Network Design – From the Physical World to Virtual Worlds

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Workshop on Social Science and Social Computing:
Steps to Integration
Honolulu, Hawaii
May 22-23, 2010

Acknowledgments

This research was supported by the John F. Smith Memorial Fund at the University of Massachusetts Amherst. This support is gratefully acknowledged.

I would like to thank Dr. Sun-Ki Chai for the invitation to present at this workshop.

Outline

- Background and Motivation
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- Network Design Through Mergers and Acquisitons
- Network Design Through the Evolution and Integration of Disparate Network Systems, Including Social Networks
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- The Algorithm and Explicit Formulae
- Numerical Examples
- Summary and Conclusions

Background and Motivation

We Are in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- *alternative and, at times, conflicting criteria* used in decision-making;
- *constraints on resources*: human, financial, natural, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- *increasing risk and uncertainty*;
- the *importance of dynamics* and realizing a timely response to evolving events.

Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *interactions among networks themselves*, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of the fragility and vulnerability of network systems;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Interdisciplinary Impact of Networks

Models and Algorithms

Networks

Economics and Finance

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

OR/MS and Engineering

Energy

Manufacturing

Telecommunications

Transportation

Supply Chains

Sociology

Social Networks

Organizational Theory

Computer Science

Routing Algorithms

Price of Anarchy

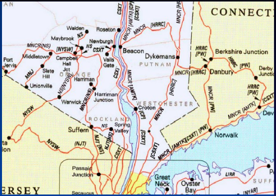
Biology

DNA Sequencing

Targeted Cancer Therapy

Subway Network

Transportation, Communication, and Energy Networks

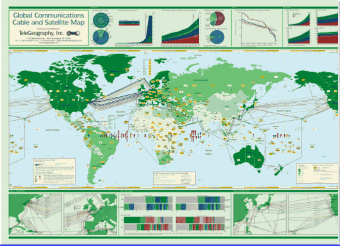


Railroad Network

Iridium Satellite Constellation Network

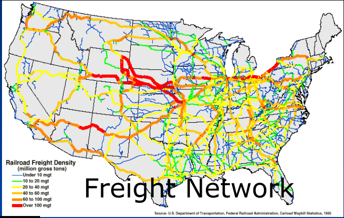
Satellite and Undersea Cable Networks

Duke Energy Gas Pipeline Network



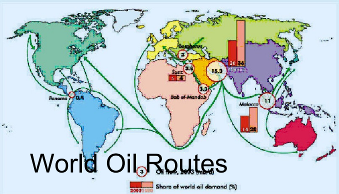
Components of Common Physical Networks

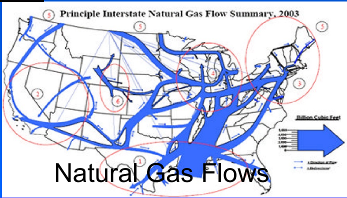
Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity



Network

Systems





Networks in Action

- Some social network websites, such as facebook.com and myspace.com, have over *300 million* users.
- Internet traffic is approximately *doubling* each year.
- In the US, the annual traveler delay per peak period (rush hour) has grown from 16 hours to *47 hours* since 1982.
- The total amount of delay reached *3.7 billion hours* in 2003.
- The wasted fuel amounted to *2.3 billion gallons* due to engines idling in traffic jams (Texas Transportation Institute 2005 Urban Mobility Report).

Why User Behavior Must be Captured in Network Design

The importance of capturing user behavior on networks will now be illustrated through a famous paradox known as the *Braess paradox* in which travelers are assumed to behave in a *user-optimizing (U-O) manner*, as opposed to a *system-optimizing (S-O) one*.

Under U-O behavior, decision-makers act independently and selfishly with no concern of the impact of their travel choices on others.

Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.

		User-Optimized
Decentralized	Selfish	U - O
vs. ⟺	vs. ⟺	vs.
Centralized	Unselfish	S - O
		System-Optimized

Flows are routed so as to minimize the total cost to society.

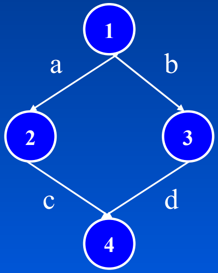
The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $\mathbf{p_1=(a,c)}$ and $\mathbf{p_2=(b,d)}$.

For a travel demand of **6,** the equilibrium path flows are $\mathbf{x_{p_1}^* = x_{p_2}^* = 3}$ and

The equilibrium path travel cost is

$\mathbf{C_{p_1} = C_{p_2} = 83.}$



$$c_a(f_a)=10\,f_a \quad c_b(f_b) = f_b+50$$

$$c_c(f_c) = f_c+50 \quad c_d(f_d) = 10\,f_d$$

Adding a Link Increases Travel Cost for All!

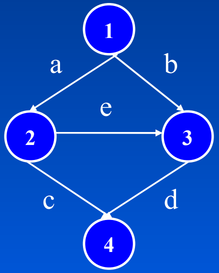
Adding a new link creates a new path **p_3=(a,e,d).**

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path **p_3, C_{p_3}=70.**

The new equilibrium flow pattern network is

$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.

The equilibrium path travel costs: **$C_{p_1} = C_{p_2} = C_{p_3} = 92$.**



$c_e(f_e) = f_e + 10$

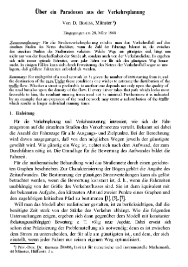
Under S-O behavior, in which case the total cost in the network is minimized, the new route p_3, under the same demand, would not be used.

The Braess paradox does not occur in S-O networks.

The 1968 Braess article has been translated from German to English and appears as:

On a Paradox of Traffic Planning

by Braess, Nagurney, Wakolbinger in the November 2005 issue of *Transportation Science.*



Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.

TRANSPORTATION SCIENCE
Vol. 39, No. 4, November 2005, pp. 446–450
issn 0041-1655 | eissn 1526-5447 | 05 | 3904 | 0446

doi 10.1287/trsc.1050.0127
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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* 12 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen ...

... that is achieved if each user attempts to optimize his own route.

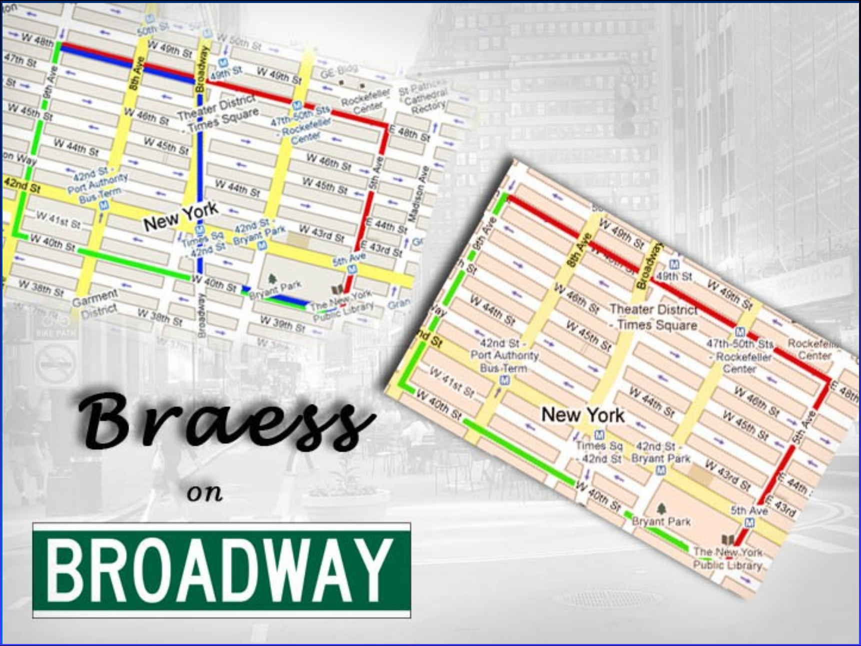
Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable ...

The Braess Paradox Around the World

1969 - Stuggart, Germany - Traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42^{nd} Street was closed and traffic flow improved.

2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.


Braess
on
BROADWAY
New York
Theater District
- Times Square
42nd St -
Port Authority
Bus Term
Times Sq
- 42nd St
42nd St -
Bryant Park
Bryant Park
The New York
Public Library
47th-50th Sts
- Rockefeller
Center
Rockefeller
Center
W 49th St
W 46th St
W 45th St
W 44th St
W 43rd St
W 41st St
W 40th St
5th Ave
8th Ave
Broadway

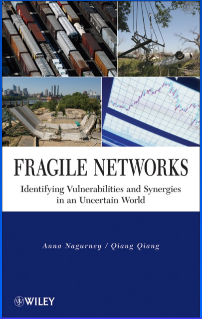
Other Networks that Behave like Traffic Networks

The Internet

Supply Chain Networks

Electric Power Generation/Distribution Networks

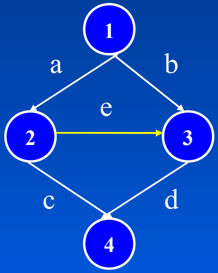
Financial Networks



This *paradox is relevant* not only to congested transportation networks but also to the Internet and electric power networks.

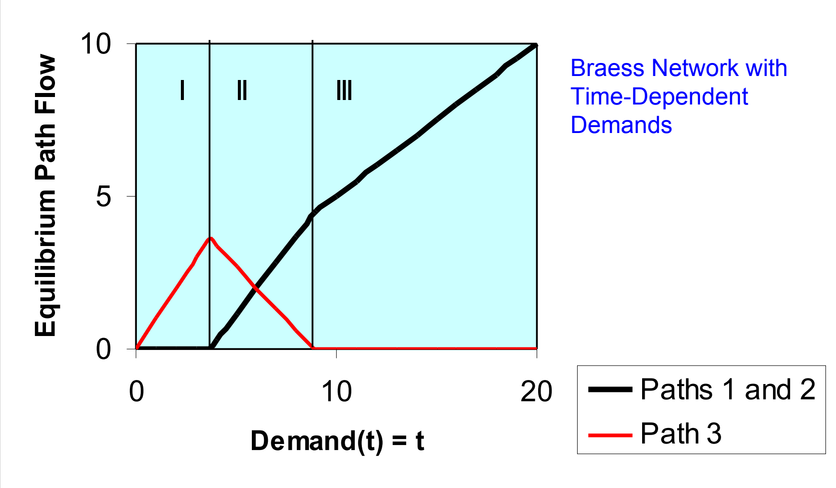
Hence, there are *huge implications* also for network design.

Recall again the Braess Network where we add the link e.

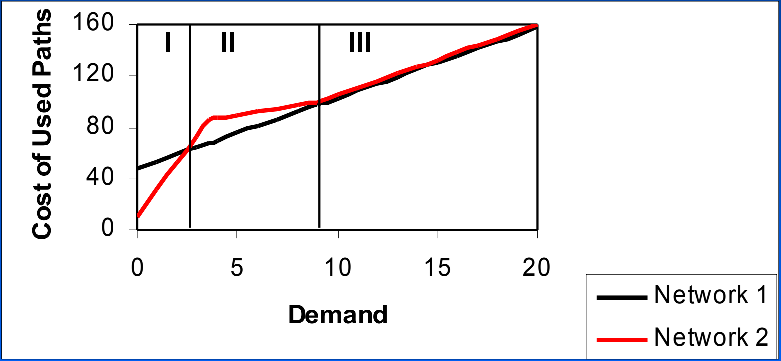


What happens if the demand varies over time?

The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands



In Demand Regime I, only the new path is used.
In Demand Regime II, the Addition of a New Link (Path) Makes Everyone Worse Off!
In Demand Regime III, only the original paths are used.



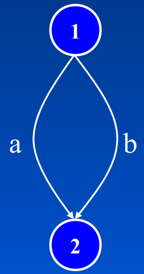
Network 1 is the Original Braess Network - Network 2 has the added link.

The new link is NEVER used after a certain demand is reached even if the demand approaches infinity.

Hence, in general, except for a limited range of demand, building the new link is a complete waste!

Another Example

Assume a network with a single O/D pair (1,2). There are 2 paths available to travelers: $\mathbf{p_1 = a}$ and $\mathbf{p_2 = b}$.



$$c_a(f_a) = f_a$$

$$c_b(f_b) = f_b + 1$$

For a travel demand of **1,** the U-O path flows are: $\mathbf{x_{p_1}^* = 1;\ x_{p_2}^* = 0}$ and

the total cost under U-O behavior is $TC_{u\text{-}o} = 1$.

The S-O path flows are: $\mathbf{x_{p_1} = 3/4;\ x_{p_2} = 1/4}$ and

the total cost under S-O behavior is $TC_{S\text{-}o} = 7/8$.

The Price of Anarchy

The price of anarchy is defined as the ratio of the TC under U-O behavior to the TC under S-O behavior:

$$\rho = TC_{U\text{-}O} / TC_{S\text{-}O}$$

See Roughgarden (2005), *Selfish Routing and the Price of Anarchy*.

Question: When does the U-O solution coincide with the S-O solution?

Answer: In a general network, with user link cost functions given by: $c_a(f_a)=c_a^0 f_a^\beta$, *for all links, with* $c_a^0 \geq 0$ *and* $\beta \geq 0$.

Note that for $c_a(f_a)=c_a^0$, *that is, in the case of uncongested networks, this result always holds.*

Network Design Must Capture the Behavior of Users.

Network Design Through Mergers and Acquisitions

Mergers and Acquisitions and Network Synergies

Recently, we introduced a system-optimization perspective for supply chains in which firms are engaged in multiple activities of production, storage, and distribution to the demand markets and proposed a cost synergy measure associated with evaluating proposed mergers:

- A. Nagurney (2009) "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

In that paper, the merger of two firms was modeled and the demands for the product at the markets, which were distinct for each firm prior to the merger, were assumed to be fixed.

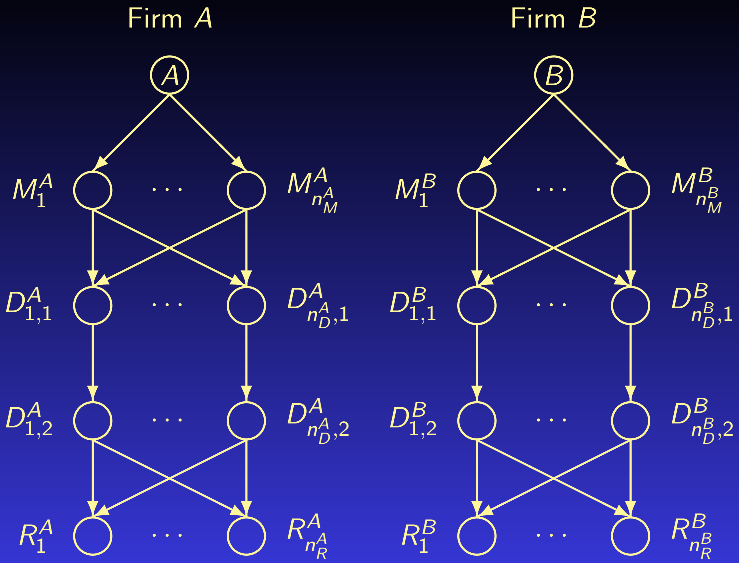


Figure 1: Case 0: Firms A and B Prior to Horizontal Merger

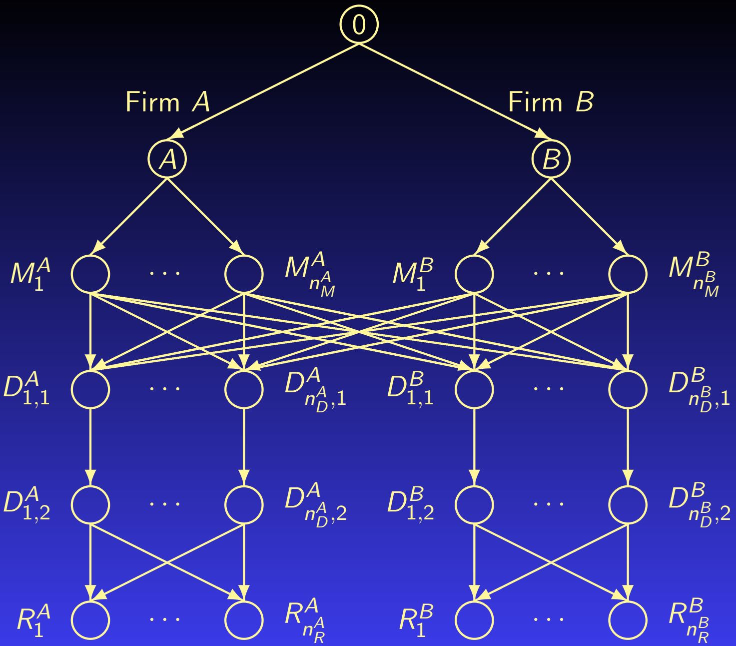


Figure 2: Case 1: Firms A and B Merge

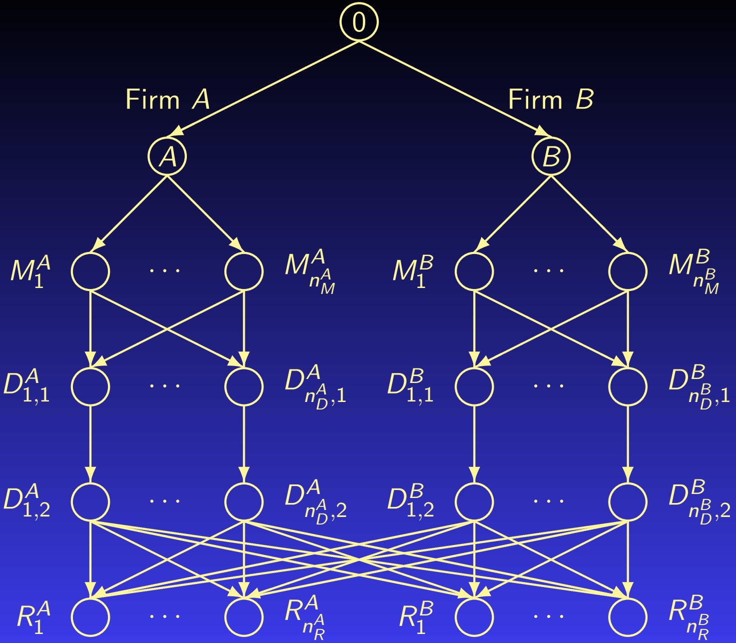


Figure 3: Case 2: Firms A and B Merge

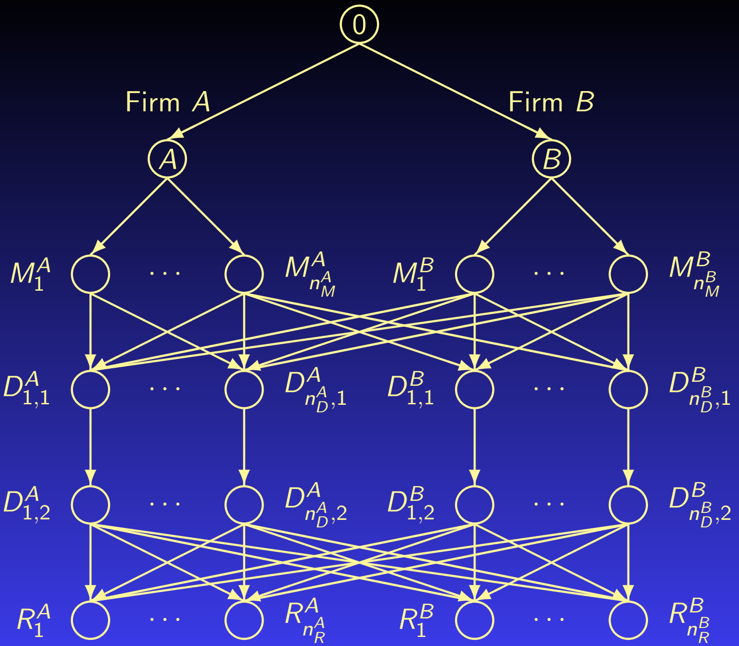


Figure 4: Case 3: Firms A and B Merge

Synergy Measure

The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case i; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0}\right] \times 100\%,$$

where TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i. Note that $\mathcal{S}^i$; $i = 1, 2, 3$ may also be interpreted as *synergy*.

This model can also be applied to the teaming of organizations in the case of humanitarian operations.

Bellagio Conference on Humanitarian Logistics

See: http://hlogistics.som.umass.edu/

The Supply Chain Network Oligopoly Model (Nagurney (2010))

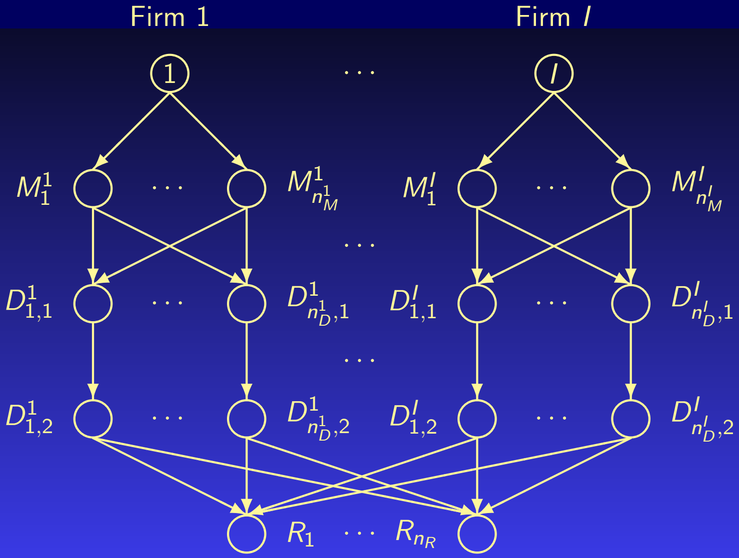


Figure 5: Supply Chain Network Structure of the Oligopoly

Mergers Through Coalition Formation

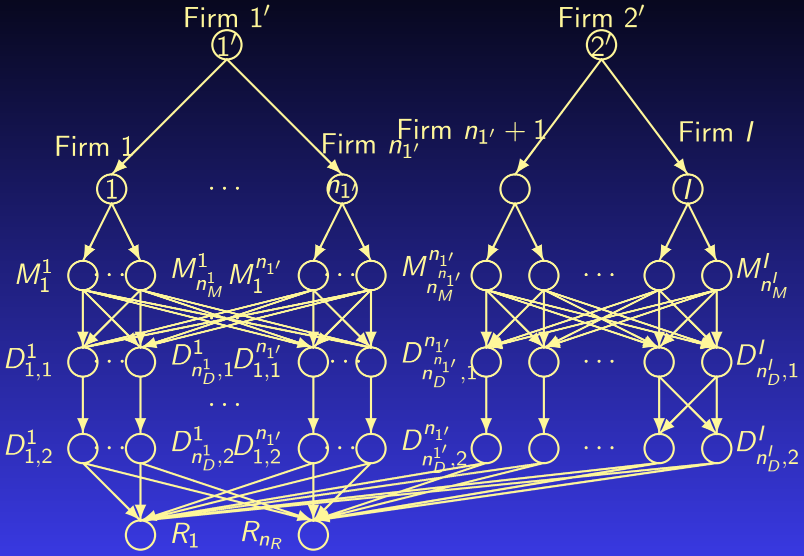


Figure 6: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

Network design (and redesign) can be accomplished through link and node additions (as well as their removals).

It can be accomplished by modifying the link capacities (expanding certain ones and, if applicable, reducing or selling off others).

It can also be accomplished through the integration of similar networks as in mergers and acquisitions.

Network Design Through the Evolution and Integration of Disparate Network Systems

In addition, network design can be accomplished through the evolution and integration of disparate network systems, including social networks.

Two References:

T. Wakolbinger and A. Nagurney (2004) "Dynamic Supernetworks for the Integration of Social Networks and Supply Chains with Electronic Commerce: Modeling and Analysis of Buyer-Seller Relationships with Computations," *Netnomics* **6**, 153-185.

A. Nagurney, T. Wakolbinger, and L. Zhao (2006) "The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels," *Computational Economics* **27**, 353-393.

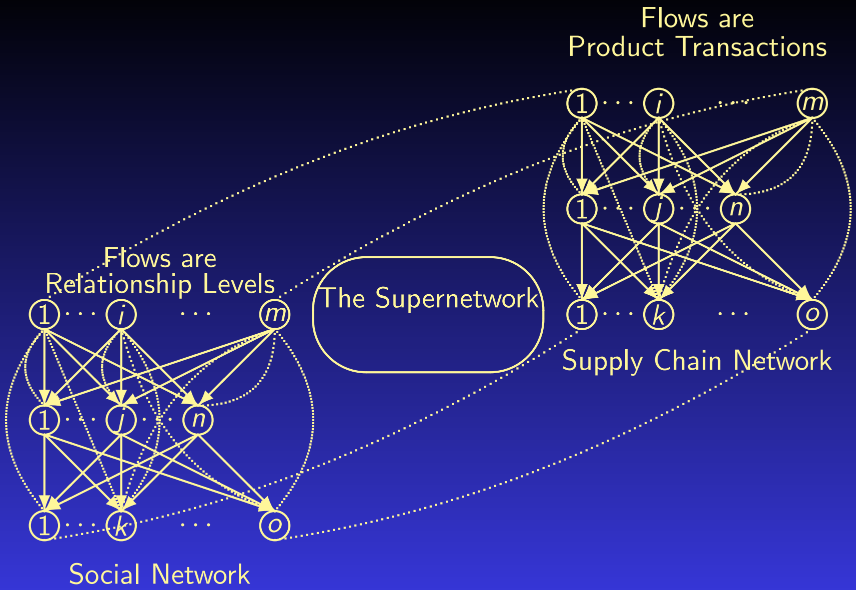


Figure 7: The Multilevel Supernetwork Structure of the Integrated Supply Chain / Social Network System

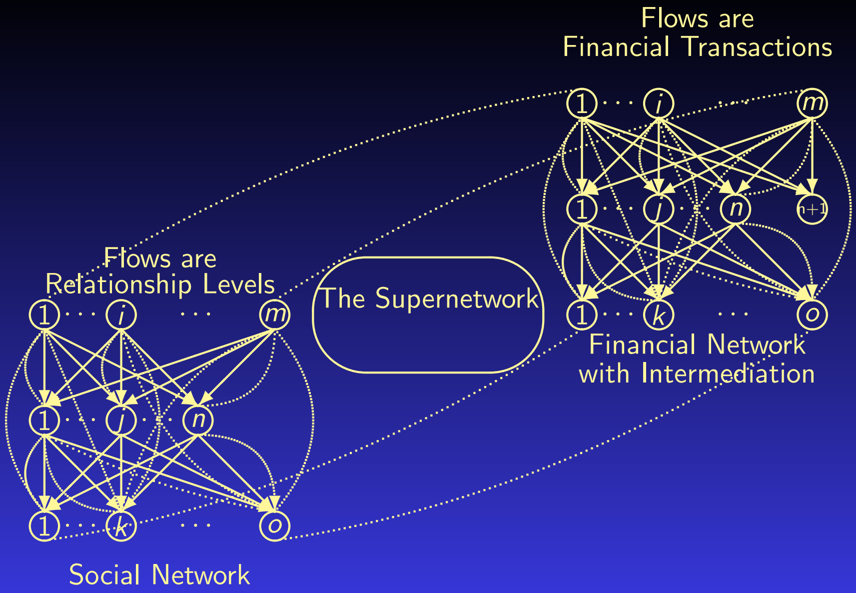


Figure 8: The Multilevel Supernetwork Structure of the Integrated Financial Network / Social Network System

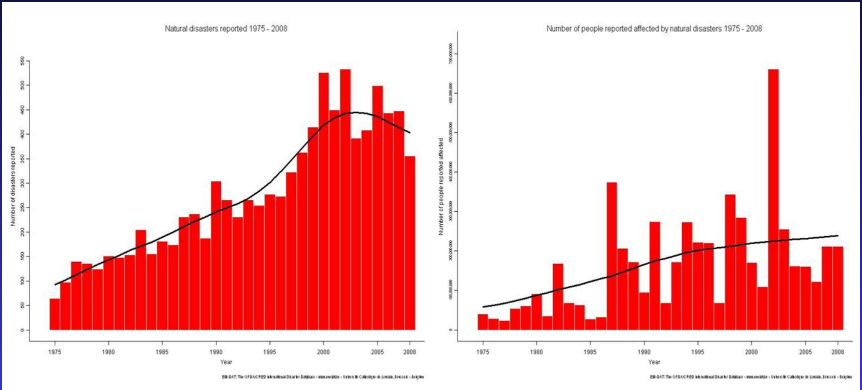
We will now focus on network design in the case of an especially challenging problem – that of supply chain network design for critical need products.

A Challenging Network Design Problem

The number of disasters is increasing globally, as is the number of people affected by disasters. At the same time, with the advent of increasing globalization, viruses are spreading more quickly and creating new challenges for medical and health professionals, researchers, and government officials.

Between 2000 and 2004 the average annual number of disasters was 55% higher than in the period 1994 through 1999, with 33% more humans affected in the former period than in the latter (cf. Balcik and Beamon (2008) and Nagurney and Qiang (2009)).

Natural Disasters (1975–2008)



However, although the average number of disasters has been increasing annually over the past decade the average percentage of needs met by different sectors in the period 2000 through 2005 identifies significant shortfalls.

According to Development Initiatives (2006), based on data in the Financial Tracking System of the Office for the Coordination of Humanitarian Affairs, from 2000-2005, the average needs met by different sectors in the case of disasters were:

- 79% by the food sector;
- 37% of the health needs;
- 35% of the water and sanitation needs;
- 28% of the shelter and non-food items, and
- 24% of the economic recovery and infrastructure needs.

Hurricane Katrina in 2005

Hurricane Katrina has been called an “American tragedy,” in which essential services failed completely (Guidotti (2006)).

Haiti Earthquake in 2010

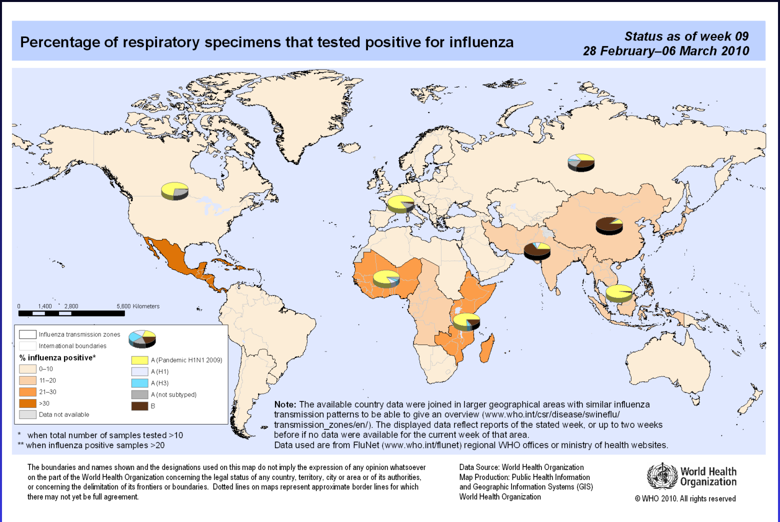
Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.

H1N1 (Swine) Flu

As of May 2, 2010, worldwide, more than 214 countries and overseas territories or communities have reported laboratory confirmed cases of pandemic influenza H1N1 2009, including over 18,001 deaths (www.who.int).

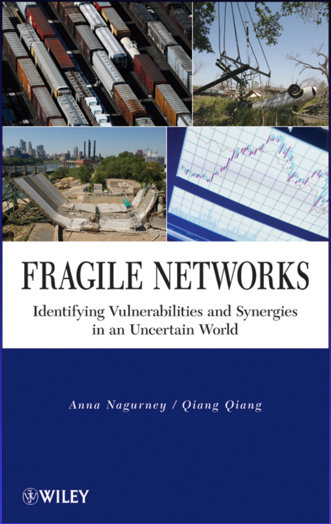
Parts of the globe experienced serious flu vaccine shortages, both seasonal and H1N1 (swine) ones, in late 2009.

Map of Influenza Activity and Virus Subtypes



Source: World Health Organization

Fragile Networks



We are living in a world of *Fragile Networks*.

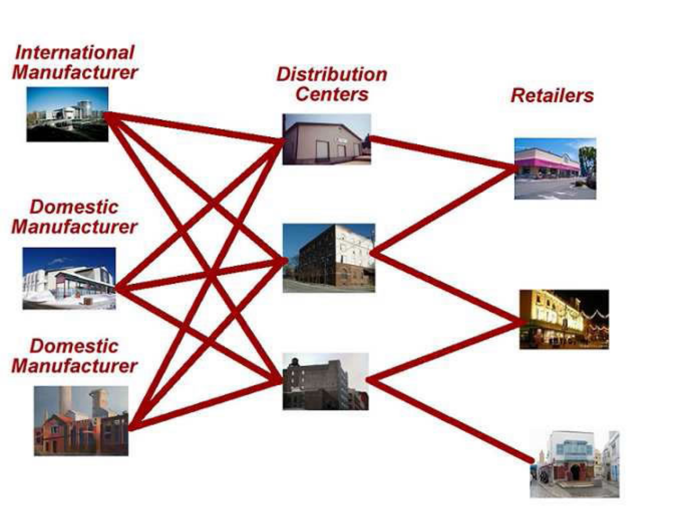
Background and Motivation

Underlying the delivery of goods and services in times of crises, such as in the case of disasters, pandemics, and life-threatening major disruptions, are supply chains, without which essential products do not get delivered in a timely manner, with possible increased disease, injuries, and casualties.

It is clear that better-designed supply chain networks would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Supply Chain Networks

Supply Chain Networks are a class of complex network. Today, supply chain networks are increasingly global in nature.



Supply Chain Networks provide the logistical backbones for the provision of products as well as services both in corporate as well as in emergency and humanitarian operations.

Here we focus on supply chains in the case of

Critical Needs Products.

Critical Needs Products

Critical needs products are those that are essential to the survival of the population, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a system-optimization approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.

An Overview of the Relevant Literature

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- D. E. Boyce, H. S. Mahmassani, and A. Nagurney (2005) "A Retrospective on Beckmann, McGuire, and Winsten's *Studies in the Economics of Transportation*," *Papers in Regional Science* **84**, 85-103.
- A. Nagurney (2009), "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

- A. Nagurney, T. Woolley, and Q. Qiang (2010) "Multiproduct Supply Chain Horizontal Network Integration: Models, Theory, and Computational Results," *International Journal of Operational Research* **17**, 333-349.
- A. Nagurney (2010) "Formulation and Analysis of Horizontal Mergers Among Oligopolistic Firms with Insights into the Merger Paradox: A Supply Chain Network Perspective," *Computational Management Science*, in press.
- A. Nagurney (2010) "Supply Chain Network Design Under Profit Maximization and Oligopolistic Competition," *Transportation Research E* **46**, 281-294.
- A. Nagurney and L. S. Nagurney (2009) "Sustainable Supply Chain Network Design: A Multicriteria Perspective," to appear in the *International Journal of Sustainable Engineering*.

This part of the presentation is based on the paper:

"Supply Chain Network Design for Critical Needs with Outsourcing,"

A. Nagurney, M. Yu, and Q. Qiang, to appear in *Papers in Regional Science*,

where additional background as well as references can be found.

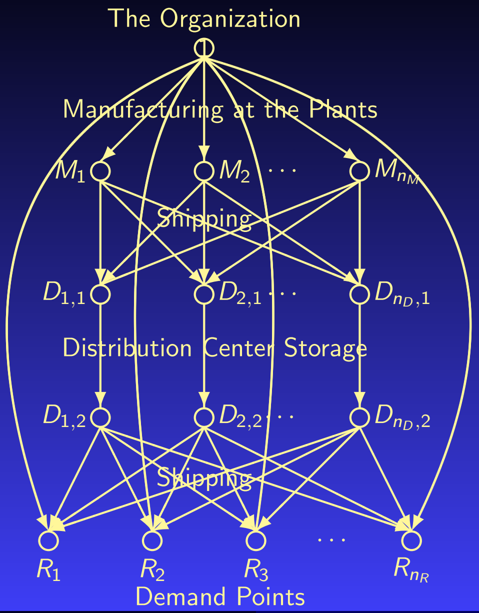
The Supply Chain Network Design Model for Critical Needs with Outsourcing

We assume that the organization (government, humanitarian one, socially responsible firm, etc.) is considering n_M manufacturing facilities/plants; n_D distribution centers, but must serve the n_R demand points.

The supply chain network is modeled as a network $G = [N, L]$, consisting of the set of nodes N and the set of links L. Let L^1 and L^2 denote the links associated with "in house" supply chain activities and the outsourcing activities, respectively. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to those in need at the demand points.

The optimization model can handle both design (from scratch) and redesign scenarios.

Supply Chain Network Topology with Outsourcing



The Links

The possible manufacturing links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$.

The possible shipment links from the manufacturing nodes, are connected to the possible distribution center nodes of the organization, denoted by $D_{1,1}, \ldots, D_{n_D,1}$.

The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links.

There are possible shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $R_1, \ldots, R_{n_R}$.

There are also outsourcing links, which may join the top node to each bottom node (or the relevant nodes for which the outsourcing activity is feasible, as in production, storage, or distribution, or a combination thereof). The organization does not control the capacities on these links since they have been established by the particular firm that corresponds to the outsource link.

The ability to outsource supply chain network activities for critical needs products provides alternative pathways for the production and delivery of products during times of crises such as disasters.

Demands, Path Flows, and Link Flows

Let d_k denote the demand at demand point k; $k = 1, \ldots, n_R$, which is a random variable with probability density function given by $\mathcal{F}_k(t)$. Let x_p represent the nonnegative flow of the product on path p; f_a denote the flow of the product on link a.

Conservation of Flow Between Path Flows and Link Flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{1}$$

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link. $\delta_{ap} = 1$ if link a is contained in path p, and $\delta_{ap} = 0$, otherwise.

Supply Shortage and Surplus

Let

$$v_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \ldots, n_R, \tag{2}$$

where v_k can be interpreted as the *projected demand* at demand market k; $k = 1, \ldots, n_R$. Then,

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \ldots, n_R, \tag{3}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \ldots, n_R, \tag{4}$$

where Δ_k^- and Δ_k^+ represent the supply shortage and surplus at demand point k, respectively. The expected values of Δ_k^- and Δ_k^+ are given by:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k)\mathcal{F}_k(t)d(t), \quad k = 1, \ldots, n_R, \tag{5}$$

$$E(\Delta_k^+) = \int_0^{v_k} (v_k - t)\mathcal{F}_k(t)d(t), \quad k = 1, \ldots, n_R. \tag{6}$$

The Operation Costs, Investment Costs and Penalty Costs

The total cost on a link is assumed to be a function of the flow of the product on the link. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L. \tag{7}$$

We denote the nonnegative existing capacity on a link a by $\bar{u}_a$, $\forall a \in L$. Note that the organization can add capacity to the "in house" link a; $\forall a \in L^1$. We assume that

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^1. \tag{8}$$

The expected total penalty at demand point k; $k = 1, \ldots, n_R$, is,

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \tag{9}$$

where λ_k^- is the unit penalty of supply shortage at demand point k and λ_k^+ is that of supply surplus. Note that $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$ is a function of the path flow vector x.

The Supply Chain Network Design Optimization Problem

The organization seeks to determine the optimal levels of product processed on each supply chain network link (including the outsourcing links) coupled with the optimal levels of capacity investments in its supply chain network activities subject to the minimization of the total cost.

The total cost includes the total cost of operating the various links, the total cost of capacity investments, and the expected total supply shortage/surplus penalty.

The Supply Chain Network Design Optimization Problem

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L^1} \hat{\pi}_a(u_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \tag{10}$$

subject to: constraints (1), (2) and

$$f_a \leq \bar{u}_a + u_a, \quad \forall a \in L^1, \tag{11}$$

$$f_a \leq \bar{u}_a, \quad \forall a \in L^2, \tag{12}$$

$$u_a \geq 0, \quad \forall a \in L^1, \tag{13}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{14}$$

The Feasible Set

We associate the Lagrange multiplier ω_a with constraint (11) for link $a \in L^1$ and we denote the associated optimal Lagrange multiplier by ω_a^*. Similarly, Lagrange multiplier γ_a is associated with constraint (12) for link $a \in L^2$ with the optimal multiplier denoted by γ_a^*. These two terms may also be interpreted as the price or value of an additional unit of capacity on link a. We group these Lagrange multipliers into the vectors ω and γ, respectively. Let K denote the feasible set such that

$$K \equiv \{(x, u, \omega, \gamma) | x \in R_+^{n_P}, u \in R_+^{n_{L^1}}, \omega \in R_+^{n_{L^1}}, \text{and}\, \gamma \in R_+^{n_{L^2}}\}.$$

Theorem

The optimization problem is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal link capacity enhancements, and the vectors of optimal Lagrange multipliers $(x^*, u^*, \omega^*, \gamma^*) \in K$, *such that:*

$$\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^1} \omega_a^* \delta_{ap} + \sum_{a \in L^2} \gamma_a^* \delta_{ap} + \lambda_k^+ P_k \left(\sum_{p \in P_{w_k}} x_p^* \right) \right.$$

$$\left. - \lambda_k^- \left(1 - P_k \left(\sum_{p \in P_{w_k}} x_p^* \right) \right) \right] \times [x_p - x_p^*]$$

$$+ \sum_{a \in L^1} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \omega_a^* \right] \times [u_a - u_a^*] + \sum_{a \in L^1} [\bar{u}_a + u_a^* - \sum_{p \in P} x_p^* \delta_{ap}] \times [\omega_a - \omega_a^*]$$

$$+ \sum_{a \in L^2} [\bar{u}_a - \sum_{p \in P} x_p^* \delta_{ap}] \times [\gamma_a - \gamma_a^*] \geq 0, \quad \forall (x, u, \omega, \gamma) \in K. \qquad (15)$$

Theorem (cont'd.)

In addition, (15) can be reexpressed in terms of links flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and link capacity enhancements, and the vectors of optimal Lagrange multipliers $(f^*, v^*, u^*, \omega^*, \gamma^*) \in K^1$*, such that:*

$$\sum_{a\in L^1}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\omega_a^*\right]\times[f_a-f_a^*]+\sum_{a\in L^2}\left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a}+\gamma_a^*\right]\times[f_a-f_a^*]$$

$$+\sum_{a\in L^1}\left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a}-\omega_a^*\right]\times[u_a-u_a^*]$$

$$+\sum_{k=1}^{n_R}\left[\lambda_k^+ P_k(v_k^*)-\lambda_k^-(1-P_k(v_k^*))\right]\times[v_k-v_k^*]+\sum_{a\in L^1}[\bar{u}_a+u_a^*-f_a^*]\times[\omega_a-\omega_a^*]$$

$$+\sum_{a\in L^2}[\bar{u}_a-f_a^*]\times[\gamma_a-\gamma_a^*]\geq 0, \quad \forall (f,v,u,\omega,\gamma)\in K^1, \qquad (16)$$

where $K^1 \equiv \{(f,v,u,\omega,\gamma)|\exists x\geq 0,$ *and* $(1),(2),(13),$ *and* (14) *hold, and* $\omega \geq 0, \quad \gamma \geq 0\}$.

Applications to Vaccine Production

Consider a vaccine manufacturer who is gearing up for next year's production of H1N1 (swine) flu vaccine. Governments around the world are beginning to contract with this company for next year's flu vaccine.

By applying the general theoretical model to the company's data, the firm can determine whether it needs to expand its facilities (or not), how much of the vaccine to produce where, how much to store where, and how much to have shipped to the various demand points. Also, it can determine whether it should outsource any of its vaccine production and at what level.

The firm by solving the model with its company-relevant data can then ensure that the price that it receives for its vaccine production and delivery is appropriate and that it recovers its incurred costs and obtains, if negotiated correctly, an equitable profit.

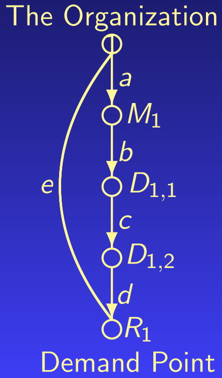
Applications to Emergency Preparedness and Humanitarian Logistics

A company can, using the model, prepare and plan for an emergency such as a natural disaster in the form of a hurricane and identify where to store a necessary product (such as food packets, for example) so that the items can be delivered to the demand points in a timely manner and at minimal total cost.

In August 2005 Hurricane Katrina hit the US and this natural disaster cost immense damage with repercussions that continue to this day. While US state and federal officials came under severe criticism for their handling of the storm's aftermath, Wal-Mart had prepared in advance and through its logistical efficiencies had dozens of trucks loaded with supplies for delivery before the hurricane even hit landfall.

Numerical Examples

Consider the supply chain network topology in which the organization is considering a single manufacturing plant, a single distribution center for storing the critical need product and is to serve a single demand point. The links are labeled, that is, a, b, c, d, and e, with e denoting the outsourcing link.



Example 1

The total cost functions on the links were:

$$\hat{c}_a(f_a) = .5f_a^2 + f_a, \quad \hat{c}_b(f_b) = .5f_b^2 + 2f_b, \quad c_c(f_c) = .5f_c^2 + f_c,$$

$$\hat{c}_d(f_d) = .5f_d^2 + 2f_d, \quad \hat{c}_e(f_e) = 5f_e.$$

The investment capacity cost functions were:

$$\hat{\pi}_a(u_a) = .5u_a^2 + u_a, \quad \forall a \in L^1.$$

The existing capacities were: $\bar{u}_a = 0, \quad \forall a \in L^1$, and $\bar{u}_e = 2$.

The demand for the product followed a uniform distribution on the interval $[0, 10]$ so that:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{10}.$$

The penalties were: $\lambda_1^- = 10, \quad \lambda_1^+ = 0.$

Example 2

Example 2 had the same data as Example 1 except that we now increased the penalty associated with product shortage from 10 to 50, that is, we now set $\lambda_1^- = 50$.

Example 3

Example 3 had the same data as Example 2 except that $\bar{u}_a = 3$ for all the links $a \in L^1$. This means that the organization does not have to construct its supply chain activities from scratch as in Examples 1 and 2 but does have some existing capacity.

Example 4

Example 4 had the total cost functions on the links given by:

$$\hat{c}_a(f_a) = f_a^2, \quad \hat{c}_b(f_b) = f_b^2, \quad c_c(f_c) = f_c^2, \quad \hat{c}_d(f_d) = f_d^2, \quad \hat{c}_e(f_e) = 100 f_e.$$

The investment capacity cost functions were: $\hat{\pi}_a(u_a) = u_a^2, \quad \forall a \in L^1.$

The existing capacities were: $\bar{u}_a = 10, \quad \forall a \in L.$

We assumed that the demand followed a uniform distribution on the interval $[10, 20]$ so that

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p - 10}{10}.$$

The penalties were: $\lambda_1^- = 1000, \quad \lambda_1^+ = 10.$

The Solutions

Example 1

The path flow solution was: $x^*_{p_1} = 0.00, \quad x^*_{p_2} = 2.00$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 0.00, f^*_e = 2.00.$$

The capacity investments were: $u^*_a = 0.00, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 1.00, \quad \forall a \in L^1$, $\gamma^*_e = 3.00$.

Since the current capacities in the "in-house" supply chain links are zero, it is more costly to expand them than to outsource. Consequently, the organization chooses to outsource the product for production and delivery.

The Solutions (cont'd.)

Example 2

The path flow solution was: $x^*_{p_1} = 2.31, \quad x^*_{p_2} = 2.00$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 2.31, f^*_e = 2.00.$$

The capacity investments were: $u^*_a = 2.31, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 3.31, \quad \forall a \in L^1$, $\gamma^*_e = 23.46$.

Since the penalty cost for under-supplying is increased, the organization increased its "in-house" capacity and product output.

The Solutions (cont'd.)

Example 3

The path flow solution was: $x_{p_1}^* = 3.23, \quad x_{p_2}^* = 2.00$, which corresponds to the link flow pattern:

$$f_a^* = f_b^* = f_c^* = f_d^* = 3.23, f_e^* = 2.00.$$

The capacity investments were: $u_a^* = 0.23, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega_a^* = 1.23, \quad \forall a \in L^1$, $\gamma_e^* = 18.84$.

Given the existing capacities in the "in-house" supply chain links, the organization chooses to supply more of the critical product from its manufacturer and distributor.

The Solutions (cont'd.)

Example 4

The path flow solution was: $x^*_{p_1} = 11.25, \quad x^*_{p_2} = 7.66$, which corresponds to the link flow pattern:

$$f^*_a = f^*_b = f^*_c = f^*_d = 11.25, f^*_e = 7.66.$$

The capacity investments were: $u^*_a = 1.25, \quad \forall a \in L^1$.
The optimal Lagrange multipliers were: $\omega^*_a = 2.50, \quad \forall a \in L^1$, $\gamma^*_e = 0.00$.

Since the penalty cost for under-supplying is much higher than that of over-supplying, the organization needs to both expand the "in-house" capacities and to outsource the production and delivery of the product to the demand point.

The Algorithm – The Euler Method

The Algorithm

At an iteration τ of the Euler method (see Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{17}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

The sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \rightarrow 0$, as $\tau \rightarrow \infty$.

Explicit Formulae for (17) to the Supply Chain Network Design Variational Inequality (15)

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{p \in P_{w_k}} x_p^\tau)) - \lambda_k^+ P_k(\sum_{p \in P_{w_k}} x_p^\tau)$$

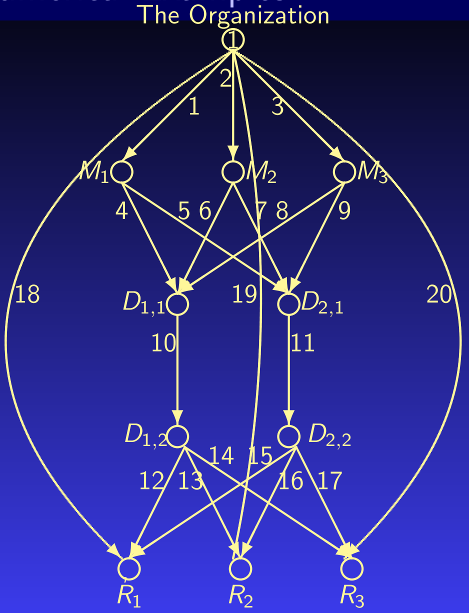
$$-\frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L^1} \omega_a^\tau \delta_{ap} - \sum_{a \in L^2} \gamma_a^\tau \delta_{ap})\}, \forall p \in P; \quad (19)$$

$$u_a^{\tau+1} = \max\{0, u_a^\tau + a_\tau(\omega_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L^1; \quad (20)$$

$$\omega_a^{\tau+1} = \max\{0, \omega_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - \bar{u}_a - u_a^\tau)\}, \quad \forall a \in L^1; \quad (21)$$

$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - \bar{u}_a)\}, \quad \forall a \in L^2. \quad (22)$$

Additional Numerical Examples



Example 5

The demands at the three demand points followed a uniform probability distribution on the intervals $[0, 10]$, $[0, 20]$, and $[0, 30]$, respectively:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{10}, \quad P_2(\sum_{p \in P_{w_2}} x_p) = \frac{\sum_{p \in P_{w_2}} x_p}{20},$$

$$P_3(\sum_{p \in P_{w_3}} x_p) = \frac{\sum_{p \in P_{w_3}} x_p}{30},$$

where $w_1 = (1, R_1)$, $w_2 = (1, R_2)$, and $w_3 = (1, R_3)$.
The penalties were:

$$\lambda_1^- = 50, \quad \lambda_1^+ = 0; \quad \lambda_2^- = 50, \quad \lambda_2^+ = 0; \quad \lambda_3^- = 50, \quad \lambda_3^+ = 0.$$

The capacities associated with the three outsourcing links were:

$$\bar{u}_{18} = 5, \quad \bar{u}_{19} = 10, \quad \bar{u}_{20} = 5.$$

We set $\bar{u}_a = 0$ for all links $a \in L^1$.

Table 1: Total Cost Functions and Solution for Example 5

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.34	1.34	2.34	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	2.47	2.47	3.47	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	2.05	2.05	3.05	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.61	0.61	1.61	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.73	0.73	1.73	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	0.83	0.83	1.83	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	1.64	1.64	2.64	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	1.67	1.67	2.67	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.37	0.37	1.37	–
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	3.11	3.11	4.11	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	2.75	2.75	3.75	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.04	0.04	1.04	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.00	0.00	0.45	–

Table 2: Total Cost Functions and Solution for Example 5 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	3.07	3.07	4.07	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.45	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.00	0.00	0.45	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	2.75	2.75	3.75	–
18	$10f_{18}$	–	5.00	–	–	14.77
19	$12f_{19}$	–	10.00	–	–	13.00
20	$15f_{20}$	–	5.00	–	–	16.96

Note that the optimal supply chain network design for Example 5 is, hence, as the initial topology but with links 13, 15, and 16 removed since those links have zero capacities and associated flows. Note that the organization took advantage of outsourcing to the full capacity available.

Example 6

Example 6 had the identical data to that in Example 5 except that we now assumed that the organization had capacities on its supply chain network activities where $\bar{u}_a = 10$, for all $a \in L^1$.

Table 3: Total Cost Functions and Solution for Example 6

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.84	0.00	0.00	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	4.51	0.00	0.00	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	3.85	0.00	0.00	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.88	0.00	0.00	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.97	0.00	0.00	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	1.40	0.00	0.00	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	3.11	0.00	0.00	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	3.47	0.00	0.00	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.38	0.00	0.00	–

Table 4: Total Cost Functions and Solution for Example 6 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	5.75	0.00	0.00	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	4.46	0.00	0.00	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.82	0.00	0.00	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.52	0.00	0.00	–
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	4.41	0.00	0.00	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.00	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.05	0.00	0.00	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.41	0.00	0.00	–
18	$10f_{18}$	–	5.00	–	–	10.89
19	$12f_{19}$	–	10.00	–	–	11.59
20	$15f_{20}$	–	5.00	–	–	11.96

Note that links 13 and 16 now have positive associated flows although at very low levels.

Example 7

Example 7 had the same data as Example 6 except that we changed the probability distributions so that we now had:

$$P_1(\sum_{p\in P_{w_1}} x_p) = \frac{\sum_{p\in P_{w_1}} x_p}{110},$$

$$P_2(\sum_{p\in P_{w_2}} x_p) = \frac{\sum_{p\in P_{w_2}} x_p}{120},$$

$$P_3(\sum_{p\in P_{w_3}} x_p) = \frac{\sum_{p\in P_{w_3}} x_p}{130}.$$

Table 5: Total Cost Functions and Solution for Example 7

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	4.23	0.00	0.00	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	9.06	0.00	0.00	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	8.61	0.00	0.00	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	2.05	0.00	0.00	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	2.18	0.00	0.00	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	3.28	0.00	0.00	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	5.77	0.00	0.00	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	7.01	0.00	0.00	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	1.61	0.00	0.00	–
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	12.34	2.34	3.34	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	9.56	0.00	0.00	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	5.82	0.00	0.00	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	2.38	0.00	0.00	–

Table 6: Total Cost Functions and Solution for Example 7 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	4.14	0.00	0.00	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	2.09	0.00	0.00	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	2.75	0.00	0.00	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.72	0.00	0.00	–
18	$10f_{18}$	–	5.00	–	–	34.13
19	$12f_{19}$	–	10.00	–	–	31.70
20	$15f_{20}$	–	5.00	–	–	29.66

The optimal supply chain network design for Example 7 has the initial topology since there are now positive flows on all the links. It is also interesting to note that there is a significant increase in production volumes by the organization at its manufacturing plants.

Summary and Conclusions

Summary and Conclusions

- We discussed a variety of network design approaches.
- We developed an integrated framework for the design of supply chain networks for critical products with outsourcing.
- The model utilizes cost minimization within a system-optimization perspective as the primary objective and captures rigorously the uncertainty associated with the demand for critical products at the various demand points.
- The supply chain network design model allows for the investment of enhanced link capacities and the investigation of whether the product should be outsourced or not.
- The framework can be applied in numerous situations in which the goal is to produce and deliver a critical product at minimal cost so as to satisfy the demand at various demand points, as closely as possible, given associated penalties for under-supply (and, if also relevant, for over-supply, which we expect to be lower than the former).

Thank You!

For more information, see: http://supernet.som.umass.edu