Supply Chain Networks: Challenges and Opportunities from Analysis to Design

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Outline

- Background and Motivation
- Why User Behavior Must be Captured in Network Design
- Methodologies for Formulation, Analysis, and Computations
- An Empirical Application to Electric Power Supply Chains
- Network Design Through Mergers and Acquisitions
- A Challenging Network Design Problem and Model for Critical Needs with Outsourcing
- Applications to Vaccine Production and Emergencies
- Extensions to Perishable Products and Blood Supply Chains for the Red Cross
- What About Sustainability and Supply Chain Network Design?
- Summary, Conclusions, and Suggestions for Future Research

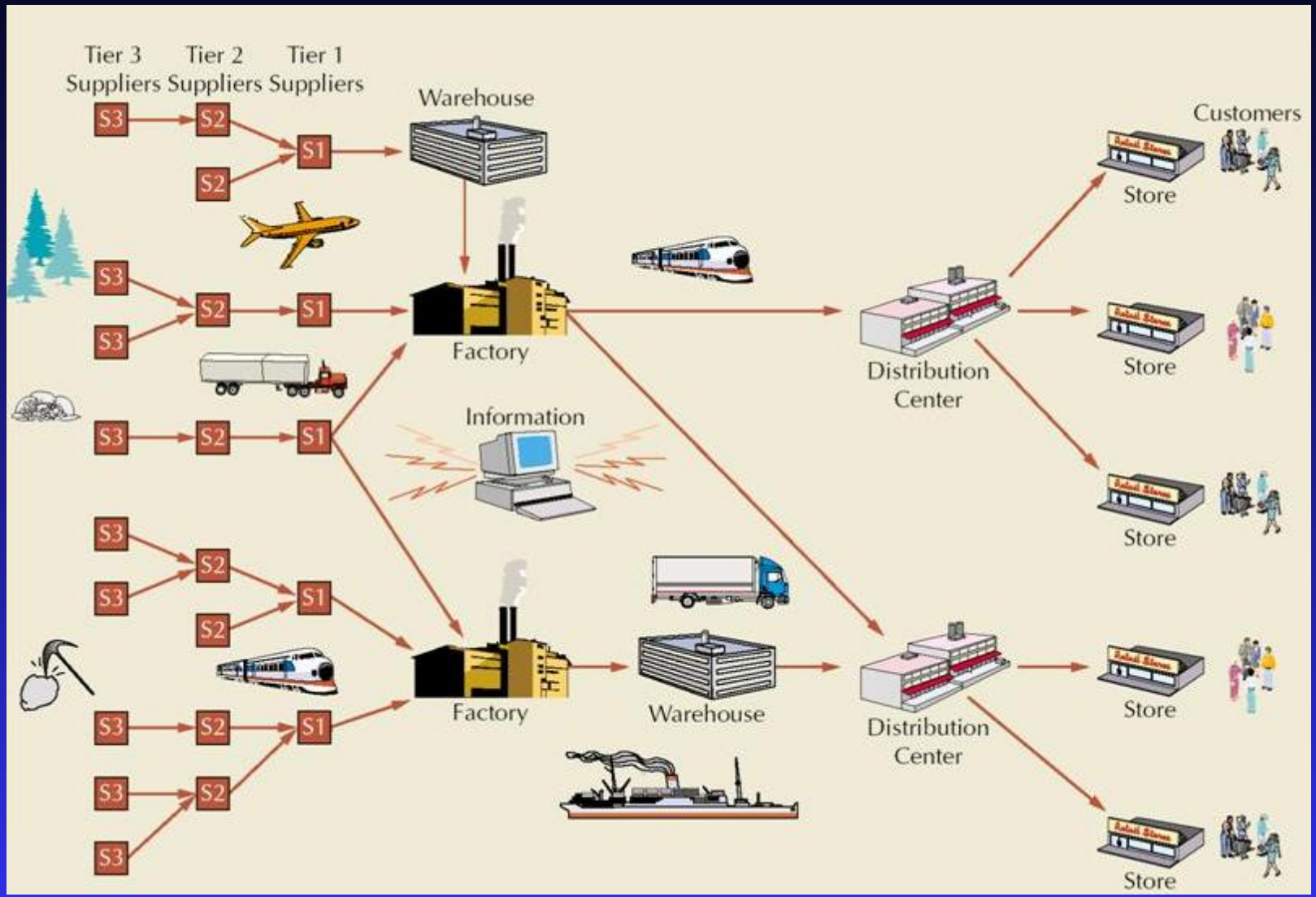
Background and Motivation

Supply chains are the *critical infrastructure and backbones* for the production, distribution, and consumption of goods as well as services in our globalized *Network Economy*.

Supply chains, in their most fundamental realization, *consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets*.

Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.

A General Supply Chain

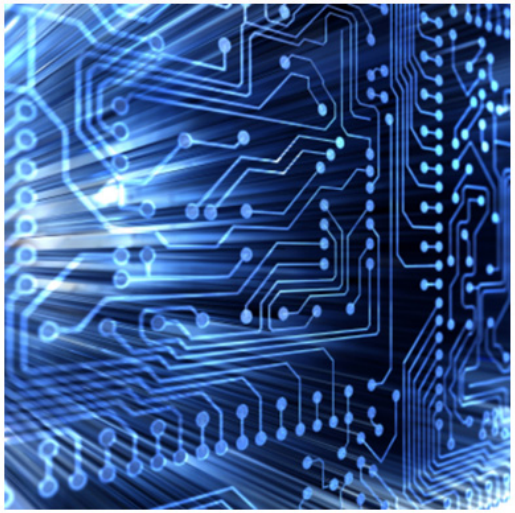


Examples of Supply Chains

- food and food products
- high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- humanitarian relief
- healthcare supply chains
- supply chains in nature.

Food Supply Chains

High Tech Products



Automotive Supply Chains

Energy Supply Chains

Clothing and Toys

Healthcare Supply Chains

Humanitarian Relief

Supply Chains in Nature

Supply chains may be characterized by *decentralized decision-making* associated with the different economic agents or by *centralized* decision-making.

Supply chains are, in fact, *Complex Network Systems*.

Hence, *any formalism that seeks to model supply chains and to provide quantifiable insights and measures must be a system-wide one and network-based.*

Indeed, such crucial issues as the stability and resiliency of supply chains, as well as their adaptability and responsiveness to events in *a global environment of increasing risk and uncertainty* can only be rigorously examined from the view of supply chains as network systems.

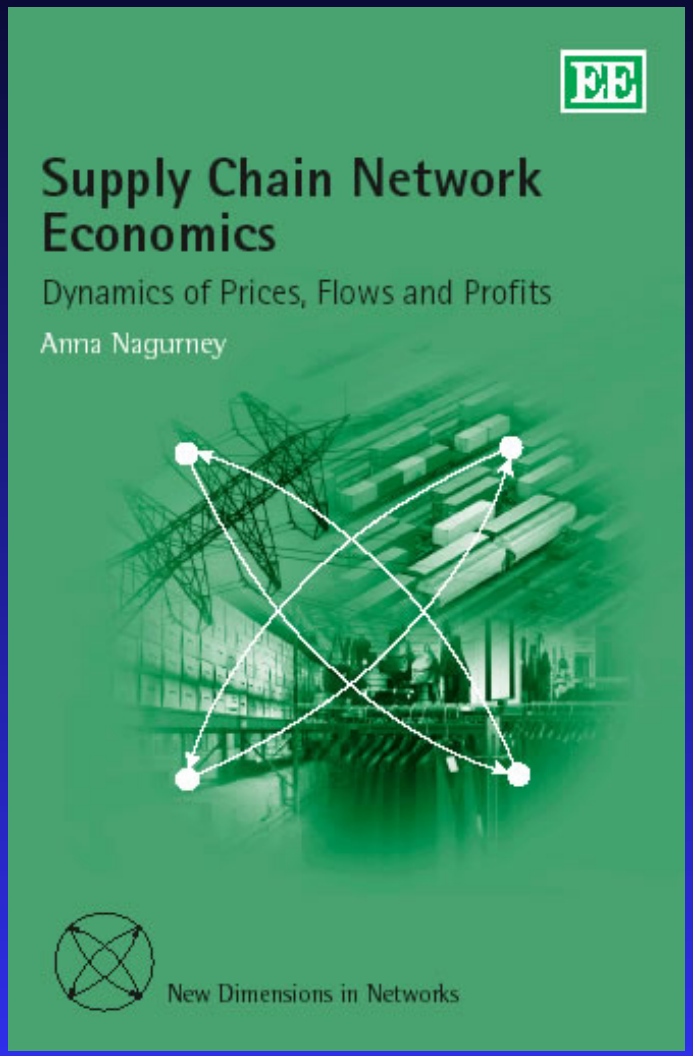
Characteristics of Supply Chains and Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *possibly conflicting criteria associated with optimization*;
- *interactions among the underlying networks themselves*, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Changes in the availability of supplies, price shocks, as well as disruptions to transportation modes or telecommunications may have negative effects and consequences that propagate throughout the supply chain.

On the other hand, *increases in demand for a product, entirely new demand markets, decreases in transaction costs, new suppliers, and even new modes of transaction, may provide new opportunities* for profit maximization for manufacturers, distributors, as well as retailers, and new linkages that were not previously possible.

Our Approach to Supply Chain Network Analysis and Design



Why User Behavior Must be Captured in Network Design

Supply Chain Network Design Must Capture the Behavior of Users

Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.

		User-Optimized
Decentralized	Selfish	U–O
vs. ⟺	vs. ⟺	vs.
Centralized	Unselfish	S–O
		System-Optimized

Flows are routed so as to minimize the total cost to society.

Two fundamental principles of travel behavior, due to Wardrop (1952), with terms coined by Dafermos and Sparrow (1969).

User-optimized (U-O) (network equilibrium) Problem – each user determines his/her cost minimizing route of travel between an origin/destination, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action (in the sense of Nash).

System-optimized (S-O) Problem – users are allocated among the routes so as to minimize the total cost in the system, where the total cost is equal to the sum over all the links of the link's user cost times its flow.

The U-O problems, under certain simplifying assumptions, possesses optimization reformulations. But now we can handle cost asymmetries, multiple modes of transport, and different classes of travelers, without such assumptions.

We Can State These Conditions Mathematically!

The U-O and S-O Conditions

Definition: U-O or Network Equilibrium – Fixed Demands

A path flow pattern x^*, with nonnegative path flows and O/D pair demand satisfaction, is said to be U-O or in equilibrium, if the following condition holds for each O/D pair $w \in W$ and each path $p \in P_w$:

$$C_p(x^*) \begin{cases} = \lambda_w, & \text{if} \quad x_p^* > 0, \\ \geq \lambda_w, & \text{if} \quad x_p^* = 0. \end{cases}$$

Definition: S-O Conditions

A path flow pattern x with nonnegative path flows and O/D pair demand satisfaction, is said to be S-O, if for each O/D pair $w \in W$ and each path $p \in P_w$:

$$\hat{C}'_p(x) \begin{cases} = \mu_w, & \text{if} \quad x_p > 0, \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p(x) = \sum_{a \in \mathcal{L}} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}$, and μ_w is a Lagrange multiplier.

The importance of behavior will now be illustrated through a famous example known as the Braess paradox which demonstrates what can happen under *U-O* as opposed to *S-O* behavior.

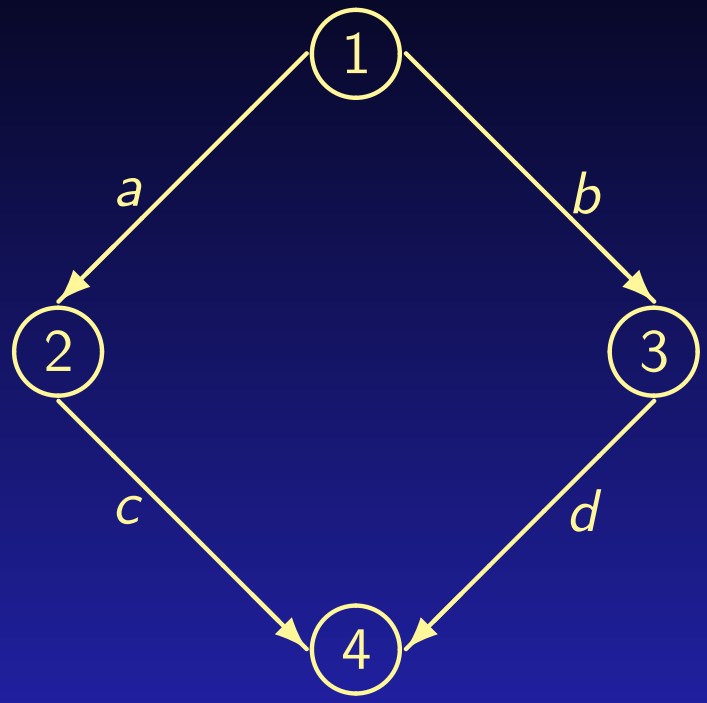
Although the paradox was presented in the context of transportation networks, it is relevant to other network systems in which decision-makers act in a noncooperative (competitive) manner.

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6,** the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

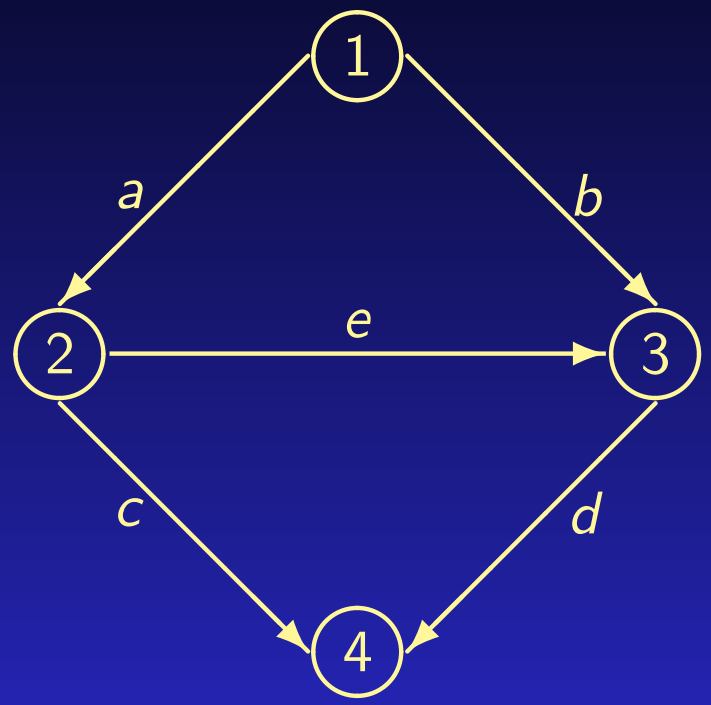
Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is
$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

The equilibrium path travel cost:
$C_{p_1} = C_{p_2} = C_{p_3} = 92$.

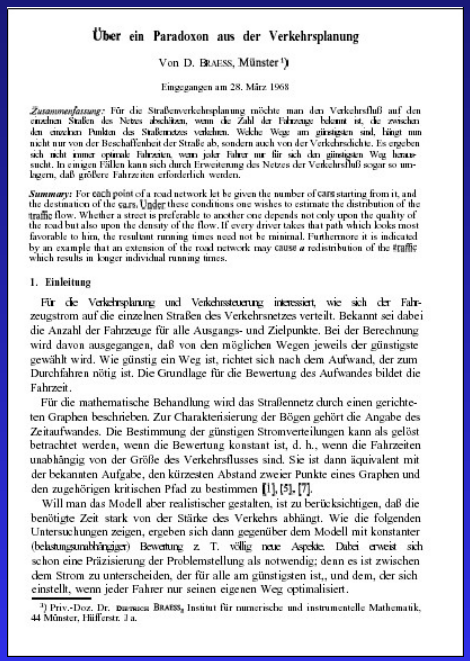


$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as:

"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) Transportation Science **39**, 446-450.



Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.

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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here morning shortest) path. See Beltrami (1968), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1964). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

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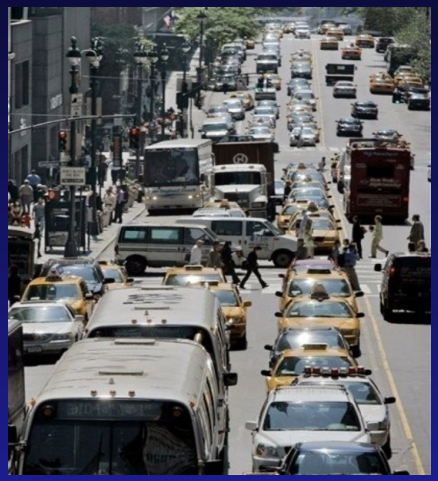
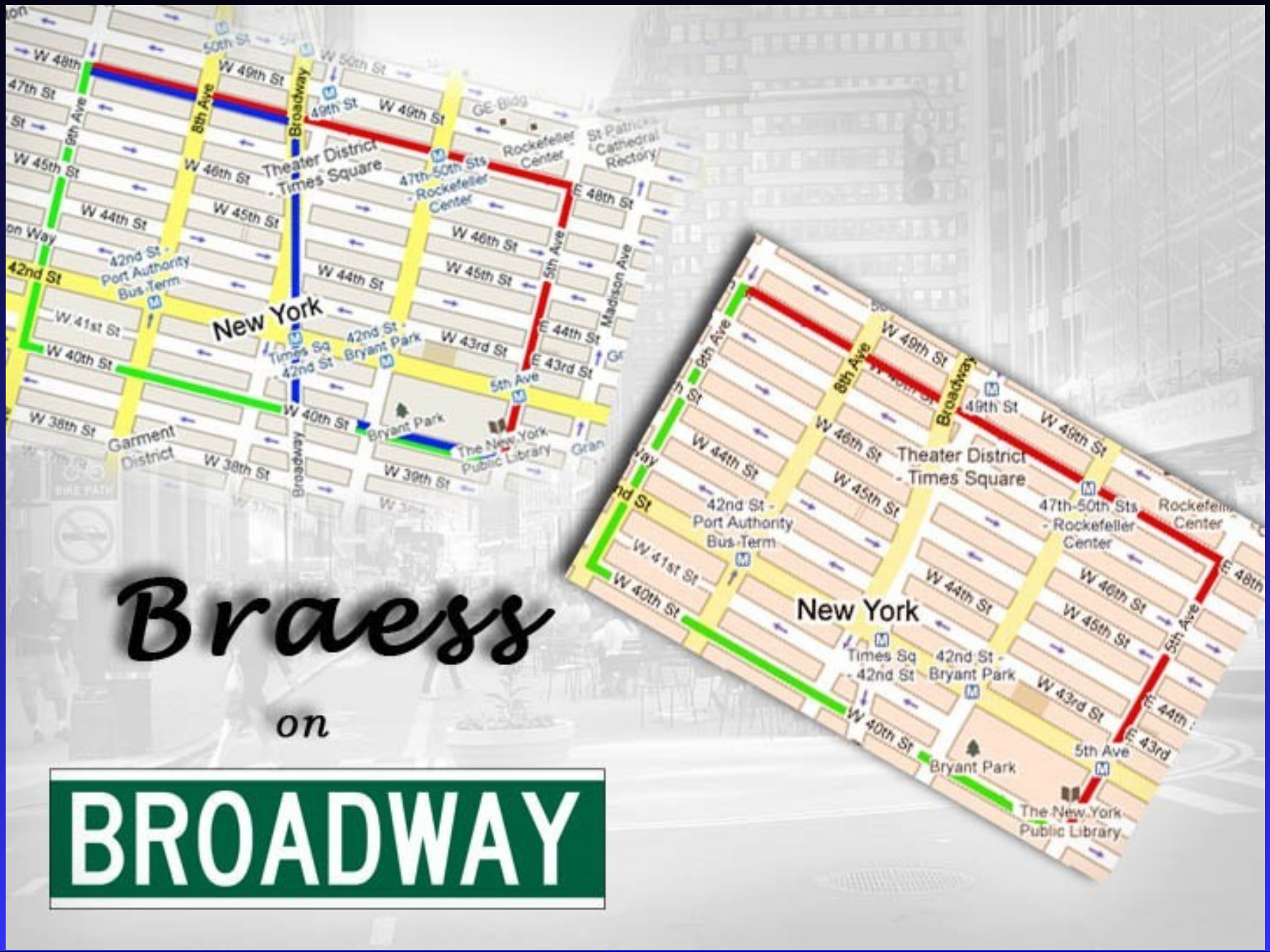
The Braess Paradox Around the World



1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42nd Street was closed and traffic flow improved.

2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.


Braess
on
BROADWAY
New York
Theater District - Times Square
47th-50th Sts - Rockefeller Center
42nd St - Port Authority Bus Term
42nd St - Bryant Park
Times Sq - 42nd St
Bryant Park
The New York Public Library
Garment District
Rockefeller Center
St Patricks Cathedral Rectory
W 48th
W 49th St
W 50th St
W 45th St
W 44th St
W 46th St
W 41st St
W 40th St
W 38th St
W 39th St
W 43rd St
E 48th St
E 44th St
E 43rd St
8th Ave
9th Ave
5th Ave
Broadway
Madison Ave

Interview on Broadway for *America Revealed* on March 15, 2011

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand, would not be used.

The Braess paradox never occurs in S-O networks.

Methodologies for Formulation, Analysis, and Computations

The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chain network problems.

Definition: The Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

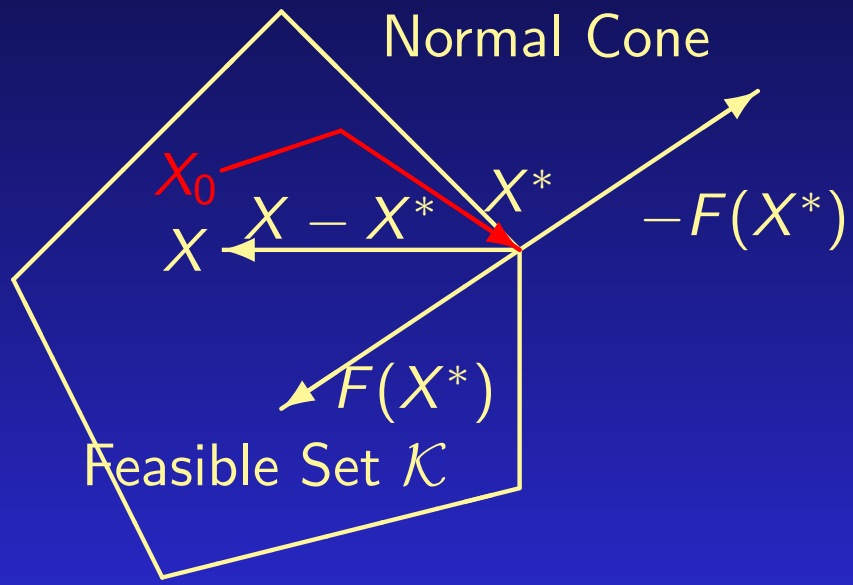
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney (1993), Nagurney and Zhang (1996))

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

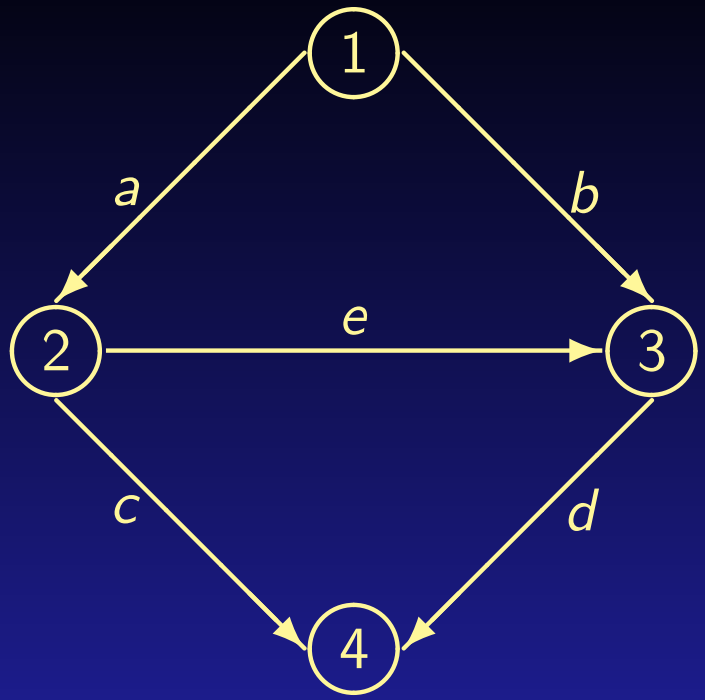
To model the *dynamic behavior of complex networks*, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in their book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even *neuroscience* (Girard et al. (2008)).



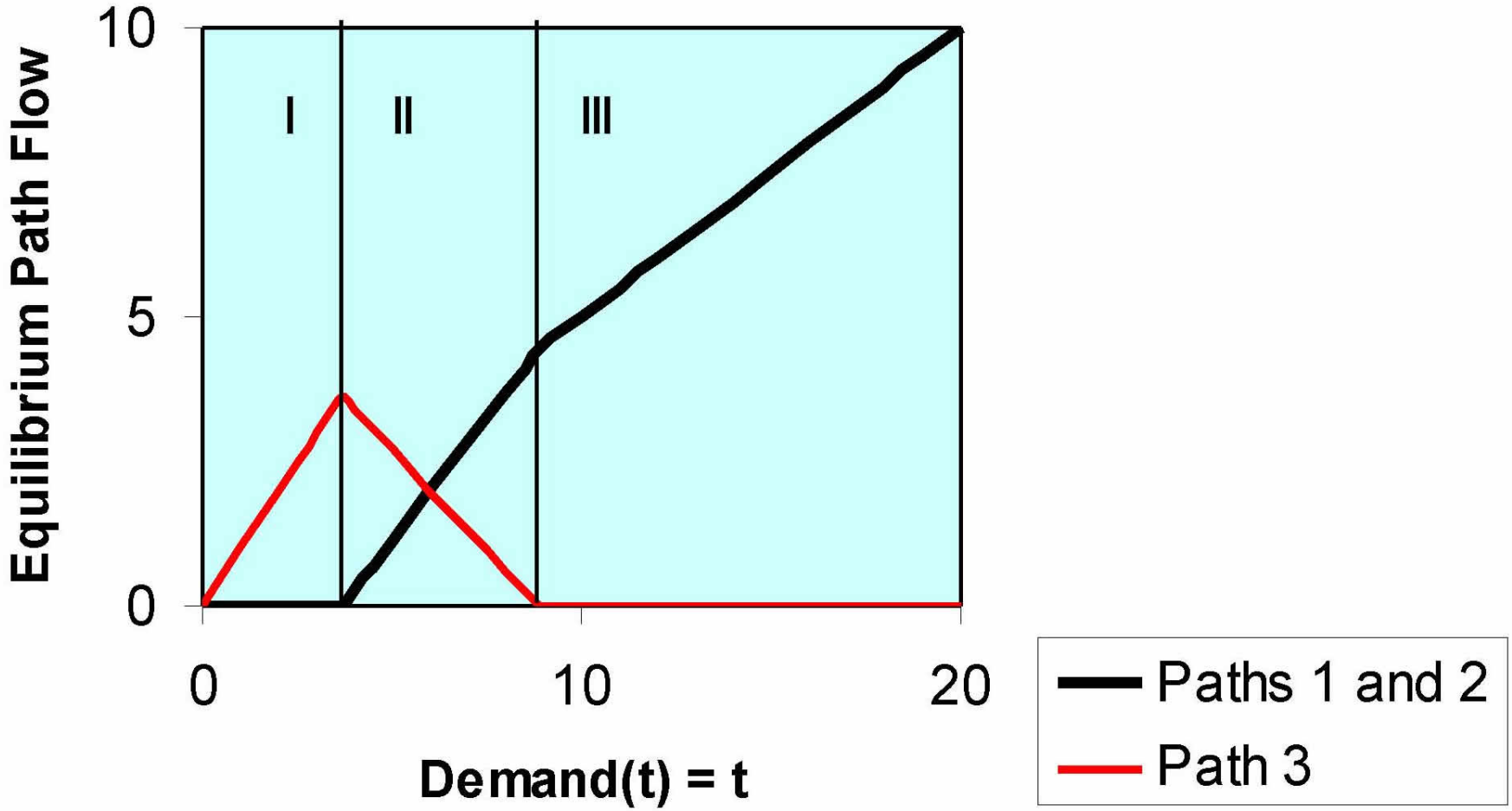
Recall the Braess network with the added link e.

What happens as the demand increases?

For Networks with Time-Dependent Demands

We Use Evolutionary Variational Inequalities

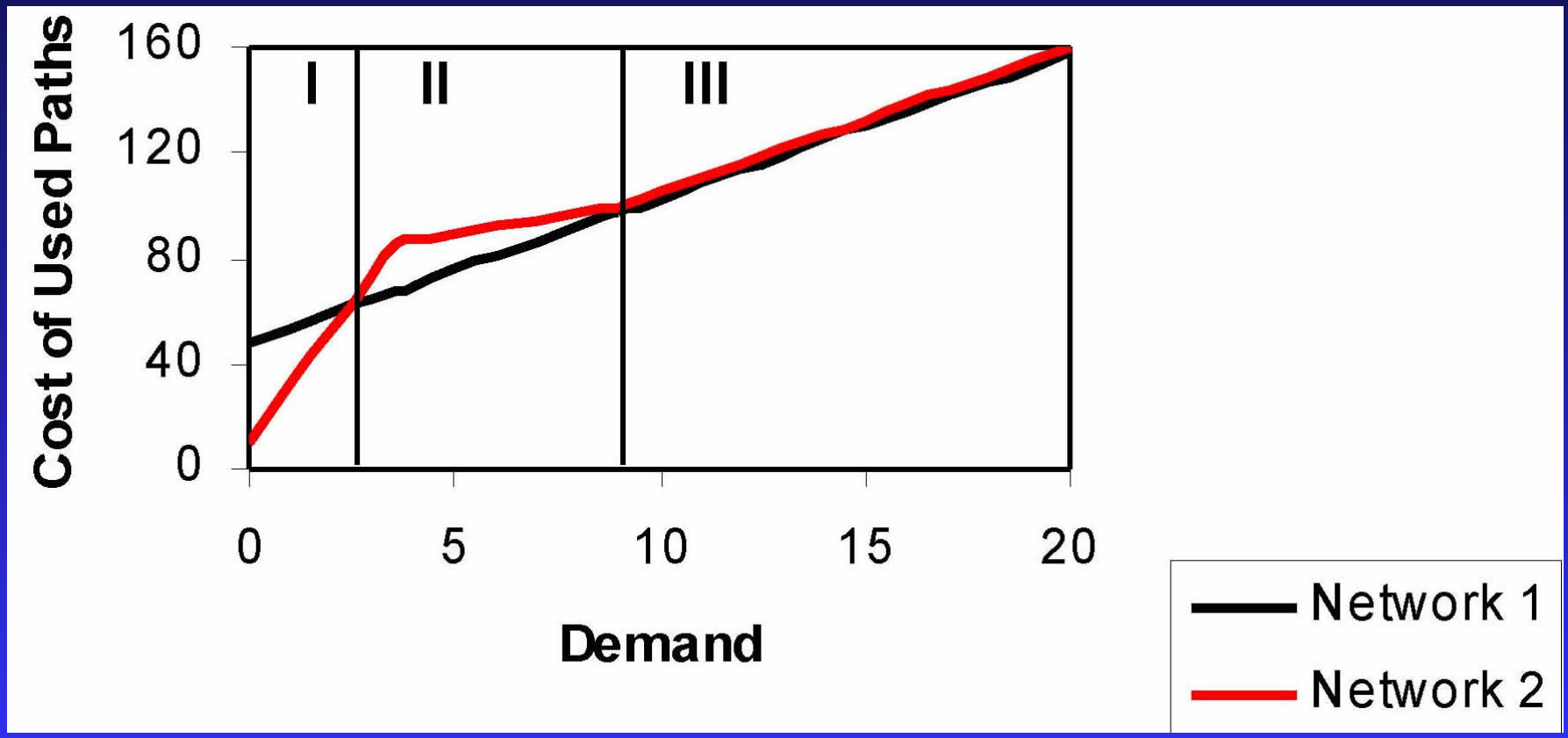
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes (2007)).



In Demand Regime I, Only the New Path is Used.

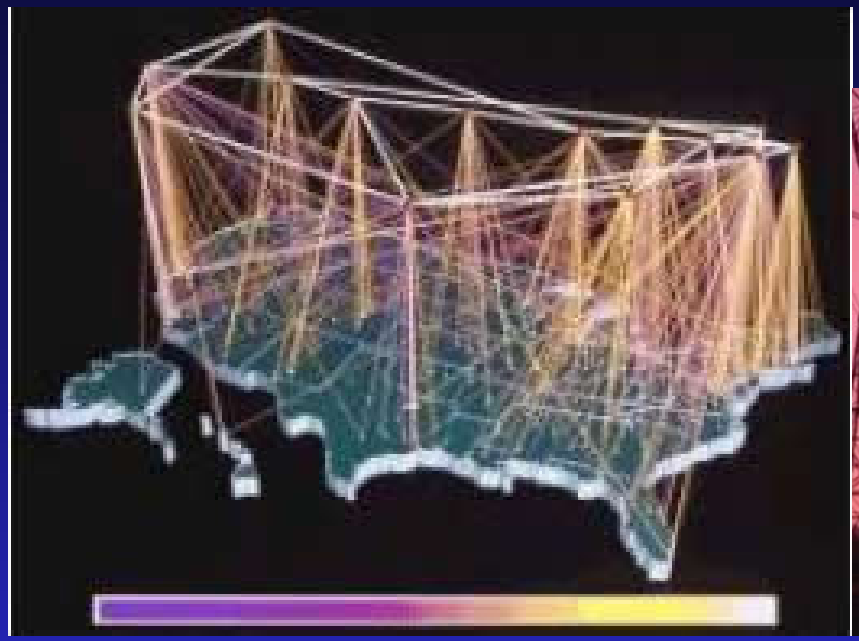
In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and *the Addition of a New Link (Path) Makes Everyone Worse Off*!

In Demand Regime III, when the travel demand exceeds 8.89, *Only the Original Paths are Used!*



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!

Other Networks that Behave like Traffic Networks



The Internet and electric power networks

Electric Power Supply Chains

We have developed *an empirical, large-scale electric supply chain network equilibrium model*, formulated it as a VI problem, and were able to solve it by *exploiting the connection between electric power supply chain networks and transportation networks* using our proof of a hypothesis posed in the classic book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten (1956).

The paper, "An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England," by Z. Liu and A. Nagurney was published in *Naval Research Logistics* (2009), **56**, 600-624.

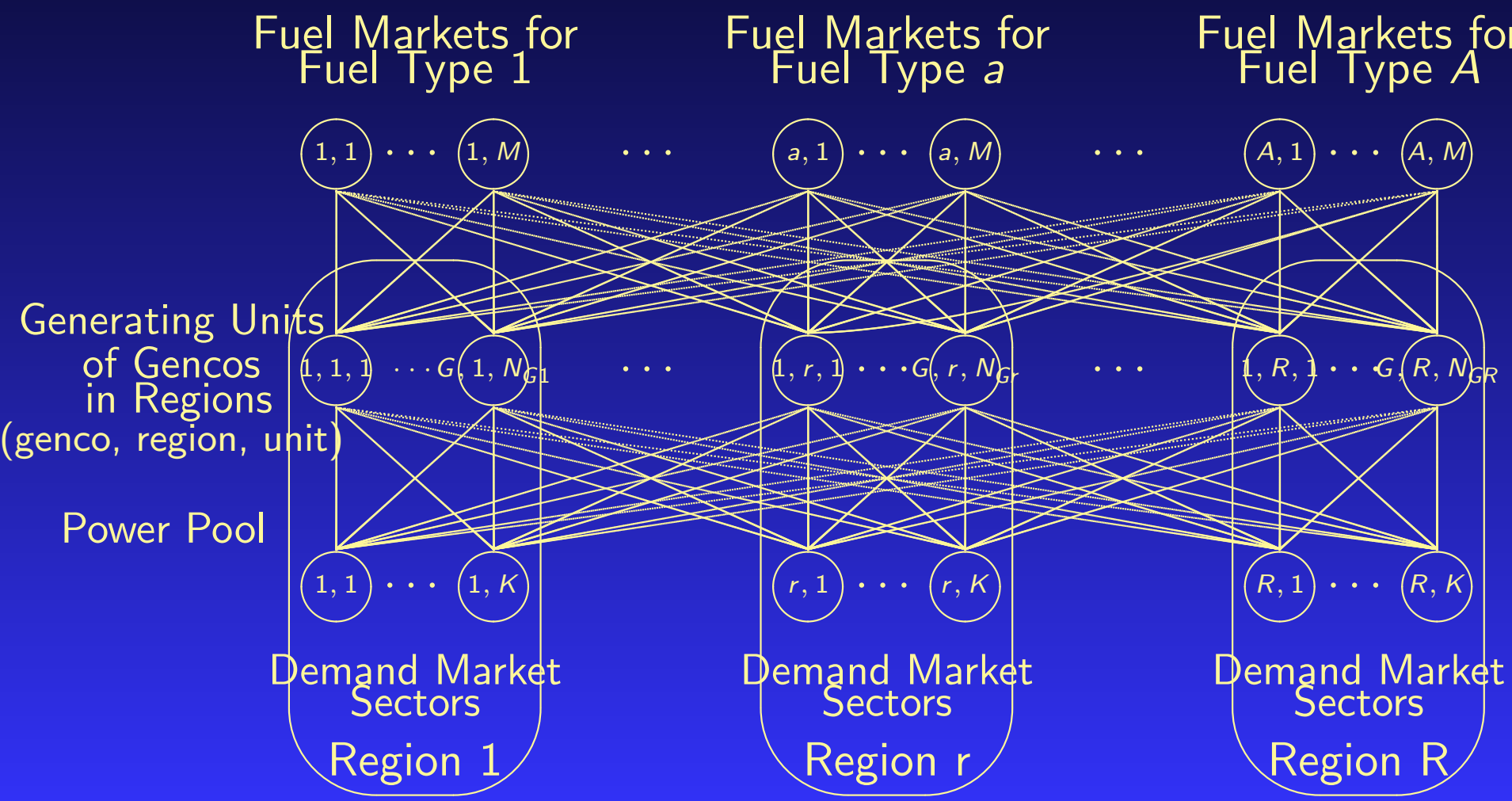
An Empirical Example of an Electric Power Supply Chain for New England

There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

1. Maine,
2. New Hampshire,
3. Vermont,
4. Connecticut (excluding Southwest Connecticut),
5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
6. Norwalk-Stamford area,
7. Rhode Island,
8. Southeastern Massachusetts,
9. Western and Central Massachusetts,
10. Boston/Northeast Massachusetts.

Graphic of New England

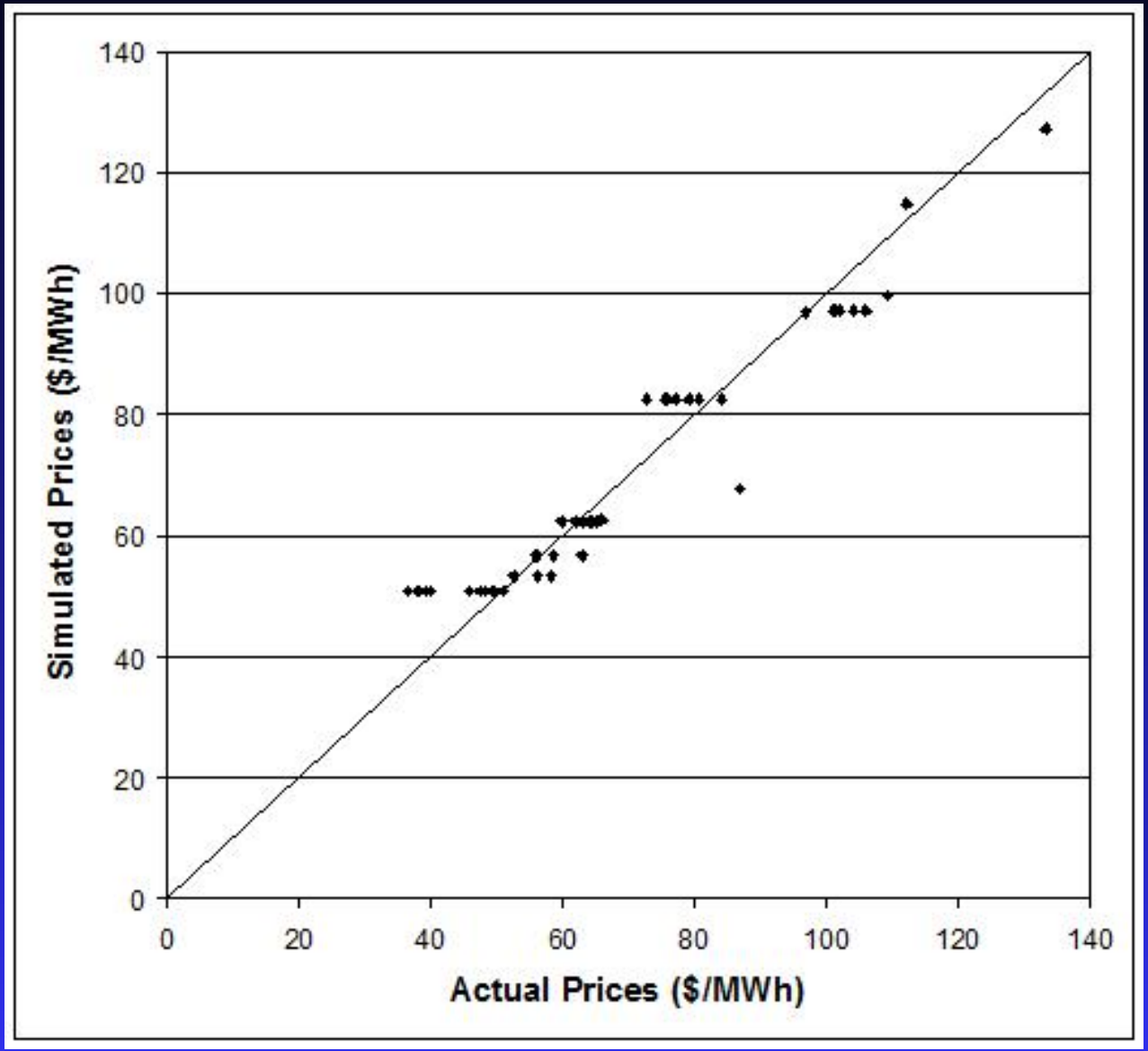
The Electric Power Supply Chain Network with Fuel Supply Markets



We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$) and calculated the average regional demands and the average weighted regional prices for each block.

The empirical model had on the order of $20,000$ *variables.*

Actual Prices Vs. Simulated Prices ($/Mwh)



Supply Chain Network Design Through Mergers and Acquisitions

Supply chain network design (and redesign) can be accomplished through link and node additions (as well as their removals).

It can be accomplished by modifying the link capacities (expanding certain ones and, if applicable, reducing or selling off others).

It can also be accomplished through the integration of networks as in mergers and acquisitions

and, as we will show later, through the design of the network from scratch.

Current Merger & Acquisition Activity

M&As totaled over $2 trillion in 2009, down 32% from full-year 2008 and down 53% from the record high in 2007, according to data from Thomson Reuters.

Mergers announced in October 2010 include Bain Capital / Gymboree, at $1.789 billion and Dynamex / Greenbriar Equity Group ($207 million).

Some of the most visible recent mergers have occurred in the airline industry with Delta and Northwest completing their merger in October 2008 and United and Continental closing on the formation of United Continental Holdings Oct. 1, 2010.

Global 2010 M&A activity is estimated to rise as much as 35% from 2009 figures (Sanford C. Bernstein research firm).

Successful mergers can add tremendous value; however, the failure rate is estimated to be between 74% and 83% (Devero (2004)).

It is worthwhile to develop tools to better predict the associated strategic gains, which include, among others, cost savings (Eccles, Lanes, and Wilson (1999)).

Mergers and Acquisitions and Network Synergies

A successful merger depends on the ability to measure the anticipated synergy of the proposed merger (cf. Chang (1988)) .

- A. Nagurney (2009) "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

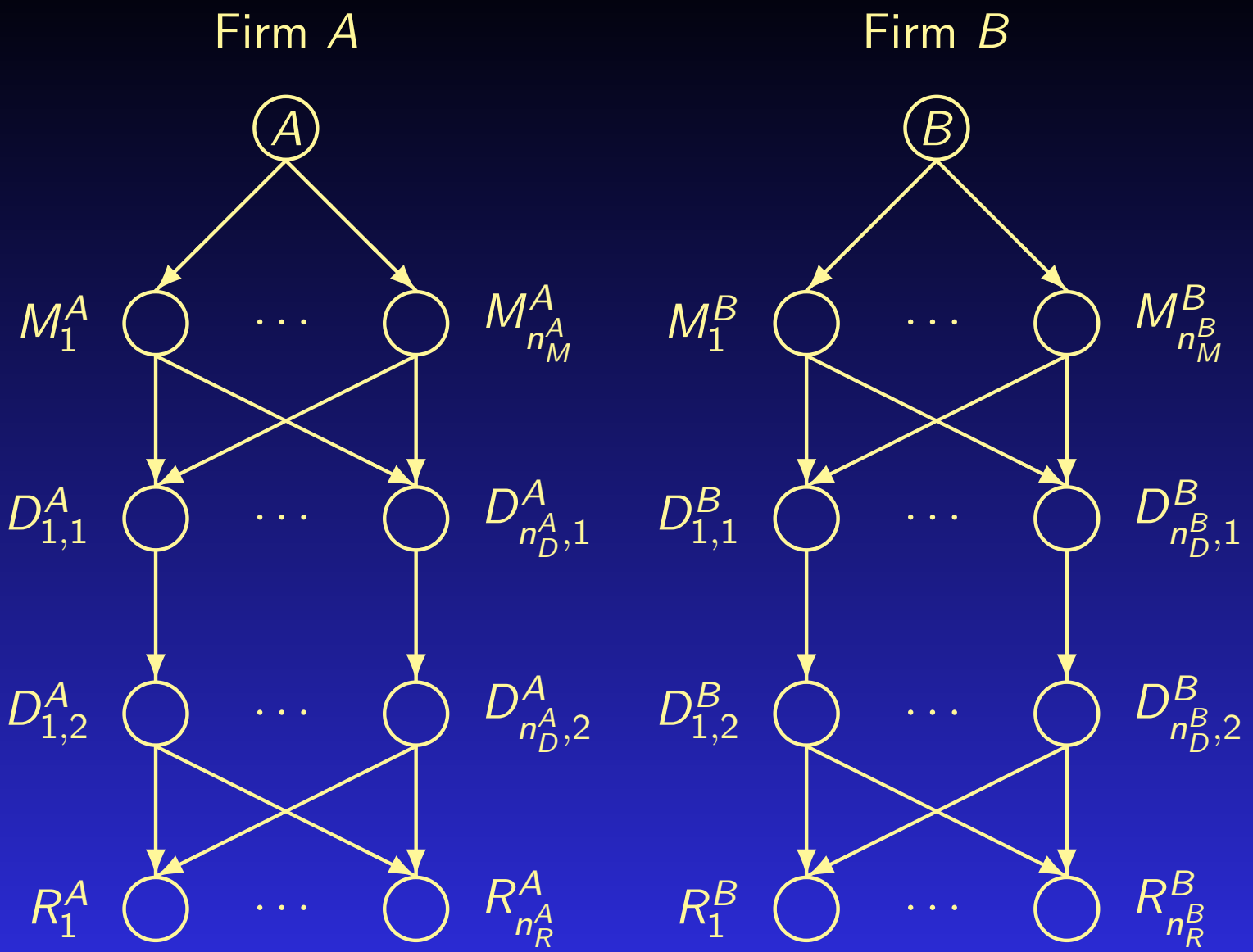


Figure 1: Case 0: Firms A and B Prior to Horizontal Merger

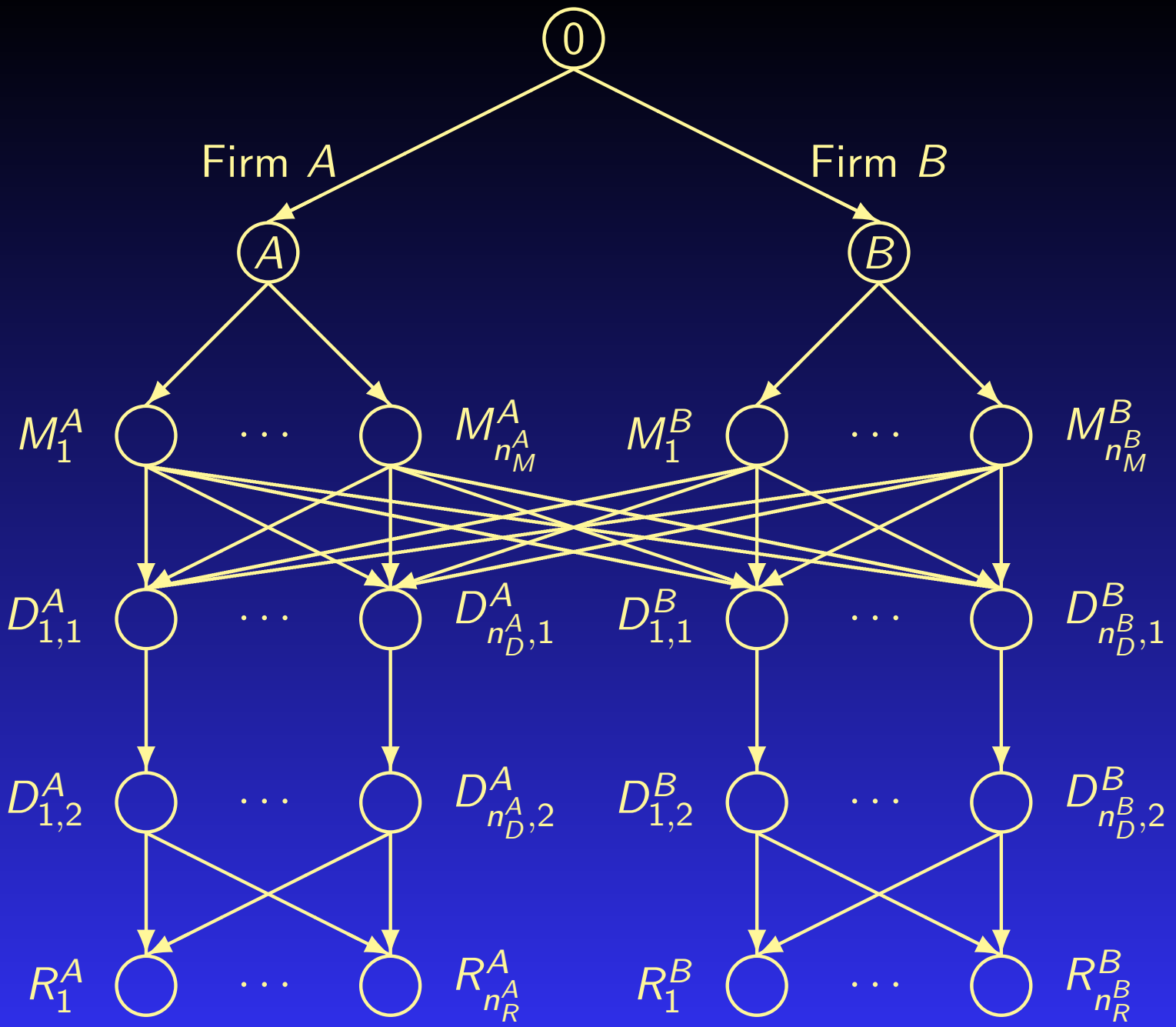


Figure 2: Case 1: Firms A and B Merge

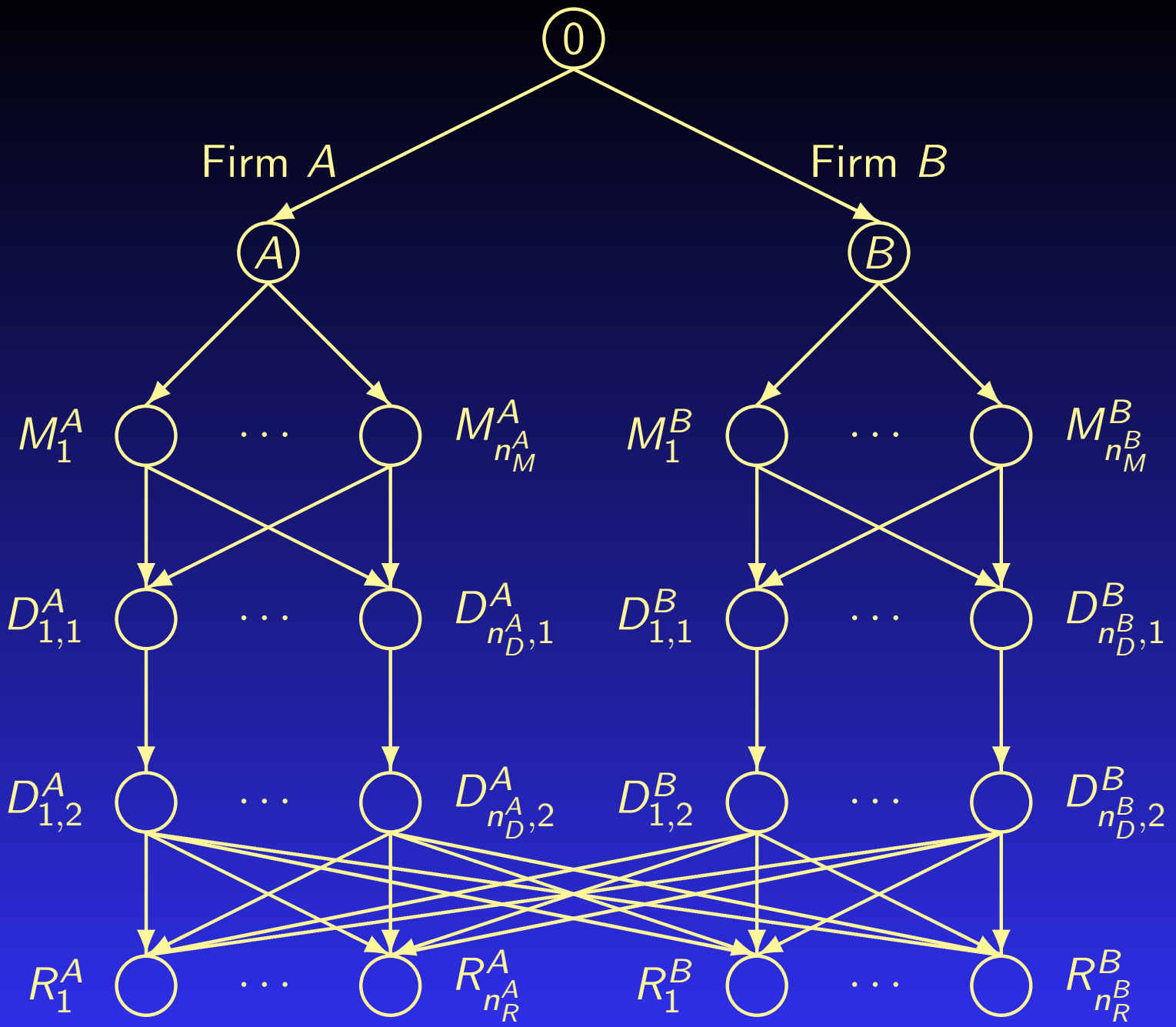


Figure 3: Case 2: Firms A and B Merge

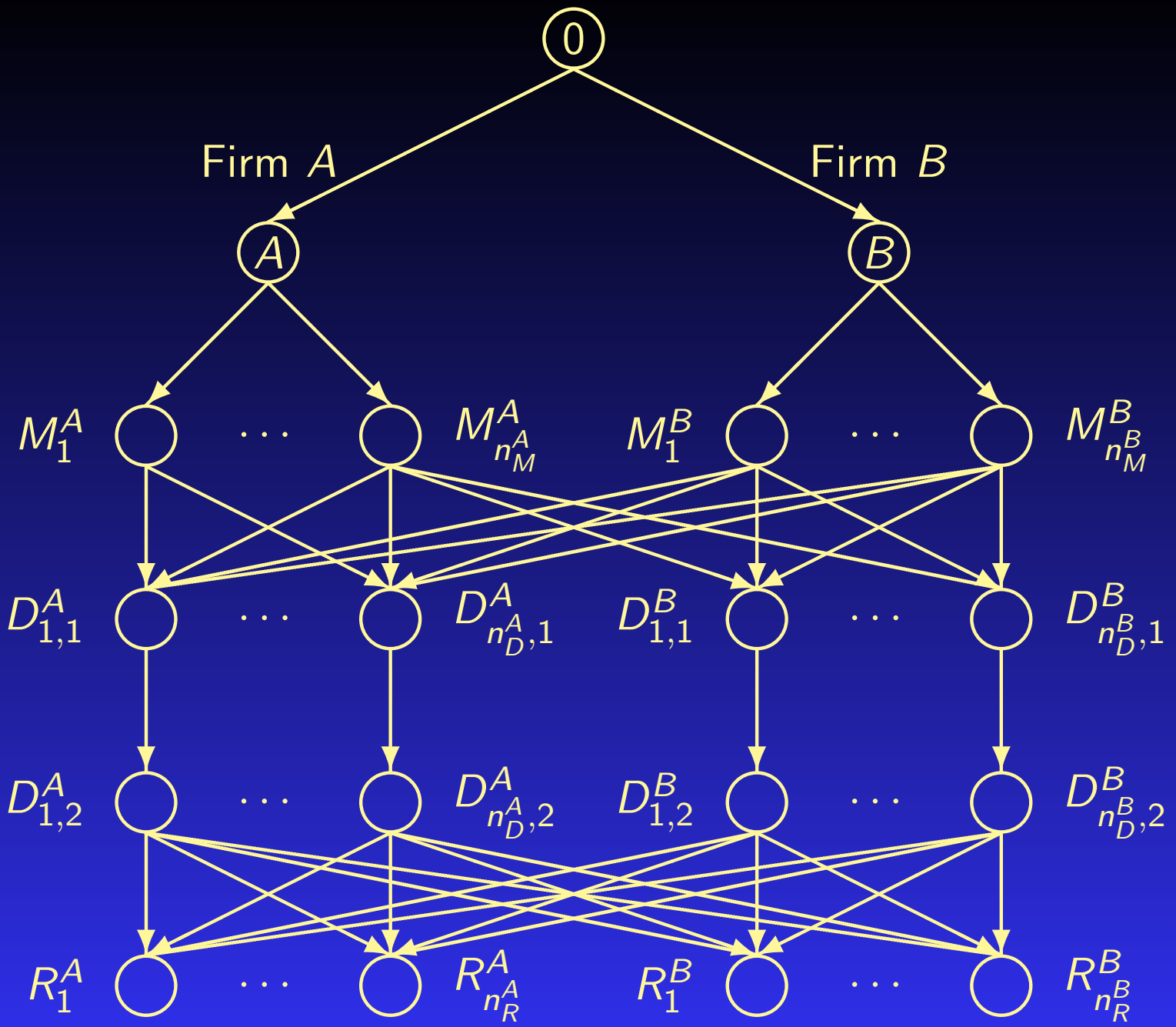


Figure 4: Case 3: Firms A and B Merge

Synergy Measure

The measure that we utilized in Nagurney (2009) to capture the gains, if any, associated with a horizontal merger Case i; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0} \right] \times 100\%,$$

where TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i. Note that $\mathcal{S}^i$; $i = 1, 2, 3$ may also be interpreted as *synergy*.

This model can also be applied to the teaming of organizations in the case of humanitarian operations.

Bellagio Conference on Humanitarian Logistics

See: http://hlogistics.som.umass.edu/

Some Examples of Oligopolies

- airlines
- freight carriers
- automobile manufacturers
- oil companies
- beer / beverage companies
- wireless communications
- fast fashion brands
- certain financial institutions.

The Supply Chain Network Oligopoly Model

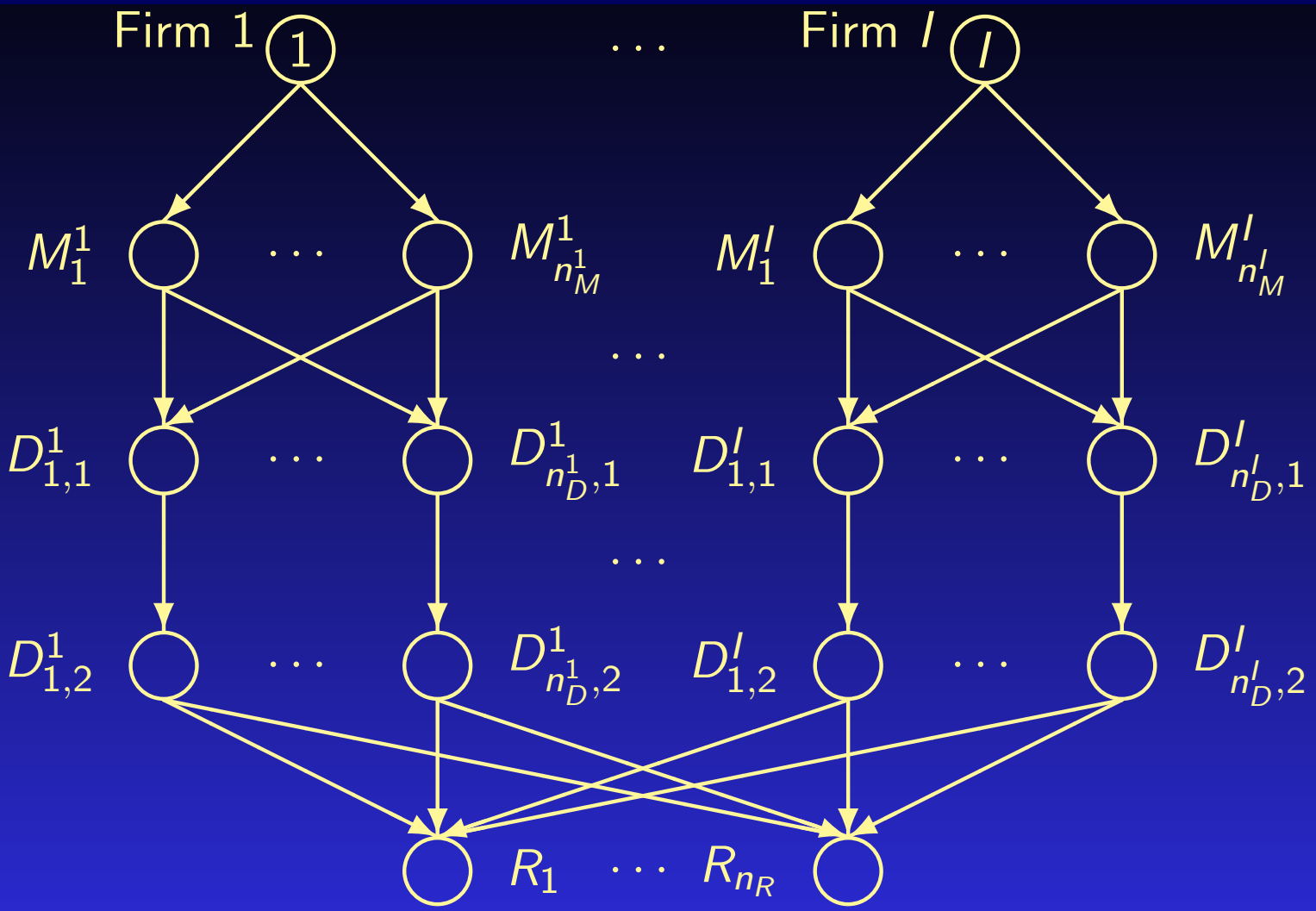


Figure 5: Supply Chain Network Structure of the Oligopoly

Nagurney *Computational Management Science* (2010) **7**, 377-401.

Mergers Through Coalition Formation

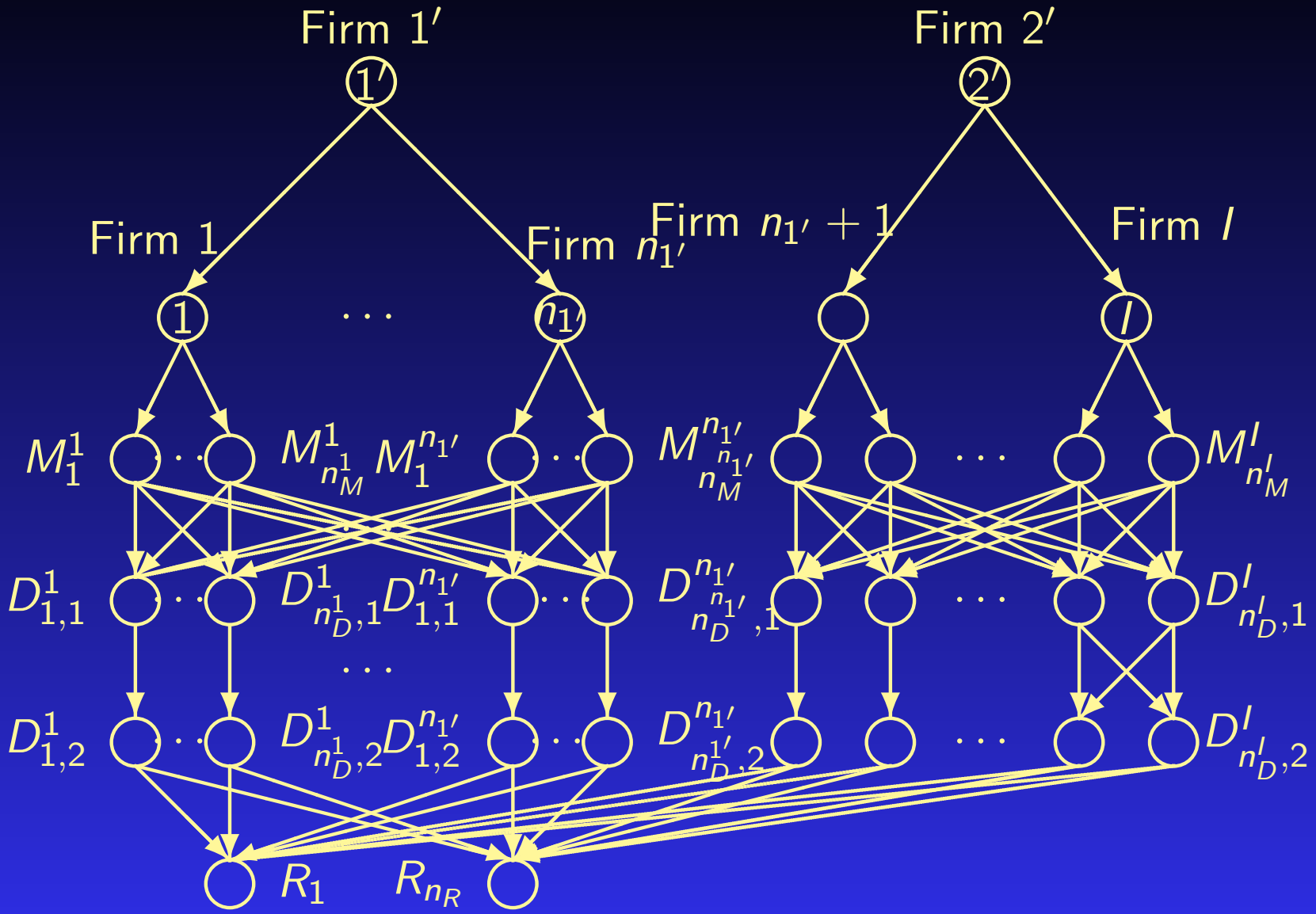


Figure 6: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

In addition, supply chain network design can be accomplished through the evolution and integration of disparate network systems, including social networks.

Two References:

A. Nagurney, T. Wakolbinger, and L. Zhao (2006) "The Evolution and Emergence of Integrated Social and Financial Networks with Electronic Transactions: A Dynamic Supernetwork Theory for the Modeling, Analysis, and Computation of Financial Flows and Relationship Levels," *Computational Economics* **27**, 353-393.

J. M. Cruz, A. Nagurney, and T. Wakolbinger (2006) "Financial Engineering of the Integration of Global Supply Chain Networks and Social Networks with Risk Management," *Naval Research Logistics* **53**, 674-696.

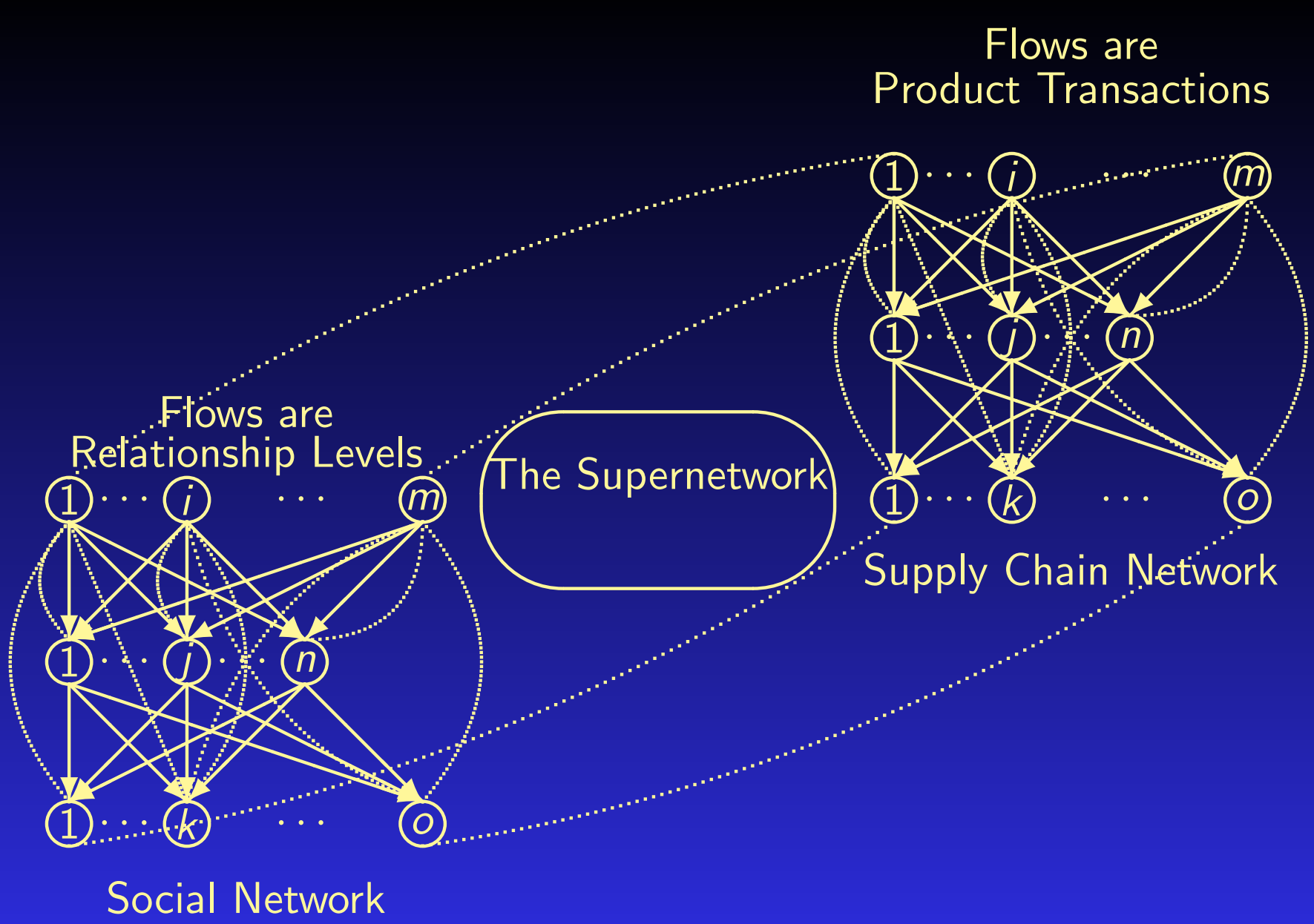


Figure 7: The Multilevel Supernetwork Structure of the Integrated Supply Chain / Social Network System

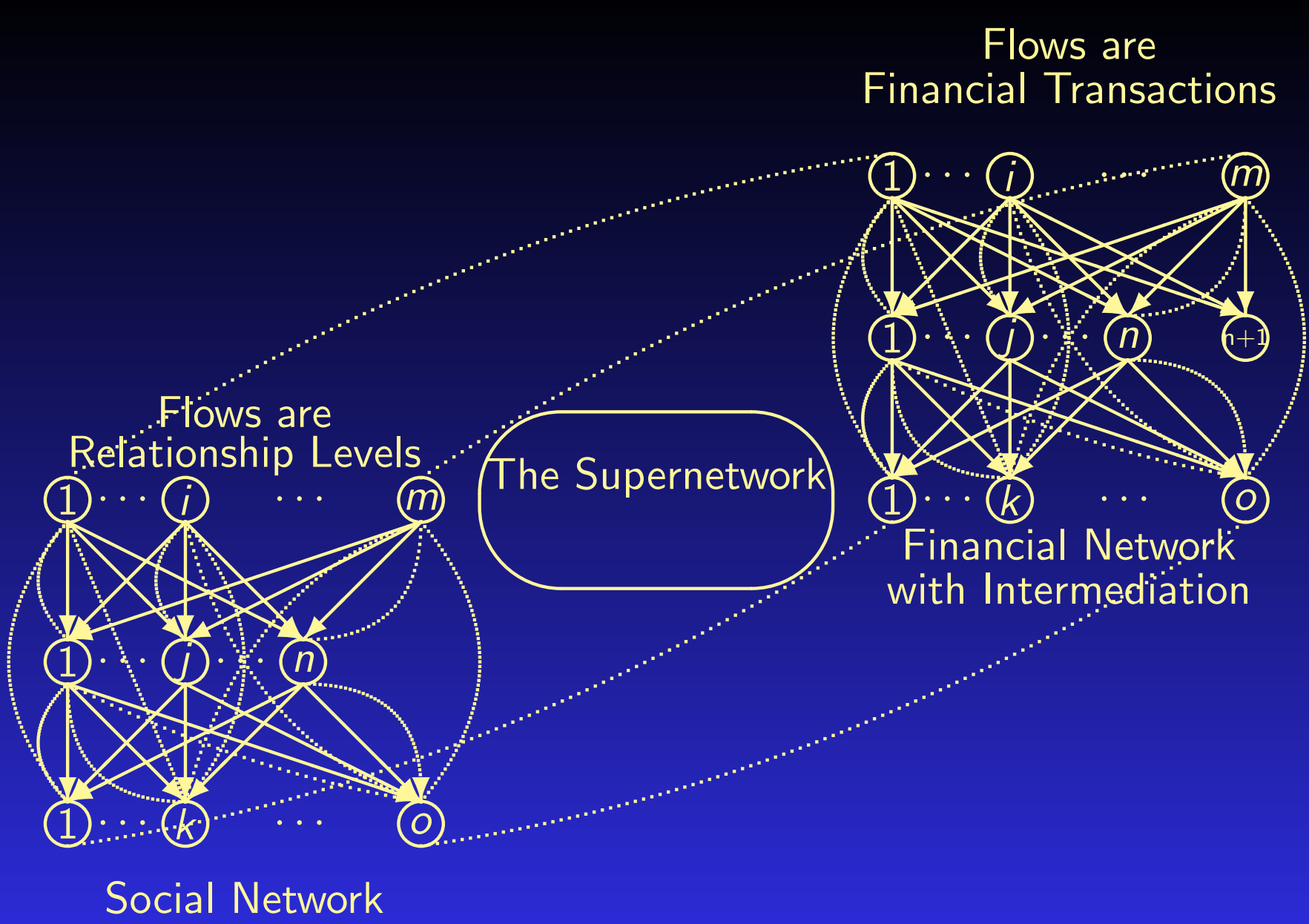


Figure 8: The Multilevel Supernetwork Structure of the Integrated Financial Network / Social Network System

A Challenging Network Design Problem

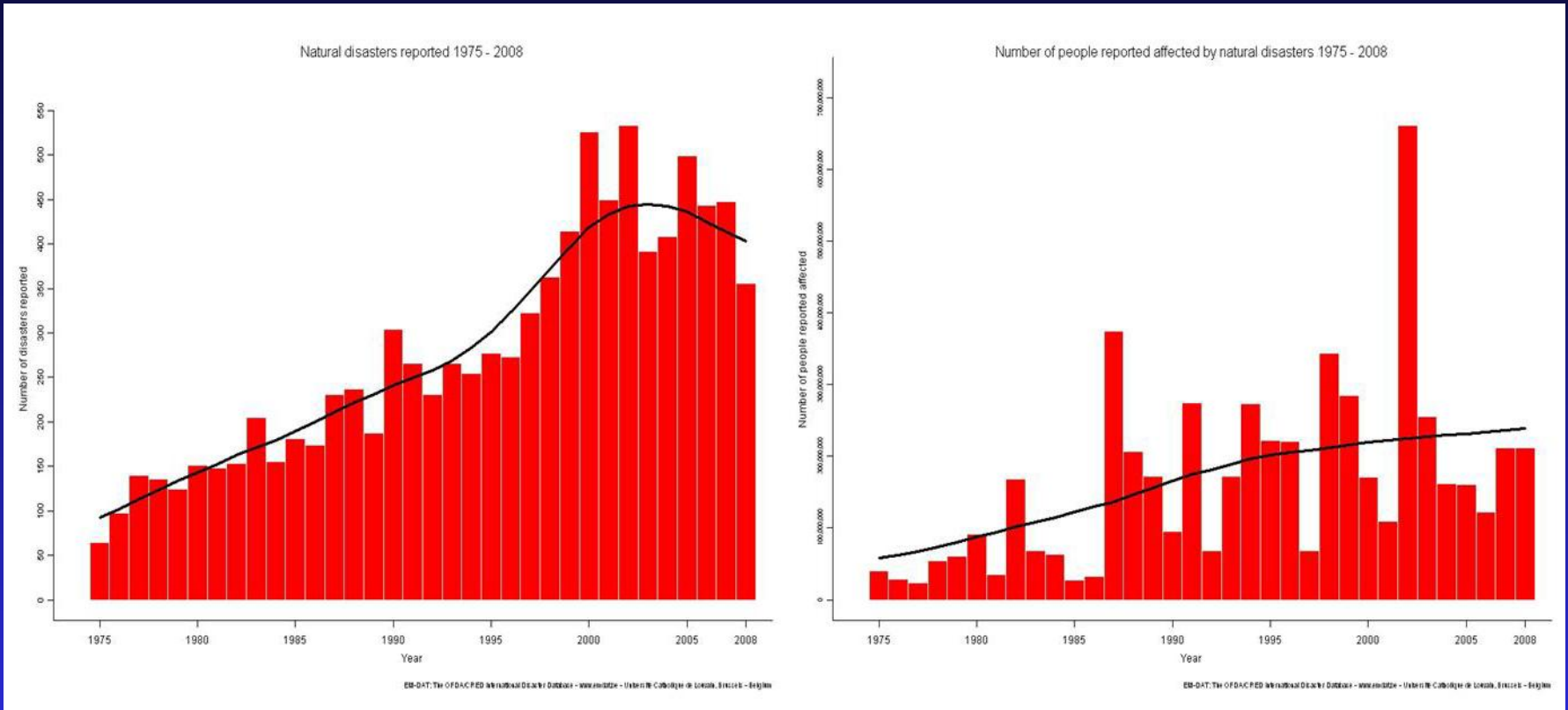
and

Model for Critical Needs with Outsourcing

The number of disasters is increasing globally, as is the number of people affected by disasters. At the same time, with the advent of increasing globalization, viruses are spreading more quickly and creating new challenges for medical and health professionals, researchers, and government officials.

Between 2000 and 2004, the average annual number of disasters was 55% higher than in the period 1994 through 1999, with 33% more humans affected in the former period than in the latter (cf. Balcik and Beamon (2008) and Nagurney and Qiang (2009)).

Natural Disasters (1975–2008)



However, although the average number of disasters has been increasing annually over the past decade the average percentage of needs met by different sectors in the period 2000 through 2005 identifies significant shortfalls.

According to Development Initiatives (2006), based on data in the Financial Tracking System of the Office for the Coordination of Humanitarian Affairs, from 2000-2005, the average needs met by different sectors in the case of disasters were:

- 79% by the food sector;
- 37% of the health needs;
- 35% of the water and sanitation needs;
- 28% of the shelter and non-food items, and
- 24% of the economic recovery and infrastructure needs.

Hurricane Katrina in 2005

Hurricane Katrina has been called an "American tragedy," in which essential services failed completely (Guidotti (2006)).

Haiti Earthquake in 2010

Delivering the humanitarian relief supplies (water, food, medicines, etc.) to the victims was a major logistical challenge.

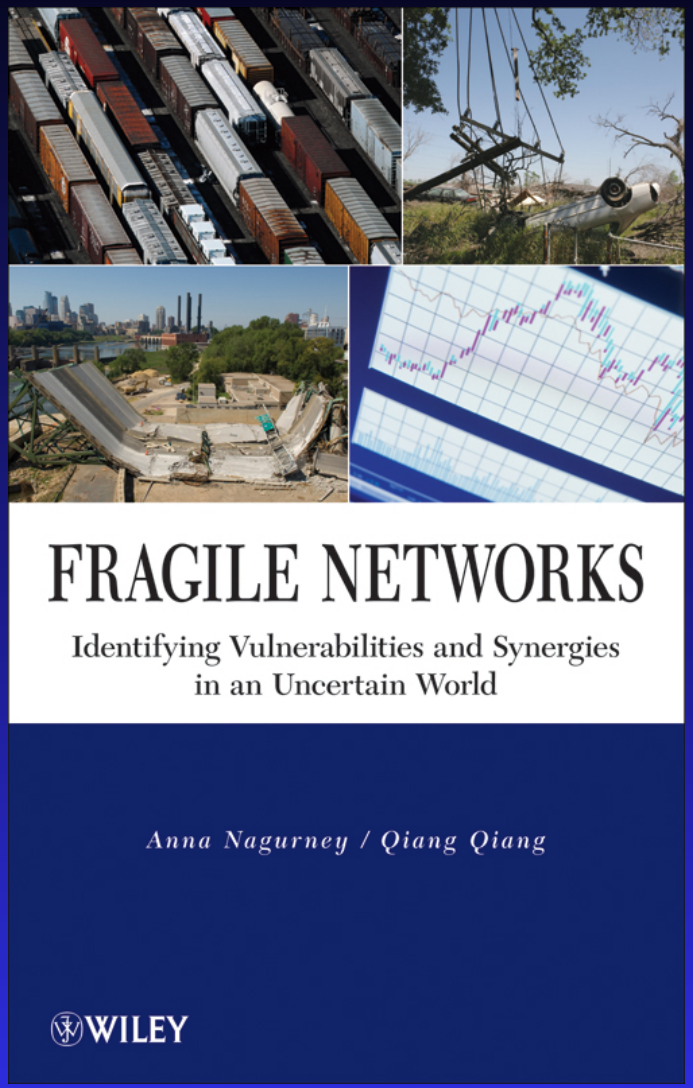
The Triple Disaster in Japan on March 11, 2011

H1N1 (Swine) Flu

As of May 2, 2010, worldwide, more than 214 countries and overseas territories or communities have reported laboratory confirmed cases of pandemic influenza H1N1 2009, including over 18,001 deaths (www.who.int).

Parts of the globe experienced serious flu vaccine shortages, both seasonal and H1N1 (swine) ones, in late 2009.

Fragile Networks



We are living in a world of *Fragile Networks*.

Background and Motivation

Underlying the delivery of goods and services in times of crises, such as in the case of disasters, pandemics, and life-threatening major disruptions, are supply chains, without which essential products do not get delivered in a timely manner, with possible increased disease, injuries, and casualties.

It is clear that better-designed supply chain networks would have facilitated and enhanced various emergency preparedness and relief efforts and would have resulted in less suffering and lives lost.

Supply chain networks provide the logistical backbones for the provision of products as well as services both in corporate as well as in emergency and humanitarian operations.

Here we focus on supply chains in the case of

Critical Needs Products.

Critical Needs Products

Critical needs products are those that are essential to the survival of the population, and can include, for example, vaccines, medicine, food, water, etc., depending upon the particular application.

The demand for the product should be met as nearly as possible since otherwise there may be additional loss of life.

In times of crises, a system-optimization approach is mandated since the demands for critical supplies should be met (as nearly as possible) at minimal total cost.

An Overview of Some of the Relevant Literature

- M. J. Beckmann, C. B. McGuire, and C. B. Winsten (1956) *Studies in the Economics of Transportation*, Yale University Press, New Haven, Connecticut.
- S. C. Dafermos and F. T. Sparrow (1969) "The Traffic Assignment Problem for a General Network," *Journal of Research of the National Bureau of Standards* **73B**, 91-118.
- D. E. Boyce, H. S. Mahmassani, and A. Nagurney (2005) "A Retrospective on Beckmann, McGuire, and Winsten's *Studies in the Economics of Transportation*," *Papers in Regional Science* **84**, 85-103.
- A. Nagurney (2009) "A System-Optimization Perspective for Supply Chain Network Integration: The Horizontal Merger Case," *Transportation Research E* **45**, 1-15.

- A. Nagurney, T. Woolley, and Q. Qiang (2010) "Multiproduct Supply Chain Horizontal Network Integration: Models, Theory, and Computational Results," *International Journal of Operational Research* **17**, 333-349.
- A. Nagurney (2010) "Formulation and Analysis of Horizontal Mergers Among Oligopolistic Firms with Insights into the Merger Paradox: A Supply Chain Network Perspective," *Computational Management Science* **7**, 377-401.
- A. Nagurney (2010) "Supply Chain Network Design Under Profit Maximization and Oligopolistic Competition," *Transportation Research E* **46**, 281-294.
- Z. Liu and A. Nagurney (2011) "Supply Chain Outsourcing Under Exchange Rate Risk and Competition," *Omega* **39**, 539-549.

This part of the presentation is based on the paper:

"Supply Chain Network Design for Critical Needs with Outsourcing,"

A. Nagurney, M. Yu, and Q. Qiang (2011), *Papers in Regional Science* **90**, 123-142.

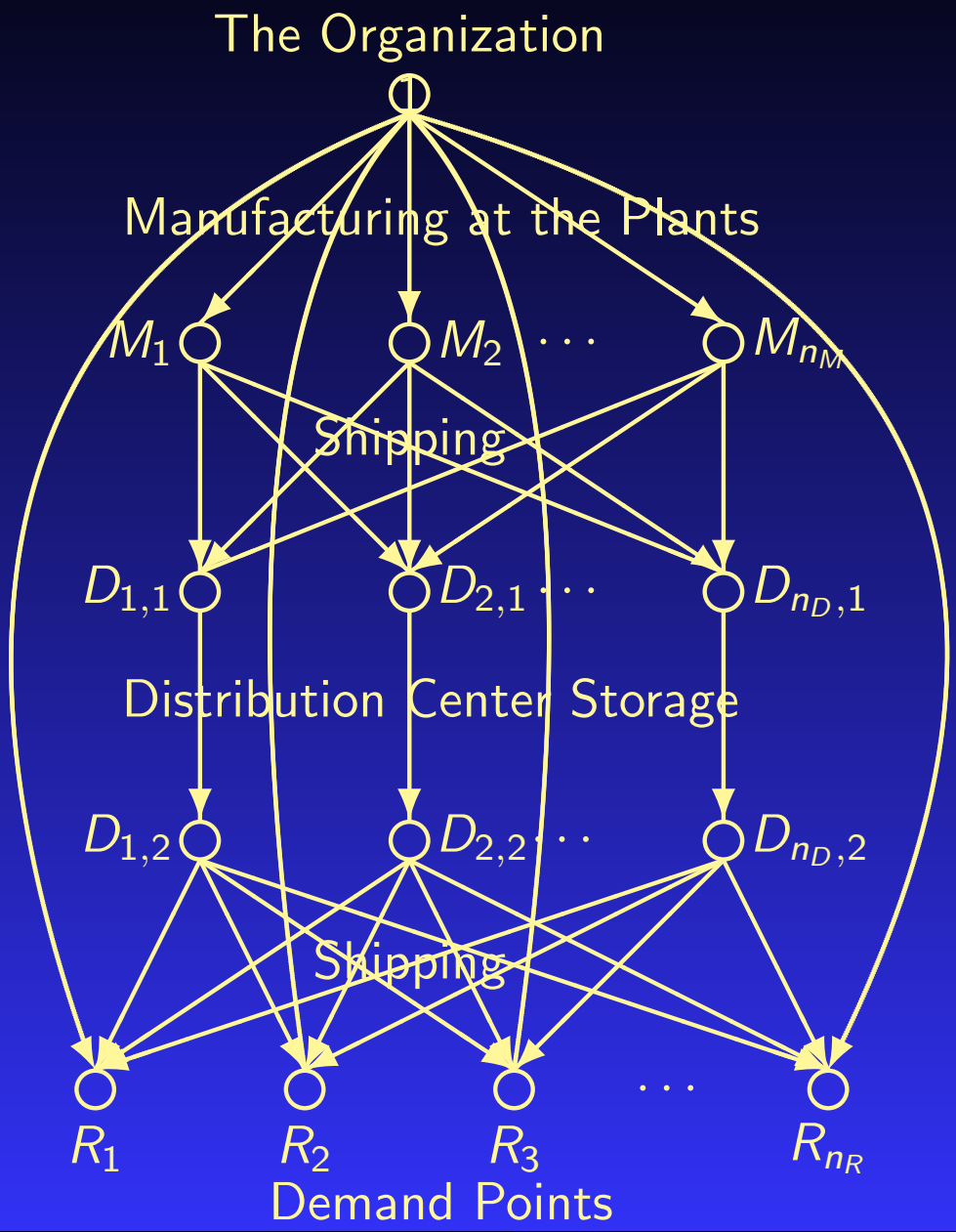
where additional background as well as references can be found.

We assume that the organization (government, humanitarian one, socially responsible firm, etc.) is considering n_M manufacturing facilities/plants; n_D distribution centers, but must serve the n_R demand points.

The supply chain network is modeled as a network $G = [N, L]$, consisting of the set of nodes N and the set of links L. Let L^1 and L^2 denote the links associated with "in house" supply chain activities and the outsourcing activities, respectively. The paths joining the origin node to the destination nodes represent sequences of supply chain network activities that ensure that the product is produced and, ultimately, delivered to those in need at the demand points.

The optimization model can handle both design (from scratch) and redesign scenarios.

Supply Chain Network Topology with Outsourcing



The Links

The possible manufacturing links from the top-tiered node 1 are connected to the possible manufacturing nodes of the organization, which are denoted, respectively, by: $M_1, \ldots, M_{n_M}$.

The possible shipment links from the manufacturing nodes, are connected to the possible distribution center nodes of the organization, denoted by $D_{1,1}, \ldots, D_{n_D,1}$.

The links joining nodes $D_{1,1}, \ldots, D_{n_D,1}$ with nodes $D_{1,2}, \ldots, D_{n_D,2}$ correspond to the possible storage links.

There are possible shipment links joining the nodes $D_{1,2}, \ldots, D_{n_D,2}$ with the demand nodes: $R_1, \ldots, R_{n_R}$.

There are also outsourcing links, which may join the top node to each bottom node (or the relevant nodes for which the outsourcing activity is feasible, as in production, storage, or distribution, or a combination thereof). The organization does not control the capacities on these links since they have been established by the particular firm that corresponds to the outsource link.

The ability to outsource supply chain network activities for critical needs products provides alternative pathways for the production and delivery of products during times of crises such as disasters.

Demands, Path Flows, and Link Flows

Let d_k denote the demand at demand point k; $k = 1, \ldots, n_R$, which is a random variable with probability density function given by $\mathcal{F}_k(t)$. Let x_p represent the nonnegative flow of the product on path p; f_a denote the flow of the product on link a.

Conservation of Flow Between Path Flows and Link Flows

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{1}$$

that is, the total amount of a product on a link is equal to the sum of the flows of the product on all paths that utilize that link. $\delta_{ap} = 1$ if link a is contained in path p, and $\delta_{ap} = 0$, otherwise.

Supply Shortage and Surplus

Let

$$v_k \equiv \sum_{p \in P_{w_k}} x_p, \quad k = 1, \ldots, n_R, \tag{2}$$

where v_k can be interpreted as the *projected demand* at demand market k; $k = 1, \ldots, n_R$. Then,

$$\Delta_k^- \equiv \max\{0, d_k - v_k\}, \quad k = 1, \ldots, n_R, \tag{3}$$

$$\Delta_k^+ \equiv \max\{0, v_k - d_k\}, \quad k = 1, \ldots, n_R, \tag{4}$$

where Δ_k^- and Δ_k^+ represent the supply shortage and surplus at demand point k, respectively. The expected values of Δ_k^- and Δ_k^+ are given by:

$$E(\Delta_k^-) = \int_{v_k}^{\infty} (t - v_k) \mathcal{F}_k(t) d(t), \quad k = 1, \ldots, n_R, \tag{5}$$

$$E(\Delta_k^+) = \int_{0}^{v_k} (v_k - t) \mathcal{F}_k(t) d(t), \quad k = 1, \ldots, n_R. \tag{6}$$

The Operation Costs, Investment Costs and Penalty Costs

The total cost on a link is assumed to be a function of the flow of the product on the link. We have, thus, that

$$\hat{c}_a = \hat{c}_a(f_a), \quad \forall a \in L. \tag{7}$$

We denote the nonnegative existing capacity on a link a by $\bar{u}_a$, $\forall a \in L$. Note that the organization can add capacity to the "in house" link a; $\forall a \in L^1$. We assume that

$$\hat{\pi}_a = \hat{\pi}_a(u_a), \quad \forall a \in L^1. \tag{8}$$

The expected total penalty at demand point k; $k = 1, \ldots, n_R$, is,

$$E(\lambda_k^- \Delta_k^- + \lambda_k^+ \Delta_k^+) = \lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+), \tag{9}$$

where λ_k^- is the unit penalty of supply shortage at demand point k and λ_k^+ is that of supply surplus. Note that $\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)$ is a function of the path flow vector x.

The Supply Chain Network Design Optimization Problem

The organization seeks to determine the optimal levels of product processed on each supply chain network link (including the outsourcing links) coupled with the optimal levels of capacity investments in its supply chain network activities subject to the minimization of the total cost.

The total cost includes the total cost of operating the various links, the total cost of capacity investments, and the expected total supply shortage/surplus penalty.

The Supply Chain Network Design Optimization Problem

$$\text{Minimize} \quad \sum_{a \in L} \hat{c}_a(f_a) + \sum_{a \in L^1} \hat{\pi}_a(u_a) + \sum_{k=1}^{n_R} (\lambda_k^- E(\Delta_k^-) + \lambda_k^+ E(\Delta_k^+)) \tag{10}$$

subject to: constraints (1), (2) and

$$f_a \leq \bar{u}_a + u_a, \quad \forall a \in L^1, \tag{11}$$

$$f_a \leq \bar{u}_a, \quad \forall a \in L^2, \tag{12}$$

$$u_a \geq 0, \quad \forall a \in L^1, \tag{13}$$

$$x_p \geq 0, \quad \forall p \in P. \tag{14}$$

The Feasible Set

We associate the Lagrange multiplier ω_a with constraint (11) for link $a \in L^1$ and we denote the associated optimal Lagrange multiplier by ω_a^*. Similarly, Lagrange multiplier γ_a is associated with constraint (12) for link $a \in L^2$ with the optimal multiplier denoted by γ_a^*. These two terms may also be interpreted as the price or value of an additional unit of capacity on link a. We group these Lagrange multipliers into the vectors ω and γ, respectively. Let K denote the feasible set such that

$$K \equiv \{(x, u, \omega, \gamma) | x \in R_+^{n_P}, u \in R_+^{n_{L^1}}, \omega \in R_+^{n_{L^1}}, \text{and}\, \gamma \in R_+^{n_{L^2}}\}.$$

Theorem

The optimization problem is equivalent to the variational inequality problem: determine the vector of optimal path flows, the vector of optimal link capacity enhancements, and the vectors of optimal Lagrange multipliers $(x^*, u^*, \omega^*, \gamma^*) \in K$, *such that:*

$$\sum_{k=1}^{n_R} \sum_{p \in P_{w_k}} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} + \sum_{a \in L^1} \omega_a^* \delta_{ap} + \sum_{a \in L^2} \gamma_a^* \delta_{ap} + \lambda_k^+ P_k \left(\sum_{p \in P_{w_k}} x_p^* \right) \right.$$

$$\left. - \lambda_k^- \left(1 - P_k \left(\sum_{p \in P_{w_k}} x_p^* \right) \right) \right] \times [x_p - x_p^*]$$

$$+ \sum_{a \in L^1} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \omega_a^* \right] \times [u_a - u_a^*] + \sum_{a \in L^1} [\bar{u}_a + u_a^* - \sum_{p \in P} x_p^* \delta_{ap}] \times [\omega_a - \omega_a^*]$$

$$+ \sum_{a \in L^2} [\bar{u}_a - \sum_{p \in P} x_p^* \delta_{ap}] \times [\gamma_a - \gamma_a^*] \geq 0, \quad \forall (x, u, \omega, \gamma) \in K. \qquad (15)$$

Theorem (cont'd.)

In addition, (15) can be reexpressed in terms of links flows as: determine the vector of optimal link flows, the vectors of optimal projected demands and link capacity enhancements, and the vectors of optimal Lagrange multipliers $(f^*, v^*, u^*, \omega^*, \gamma^*) \in K^1$, *such that:*

$$\sum_{a \in L^1} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \omega_a^* \right] \times [f_a - f_a^*] + \sum_{a \in L^2} \left[\frac{\partial \hat{c}_a(f_a^*)}{\partial f_a} + \gamma_a^* \right] \times [f_a - f_a^*]$$

$$+ \sum_{a \in L^1} \left[\frac{\partial \hat{\pi}_a(u_a^*)}{\partial u_a} - \omega_a^* \right] \times [u_a - u_a^*]$$

$$+ \sum_{k=1}^{n_R} \left[\lambda_k^+ P_k(v_k^*) - \lambda_k^- (1 - P_k(v_k^*)) \right] \times [v_k - v_k^*] + \sum_{a \in L^1} [\bar{u}_a + u_a^* - f_a^*] \times [\omega_a - \omega_a^*]$$

$$+ \sum_{a \in L^2} [\bar{u}_a - f_a^*] \times [\gamma_a - \gamma_a^*] \geq 0, \quad \forall (f, v, u, \omega, \gamma) \in K^1, \qquad (16)$$

where $K^1 \equiv \{(f, v, u, \omega, \gamma) | \exists x \geq 0$, *and* (1), (2), (13), *and* (14) *hold, and* $\omega \geq 0, \quad \gamma \geq 0\}$.

Applications to Vaccine Production and Emergencies

Applications to Vaccine Production

By applying the general theoretical model to the company's data, the firm can determine whether it needs to expand its facilities (or not), how much of the vaccine to produce where, how much to store where, and how much to have shipped to the various demand points. Also, it can determine whether it should outsource any of its vaccine production and at what level.

The firm by solving the model with its company-relevant data can then ensure *that the price that it receives for its vaccine production and delivery is appropriate* and that it recovers its incurred costs and obtains, if negotiated correctly, an equitable profit.

Applications to Emergencies

A company can, using the model, prepare and plan for an emergency such as a natural disaster in the form of a hurricane and identify where to store a necessary product (such as food packets, for example) so that the items can be delivered to the demand points in a timely manner and at minimal total cost.

The Algorithm, Explicit Formulae, and Numerical Examples

The Algorithm

At an iteration τ of the Euler method (see Dupuis and Nagurney (1993) and Nagurney and Zhang (1996)) one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})), \tag{17}$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem: determine $X^* \in \mathcal{K}$ such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ is the inner product in n-dimensional Euclidean space, $X \in R^n$, and $F(X)$ is an n-dimensional function from $\mathcal{K}$ to R^n, with $F(X)$ being continuous.

The sequence $\{a_{\tau}\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_{\tau} = \infty$, $a_{\tau} > 0$, $a_{\tau} \rightarrow 0$, as $\tau \rightarrow \infty$.

Explicit Formulae for (17) to the Supply Chain Network Design Variational Inequality (15)

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\lambda_k^-(1 - P_k(\sum_{p \in P_{w_k}} x_p^\tau)) - \lambda_k^+ P_k(\sum_{p \in P_{w_k}} x_p^\tau)$$

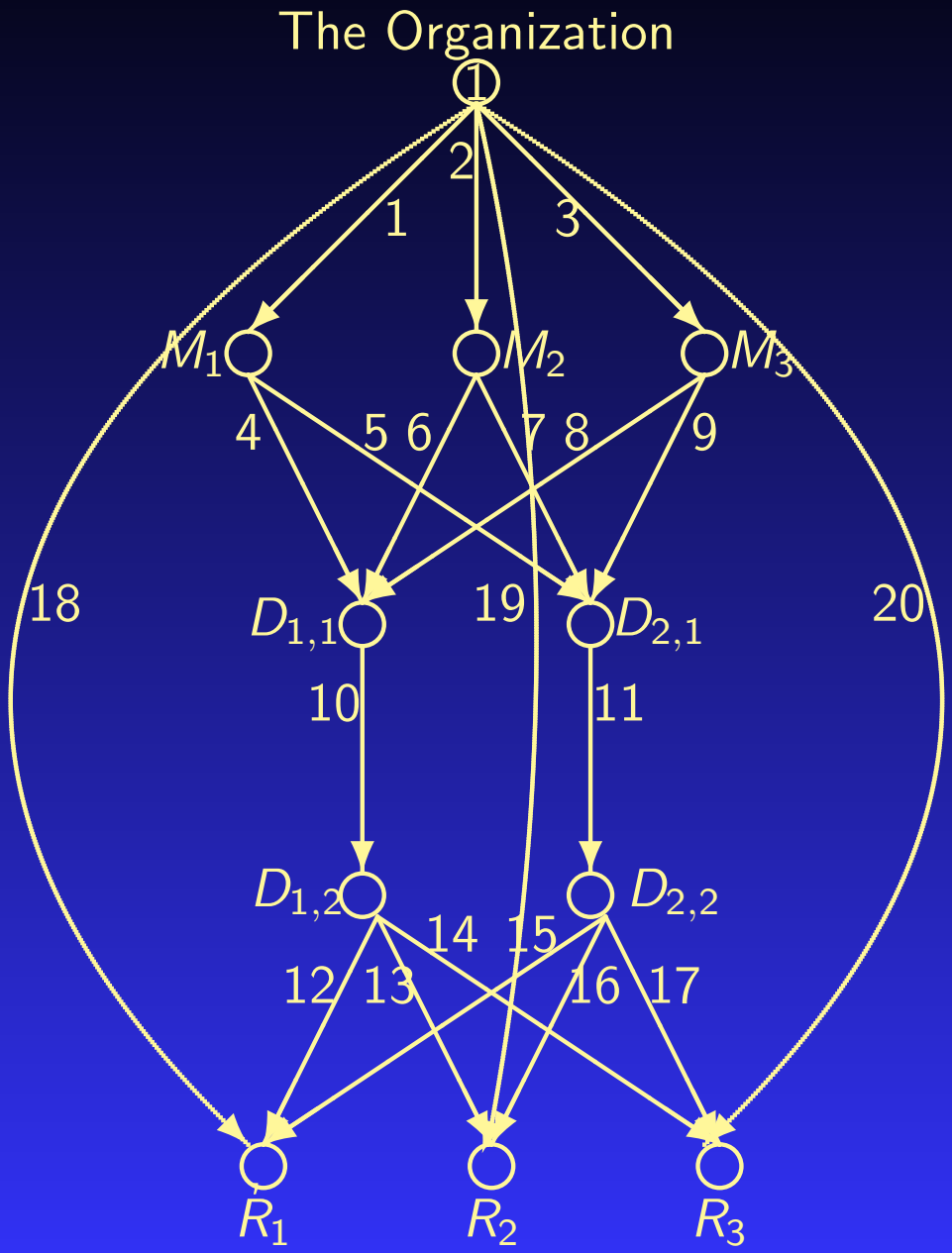
$$-\frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L^1} \omega_a^\tau \delta_{ap} - \sum_{a \in L^2} \gamma_a^\tau \delta_{ap})\}, \ \forall p \in P; \quad (19)$$

$$u_a^{\tau+1} = \max\{0, u_a^\tau + a_\tau(\omega_a^\tau - \frac{\partial \hat{\pi}_a(u_a^\tau)}{\partial u_a})\}, \quad \forall a \in L^1; \quad (20)$$

$$\omega_a^{\tau+1} = \max\{0, \omega_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - \bar{u}_a - u_a^\tau)\}, \quad \forall a \in L^1; \quad (21)$$

$$\gamma_a^{\tau+1} = \max\{0, \gamma_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - \bar{u}_a)\}, \quad \forall a \in L^2. \quad (22)$$

Numerical Examples



Example 1

The demands at the three demand points followed a uniform probability distribution on the intervals $[0, 10]$, $[0, 20]$, and $[0, 30]$, respectively:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{10}, \quad P_2(\sum_{p \in P_{w_2}} x_p) = \frac{\sum_{p \in P_{w_2}} x_p}{20},$$

$$P_3(\sum_{p \in P_{w_3}} x_p) = \frac{\sum_{p \in P_{w_3}} x_p}{30},$$

where $w_1 = (1, R_1)$, $w_2 = (1, R_2)$, and $w_3 = (1, R_3)$.
The penalties were:

$$\lambda_1^- = 50, \quad \lambda_1^+ = 0; \quad \lambda_2^- = 50, \quad \lambda_2^+ = 0; \quad \lambda_3^- = 50, \quad \lambda_3^+ = 0.$$

The capacities associated with the three outsourcing links were:

$$\bar{u}_{18} = 5, \quad \bar{u}_{19} = 10, \quad \bar{u}_{20} = 5.$$

We set $\bar{u}_a = 0$ for all links $a \in L^1$.

Table 1: Total Cost Functions and Solution for Example 1

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.34	1.34	2.34	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	2.47	2.47	3.47	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	2.05	2.05	3.05	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.61	0.61	1.61	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.73	0.73	1.73	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	0.83	0.83	1.83	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	1.64	1.64	2.64	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	1.67	1.67	2.67	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.37	0.37	1.37	–
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	3.11	3.11	4.11	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	2.75	2.75	3.75	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.04	0.04	1.04	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.00	0.00	0.45	–

Table 2: Total Cost Functions and Solution for Example 1 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	3.07	3.07	4.07	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.45	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.00	0.00	0.45	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	2.75	2.75	3.75	–
18	$10f_{18}$	–	5.00	–	–	14.77
19	$12f_{19}$	–	10.00	–	–	13.00
20	$15f_{20}$	–	5.00	–	–	16.96

Note that the optimal supply chain network design for Example 1 is, hence, as the initial topology but with links 13, 15, and 16 removed since those links have zero capacities and associated flows. Note that the organization took advantage of outsourcing to the full capacity available.

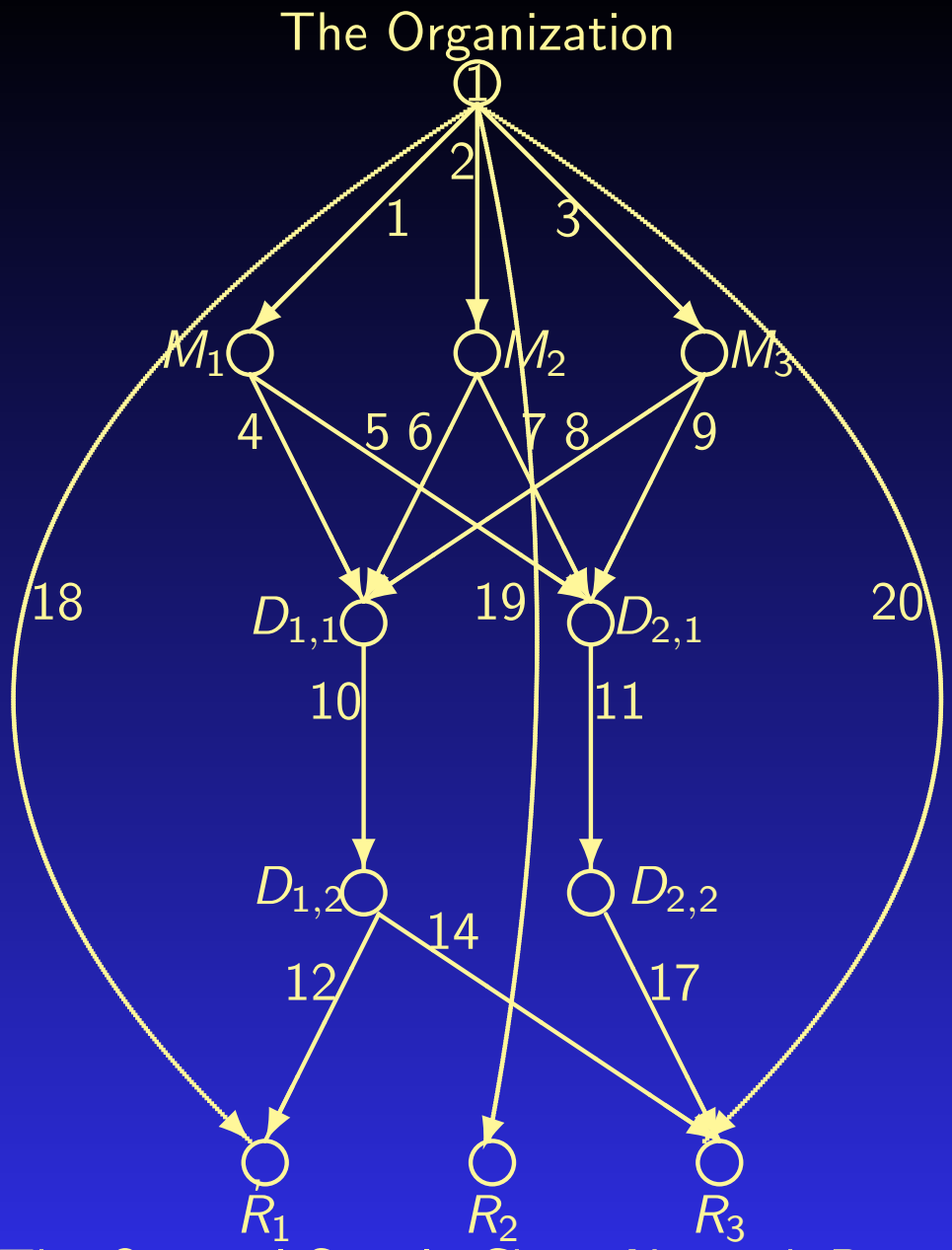


Figure 9: The Optimal Supply Chain Network Design for Example 1

Example 2

Example 2 had the identical data to that in Example 1 except that we now assumed that the organization had capacities on its supply chain network activities where $\bar{u}_a = 10$, for all $a \in L^1$.

Table 3: Total Cost Functions and Solution for Example 2

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	1.84	0.00	0.00	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	4.51	0.00	0.00	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	3.85	0.00	0.00	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	0.88	0.00	0.00	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	0.97	0.00	0.00	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	1.40	0.00	0.00	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	3.11	0.00	0.00	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	3.47	0.00	0.00	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	0.38	0.00	0.00	–

Table 4: Total Cost Functions and Solution for Example 2 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	5.75	0.00	0.00	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	4.46	0.00	0.00	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	0.82	0.00	0.00	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	0.52	0.00	0.00	–
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	4.41	0.00	0.00	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	0.00	0.00	0.00	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	0.05	0.00	0.00	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.41	0.00	0.00	–
18	$10f_{18}$	–	5.00	–	–	10.89
19	$12f_{19}$	–	10.00	–	–	11.59
20	$15f_{20}$	–	5.00	–	–	11.96

Note that links 13 and 16 now have positive associated flows although at very low levels.

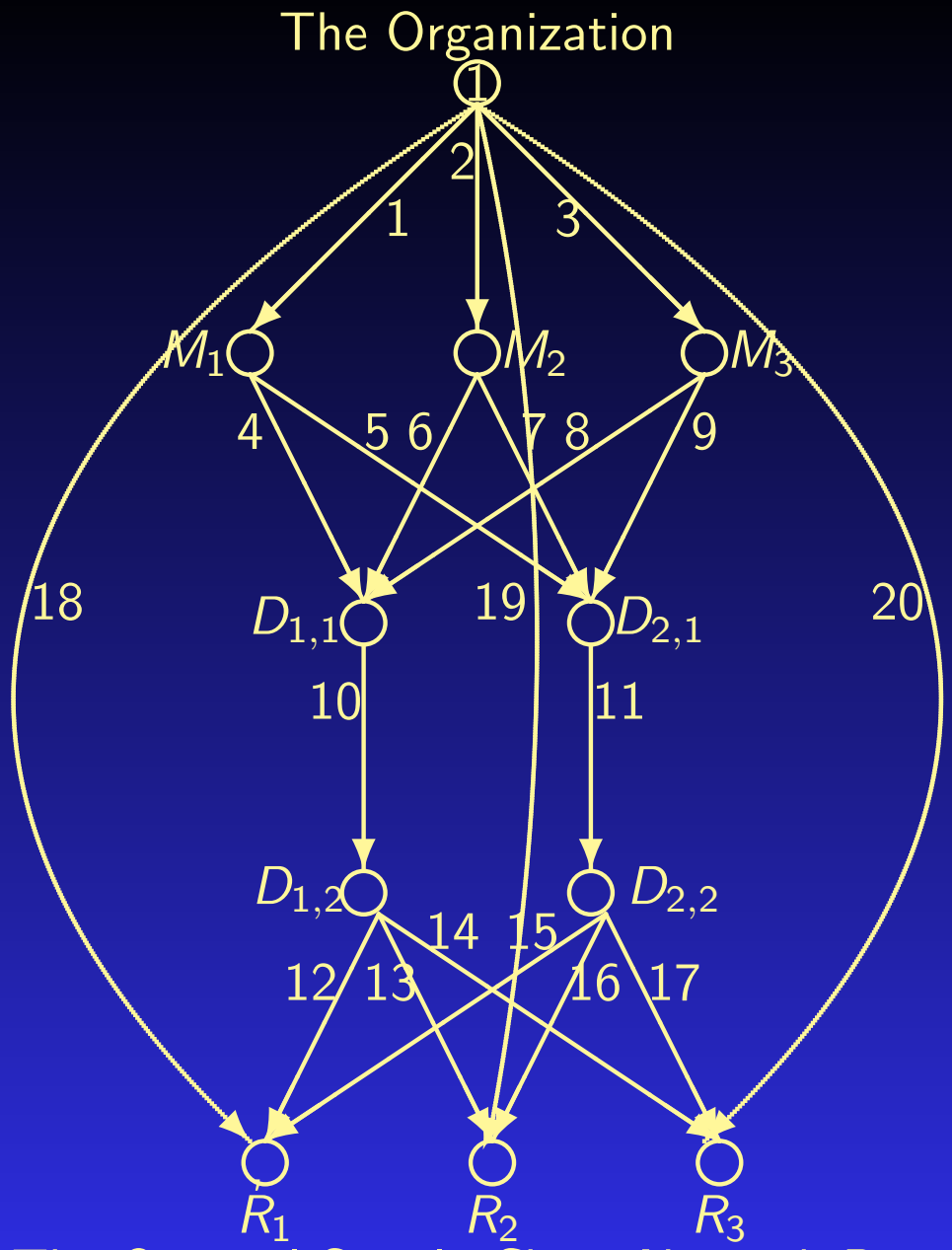


Figure 10: The Optimal Supply Chain Network Design for Example 2

Example 3

Example 3 had the same data as Example 2 except that we changed the probability distributions so that we now had:

$$P_1(\sum_{p \in P_{w_1}} x_p) = \frac{\sum_{p \in P_{w_1}} x_p}{110},$$

$$P_2(\sum_{p \in P_{w_2}} x_p) = \frac{\sum_{p \in P_{w_2}} x_p}{120},$$

$$P_3(\sum_{p \in P_{w_3}} x_p) = \frac{\sum_{p \in P_{w_3}} x_p}{130}.$$

Table 5: Total Cost Functions and Solution for Example 3

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
1	$f_1^2 + 2f_1$	$.5u_1^2 + u_1$	4.23	0.00	0.00	–
2	$.5f_2^2 + f_2$	$.5u_2^2 + u_2$	9.06	0.00	0.00	–
3	$.5f_3^2 + f_3$	$.5u_3^2 + u_3$	8.61	0.00	0.00	–
4	$1.5f_4^2 + 2f_4$	$.5u_4^2 + u_4$	2.05	0.00	0.00	–
5	$f_5^2 + 3f_5$	$.5u_5^2 + u_5$	2.18	0.00	0.00	–
6	$f_6^2 + 2f_6$	$.5u_6^2 + u_6$	3.28	0.00	0.00	–
7	$.5f_7^2 + 2f_7$	$.5u_7^2 + u_7$	5.77	0.00	0.00	–
8	$.5f_8^2 + 2f_8$	$.5u_8^2 + u_8$	7.01	0.00	0.00	–
9	$f_9^2 + 5f_9$	$.5u_9^2 + u_9$	1.61	0.00	0.00	–
10	$.5f_{10}^2 + 2f_{10}$	$.5u_{10}^2 + u_{10}$	12.34	2.34	3.34	–
11	$f_{11}^2 + f_{11}$	$.5u_{11}^2 + u_{11}$	9.56	0.00	0.00	–
12	$.5f_{12}^2 + 2f_{12}$	$.5u_{12}^2 + u_{12}$	5.82	0.00	0.00	–
13	$.5f_{13}^2 + 5f_{13}$	$.5u_{13}^2 + u_{13}$	2.38	0.00	0.00	–

Table 6: Total Cost Functions and Solution for Example 3 (continued)

Link a	$\hat{c}_a(f_a)$	$\hat{\pi}_a(u_a)$	f_a^*	u_a^*	ω_a^*	γ_a^*
14	f_{14}^2	$.5u_{14}^2 + u_{14}$	4.14	0.00	0.00	–
15	$f_{15}^2 + 2f_{15}$	$.5u_{15}^2 + u_{15}$	2.09	0.00	0.00	–
16	$.5f_{16}^2 + 3f_{16}$	$.5u_{16}^2 + u_{16}$	2.75	0.00	0.00	–
17	$.5f_{17}^2 + 2f_{17}$	$.5u_{17}^2 + u_{17}$	4.72	0.00	0.00	–
18	$10f_{18}$	–	5.00	–	–	34.13
19	$12f_{19}$	–	10.00	–	–	31.70
20	$15f_{20}$	–	5.00	–	–	29.66

The optimal supply chain network design for Example 3 has the initial topology since there are now positive flows on all the links. It is also interesting to note that there is a significant increase in production volumes by the organization at its manufacturing plants.

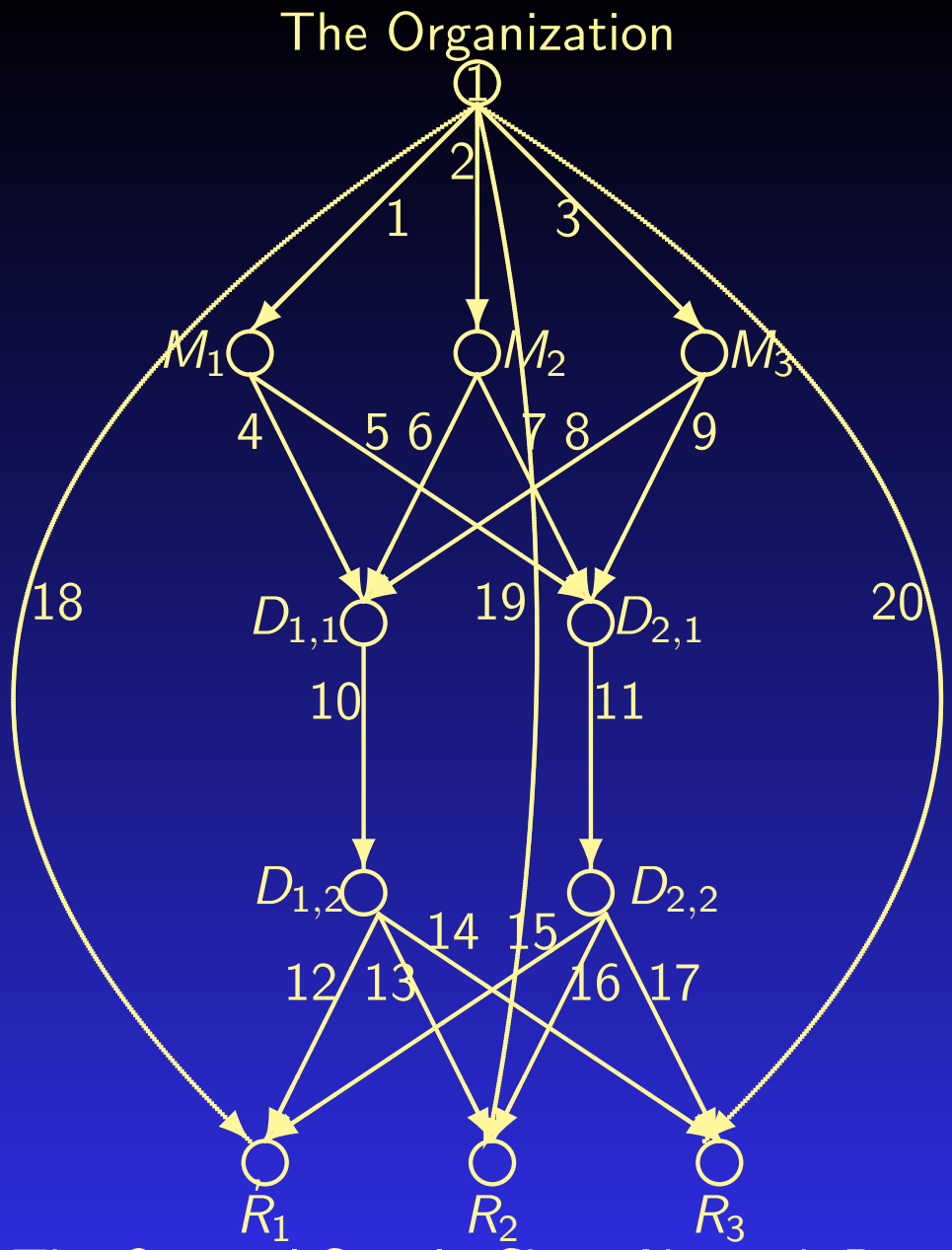


Figure 11: The Optimal Supply Chain Network Design for Example 3

Extensions to Perishable Products

and

Blood Supply Chains for the Red Cross

A. Nagurney, A. Masoumi, and M. Yu (2010) "Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization."

- Over 39,000 donations are needed everyday in the United States, and the blood supply is frequently reported to be just 2 days away from running out (American Red Cross (2010)).
- Hospitals with as many days of surgical delays due to blood shortage as 120 a year have been observed (Whitaker et al. (2007)).
- The national estimate for the number of units blood products outdated by blood centers and hospitals was 1,276,000 out of 15,688,000 units (Whitaker et al. (2007)).

The American Red Cross is the major supplier of blood products to hospitals and medical centers satisfying over 45% of the demand for blood components nationally (Walker (2010)).

Supply Chain Network Topology for a Regionalized Blood Bank

ARC Regional Division

Blood Collection Sites

Blood Centers

Component Labs

Storage Facilities

Distribution Centers

Demand Points

We developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of a perishable product – that of human blood.

Novel features of the model include:

- It captures *perishability of this life-saving product* through the use of arc multipliers;
- It contains *discarding costs* associated with waste/disposal;
- It handles *uncertainty* associated with demand points;
- It assesses *costs associated with shortages/surpluses at the demand points*, and
- It quantifies the *supply-side risk* associated with procurement.

What About Sustainability

and

Supply Chain Network Design?

Sustainability of supply chains has also emerged as a major theme in both research and practice since the impacts of climate change have made both producers and consumers more cognizant of their decision-making and how their decisions affect the environment.

Photos of oil spill crisis in Gulf of Mexico, May 2010

Recent results in this dimension:

"Sustainable Supply Chain Network Design: A Multicriteria Perspective,"

A. Nagurney, and L. S. Nagurney, *International Journal of Sustainable Engineering* (2010) **3**, 189-197.

Summary, Conclusions,

and

Suggestions for Future Research

- We emphasized the *importance of capturing behavior* in supply chain modeling, analysis, and design.
- We discussed a *variety of network design approaches*: the addition of links; the integration of networks as in mergers and acquisitions; and the design from scratch (and redesign).
- We developed an *integrated framework for the design of supply chain networks for critical products* with outsourcing.
- The model utilizes cost minimization within a system-optimization perspective as the primary objective and captures rigorously the uncertainty associated with the demand for critical products at the various demand points.

- The supply chain network design model allows for the investment of enhanced link capacities and the investigation of whether the product should be outsourced or not.

- The framework *can be applied in numerous situations* in which the goal is to produce and deliver a critical product at minimal cost so as to satisfy the demand at various demand points, as closely as possible, given associated penalties for under- and over-supply.

- We noted the *importance of sustainable supply chain network design* but presented only highlights, due to time limitations.

- Our recent research in supply chain network design has also considered oligopolistic markets.
- In addition, we have been heaving involved in *constructing mathematical models that capture the impacts of foreign exchange risk and competition intensity* on supply chain companies who are involved in offshore outsourcing activities.
- Our research in supply chains has also led us to other time-sensitive products, such as *fast fashion*, and
- Finally, we are now working on modeling disequilibrium dynamics and equilibrium states in ecological predator-prey networks, that is, supply chains in nature.

- We expect that future research will include design for robustness and resiliency.

- Some recent research that we have begun in this direction: "Modeling of Supply Chain Risk Under Disruptions with Performance Measurement and Robustness Analysis," Q. Qiang, A. Nagurney, and J. Dong (2009), in *Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers*, T. Wu and J. Blackhurst, Editors, Springer, London, England, 91-111.

THANK YOU!

For more information, see: http://supernet.som.umass.edu