Linear Programming and Environmental Quality

Trisha Woolley
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Professor Nagurney
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Topics

- Introduction
- Background
- Air
- Land
- Water

Introduction

- "The United States spends more than 2% of its gross domestic product on pollution control, and this is more than any other country (Greenberg, 1995)."

- U.S. Environmental Protection Agency (http://www.epa.gov/)
  - Air quality – outdoor (e.j. SO2 emissions which causes acid rain), indoor (e.j. tobacco smoke and radon) and greenhouse gases emitted (e.j. CO2 emissions), for example, by automobiles, power generation, and industry activities

  - Land Quality – land cover (the six major classes, forestland, grassland, shrubland, developed land, agricultural land, and other, have equilibrium ecological effects), land use (purpose to which a unit of land is being used; to differentiate from land cover, for example, "a unit of land designated for use as timberland may appear identical to an adjacent unit of protected forestland"), chemicals (e.j. fertilizers), contaminated land, and waste (e.j. garbage)

  - Water Quality – water purification models to address pollution in fresh surface water, groundwater, wetlands, coastal water, and drinking water quality, recreation in and on water, consumption of fish and shellfish, and sewage treatment models.

- Lynn, Logan, and Charnes, in 1962, developed the first linear program to control the quality of the environment, which controlled wastewater treatment plant design and minimize the cost of sewage treatment. (Greenberg, 1995).

Table I
Journals Cited in this Survey for Air Quality Control

Journal	Number	Earliest	Newest
American Economic Review	1	1974 (Tietenberg)	
Applied Mathematical Modeling	1	1993 (Wanatabe and Ellis)	
ASCE Journal of Environmental Engineering	5	1974 (Darby et al.)	1994 (Ellis and Bowman)
Atmospheric Environment	7	1972 (Seinfeld)	1992 (Trujillo-Ventura & Ellis)
Computers and Operations Research	1	1993 (Wanatabe and Ellis)	
Econometrica	1	1971 (Kohn)	
Energy Research	1	1981 (Fishbone and Abilock)	
Engineering Optimization	1	1992 (Ellis)	
Environment and Planning[a]	1	1972 (Gorr et al.)	
Environmental Science and Technology	2	1974 (Trijonis)	1988 (Ellis)
European Journal of Operational Research	1	1990 (Ellis)	
Geographical Analysis	1	1986 (Guldmann)	
Journal of the Air Pollution Control Association	3	1977 (Ott)	1985 (Morrison and Rubin)
Journal of Environmental Economics and Management	6	1976 (Atkinson and Lewis)	1993 (Welsch)
Journal of Resource Management and Technology	1	1993 (Chang et al.)	
Management Science	2	1971 (Kohn)	1976 (Carbone and Sweigart)
Operations Research	2	1972 (Blumstein et al.)	1973 (Kohn)
Papers of the Regional Science Association	1	1974 (Werczberger)	
Socio-Economic Planning Sciences	4	1971 (Seinfeld and Kyan)	1978 (Guldmann)
The Energy Journal	1	1992 (Peck and Teisberg)	
Water Resources Bulletin	1	1992 (Okada and Mikami)	

[a]In 1974, this split into parts A and B (part B is planning and design, which has no mathematical programming models for environmental quality control).

Table III
Journals Cited in this Survey for Water Quality Control

Journal	Number	Earliest	Newest
Advances in Water Resources	1	1986 (Ahlfeld et al.)	
Annals of Operations Research	1	1991 (Pinter)	
ASCE Journal of Environmental Engineering	14	1977 (Grady)	1993 (Mhaisalkar et al.)
ASCE Journal of Hydraulics	3	1974 (Aguado and Remson)	1976 (Futagami et al.)
ASCE Journal of Sanitary Engineering	8	1966 (Goodman and Dobbins)	1971 (Bishop and Hendricks)
ASCE Journal of Water Resources Planning and Management	7	1986 (Tung)	1994 (Chan)
Biotechnology and Bioengineering	1	1974 (Middleton and Lawrence)	
Canadian Operational Research Society Journal	1	1968 (Clough and Bayer)	
CRC Critical Reviews in Environmental Control	1	1977 (Tyteca et al)	
Ground Water	1	1974 (Remson et al)	
IEEE Transactions on Systems Science and Cybernetics	1	1970 (Dysart and Hines)	
International Journal of Water Resource Development	1	1983 (Lohani and Lee)	
Journal of Environmental Economics and Management	2	1974 (Russell and Vaughan)	1976 (Herzog)
Journal of the Water Pollution Control Federation[a]	12	1962 (Lynn et al.)	1976 (Middleton and Lawrence)
Management Science	3	1967 (Loucks et al.)	1975 (Ecker)
Mathematical Programming	1	1990 (Gorelick)	
Operations Research	3	1978 (Jarvis et al.)	1982 (Fiacco and Ghaemi)
Water Resources Bulletin	9	1970 (Keegan and Leeds)	1984 (Colarullo et al.)
Water Research	1	1971 (Fan et al.)	
Water Resources Research	35	1967 (Johnson)	1993 (Whiffen and Shoemaker)

[a]In 1989, this split into *Water Environment & Technology* and *Research Journal of the Water Pollution Control Federation* (also called *Water Environmental Research*).

Table II
Journals Cited in this Survey for Land Quality Control

Journal	Number	Earliest	Newest
American Journal of Agricultural Economics	3	1974 (Hueth and Regev)	1977 (Taylor and Frohberg)
Canadian Journal of Economics	1	1972 (Plourde)	
Journal of Environmental Economics and Management	1	1990 (Stavins)	
Journal of Resource Management and Technology	1	1993 (Chang et al.)	
Journal of Soil and Water Conservation	1	1978 (Taylor et al.)	
Land Economics	1	1979 (Seitz et al.)	
Natural Resources Journal	1	1970 (Edwards et al.)	
Transportation Science	1	1991 (ReVelle et al.)	

Distribution of Publications

	Air					Land					Water				
	LP	MIP	NLP	DP	Total	LP	MIP	NLP	DP	Total	LP	MIP	NLP	DP	Total
1962–69	1	0	0	0	1	0	0	0	0	0	13	2	4	3	22
1970–79	13	5	17	0	35	7	3	2	0	12	24	7	21	12	64
1980–89	17	1	8	0	26	2	1	1	0	4	7	7	22	6	42
1990–94	8	1	17	0	26	7	2	5	0	14	5	3	8	10	26
Total	39	7	42	0	88	16	6	8	0	30	49	19	55	31	154

(LP = Linear Programming, MIP = Mixed Integer Programming, NLP = Nonlinear Programming, DP = Dynamic Programming)

Linear Programming	Mixed Integer Programming	Nonlinear Programming	Dynamic Programming
Linear programming "tends to be the mathematical programming model of choice when first addressing a problem with many decision variables and relations." (Greenberg, 1995)	Used as "an extension of LP models, to represent capacity expansion (e.g. treatment plants) or location decisions (e.g. wells), and combinatorial optimization problems (e.g. finding routes for complex transport problems) (Greenberg,	"Used to improve a model's validity, or accuracy. One source of nonlinearity is the cost function…and the approximation of the differential equations that describe hydraulic and aerodynamic phenomena" (Greenberg, 1995)	Solving sequential decision problems… "used for computational efficiency when the state space can be defined appropriately". Mostly used for water quality control. (Greenberg, 1995)

Early Air Quality Models

- Early Air Quality Models focused on the impact of emissions within particular airsheds (or receptor points) while later models were aggregate and dealt with global issues (e.j. greenhouse effect).

- First Linear Program to control air pollution was developed in 1968 by Teller, which minimized cost with decision variables being tons of each of two types of fuels used at different sources of which each emits pollutants at known rates. Constraints limit the amount of each pollutant emitted and require energy demand to be met.

- This paper was then executed for the Environmental Protection Agency in 1972 by Chilton et al. and also Gass.

- Kohn applied welfare economics to air sheds in an LP model developed for his PhD thesis in 1969. “He generates a set of alternative air quality levels that have the same total cost. The frontier tradeoff is compared to a social indifference curve, based on medical considerations, for the St. Louis airshed.” (Greenberg, 1995)

Some Additional Air Quality Research

Blumstein, A., R.G. Cassidy, W.L. Gorr and A.S. Walters. 1972. Optional Specifications of Air-Pollution-Emission Regulations Including Reliability Requirements. This LP incorporates reliability with random breakdowns of pollution control devices.

DINKEL, J. J., G. B. KLEINDORFER, G. A. KOCHENBERGER AND S. N. WONG. 1976. Environmental Inspection Routes and the Constrained Travelling Salesman Problem. *Comput. and Opns. Res.* 3(4), 269–283.

The problem is to find a route for an inspector to visit plants and return home in the least time. It is a traveling salesman problem with an added time constraint. The authors discuss their experience with heuristics and with data acquisition.

Batterman, S.A. 1989. Selection of Receptor Sites for Optimized Acid Rain Control Strategies. ASCE J. Environmental Engineering. 115(5), 1046-1058. Uses LP to select sites for monitoring acid rain by deciding if a receptor point is "inactive' versus "influential".

Air Quality Model

Sulfur Emissions Taxes and Coal Resources

Alan Schlottmann; Lawrence Abrams

The Review of Economics and Statistics, Vol. 59, No. 1. (Feb., 1977), pp. 50-55.

- Determine the most efficient (minimum cost) network of coal extraction, distribution and use for production in steam electric generating plants while meeting sulfur emissions requirements.
- Coal is burned which creates steam that runs through a turbine to generate electricity
- Motivation: “The four largest power regions in the U.S., East North Central, South Atlantic, East South Central, and the Middle Atlantic – are responsible for 86% of the coal used in electric generation”.
- Clean Air Act of 1970, its requirements, and if they will be met.
- Used 52 demand regions, 23 supply districts, 267 electric utility companies with 744 plants, 2 types of coal (low sulfur and high sulfur)

Air Quality Model

$$\text{minimize } \sum_i \sum_j \sum_k \sum_l \sum_m (C_{ijm} + t_{il}) X_{ijklm} + \phi(b_{ijkm} S_{ijkm}) X_{ijklm}$$

Objective: Minimize Total Cost of extraction, distribution, and sulfur tax imposed. This includes the cost to extract the coal times plus shipment costs, times the tons of coal extracted, plus the tax times the amount of coal extracted times the energy value times the average emitted sulfur per million Btu's.

$$\sum_i \sum_j \sum_k \sum_m b_{ijkm} X_{ijklm} \geqslant G_l \text{ for each } l$$

$$l = 1, \ldots, 52$$

$$G_l = f(P_c, P_o, P_g, D_l(t))$$

$$D_l(t) = g(P_e, Y, D_l(t-1))$$

Meet demand at each market. The demand at each demand region will be met as it is greater than or equal to the multiplication of the amount of coal extracted that is delivered to each demand region times its associated energy value

$$\sum_l X_{ijklm} \leqslant K_{ijkm} \text{ for each } i, j, k, m$$

$$i = 1, \ldots, 23; \quad j = 1, 2;$$
$$k = 1, \ldots, 5; \quad m = 1, 2$$

Capacity constraint: the amount of coal extracted cannot exceed the physical capacity at each district .

$$\sum_i \sum_j \sum_k \sum_m S_{ijklm} X_{ijklm} \leqslant S'_l \text{ for each } l$$

$$X_{ijklm} \geqslant 0 \text{ for all } i, j, k, l, m$$

Sulfur Emissions constraint: average emitted sulfur per million Btu's times the amount of coal extracted is less than or equal to the allowable sulfur that can be emitted at each demand region

Conclusions of Paper

TABLE 1.— REGIONAL SHIPMENTS OF HIGH SULFUR COAL WITH A SULFUR EMISSIONS TAX (millions of tons)

Region[a]	Sulfur Emissions Tax ($/lb.)[b] 0	.05	.10	.15	.20
Northern Appalachia	61.46	46.78	39.37	37.37	36.99
Central Appalachia	2.93	2.88	2.08	1.67	0
Southern Appalachia	2.38	2.38	1.95	1.73	1.54
Total, Appalachia	66.77	62.04	43.40	40.77	38.53
Midwest	125.84	105.83	72.76	56.08	55.37
West	0	0	0	0	0
United States	192.61	167.87	116.16	96.85	93.90

- **Except for the West (which only ships low sulfur coal), shipments for all regions of high sulfur coal decreased as the tax rate was increased.**
- **At a tax rate of $0.15 per pound of emitted sulfur, the use of high sulfur coal is reduced by 50%**

Conclusions of Paper

TABLE 2. — THE REGIONAL EFFECTS OF A SULFUR EMISSIONS TAX ON COAL EXTRACTION (millions of tons)

Region	Sulfur Emissions Tax ($/lb.)				
	0	.05	.10	.15	.20
Northern Appalachia	165.7	151.3	142.9	128.8	113.2
Central Appalachia	122.1	129.6	129.3	128.4	126.5
Southern Appalachia	20.1	20.1	19.7	19.4	19.3
Total, Appalachia	307.9	301.0	291.9	276.6	259.0
Midwest	150.0	135.1	93.3	88.8	77.7
West	48.9	55.7	73.8	83.5	89.8
United States	506.8	491.8	459.6	448.9	426.5

- **Except for the West, total coal extraction (low and high sulfur) for all other regions decreases as the tax is increased.**

- **In the Midwest, the switch to low sulfur coal, is not economically efficient due to transportation cost disadvantages. However, the West has an advantage over the transportation cost to ship low sulfur coal, so total coal extracted increased.**

- **Specifically, Midwestern production fell by 56.7 million tons while the Western shipments increased by 24.9 million tons.**

Conclusions of Paper

TABLE 3.—TOTAL EFFECTS OF THE SULFUR EMISSIONS TAX

	Sulfur Emissions Tax ($/lb.)				
	.00	.05	.10	.15	.20
Total emissions (million tons)	11.41	10.02	8.45	7.45	6.65
Benefits of abatement (billions of dollars)	—	1.05	2.24	2.99	3.60
Increased oil use (billions of dollars)	—	.63	2.01	2.43	3.40
Total steam coal costs (billions of dollars)	5.11	5.04	4.91	4.82	4.70

Note: Comparisons at the alternative emissions tax levels are relative to the no tax level.

- **Sulfur emissions decreased as the tax rate increases.**
- **For example, with a $0.20 tax rate, emissions are reduced by 42% to 6.65 million tons as compared to when a tax is not imposed.**
- **The cost to ship the coal slightly increases while overall shipments decline.**

Early Land Quality Models

- Used to model and understand the effects of, for example, controls on pesticides and soil erosion, land use, storage of crops and livestock growth.

- First LP developed by Edwards, Langham and Headley in 1970, applying welfare economics to the agricultural sector in Dade County, Florida. "The decision variables are acres of land allocated to each of several crops," constraints include the level of chemical treatment of each crop, and the "objective is to maximize a net benefit function, which includes damage caused by pesticide residues."

- In 1977, Taylor and Frohberg applied a LP to the corn belt (Midwestern U.S.) to analyze pollution controls such as bans on herbicides, bans on insecticides, soil erosion limits, nitrogen restriction, and soil erosion taxes.

Some Additional Land Quality Research

Russell, C. S., and W. J. Vaughan. 1974. A Linear Programming Model of Residuals Management for Integrated Iron and Steel Production. *J. Environ. Econ. and Mgmt.* **1,** 17–42.

This applies the LP described by Russell (1971, 1973) to consider how waste discharges from iron and steel production into a stream or into the air are affected by effluent taxes. Among their conclusions, they show that continuous casting results in less water pollution, and an increase in the price of scrap iron results in less scrap at steel mills, which increases water pollution.

Wade, J. C., and E. O. Heady. 1977. Controlling Nonpoint Sediment Sources eith Cropland Management: A National Economic Assessment. *Am. J. Agric. Econ.* **59**(1), 13–24.

This is an LP concerned with adjustments in crop production to achieve sediment quality goals.

Seitz, W. D., C. R. Taylor, R. G. F. Spitze, C. Osteen and M. C. Nelson. 1979. Economic Aspects of Soil Erosion. *Land Econ.* **55**(1), 28–42.

This uses an LP model whose objective is to maximize the total producer and consumer surplus in the corn and soybean market. The activities include land allocations to crops having different characteristics for soil erosion. The basic model is short term, but these authors also applied it to analyze long-term effects.

Land Quality Models

Mathematical Programming Models for Environmental Quality Control

Harvey J. Greenberg

Operations Research, Vol. 43, No. 4. (Jul. - Aug., 1995), pp. 578-622.

- Generic land use model that can be used to evaluate environmental impact and soil erosion.
- Based on collection of papers by Heady and Vocke (1992).

Objective Function: minimize the cost to produce and transport each product

Regions

i = producers; j = markets;

Classes

k = methods of production (e.g., tilling);
s = soils;
h = chemicals (including pesticides and fertilizers);
p = products (crops and livestock commodities).

Production

X_{ipk} allocates land in region i to make product p by method k;

Distribution

T_{pij} transports product p from region i to market j.

Equations

Cost

$$Z = \sum_{i,p,k} (CX)_{ipk} X_{ipk} + \sum_{p,i,j} (CT_{pij}) T_{pij};$$

$(CX)_{ipk}$ = production cost, which could include taxes;
$(CT)_{pij}$ = the transportation cost, which could include taxes.

Land Quality Models

Land use for each producer: sum of all products and all methods of production for each.

Conservation of Flow: ship all that produce

Meet Demand: shipments must be greater than or equal to demand at each market.

Soil Damage: is equal to that caused by each producer to produce each product.

Contamination level: is equal to chemical used times amount produced of each product.

Land Use

$$L_i = \sum_{p,k} X_{ipk}.$$

Balance

$$\sum_k R_{ipk} X_{ipk} - \sum_j T_{pij} = 0,$$

R_{ipk} = the rate of product p produced per acre in region i using method k.

Demand

$$\sum_i T_{pij} \geq d_{pj},$$

d_{pj} = the demand for product p in market j.

Damage

$$D_i = \sum_{p,s,k} a_{psk} \alpha_{is} X_{ipk};$$

a_{psk} = the rate of soil damage when producing p with soil class s;

α_{is} = 1 if region i has soil class s (else, $\alpha_{is} = 0$);

$$C_{ih} = \sum_{p,k} b_{phk} X_{ipk};$$

b_{phk} = chemical h used by, or produced from, method k to make p ($b_{phk} < 0$ if used, such as a pesticide; $b_{phk} > 0$ if produced, such as nitrogen in cow manure, which can then be used as fertilizer for a crop).

Land Quality Models

$L_i \leq$ available land in producer region i.

Available Land

$D_i \leq$ soil loss limit.

Maximum Soil Loss Permitted

$C_{ih} \leq$ contamination level.

Maximum contamination level permitted

Early Water Quality Models

- First LP, Lynn, Logan, and Charnes in 1962 to control wastewater treatment plant design and minimize the cost of sewage treatment.

- In 1964 Thomann and Sobel developed a LP to control stream pollution.

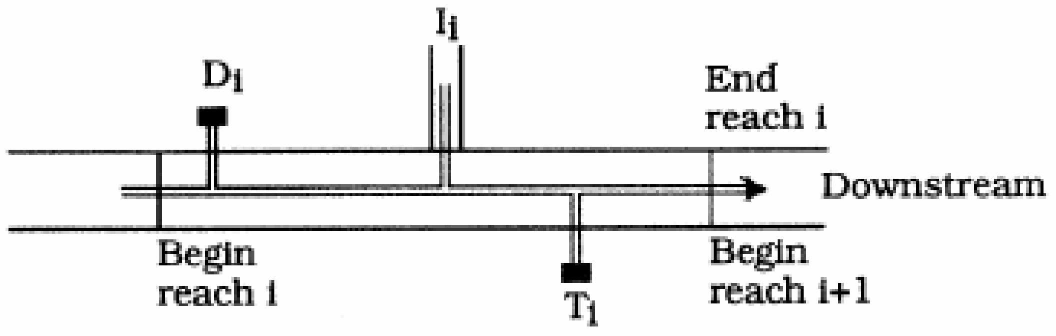

Figure 1. Flows in a stream.

Some Additional Water Quality Research

LIEBMAN, J. C. 1968. A Branch-and-Bound Algorithm for Minimizing the Costs of Waste Treatment, Subject to Equity Constraints. Number 10 in IBM, 193–202.

Allowed pollution to flow upstream as well as downstream

AGUADO, E., I. REMSON, M. F. PIKUL AND W. A. THOMAS. 1974. Optimal Pumping for Aquifer Dewatering. *ASCE J. Hydraul.* **100**(7), 869–877.

Using a finite difference approximation of the Streeter–Phelps equations, this is an LP to determine the number of wells, their locations and pumping rates to minimize cost.

GORELICK, S. M., I. REMSON AND R. W. COTTLE. 1979. Management Model of a Groundwater System With a Transient Pollutant Source. *Water Resour. Res.* **15**(5), 1243–1249.

The model is a linear program, and the paper uses parametric programming to show how this applies to such questions as: What river concentration would be permitted if the most restrictive local groundwater quality limit were removed?

HUDAK, P. F., AND H. A. LOAICIGA. 1993. An Optimization Method for Monitoring Network Design in Multilayered Groundwater Flow Systems. *Water Resour. Res.* **29**(8), 2835–2845.

The problem is to locate wells for monitoring groundwater quality in a region that contains a contaminant. A network is defined by discretizing the region, calling each location a node.

Water Quality Models

Linear Programming Models for Water Pollution Control

Daniel P. Loucks; Charles S. Revelle; Walter R. Lynn

Management Science, Vol. 14, No. 4, Application Series. (Dec., 1967), pp. B166-B181.

"Model presented can be used to determine the minimum total cost of any particular dissolved oxygen control policy in a river basin".

- **Organic material is a large portion of the waste released into a stream, which organisms feed on.**

- **"Dissolved oxygen contained in the stream is withdrawn by these organisms in the process of utilizing these wastes". Thus, waste concentration is measured by its biochemical oxygen demand (BOD).**

- **"Fish and other aquatic animals and plants require certain minimum concentrations of dissolved oxygen if they are to survive in the stream".**

- **Thus a certain level of organics must be removed to ensure a minimum allowable dissolved oxygen concentration in each section (or reach) of the stream.**

Model

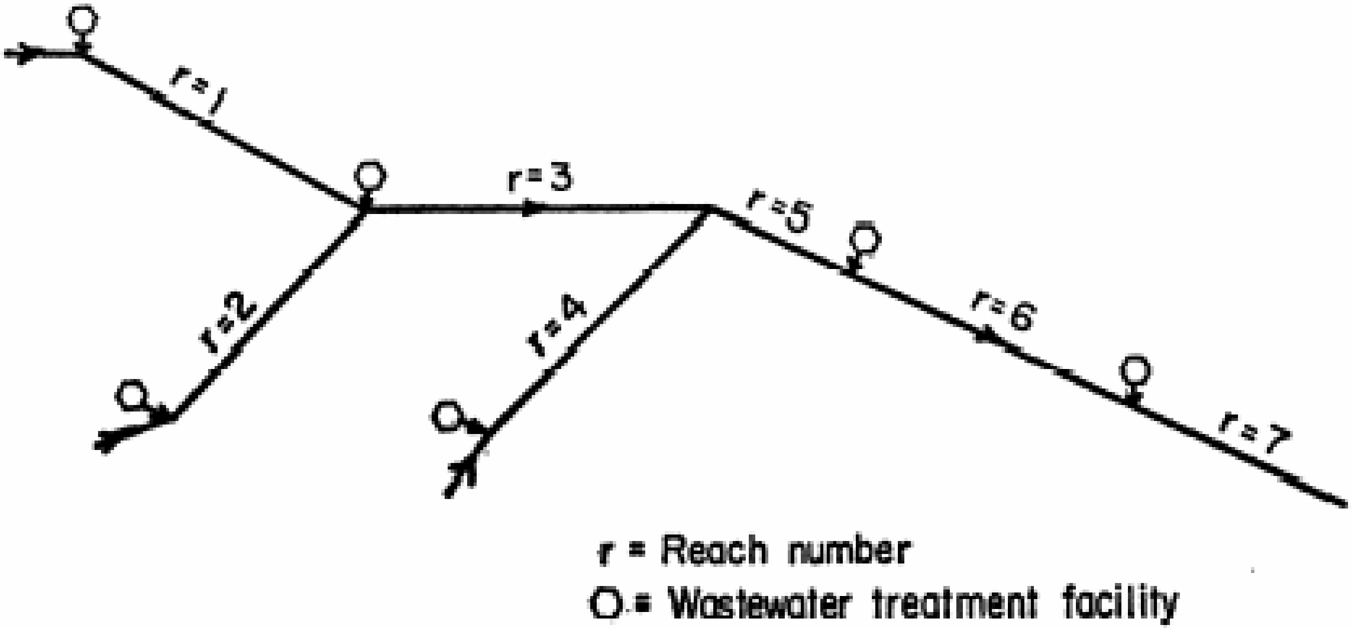

FIGURE 3. Hypothetical river basin

$$\text{MINIMIZE} \quad \sum_r (\Psi_r - C_r B W_r).$$

Total cost of wastewater treatment: cost of removing all pollutants less those pollutants that need not be removed

$$QS_r = QS_{r-1} + QT_r + QW_r \,.$$

Total flow at each reach point is the sum of flow from previous reach, the entering tributary flow and wastewater inflow.

Model

$$BB_r = \frac{BE_{r-1}QS_{r-1} + BT_rQT_r + BW_rQW_r}{QS_r}.$$

Waste concentration (as measured by biochemical oxygen demand) at beginning of each reach point is the sum of BOD concentration from previous period, in the tributary, and in the wastewater, times their respective flows, divided by the total flow.

$$CB_r = \frac{CE_{r-1}QS_{r-1} + CT_rQT_r + CW_rQW_r}{QS_r}.$$

Dissolved oxygen concentration at beginning of each reach point is the sum of the concentrations from previous period, tributary, and wastewater, divided by the total flow.

$$DB_r = CS_r - CB_r .$$

Dissolved oxygen deficit at beginning of each reach is the difference between the saturation concentration and the initial dissolved oxygen concentration

$$BE_r = \lambda_r BB_r + \mu_r .$$

Waste concentration (as measured by biochemical oxygen demand) at the end of each reach point

$$DE_r = \alpha_r DB_r + \gamma_r BB_r + \rho_r .$$

Dissolved oxygen deficit at the end of each reach

Model

$$CE_r = CS_r - DE_r \, .$$

Dissolved oxygen concentration at the end of each reach

$$D_{rt} \leqq D_r^{\max} \quad \text{for various } t: 0 \leqq t \leqq T_r \, .$$

Dissolved oxygen deficit constrained to be less than or equal to the maximum allowable deficit for each point, t, along the reach.

$$BB_r \leqq f_r(DB_r).$$

BOD concentration that can exist at the beginning of each reach can not violate the standard is a function of the initial dissolved oxygen deficit.

$$BB_r \leqq \sigma_r + \phi_r DB_r \, .$$

Dissolved oxygen standard in which the BOD concentration that can exist at the beginning of each reach must be less than or equal to the maximum allowable BOD concentration.

$$0 \leqq BW_r \leqq BW_r^{\max}.$$

No more than the total amount of BOD available for release can be released into any reach.

Conclusions of Paper

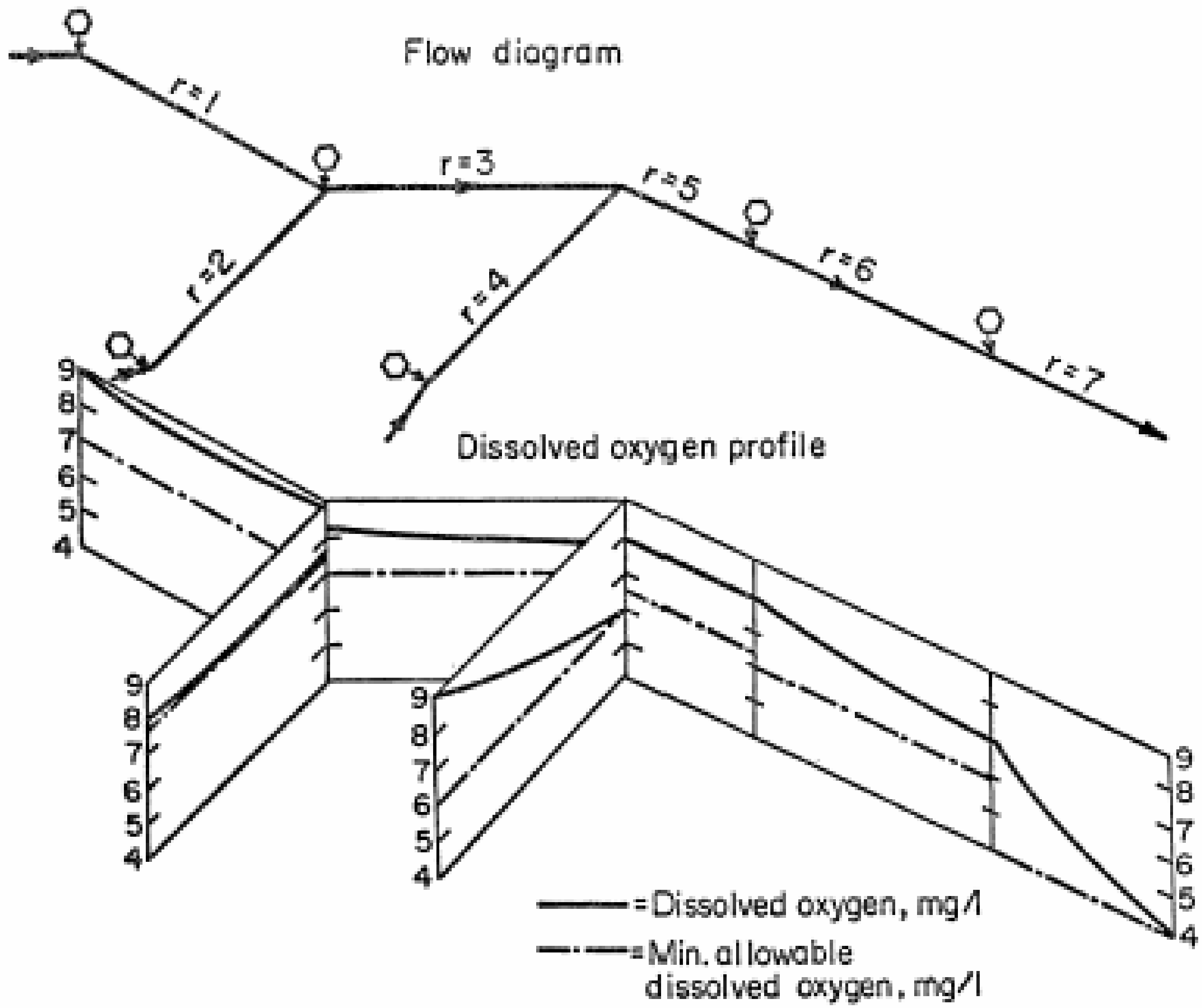

FIGURE 5. Dissolved oxygen profile for minimum cost solution

Conclusions of Paper

- "Model presented can be used to determine the minimum total cost associated with any particular set of minimum allowable dissolved oxygen concentrations in a river basin".

- "Model can be used to determine the sensitivity of both the cost and the actual minimum dissolved oxygen concentrations in each reach to changes in the minimum allowable concentration in any particular reach".

- "A change in the minimum allowable concentration in a single reach may or may not affect the oxygen profile in every other reach".

- A reach may be critical, in which the dissolved oxygen concentration will affect the total cost.

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Thank you!