

# Refugee Migration Networks with Regulations

**Anna Nagurney**

**Eugene M. Isenberg Chair in Integrative Studies  
Director – Virtual Center for Supernetworks  
Isenberg School of Management  
University of Massachusetts Amherst**

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# Acknowledgments

**Thanks to Dr. Geri Louise Dimas, Dr. Daniel Reichman, and Dr. Andrew C. Trapp for organizing this important conference!**



**The lifting of Title 42 on May 11, 2023 is likely to spur a significant increase in the number of migrants trying to cross into the United States and, hence, this conference is presciently timed.**

# Outline of Presentation

- **Motivation and Some Background**
- **Human Migration and Refugees**
- **The Refugee Migration Models and Variational Inequality Formulations**
- **Some Additional Research**
- **Summary and Conclusions**

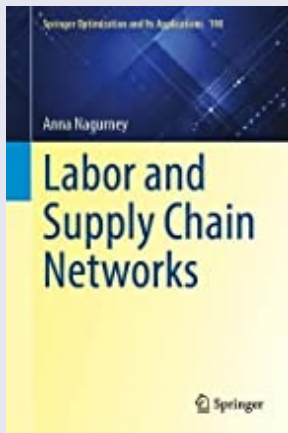
# Motivation and Some Background

# I Work on the Modeling of Network Systems



# Some of My Books





# It's All About People

A major research theme of ours in the COVID-19 pandemic has been the inclusion of labor in supply chains, using optimization and game theory, as well as expanding the scope of human migration networks. Such research is also very relevant to Russia's war against Ukraine with the immense refugee crisis.

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NEWS FEATURES PODCASTS

January 29, 2021 in Supply Chain Networks

## In the End, It's All About People

*COVID-19 vaccine production reveals dependency on supply chains, labor workforce in the U.S.*

By Anna Nagurney

SHARE: [f](#) [in](#) [t](#) PRINT ARTICLE: [🖨](#) <https://doi.org/10.1287/orms.2021.01.17>

A photograph showing several healthcare workers in full personal protective equipment (PPE), including blue gowns, masks, and face shields, working in a clinical setting. They are huddled together, possibly performing a procedure or examining a patient.

The COVID-19 pandemic has dramatically revealed how dependent we are on supply chains and the availability of labor. Without the human element, meatpacking plants cannot function; fresh produce cannot be picked; grocery stores cannot be shelved; PPEs cannot be produced and distributed; and products cannot be delivered to our homes through e-commerce. Also, COVID-19 vaccine production may lack the human resources to ensure product quality and efficacy as well as its distribution and ultimate administration into our arms. Without healthcare workers to



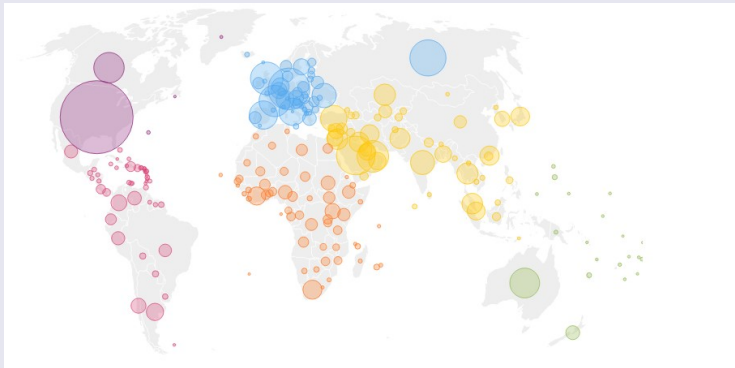
# Human Migration and Refugees

# Human Migration and Refugees

- **Reasons for human migration are numerous**, from individuals **seeking better economic opportunities and enhanced prosperity for themselves and their families**, to **those fleeing conflict, violence, and persecution**. With climate change and the increasing number and severity of disasters, including hurricanes, floods, tornados, earthquakes, etc., some migrants are seeking locations of greater expected safety and security.
- **The current global estimate is that there were around 281 million international migrants in the world in 2020, about 3.6% of the global population. This is over three times the estimated number in 1970!**
- **The number of international migrants is growing faster than the global population.**

# International Migrants

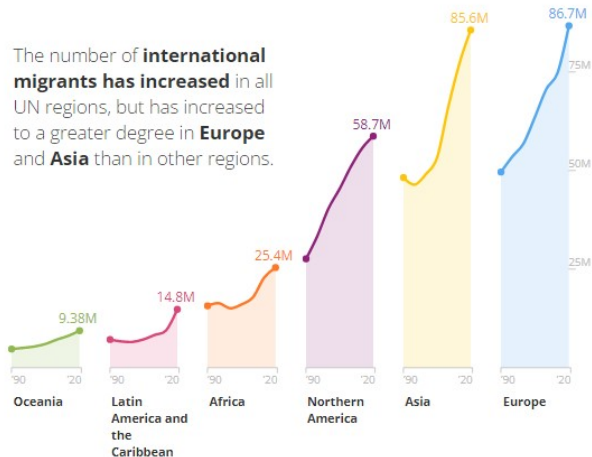
**The total number of international migrants within each country:**



**United Nations (2021)**

# International Migrants

The number of **international migrants** has increased in all UN regions, but has increased to a greater degree in **Europe** and **Asia** than in other regions.



UN DESA 2021.

United Nations (2021)

# Definition of a Refugee

**The United Nations Convention Relating to the Status of Refugees in 1951 defined a refugee as an individual living outside his or her country of nationality, who is unable or unwilling to return because of a well-substantiated fear of persecution due to race, religion, nationality, membership in a political social group, etc.**

**In our research, we also consider humans adversely affected by climate change, as refugees,** and note that, as emphasized by Hebert, Perez, and Harati (2018), among the most studied causes of human migration are climate issues and conflicts, as well as economic reasons.

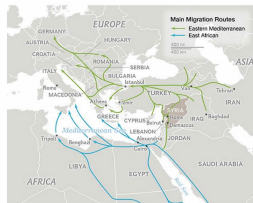
# Refugees and the Pandemic

Refugees have historically always been part of human migration, seeking locations of greater safety and security for themselves and their families.

With the COVID-19 pandemic, declared by the World Health Organization on March 11, 2020, Dolmans et al. (2020) reported that COVID-19 likely exacerbated refugee flows and increased the difficulty in seeking asylum due to measures imposed by governments in response to the pandemic.

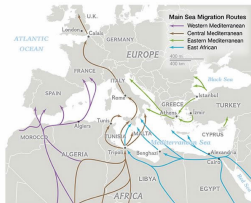
# Some Migration Routes

**Eastern Mediterranean Route**



NG-SDNY  
SOURCE: MISSING MIGRANTS PROJECT, INTERNATIONAL ORGANIZATION FOR MIGRATION, UNHCR, I.M.P.  
REGIONAL MIXED MIGRATION SECRETARIAT

**Mediterranean Sea Route**



NG-SDNY  
SOURCE: MISSING MIGRANTS PROJECT, INTERNATIONAL ORGANIZATION FOR MIGRATION, UNHCR, I.M.P.  
REGIONAL MIXED MIGRATION SECRETARIAT

**Central American Route**



NG-SDNY  
SOURCE: MISSING MIGRANTS PROJECT, INTERNATIONAL ORGANIZATION FOR MIGRATION

**Southeast Asian Route**



NG-SDNY  
SOURCE: MISSING MIGRANTS PROJECT, INTERNATIONAL ORGANIZATION FOR MIGRATION

Source: National Geographic via IOM UN Migration Blog - 2015 data

# The Venezuelan Migration Crisis

More than seven million Venezuelans have left their homeland since 2015 amid an ongoing economic and political crisis, according to recent United Nations data.



Source: United Nations, September 2022

B B C

Navigation icons: back, forward, search, and other controls.



# The Largest Refugee Crisis Since World War II

- Russia's invasion of Ukraine on February 24, 2022, has resulted in the largest refugee crisis since World War II.
- In October 2022, the United Nations listed 7.6 million refugees across Europe from Ukraine.
- In all, nearly one-third of Ukrainians have been displaced. About 13 million are stranded in Ukraine due to fighting, impassable routes, or the lack of resources to move.



- About half of the refugees are children.

# Refugees from Ukraine



## UKRAINE SITUATION Refugees from Ukraine across Europe (as of 23 June 2022)

### SITUATION OVERVIEW

The majority of refugees from Ukraine initially fled to countries in the immediate vicinity. However, border policies applicable to Ukrainian nationals have allowed refugees to travel. Refugees may choose particular destination countries. Others have decided to stay closer to home, waiting for the security situation to improve.

### KEY FIGURES

**5.3M**  
Individual refugees from Ukraine recorded across Europe<sup>1</sup>

**8.0M**  
Border crossings from Ukraine<sup>2</sup>

**3.5M**  
Refugees from Ukraine registered for Temporary Protection or similar national protection schemes in Europe<sup>3</sup>

**2.8M**  
Border crossings to Ukraine<sup>4</sup>

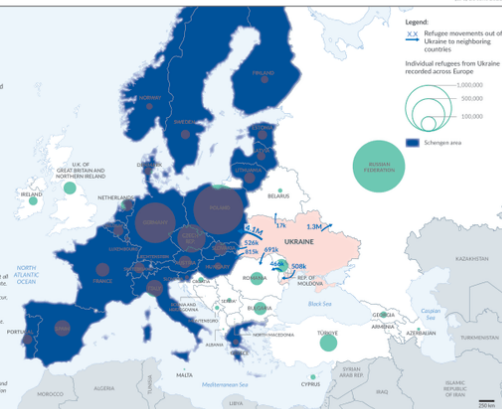
Statistics are compiled mainly from data provided by authorities. While every effort has been made to ensure that all statistical information is verified, figures represent an estimate. Triangulation of information and sources is performed on a continuous basis. Therefore, amendments to figures may occur, including retroactively.

<sup>1</sup> Estimate based on most recent data available as of 23 June.

<sup>2</sup> This figure reflects cross-border movements (and not individuals). An additional 103,000 people moved to the Russian Federation from the Donetsk and Luhansk regions between 18 and 23 February.

<sup>3</sup> This figure reflects cross-border movements (and not individuals). Movements back to Ukraine may be pendular, and do not necessarily indicate sustainable returns as the situation across Ukraine remains highly volatile and unpredictable.

Creation date: 23 June 2022 Sources: UNHCR



The boundaries and names shown and the designations used on this map do not imply official endorsement or acceptance by the United Nations.  
\*Border and Name (S-RE3-1244) (2019)

Source: UNHCR, June 27, 2022

# Human Migrants and Refugees

**Vivid depictions of people fleeing their origin locations permeate the news**, whether attempting to escape the great strife and suffering in Syria; the violence in parts of Central America, the economic collapse of Venezuela, and even flooding in parts of Asia as well as droughts in parts of Africa.



# Refugees

**At times, refugees will travel in extremely dangerous conditions to escape the dire circumstances at their origin nodes.**



In 2015, the UN Refugee Agency reported a maritime refugee crisis with, in the first half of that year, 137,000 refugees crossing the Mediterranean Sea to Europe, via very risky transport modes, and with many more unsuccessfully attempting such a passage. 800 died in the largest refugee shipwreck on record that April.

# Refugees from Afghanistan

**Prior to Russia's war on Ukraine, the largest number of refugees were from Syria, Venezuela, and Afghanistan.**



Afghan evacuees boarding American aircraft during Operation Allies Refuge in August 2021.

**Governments of various nations, hence, are increasingly being faced with multiple challenges associated with human migration flows. In response to challenges, they are adopting different regulations.**

According to the United Nations (2013), **migration policies in both origin and destination countries play an important role in determining the migratory flows**. In managing international migration flows, governments usually focus on different types of migrants, of which the most salient are highly skilled workers, dependents of migrant workers, irregular migrants, and refugees and asylum seekers (cf. Karagiannis (2016)).

**Between 11 March 2020, when the WHO declared COVID-19 a pandemic, and 22 February 2021, nearly 105,000 movement restrictions were implemented around the world, according to the International Organization for Migration.**

We can expect refugee migratory flows to continue to increase, posing a critical need for the provision of rigorous tools for policy-makers and decision-makers for the quantification of refugee migratory flows and the impacts of various regulations.

The construction of a relevant model with policy implications was one of our goals.

# The Refugee Migration Models and Variational Inequality Formulations



This part of my presentation is based on the paper:

**A. Nagurney, P. Daniele, and L.S. Nagurney, “Refugee Migration Networks and Regulations: A Multiclass, Multipath Variational Inequality Framework,” *Journal of Global Optimization* 78 (2020), pp 627-649.**

Journal of Global Optimization  
<https://doi.org/10.1007/s10898-020-08936-6>



**Refugee migration networks and regulations: a multiclass, multipath variational inequality framework**

Anna Nagurney<sup>1</sup> · Patrizia Daniele<sup>2</sup> · Ladimer S. Nagurney<sup>3</sup>

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**Abstract**

In this paper, we take up the timely topic of the modeling, analysis, and solution of refugee migration networks. We construct a general, multiclass, multipath model, determine the governing equilibrium conditions, and provide alternative variational inequality formulations in path flows and in link flows. We also demonstrate how governmental imposed regulations associated with refugees can be captured via constraints. We provide qualitative properties and then establish, via a supernetwork transformation, that the model(s) are isomorphic to traffic network equilibrium models with fixed demands. Illustrative examples are given, along with numerical examples, inspired by a refugee crisis from Mexico to the United States, which are solved using the Euler method embedded with exact equilibration. The work sets the foundation for the development of additional models and algorithms and also provides insights as to who wins and who loses under certain refugee regulations.

**Keywords** Refugees · Human migration · Networks · Variational inequalities · Regulations

**1 Introduction**

Refugees have historically always been part of human migration, seeking locations of greater safety and security for themselves and their families. Now, with the increasing impacts of climate change, and associated increases in the severity, as well as the frequency, of many disasters, including hurricanes and floods, coupled with numerous conflicts, strife, and violence in many parts of the globe, the number of refugees and asylum seekers is escalating. According to the United Nations [52], the number of international migrants was approximately 258 million (3.4% of the global population) in the year 2017, with the total number of international migrants increasing by almost 50% since 2000 (United Nations [52]). Also,

✉ Anna Nagurney  
[nagurney@isee.engg.umn.edu](mailto:nagurney@isee.engg.umn.edu)

<sup>1</sup> Department of Operations and Information Management, Leeborg School of Management, University of Massachusetts, Amherst, MA 01003, USA

# Literature Review with a Focus on Networks and Migration

- Nagurney (1989) introduced a multiclass migration equilibrium model, which did not include migration/movement costs, and **was isomorphic to a traffic network equilibrium with special structure**. The model was then extended to include flow-dependent migration costs and an expanded set of equilibrium conditions in Nagurney (1990).
- Nagurney, Pan, and Zhao (1992a) proposed a **multiclass human migration model**, which further generalized to include class transformations in Nagurney, Pan, and Zhao (1992b).
- Pan and Nagurney (1994) considered **chain migration (unlike the earlier work) and introduced a multi-stage (but single class) Markov chain model**. The authors established a connection between a sequence of variational inequalities and a non-homogeneous Markov chain. They also proved that, under certain assumptions, the stability of the one-step transition matrix guarantees the stability of the  $n$ -step transition matrix.

# Literature Review with a Focus on Networks and Migration

- Pan and Nagurney (2006) utilized **evolution variational inequalities for the first time to model the dynamic adjustment of a socio-economic process in the context of human migration**. Convergence of algorithms in this framework, which is infinite-dimensional, was addressed (see Daniele (2006)).
- Interestingly, many of the network equilibrium models of human migration have **found application to the migration of animals in ecology with a focus on fish and maritime ecosystems** (see Mullon and Nagurney (2012), Mullon (2014), Mariani et al. (2016)).
- Kalashnikov et al. (2008) constructed a human migration model with a **conjectural variations equilibrium (CVE)**.
- Capello and Daniele (2019) developed a **Nash equilibrium model of human migration** with features of CV and provided examples on the flow of migrants from Africa through the Mediterranean sea to Italy in 2018.

# Literature Review with a Focus on Networks and Migration

The paper by Nagurney and Daniele (2020) was the first to include regulations within a human migration network framework.



## Human migration networks and policy interventions: bringing population distributions in line with system optimization

Anna Nagurney<sup>a</sup>, Patricia Daniele<sup>a,\*</sup> and Giorgio Cappelletti<sup>b</sup>

<sup>a</sup>Department of Operations and Information Management, Bentley School of Management, University of Massachusetts, Amherst, MA 01003, USA

<sup>b</sup>Department of Mathematics and Computer Science, University of Catania, Catania, CT 95131, Italy  
(Email: [andanie@unict.it](mailto:andanie@unict.it))

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### Abstract

In this paper, we demonstrate that, through policy interventions, in the form of subsidies, a system-optimist for a multiclass human migration network can be achieved, despite the migrants, who can be refugees, behaving in a user-optimist manner. The formulation and analysis are conducted using variational inequality theory. The policy intervention allows governmental decision-makers to moderate the flow of migrants while obtaining societal welfare. An algorithm is proposed and applied to compare the solution to a series of numerical examples, with changes in initial populations and utility functions, inspired by a pandemic, followed by a natural disaster.

**Keywords:** human migration networks; variational inequality; policy intervention; system-optimization; user optimization; subsidies

### 1. Introduction

Massive human migrations are posing major challenges to national governments across the globe. The reasons for recent migrations include violence, wars, and persecution, climate change, a variety of disasters (earthquakes, hurricanes, floods, and tornadoes), and poverty and economic inequality, with the latter driving humans to seek better lives for their families and themselves. According to the UNHCR (2020), 70.8 million have fled their home worldwide, the highest level of displacement on record. The United Nations (2017) is reporting that the number of international migrants was an estimated 281 million individuals in 2017, with the total number of international migrants increasing by almost 50% since the year 2000. The number of refugees and asylum seekers during this period has increased from 16 to 26 million, approximately 10% of the international migrants.

The media has been filled with news and images of migrants, including refugees, often undertaking dangerous journeys on land and sea to flee their compromised situations. The economic

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## Modeling of Covid-19 trade measures on essential products: a multiproduct, multicountry spatial price equilibrium framework

Anna Nagurney<sup>a,\*</sup>, Mojtaba Salarpour<sup>b</sup> and Jane Dong<sup>c</sup>

<sup>a</sup>Department of Operations and Information Management, Bentley School of Management, University of Massachusetts, Amherst, Amherst, MA, USA

<sup>b</sup>Department of Marketing and Management, School of Business, State University of New York, Oswego, NY, USA  
(Email: [nagurney@bentley.umass.edu](mailto:nagurney@bentley.umass.edu))

<sup>c</sup>Department of Marketing and Management, School of Business, State University of New York, Oswego, NY, USA  
(Email: [nsalarpour@bentley.umass.edu](mailto:nsalarpour@bentley.umass.edu))

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### Abstract

In this paper, we develop a unified variational inequality framework in the context of spatial price network equilibrium problems that handles multiple products with multiple demand and supply markets in multiple countries as well as multiple transportation modes. The model incorporates a plethora of distinct trade measures, which is particularly important in the pandemic, as PPEs and other essential products are in high demand, but short in supply globally. In the model, product flows as well as prices at the supply markets and the demand markets in different countries are variables that allow us to accurately introduce various trade measures, including tariffs, quotas, as well as price floors and ceilings. Qualitative properties are analyzed. Numerical examples are provided to illustrate the impacts of the trade measures on equilibrium product path and link flows, and on prices, and demand and supply quantities. Given the relevance of the trade measures in the world today and discussions concerning the impacts, the framework constructed in this paper is especially timely.

**Keywords:** Covid-19; essential supplies; trade measures; spatial price equilibrium; networks

### 1. Introduction

The World Health Organization (WHO) declared the Covid-19 pandemic on March 11, 2020 (WHO, 2020a). The ensuing global health-care disaster has endangered and disrupted the lives of billions around the world, resulting in illness and deaths, and has also generated secondary crises. No one knows with certainty when the pandemic will end. According to Johns Hopkins

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# Contributions in Our Paper

1. An international human migration network model is constructed, which allows for route choices by the migrants, which are refugees.
2. The routes consist of one or more links, with cost functions that capture congestion, a factor that has been seen in practice.
3. The model is then extended to include regulations that can be imposed by distinct multiple countries. In previous work (cf. Nagurney and Daniele (2020)), it was assumed that a single country imposes the regulations on migrants.
4. A supernetwork transformation into a traffic network equilibrium problem with fixed demands is constructed. This identification enables the transfer of algorithmic schemes for the TNE problem, which has had a long history, to the novel application domain of refugee/migration networks.
5. Theoretical results are presented plus an algorithm.
6. Numerical examples reveal insights for policy-makers and decision-makers.

# The Multiclass, Multipath Refugee Migration Models

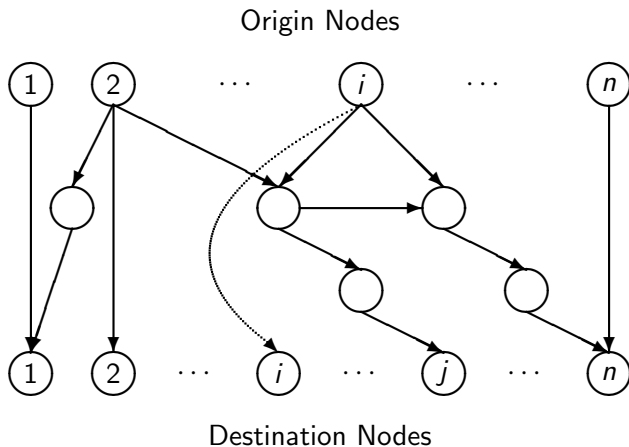


Figure: Sample Refugee Network Topology

**Table:** Common Notation for the Refugee Migration Models

Notation	Definition
$x_r^k$	flow of refugees of class $k$ on route/path $r$ . The $\{x_r^k\}$ elements are grouped into vector $x^k \in R_+^{n_P}$ , where $n_P$ denotes the number of paths in the migration network. We further group the $x^k$ vectors; $k = 1, \dots, J$ , into vector $x \in R_+^{Jn_P}$ .
$f_a^k$	flow of refugees of class $k$ on link $a$ . We group the link flows for class $k$ for all links $a \in L$ into vector $f^k \in R^{n_L}$ where $n_L$ is the number of links. We then group the link flows for all classes into vector $f \in R^{Jn_P}$ .
$p_i^k$	nonnegative population of refugee class $k$ at origin node $i$ . We group the populations of class $k$ ; $k = 1, \dots, J$ , into vector $p^k \in R_+^n$ . We further group all such vectors into vector $p \in R_+^{Jn}$ .
$\bar{p}_i^k$	initial fixed population of class $k$ at origin node $i$ ; $i = 1, \dots, n$ ; $k = 1, \dots, J$ .
$u_i^k(p)$	utility perceived by refugee class $k$ at node $i$ ; $i = 1, \dots, n$ ; $k = 1, \dots, J$ . We group the utility functions for each $k$ into vector $u^k \in R^n$ and then group all such vectors for all $k$ into vector $u \in R^{Jn}$ .
$c_a^k(f)$	migration cost associated with traversing link $a$ by refugees of class $k$ . Here we interpret the migration cost as a travel cost. We group link costs for each $k$ into vector $c^k \in R^{n_L}$ and then group all such vectors into vector $c \in R^{Jn_L}$ .
$C_r^k(x)$	cost of migration, that is, the travel cost, encumbered by class $k$ in migrating on route $r$ associated with an O/D pair $w_{ij}$ ; $i, j = 1, \dots, n$ ; $k = 1, \dots, J$ .



# Conservation of Flow

Since the route flows must be nonnegative, we have that

$$x_r^k \geq 0, \quad \forall r \in P, \forall k. \quad (1)$$

Furthermore, the refugee flows out of an origin node  $i$  must satisfy:

$$\bar{p}_i^k = \sum_{j=1}^n \sum_{r \in P_{wij}} x_r^k, \quad \forall i, \forall k. \quad (2)$$

The volume of population of each class  $k$  at each destination node  $j$ , after migration takes place, must satisfy the following equation:

$$p_j^k = \sum_{i=1}^n \sum_{r \in P_{wij}} x_r^k, \quad \forall j, \forall k. \quad (3)$$

The link flows are related to the route flows according to:

$$f_a^k = \sum_{r \in P} x_r^k \delta_{ar}, \quad \forall a, \forall k, \quad (4)$$

where  $\delta_{ar} = 1$ , if link  $a$  is contained in route  $r$  and 0, otherwise.

# Additional Constructs

In view of the conservation of flow equations (4), we may define link cost functions in route/path flows, such that  $\hat{c}_a^k = \hat{c}_a^k(x) \equiv c_a^k(f)$ , for all links  $a$  and for all classes of refugees  $k$ .

The cost on a route  $r$  is equal to the sum of costs on the links that make up the route, that is,

$$C_r^k(x) = \sum_{a \in L} \hat{c}_a^k(x) \delta_{ar}, \quad \forall k, \forall r. \quad (5)$$

We define the feasible set

$$K^1 \equiv \{(p, x) | x \geq 0, \text{ and (2) and (3) hold}\}.$$

# Equilibrium Conditions for the Multiclass, Multipath Refugee Migration Model without Regulations

## Definition 1: Multiclass, Multipath Refugee Migration Equilibrium without Regulations

A vector of populations and refugee migration flows  $(p^*, x^*) \in K^1$  is in equilibrium if it satisfies the following conditions: For each class  $k$ ;  $k = 1, \dots, J$ , and each pair of origin/destination nodes  $i, j$ ;  $i, j = 1, \dots, n$ , and all routes  $r \in P_{wij}$  we have that

$$u_i^k(p^*) + C_r^k(x^*) \begin{cases} = u_j^k(p^*) - \lambda_i^{k*}, & \text{if } x_r^{k*} > 0, \\ \geq u_j^k(p^*) - \lambda_i^{k*}, & \text{if } x_r^{k*} = 0, \end{cases} \quad (6)$$

and

$$\lambda_i^{k*} \begin{cases} \geq 0, & \text{if } \sum_{j=1}^n \sum_{r \in P_{wij}, j \neq i} x_r^{k*} = \bar{p}_i^k, \\ = 0, & \text{if } \sum_{j=1}^n \sum_{r \in P_{wij}, j \neq i} x_r^{k*} < \bar{p}_i^k. \end{cases} \quad (7)$$

# Variational Inequality Formulations

## Theorem 1: Variational Inequality Formulation of the Refugee Migration Model without Regulations in Path Flows

*A population and refugee flow pattern  $(p^*, x^*) \in K^1$  is a refugee migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem in path flows*

$$-\langle u(p^*), p - p^* \rangle + \langle C(x^*), x - x^* \rangle \geq 0, \quad \forall (p, x) \in K^1, \quad (8)$$

*where  $\langle \cdot, \cdot \rangle$  denotes the inner product in the appropriately dimensioned Euclidean space.*

Existence of a solution follows from the standard theory of variational inequalities (see Kinderlehrer and Stampacchia (1980) Theorem 3.1) under the assumption of continuity of the utility functions  $u$  and the migration cost functions  $c$ , since the feasible convex set  $K^1$  is compact.

# Variational Inequality Formulations

Alternative variational inequality formulations can induce distinct algorithmic schemes. Hence, for completeness, we now provide a link flow variational inequality formulation equivalent to the path flow one in (8). We first define the feasible set  $K^2 \equiv \{(p, f) | \exists x \text{ such that (1) – (4) hold}\}$ .

## Corollary 1: Variational Inequality Formulation of the Refugee Migration Model without Regulations in Link Flows

*A population and refugee link flow pattern  $(p^*, f^*) \in K^2$  is a refugee migration equilibrium without regulations according to Definition 1, if and only if it satisfies the variational inequality problem in link flows*

$$-\langle u(p^*), p - p^* \rangle + \langle c(f^*), f - f^* \rangle \geq 0, \quad \forall (p, f) \in K^2. \quad (9)$$

# Variational Inequality Formulation of the Refugee Migration Model with Regulations

We denote a specific country by  $h$ , where  $h = 1, \dots, H$  and define the set  $O^h$  consisting of origin nodes of refugees from countries/locations that the country  $h$  imposes a regulation on, and let  $D^h$  denote the set of destination nodes, which lie in country  $h$ .  $C^h$  denotes the set of refugee classes that country  $h$  imposes the regulations on and  $U^h$  is the nonnegative upper bound imposed by country  $h$  on refugee migratory flows.

The constraints can then be stated as follows:

$$\sum_{i \in O^h} \sum_{j \in D^h} \sum_{k \in C^h} \sum_{r \in P_{wij}} x_r^k \leq U^h, \quad h = 1, \dots, H. \quad (10)$$

**The set of constraints (10) is sufficiently general to capture specific, distinct migration regulations in practice. such as: an upper bound (which may be zero) of all classes from a certain country or countries; or an upper bound on a single class or several classes from a specific country or countries.**

# Variational Inequality Formulation of the Refugee Migration Model with Regulations

For the refugee migration model with regulations, the equilibrium conditions (6) and (7) are still relevant but with a new feasible set  $K^3$  defined as below to include the constraints (10):

$$K^3 \equiv K^1 \cap \{x | (10) \text{ is satisfied}\}. \quad (11)$$

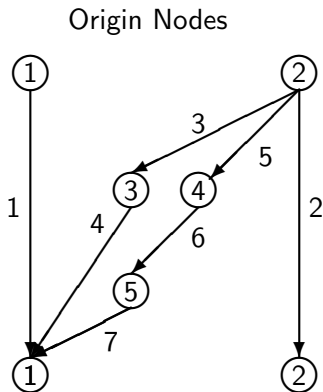
## Theorem 2: Variational Inequality Formulation of the Refugee Migration Model with Regulations in Path Flows

*A population and refugee migration flow pattern  $(p^*, x^*) \in K^3$  is a refugee migration equilibrium with regulations in path flows, if and only if it satisfies the variational inequality problem*

$$-\langle u(p^*), p - p^* \rangle + \langle C(x^*), x - x^* \rangle \geq 0, \quad \forall (p, x) \in K^3. \quad (12)$$

# Illustrative Examples

According to the network in the Figure below, refugees residing in country 1 are not interested in migrating to country 2. On the other hand, refugees residing in country 2 are interested in the possibility of migrating to country 1. There are two available paths joining country 2 with country 1.





# Illustrative Example - No Regulation

The routes are comprised of links and are enumerated as follows:

$$r_1 = (1), \quad r_2 = (2), \quad r_3 = (3, 4), \quad r_4 = (5, 6, 7).$$

We consider a single class of migrant. Hence, we suppress the superscript 1 in the notation. The data are:  $\bar{p}_1 = 100$  and  $\bar{p}_2 = 200$  with the utility functions:  $u_1(p) = -p_1 + 1000$  and  $u_2(p) = -p_2 + 500$ .

The link migration cost functions are:

$$c_1 = c_2 = 0,$$

$$c_3(f) = f_3 + 200, \quad c_4(f) = f_4 + 100,$$

$$c_5(f) = f_5 + 30, \quad c_6(f) = .5f_6 + 40, \quad c_7(f) = f_7 + 30.$$

# Illustrative Example - No Regulation

We first consider the case without regulations. It is easy to compute the equilibrium solution, using simple algebra. Indeed, we find that:

$$x_{r_1}^* = 100, \quad x_{r_2}^* = 75, \quad x_{r_3}^* = 25, \quad x_{r_4}^* = 100;$$

hence,

$$p_1^* = 225, \quad p_2^* = 75,$$

with associated utilities being:

$$\hat{u}_1(x^*) = u_1(p^*) = 775, \quad \hat{u}_2(x^*) = u_2(p^*) = 425,$$

and the incurred migration costs on the routes at equilibrium:  
 $C_{r_1} = 0, C_{r_2} = 0, C_{r_3} = C_{r_4} = 350$ . Moreover,  $\lambda_1^* = \lambda_2^* = 0$ .

**It is clear that the equilibrium conditions (6) and (7) hold.**

# Illustrative Example - No Regulation

Observe that, initially, before the refugee migration takes place,  $u_1(\bar{p}_1) = 900$  and  $u_2(\bar{p}_2) = 300$ .

Once equilibrium is achieved, those who have migrated from country 2 to country 1 more than double their utility (from 300 to 775), whereas those who remain in country 2 experience a gain in utility of over 33% (from 300 to 425).

Those in country 1, because of the increase in the number of refugees and that they are not migrating, suffer a reduction in utility of approximately 14% (from 900 to 775).

# Illustrative Example with Regulation

We now suppose that a regulation is imposed on destination node 1 by country 1 of the following form:

$$x_{r_3} + x_{r_4} \leq U^1 = 25.$$

The new equilibrium solution is:

$$x_{r_1}^* = 100, \quad x_{r_2}^* = 175, \quad x_{r_3}^* = 0, \quad x_{r_4}^* = 25;$$

hence,

$$p_1^* = 125, \quad p_2^* = 175,$$

with associated utilities being:

$$\hat{u}_1(x^*) = u_1(p^*) = 875, \quad \hat{u}_2(x^*) = u_2(p^*) = 325,$$

and the migration costs on the routes:  $C_{r_1} = 0$ ,  $C_{r_2} = 0$ ,  $C_{r_3} = 300$ ,  $C_{r_4} = 162.50$ . Moreover,  $\lambda_1^* = \lambda_2^* = 0$ . The optimal Lagrange multiplier associated with the regulation constraint is:  $\mu_1^* = 387.50$ . One can see that route  $r_3$  is too expensive and will not be used under the refugee migratory flow pattern.

# Illustrative Example with Regulation

Now, refugees in country 1, at equilibrium, enjoy a higher utility of 875 than before the regulation was imposed (775), an increase of about 13%.

On the other hand, refugees in country 2 now experience a lower utility (325), than before the regulation was imposed (425), a drop of about 30%. They are no longer all free to migrate because of the imposed regulatory upper bound of 25 limiting the migration from country 2 to country 1.

# Computation of Solutions to Larger Examples

**Our numerical examples are inspired by the refugee flows from Mexico to the United States, an issue that has been receiving a lot of attention in the press.**

The baseline network for the numerical examples is depicted in the next Figure.

**We consider a single class of refugee.**

# Computation of Solutions to Larger Examples

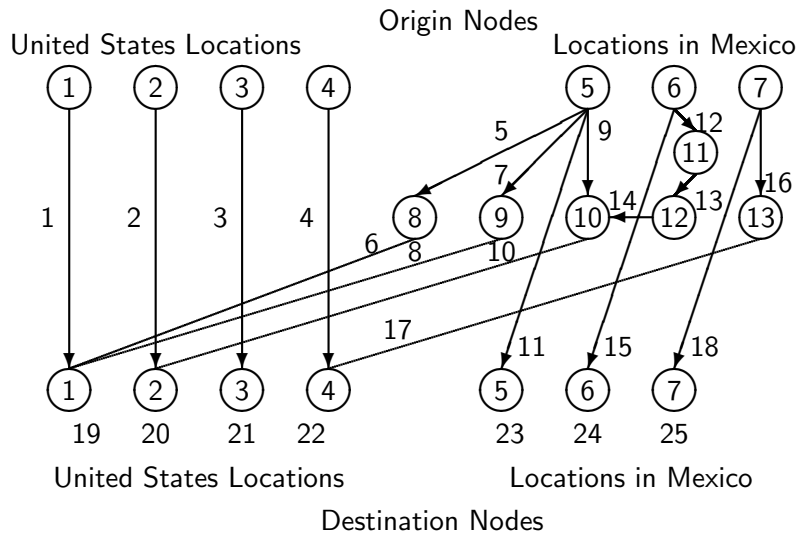


Figure: The Baseline Refugee Network Topology for the Larger Examples

# Computation of Solutions to Larger Examples

We consider four examples: Example 1 through Example 4.

## Example 1: No Regulations

The initial populations at the origin nodes are:

$$\bar{p}_1 = 1,400,000, \quad \bar{p}_2 = 20,000, \quad \bar{p}_3 = 70,000, \quad \bar{p}_4 = 260,000, \\ \bar{p}_5 = 50,000, \quad \bar{p}_6 = 225,000, \quad \bar{p}_7 = 30,000.$$

The utility functions associated with these locations are:

$$u_1(p) = -p_1 + 3,000,000, \quad u_2(p) = -2p_2 + 200,000, \\ u_3(p) = -p_3 + 1,500,000, \quad u_4(p) = 3p_4 + 900,000, \\ u_5(p) = -p_5 + 100,000, \quad u_6(p) = -p_6 + 300,000, \\ u_7(p) = -p_7 + 100,000.$$

From the above utility functions, one can see that the locations in the United States are more attractive than those in Mexico, due to the significantly larger fixed utility term in the corresponding utility functions.



# Computation of Solutions to Larger Examples

The link costs associated with remaining at one's location at nodes 1 through 7, respectively, are, all equal to 0.00:

$$c_1(f) = c_2(f) = c_3(f) = c_4(f) = c_{11}(f) = c_{15}(f) = c_{18}(f) = 0.00.$$

The costs associated with the refugee migrations are, in turn, as follows:

$$c_5(f) = .00006f_5^4 + 6f_5 + 4f_6 + 200, \quad c_6(f) = 7f_6 + 3f_8 + 300,$$

$$c_7(f) = .00008f_7^4 + 8f_7 + 2f_8 + 400,$$

$$c_8(f) = .00004f_8^4 + 5f_8 + 2f_{10} + 450, \quad c_9(f) = .00001f_9^4 + 6f_9 + 2f_{10} + 300,$$

$$c_{10}(f) = 4f_{10} + f_{12} + 400,$$

$$c_{12}(f) = 8f_{12} + 2f_{13} + 100, \quad c_{13}(f) = .00001f_{13}^4 + 7f_{13} + 3f_9 + 50,$$

$$c_{14}(f) = 8f_{14} + 3f_9 + 100,$$

$$c_{16}(f) = 3f_{16} + f_{12} + 100, \quad c_{17}(f) = .00003f_{17} + 3f_{17} + 50.$$

# Computation of Solutions to Larger Examples

The routes are enumerated as follows:

$$\begin{aligned}r_1 &= (1), & r_2 &= (2), & r_3 &= (3), & r_4 &= (4), \\r_5 &= (5, 6), & r_6 &= (7, 8), & r_7 &= (9, 10), & r_8 &= (11), \\r_9 &= (12, 13, 14, 10), & r_{10} &= (15), \\r_{11} &= (16, 17), & r_{12} &= (18).\end{aligned}$$

For the solution of the problem, we first construct the supernetwork equivalence with the supernetwork topology as in the Figure and the O/D pairs, the links and paths, the link costs on the new links, the demands, and the path costs, defined accordingly.

# Supernetwork Transformation of Examples

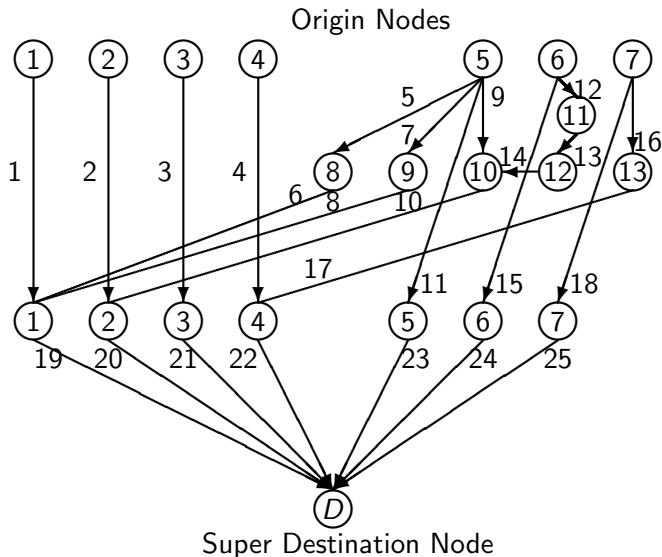


Figure: The Supernetwork Topology of the Examples

# Computation of Solutions to Larger Examples

We implemented the Euler method, embedded with the exact equilibration algorithm as described in Nagurney and Zhang (1997); see also the book by Nagurney and Zhang (1996). We initialized the algorithm as follows. The initial populations were equally distributed among all the paths. The sequence  $\{a_\tau\}$  in the Euler method satisfied the conditions required for convergence and was set to:  $.1\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots\}$ .

The algorithm was deemed to have converged if the absolute value of the difference between each successively computed path flow differed by no more than  $10^{-7}$ . The computer system utilized was a Linux system at the University of Massachusetts Amherst.

# Solution to Example 1

The computed equilibrium path flow pattern for Example 1 is:

$$\begin{aligned}x_{r_1}^* &= 1,400,000.00, x_{r_2}^* = 20,000.00, x_{r_3}^* = 700,000.00, \\x_{r_4}^* &= 260,000.00, x_{r_5}^* = 400.25, x_{r_6}^* = 336.64, x_{r_7}^* = 317.17, \\x_{r_8}^* &= 48,945.95, x_{r_9}^* = 290.25, x_{r_{10}}^* = 224,709.75, x_{r_{11}}^* = 199.54, \\x_{r_{12}}^* &= 29,800.46.\end{aligned}$$

The computed equilibrium populations at the four locations in the United States and at the three locations in Mexico are:

$$\begin{aligned}p_1^* &= 1,400,736.88, p_2^* = 20,607.42, p_3^* = 700,000.00, \\p_4^* &= 260,199.55, p_5^* = 48,945.95, p_6^* = 224,709.75, p_7^* = 29,800.46.\end{aligned}$$

# Solution to Example 1

The associated incurred utilities at the equilibrium at the locations are:

$$\begin{aligned}u_1(p^*) &= 1,599,263.13, \quad u_2(p^*) = 158,785.17, \quad u_3(p^*) = 800,000.00, \\u_4(p^*) &= 119,401.38, \quad u_5(p^*) = 51,054.05, \quad u_6(p^*) = 75,290.25, \\u_7(p^*) &= 70,199.55.\end{aligned}$$

We also, for completeness, report the path costs at the computed equilibrium flows since it is easy to then verify that the equilibrium conditions, as stated in Definition 1, hold. Specifically, the incurred path costs at the equilibrium are:

$$\begin{aligned}C_{r_1}(x^*) &= C_{r_2}(x^*) = C_{r_3}(x^*) = C_{r_4}(x^*) = 0.00, \\C_{r_5}(x^*) &= 1,548,207.50, \quad C_{r_6}(x^*) = 1,548,196.38, \quad C_{r_7}(x^*) = 107,729.39, \\C_{r_8}(x^*) &= 0.00, \quad C_{r_9}(x^*) = 83,500.69, \quad C_{r_{10}}(x^*) = 0.00, \\C_{r_{11}}(x^*) &= 49,200.66, \quad C_{r_{12}}(x^*) = 0.00.\end{aligned}$$

# Solutions to Examples 1 Through 4

**Examples 2 through 4 are analogues of Example 1, but with regulations.**

- In Example 2, we considered the following scenario: The US government has told the Mexican government that it is restricting the flow on route  $r_5 = (5, 6)$  to zero; essentially resulting in the elimination of this path, since its processing facilities are experiencing delays due to congestion. The rest of the data remain as in Example 1.
- In Example 3, we studied the following scenario: Route  $r_6 = (7, 8)$  is now unavailable to refugees, but the other routes and data remain as in Example 1.
- And, in Example 4, we investigate the scenario that the United States is concerned about the influx of refugees and both routes  $r_5$  and  $r_6$  from Mexico are, in effect, banned/eliminated.

# Solutions to Examples 1 Through 4

In order to enable cross comparisons with the examples with regulations and with the baseline Example 1, in the following Tables, we report, respectively, the associated computed equilibrium populations, whereas the incurred utilities at the locations are reported in the final Table.



# Solutions to Examples 1 Through 4

Node	Equilibrium Populations			
	Example 1	Example 2	Example 3	Example 4
1	1,400,736.88	1,400,336.63	1,400,400.38	1,400,000.00
2	20,607.42	20,607.72	20,607.67	20,607.67
3	700,000.00	700,000.00	700,000.00	700,000.00
4	260,199.55	260,199.55	260,199.55	260,199.55
5	48,945.95	49,345.84	49,282.21	49,682.27
6	224,709.75	224,709.75	224,709.75	224,709.75
7	29,800.46	224,709.75	224,709.75	29,800.46

# Solutions to Examples 1 Through 4

Node	Utility at Equilibrium			
	Example 1	Example 2	Example 3	Example 4
1	1,599,263.13	1,599.663.38	1,599,599.63	1,600,000.00
2	158,785.17	158,784.56	158,784.66	158,784.00
3	800,000.00	800,000.00	800,000.00	800,000.00
4	119,401.38	119,401.38	119,401.38	119,401.38
5	51,054.05	50,654.16	50,717.79	50,317.73
6	75,290.25	75,290.25	75,290.25	75,290.25
7	70,199.55	70,199.55	70,199.55	70,199.55

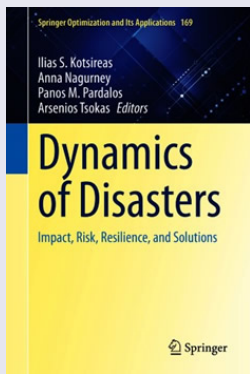
One can see, from the preceding Table, that (as occurred also in the illustrative examples), under the regulations (as in Examples 2 through 4), the utility of those subject to regulations, as for those in node 5, is reduced.

On the other hand, those in location 1, which now has a lower flow of refugees, experience a higher utility.

Also, with both refugee routes blocked to the US, the population that remains at node 5 is the highest in Example 4, as compared to the value in Examples 2 and 3.

## Some Additional Research

# System-Optimization versus User-Optimization



In our edited volume, we have a paper with Daniele and Cappello, "Capacitated Human Migration Networks and Subsidization."

This work demonstrates that one can make system-optimized solutions for human migration sustainable through the imposition of subsidies at locations - in this way, migrants' user-optimizing behavior with also be system-optimizing.

# Migrant Labor to Alleviate Labor Shortages

## A. Nagurney, "Attracting International Migrant Labor: Investment Optimization to Alleviate Supply Chain Labor," *Operations Research Perspectives* 9 (2022), 100233.



Operations Research Perspectives 9 (2022) 100233

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### Attracting international migrant labor: Investment optimization to alleviate supply chain labor shortages

Anna Nagurney

Department of Operations and Information Management, Isenberg School of Management, University of Massachusetts, Amherst, MA 01003, United States of America

#### ARTICLE INFO

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#### ABSTRACT

The COVID-19 pandemic has disrupted supply chains globally with a major disruption being that of labor shortages from production through distribution activities. In this paper, we introduce a new supply chain design optimization model that includes both domestic labor and cross-national migrant labor into decision scenarios, with the latter made possible through investments in attracting labor within a budget constraint. We allow for an arbitrary wage setting for domestic cross-national migration and also focus on the benefit of providing cross information as in the shape of expertise or not. No discrete national inequality limitations of the model along with qualitative properties and generic sub-optimalities for the public should have implications for the underlying problem variables at each location. The model is one of the very few national inequality constraints, namely models with nonlinear constraints. These sorts of experimentally solved non-linear examples, motivated by a high value agricultural product – that of coffee, demonstrate the impact in terms of profits, sales, product price flows and investments, with variations in the data including that of model and variable wages being used to attract migrant labor.

#### 1. Introduction

The COVID-19 pandemic has demonstrated the importance of supply chain and their effective and efficient operation, with disruptions adversely affecting product prices and deliveries, as well as the connectivity of companies and the health and well-being of consumers [1]. The extent for the disruption has been established with shocks both on the demand side as well as on the supply side and challenges associated with transport [2–4]. One of the major characteristics of the pandemic has been that of labor shortages. Workers throughout the pandemic have been falling ill, some, with have lost their lives, whereas others chose to switch jobs or to leave for labor force [5]. Furthermore, various countries imposed restrictions further expediting the flow of workers [6]. Even with vaccination continuing, the challenge of overcoming labor shortages throughout many supply chains remain [7]. Employees have had difficulties returning workers not only with advanced technical skills, but also those with low end middle level skills. Countries are increasingly seeking towards immigration policy to mitigate the labor shortage issues [8,9].

With the current COVID-19 pandemic [10,11], Attracting workers from other countries, who can leverage international migrant labor, may assist in alleviating labor shortages.

According to [12], noting the most recent estimate by the International Labor Organization, there are 104 million migrant laborers globally and, in many countries, they are a major proportion of the workforces, with contributions to economic both where they work and in terms of remittances to their countries of origin. Nevertheless, many migrant workers face inequality in terms of a wage gap (being paid less than the workers from the particular nation) among other discriminatory practices [13]. For example, migrant laborers in high income countries (OECD) earn, on the average, approximately 22% less than nationals, with the wages earned by migrants widening over the past 10 years or so to more than 30% [12]. Furthermore, employment is among the most negatively affected workers by the economic recovery due to the COVID-19 pandemic, in terms of job losses and a decrease in earnings for those who have been able to stay employed. This is happening despite the fact that the United Nations Sustainable Development Goals (SDGs), in the framework of the UN agenda for 2030, have as their targets 8.5 and 8.8, having equal pay for work of equal value and protected labor rights for all workers, including migrant workers [13,14].

This paper aims to integrate and advance two streams of literature, which have received significant attention in the pandemic: that of

E-mail address: [nagurney@iseng.umass.edu](mailto:nagurney@iseng.umass.edu).

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# Coalition Formation Among Countries

## M. Passacantando, F. Raciti, and A. Nagurney, “International Migrant Flows: Coalition Formation Among Countries and Social Welfare,” *EURO Journal on Computational Optimization* 11 (2023), 100062.




## Summary and Conclusions



# Summary and Conclusions

- In this presentation, I have highlighted some of our Operations Research contributions to address societal issues of human migration networks in the context of refugees as well related issues.
- The work is inspired by the need to include “people” in various network systems including supply chain networks.
- The world has benefited greatly from the developments in our profession of models, methodologies, associated insights, and relevance to both practice and policy making.
- Together we can continue to accomplish what needs to be done in these challenging times.


# Thank You Very Much!



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