationships and unification.

This allows us to model dynamic networks with:

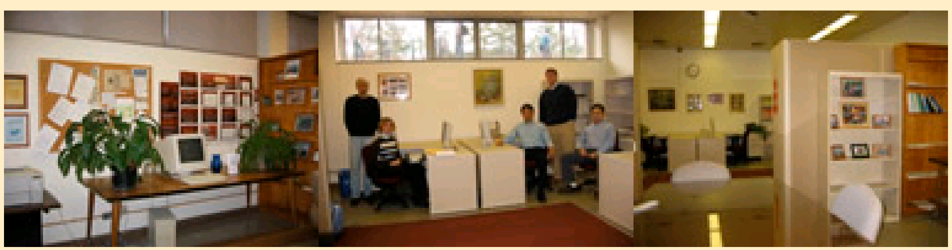
- ***dynamic (time-dependent)* supplies and demands**
- ***dynamic (time-dependent)* capacities**
- ***structural changes* in the networks themselves.**

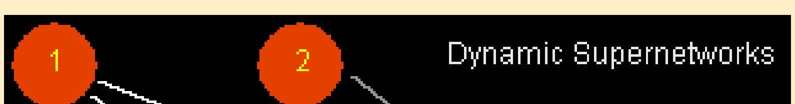
The Virtual Center for Supernetworks

Supernetworks for Optimal Decision-Making and Improving the Global Quality of Life

The Virtual Center for Supernetworks at the Isenberg School of Management, under the directorship of Anna Nagurney, the John F. Smith Memorial Professor, is an interdisciplinary center, and includes the Supernetworks Laboratory for Computation and Visualization.

Supernetworks Lab Page and Virtual Tour





Anna Nagurney will deliver her Fellowship Lecture, *Dynamic Networks with Applications*, at the Radcliffe Institute for Advanced Studies, Harvard University, November 9, 2005

NEW!

Special Sessions on the 50th Anniversary of Beckmann, McGuire, and Winsten's *Studies in the Economics of Transporation* at the November 2005 INFORMS Meeting in San Francisco

NEW!

Fall 2005 INFORMS Student Chapter Speaker Series Announced!!!

Thank you!

For more information, see
http://supernet.som.umass.edu

Eugene M. Isenberg School of Management

The Virtual Center for Supernetworks

RADCLIFFE INSTITUTE FOR ADVANCED STUDY
HARVARD UNIVERSITY