A Variational Inequality Formulation of Equilibrium Models for End-of-Life Products with Nonlinear Constraints

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Outline

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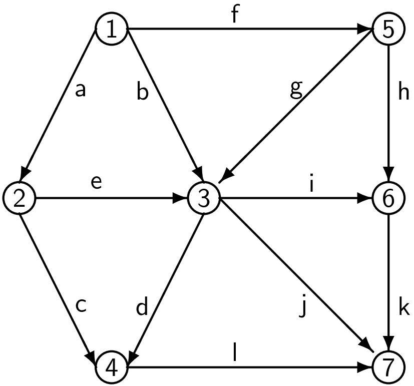
Time-dependent traffic equilibrium networks (1999)



Traffic Networks

Definition (Wardrop, 1952)

$H \in \mathbb{K} = \{F \in \mathbb{R}^m : F \geq 0$ and $\Phi F = \rho\}$ is an equilibrium distribution (user's equilibrium) if the following condition holds:

$$\forall w_j \in W,\ \forall R_q,\ R_s \in \mathcal{R}_j \text{ if } C_q(H) < C_s(H),\ \text{then } H_s = 0$$

Theorem (Smith, 1979)

H is an equilibrium distribution if and only if it is a solution to

$$\langle C(H), F - H \rangle \geq 0\ \forall F \in \mathbb{K}$$

Dynamic Traffic Networks

Definition (Daniele - Maugeri - Oettli, 1999)

$$H \in \mathbb{K} = \{F \in L^2([0,T], \mathbb{R}^m) : \lambda(t) \le F(t) \le \mu(t) \text{ a.e. in } [0,T]$$

$$\text{and } \Phi F(t) = \rho(t) \text{ a.e. in } [0,T]\}$$

is a dynamic equilibrium distribution (user's dynamic equilibrium) if the following condition holds:

$$\forall w_j \in W, \; \forall R_q, R_s \in \mathcal{R}_j \text{ and a.e. in } [0,T]$$

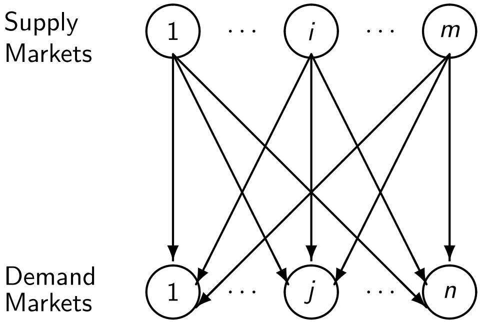
$$\text{if } C_q(H(t)) < C_s(H(t)), \text{ then } H_q(t) = \mu_q(t) \text{ or } H_s(t) = \lambda_s(t).$$

Theorem (Variational Formulation)

H is a dynamic equilibrium distribution if and only if it is a solution to

$$\ll C(H), F - H \gg \ge 0 \; \forall F \in \mathbb{K}$$

Time-dependent spatial price equilibrium problem (2004)



Spatial Price Model

Conservation Laws

$$g_i(p(t)) = \sum_{j=1}^{n} x_{ij}(t) + s_i(t), \quad i = 1, \ldots, m$$

$$f_j(q(t)) = \sum_{i=1}^{m} x_{ij}(t) + d_j(t), \quad j = 1, \ldots, n$$

Spatial Price Model

Definition (Dynamic Market Equilibrium)

$$u(t) = (p(t), q(t), x(t)) \in \mathcal{L}$$

where

$$\mathcal{L} = L^2([0,T], \mathbb{R}^m) \times L^2([0,T], \mathbb{R}^n) \times L^2([0,T], \mathbb{R}^{mn})$$

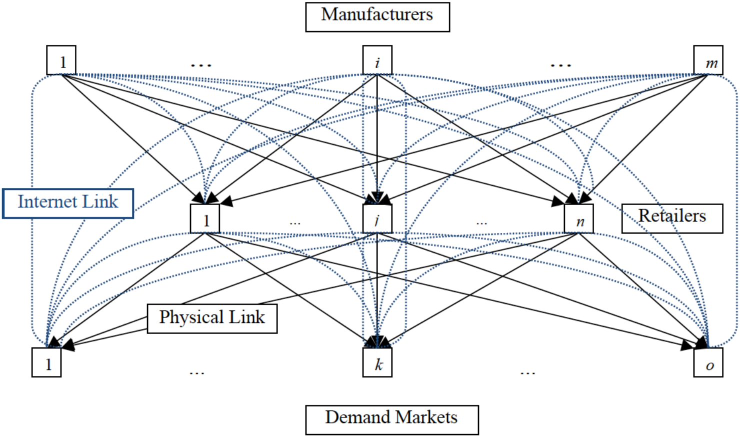
is a dynamic market equilibrium if for each $i = 1, \dots, m$, $j = 1, \dots, n$ *and a.e. in* $[0, T]$ *there hold:*

$$s_i(t) > 0 \Rightarrow p_i(t) = \underline{p}_i(t), \quad \underline{p}_i(t) < p_i(t) \leq \overline{p}_i(t) \Rightarrow s_i(t) = 0$$

$$d_j(t) > 0 \Rightarrow q_j(t) = \overline{q}_j(t), \quad \underline{q}_j(t) \leq q_j(t) < \overline{q}_j(t) \Rightarrow d_j(t) = 0$$

$$p_i(t) + c_{ij}(x(t)) \begin{cases} > q_j(t) & \text{if} \quad x_{ij}(t) = \underline{x}_{ij}(t) \\ = q_j(t) & \text{if} \quad \underline{x}_{ij}(t) \leq x_{ij}(t) \leq \overline{x}_{ij}(t) \\ < q_j(t) & \text{if} \quad x_{ij}(t) = \overline{x}_{ij}(t) \end{cases}$$

Evolutionary supply chain network (2010)



Substainable Supply Chain Management

F. Toyasaki, P. D., T. Wakolbinger, *A variational inequality formulation of equilibrium models for end-of-life products with nonlinear constraints*, EJOR **236**, 2014, 340-350

Measures

- Limiting precious and hazardous contents
- Increasing the amount of recyclable materials
- Increasing the use of recycled materials

⇓

Consequences

- Improvement of ecological effects of products
- Reduction of dependency on virgin material availability and prices

WEEE

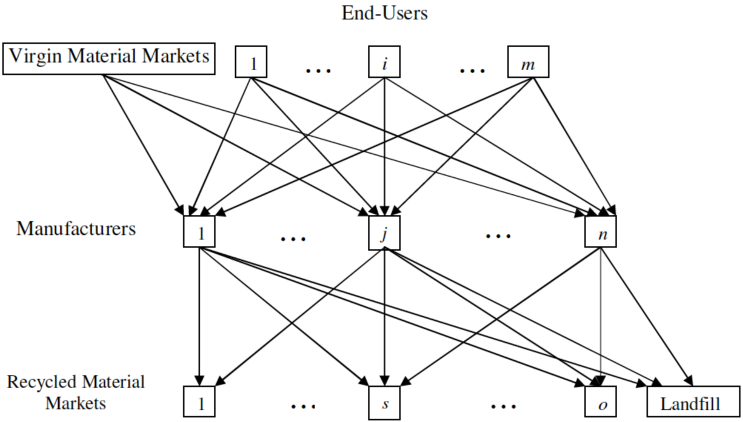
Electrical and Electronic Equipment Waste (WEEE) Categories:

- Large household appliances (washing machines, driers, . . .)
- Cooling and freezing appliances
- Small household appliances (telephones, vacuum cleaners, . . .)
- TV sets and monitors
- Lighting equipment

Legislation and regulations for reducing WEEE

- 27 Member States in EU
- Japan
- 25 states in USA
- Canada
- China

Supply chain network with recycled material (2014)



The Model

Decision Makers

- m end-users: hold end-of-life products and decide on the quantity of products that they return to manufacturers;
- n manufacturers: produce new products and are responsible for recycling end-of-life products

Virgin and Recycled materials

- Virgin materials are obtained from virgin material markets
- Recycled materials are extracted from returned products
- $\overline{\alpha}^s_{js}$: rate of recyclability, that is the amount of recycled materials that can be taken from one unit of returned products

Rhole of Manufacturers

- Manufacturers seek to extract materials that have no quality difference between recycled ones and virgin ones, such as rare metals and high purity plastics
- Manufacturers make decisions in the forward and reverse supply chain
- Manufacturers decide on the amounts of products collected from end-users and the amounts of virgin materials acquired.

The Model

Data

- $\bar{z}_j$: minimum quantity of new products that manufacturer j produces
- q^v_{js} : quantity of virgin material s acquired by manufacturer j
- q^r_{ij} : quantity of end-of-life products collected by j from i
- x_{js} : quantity of recycled material s sold by manufacturer j
- $\rho_{js}(X_s)$: unit price for recycled material s sold by j
- $\frac{1}{\beta_{js}}$: amount of material s included in one product
- $\sum_{i=1}^{m}\left(1-\sum_{s=1}^{o}\bar{\alpha}^r_{ijs}\right)\cdot q^r_{ij}$: amount of materials that will be sent to landfills is the difference between the returned product flow and the amount of products used in the recycling process

Aim of Manufacturers

- Each manufacturer j seeks to determine the level of q^r_{ij}, x_{js}, and β_{js} in order to maximize his/her profit
- Each manufacturer tries to minimize total costs incurred in the reverse logistics process for returned products and the production process for new products minus revenue from sales of recycled materials

Total Costs

- Prices paid to end-users for returning their products
- Costs for acquiring virgin materials
- Transaction costs for returned products (including inspection and disassembly costs)
- Recycling costs
- Costs for dumping residual waste in landfills
- Production costs

Aim of Manufacturers

Optimality Conditions for Manufacturer j

$$\min_{(Q_j^r, Q_j^v, B_j, X_j) \in \mathbb{K}_1^j} \left\{ \sum_{i=1}^{m} \left(\rho_{ij}^{r*} \cdot q_{ij}^r\right) + \sum_{s=1}^{o} f_{js}\left(Q_s^v\right) + \tilde{c}_j\left(Q_j^r\right) + \sum_{s=1}^{o} r_{js}\left(\bar{\alpha}_{js}^r, Q^r\right) + c_j\left(\sum_{i=1}^{m}\left(\bar{w}_{ij} \cdot q_{ij}^r\right)\right) + t_j(Q^v, Q^r, B_j, X_j) - \sum_{s=1}^{o} \rho_{js}(X_s) \cdot x_{js} \right\}$$

Aim of Manufacturers

Constraint set

$$\mathbb{K}_1^j = \Big\{ (Q_j^r, Q_j^v, B_j, X_j) \in \mathbb{R}^{m+3o} : \ \bar{\tau}_j \leq \sum_{i=1}^m q_{ij}^r, \tag{1}$$

$$\frac{\bar{z}_j}{\beta_{js}} - q_{js}^v - \sum_{i=1}^m \left(\bar{\alpha}_{ijs}^r q_{ij}^r \right) + x_{js} \leq 0 \quad \forall s = 1, \ldots, o \tag{2}$$

$$\sum_{i=1}^m \left(\bar{\alpha}_{ijs}^r q_{ij}^r \right) - x_{js} \geq 0 \quad \forall s = 1, \ldots, o \tag{3}$$

$$\bar{b}_{js}^l \leq \beta_{js} \leq \bar{b}_{js}^u \quad \forall s = 1, \ldots, o \tag{4}$$

$$q_{js}^v, q_{ij}^r, x_{js} \geq 0 \quad \forall i = 1, \ldots, m, \ s = 1, \ldots, o \Big\}. \tag{5}$$

Assumptions

Hp. 1

Let all the involved functions be continuously differentiable and convex

Convexity of the constraint set

$$\frac{\bar{z}_j}{\beta_{js}} - q^v_{js} - \sum_{i=1}^{m} \left(\bar{\alpha}^r_{ijs} q^r_{ij} \right) + x_{js} \leq 0 \quad \forall s = 1, \dots, o:$$

$\frac{\bar{z}_j}{\beta_{js}}$ is convex with respect to β_{js} because $\bar{z}_j$ is a positive parameter and the other terms are linear with respect to the manufacturers' variables

Variational Formulation

Find $(Q^{r*}, Q^{v*}, B^*, X^*) \in \mathbb{K}_1$ such that

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\rho_{ij}^{r*} + \frac{\partial \tilde{c}_j\left(Q_j^{r*}\right)}{\partial q_{ij}^r} + \frac{\partial r_{js}\left(\bar{\alpha}_{js}^r, Q^{r*}\right)}{\partial q_{ij}^r} + \frac{\partial c_j\left(\sum_{i=1}^{m}\left(\bar{w}_{ij}\cdot q_{ij}^{r*}\right)\right)}{\partial q_{ij}^r}\right.$$

$$\left. + \frac{\partial t_j\left(Q^{v*}, Q^{r*}, B_j^*, X_j^*\right)}{\partial q_{ij}^r}\right]\cdot\left(q_{ij}^r - q_{ij}^{r*}\right)$$

$$+\sum_{j=1}^{n}\sum_{s=1}^{o}\left[\frac{\partial f_{js}\left(Q_s^{v*}\right)}{\partial q_{js}^v} + \frac{\partial t_j\left(Q^{v*}, Q^{r*}, B_j^*, X_j^*\right)}{\partial q_{js}^v}\right]\cdot\left(q_{js}^v - q_{js}^{v*}\right)$$

$$+\sum_{j=1}^{n}\sum_{s=1}^{o}\left[\frac{\partial t_j\left(Q^{v*}, Q^{r*}, B_j^*, X_j^*\right)}{\partial x_{js}} - \rho_{js}(X_s^*) - \frac{\partial \rho_{js}(X_s^*)}{\partial x_{js}}\cdot x_{js}^*\right]\cdot\left(x_{js} - x_{js}^*\right)$$

$$+\sum_{j=1}^{n}\sum_{s=1}^{o}\frac{\partial t_j\left(Q^{v*}, Q^{r*}, B_j^*, X_j^*\right)}{\partial \beta_{js}}\cdot\left(\beta_{js} - \beta_{js}^*\right) \geq 0 \quad \forall (Q^r, Q^v, B, X) \in \mathbb{K}_1$$

Behavior of end-users

In the reverse chain, end-users have the options of either returning the product or not

- ρ_{ij}^r : manufacturer j's buy-back price from end-user i

Equilibrium Laws

$$\underbrace{a_i(q_i^{r*})}_{\text{aversion to product returns}} - \underbrace{\varepsilon_i\left(\bar{q}_i, q_i^{r*}\right)}_{\text{disutility of holding end-of-life products}} + \underbrace{\hat{c}_{ij}(q_{ij}^{r*})}_{\text{transaction costs}} + \underbrace{\lambda_i^*}_{\text{marginal value}} \begin{cases} = \rho_{ij}^{r*} & \text{if} \quad q_{ij}^{r*} > 0 \\ \geq \rho_{ij}^{r*} & \text{if} \quad q_{ij}^{r*} = 0 \end{cases}$$

$$\underbrace{\sum_{j=1}^{n} q_{ij}^{r*}}_{\text{products returned from end-user}} \begin{cases} = \underbrace{\bar{q}_i}_{\text{end-of-life product hold by end-user}} & \text{if} \quad \lambda_i^* > 0 \\ \leq \bar{q}_i, & \text{if} \quad \lambda_i^* = 0 \end{cases}$$

Equilibrium Conditions for all end-users

Variational Formulation

$$\text{Find } (Q^{r*}, \lambda^*) \in \mathbb{K}_2 \text{ such that}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[a_i\left(q_i^{r*}\right) - \varepsilon_i\left(\bar{q}_i, q_i^{r*}\right) + \hat{c}_{ij}\left(q_{ij}^{r*}\right) + \lambda_i^* - \rho_{ij}^{r*}\right]\cdot\left(q_{ij}^{r} - q_{ij}^{r*}\right)$$

$$+\sum_{i=1}^{m}\left(\bar{q}_i - q_i^{r*}\right)\cdot\left(\lambda_i - \lambda_i^*\right) \geq 0 \quad \forall\left(Q^r, \lambda\right) \in \mathbb{K}_2$$

where

$$\mathbb{K}_2 = \left\{\left(Q^r, \lambda\right) \in \mathbb{R}_+^{mn+m}\right\}.$$

Equilibrium Distribution

Forward Chain

Shipments that end-users and virgin material markets ship to manufacturers	=	Shipments that manufacurers accept from end-users and virgin material markets

Reverse Chain

Shipments of recycled materials to recycled material markets and shipments of materials that are not recycled and sent to landfills	=	Shipments that recycled material markets and landfills accept from manufacturers

Equilibrium Definition

Definition

The equilibrium state for the economic network with end-of-life products, material transformation factors, recycled and virgin materials in the forward and reverse chain is the vector $(Q^{r*}, Q^{v*}, \lambda^*, B^*, X^*) \in \mathbb{K}$ *for which the flows between the tiers of the supernetwork coincide and which satisfies the sum of the previous variational inequalities.*

Existence Result

Standard VI

Find $u \in \mathbb{K}_3$ such that: $\langle V(u^*), u - u^* \rangle \geq 0 \quad \forall u \in \mathbb{K}_3$

where

$$\mathbb{K}_3 = \{u \in \mathbb{R}^{mn+3no+m} : G(u) \leq 0\}$$

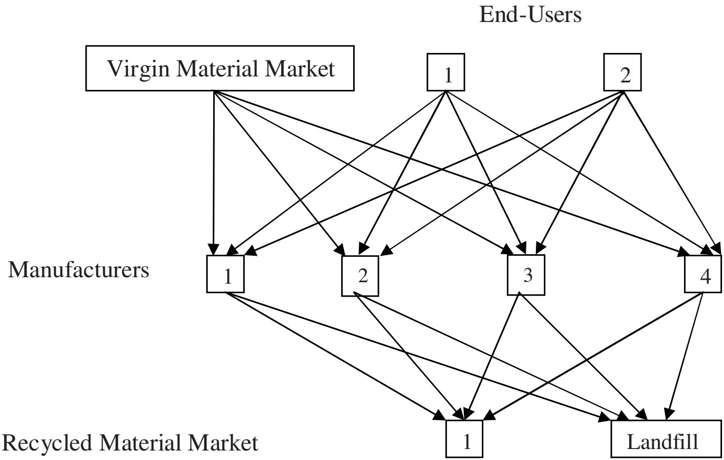
Theorem (Existence)

If $V : \mathbb{K}_3 \to \mathbb{R}_+^{mn+3no+m}$ is coercive, namely:

$$\lim_{\substack{u \in \mathbb{K}_3 \\ \|u\| \to +\infty}} \frac{\langle V(u), u \rangle}{\|u\|} = +\infty,$$

then the previous variational inequality admits a solution.

Numerical Examples



Data of Dell (EPA, 2011)

- 72 million computers were sold in the US
- 18 million were collected for recycling (25% of computers sold).
- 70.5 million were in storage (57.9 million residential storage and 12.6 commercial storage)
- 0.9 million were disposed or collected for recycling per week

$\Downarrow$

71.4 million available for end-of-life management at any given week

$\Downarrow$

amount available for end-of-life-management $\simeq$ **annual production**

Base Model

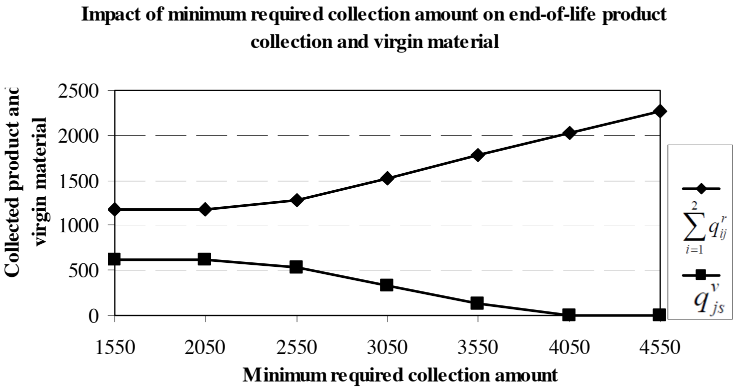
- $x_{js} = 0; \forall j, \forall s$: manufacturers use recycled materials only in their own production processes

This type of recycling system is used by many electronic manufacturers in Japan, for example, Sharp, Sony, Mitsubishi Electronic

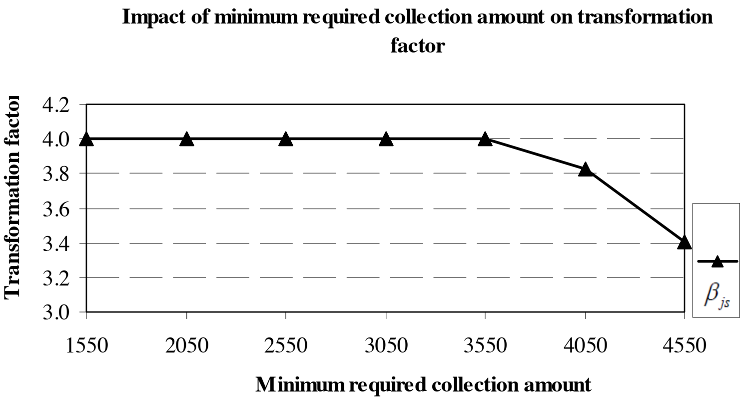
$\bar{q}_i$	645,160	EPA (2011), Kenney (2007)
$\bar{z}_j$	6,200 {6,200; 6,700;...; 8,200}	Kenney (2007)
$\bar{\tau}_j$	1,550 {1,550; 2,550;...; 4,550}	Kenney (2007), EPA (2011)
$\bar{\alpha}^r_{ijs}$	0.4 {0.35; 0.3;...; 0.1}	Sodhi and Reimer (2001)
$\bar{b}^u_{js}$	0	
$\bar{b}^l_{js}$	4	
p_1	0.1{0.1; 0.2; ...; 0.6}	
p_2	0.01{0.01; 0.012;...;0.022}	
p_3	17.2	Boon, Isaacs and Gupta (2002)
p_{4s}	$8,000 + 0.1 \cdot \sum_{j=1}^{4} q^v_{j1}$	Sodhi and Reimer (2001)
p_5; p_6	1.72; 3.24	Atasu, Toktay and Van Wasenhove (2008)
p_7	140	Boon, Isaacs and Gupta (2002)
p_8	315	Boon, Isaacs and Gupta (2002)
p_{9s}	640	Ontario Electronic Stewardship (2008)
p_{10}	53,000	Edge Work Group Report (2000)
p_{11}	53,000	Edge Work Group Report (2000)
p_{12}	10,000 {10,000; 20,000;...; 80,000}	
p_{13}	233	Boon, Isaacs and Gupta (2002)

Table : Basis data of the parameters for the numerical examples

Numerical Examples



Numerical Examples



Results

Impacts of Required Minimum Collection Amount

- Recycled material is cheaper than virgin material up to the collection amount of 2,348 tons
- Total costs that manufacturers incur remain the same up to that level of the minimum required collection amount
- If the minimum collection amount is raised above 2,348 tons, then manufacturers are forced to collect more end-of-life products and their total costs increase
- Recycled materials substitute for virgin materials and the quantities of virgin materials bought decrease
- Once the quantity of virgin materials purchased becomes zero, then manufactures start reducing the transformation factor

Results

Impacts of the Percentage of Recyclable Materials Extracted

Two trends:

↙ ↘

Manufacturers increase the amount of materials that can be extracted by providing product parts with labels that indicate their material compositions and by designing their products in such a way that they can be easily disassembled (Sony)

Manufacturers develop technologies to lighten products and use fewer raw materials

Results

$\bar{\alpha}^r_{ijs}$	0.4	0.35	0.3	0.25	0.2	0.15	0.1
q^r_i	2275	2275	2275	2275	2275	2275	2275
q^v_{j1}	0	0	185	413	640	868	1095
β_{j1}	3	3.9	4	4	4	4	4

Table : Impact of the percentage of recyclable materials

- A reduction of $\bar{\alpha}^r_{ijs}$ (percentage of material s extracted by manufacturer j from waste of end-user i) first leads to an increase in the transformation factor and then it leads to an increase in the amount of virgin materials acquired
- Improving product design is less costly than acquiring virgin materials in this case
- Due to high transaction costs (i.e., collection, sorting and dismantling) associated with the end-of-life products, manufacturers do not increase recycled material usage to reach their production goals.

Results

z_j	6200	6700	7200	7700	8200	8700	9200
q_i^r	2275	2275	2275	2275	2275	2275	2275
q_{j1}^v	0	0	0	105	230	355	480
β_{j1}	3	3.68	3.96	4	4	4	4

Table : Impact of demand for new products

- The amount of products that is recycled is stable and stays at the level required by the collection target
- An increase of demand for new products first increases the transformation factor and then leads to more virgin material being acquired and used because improving product designs is more cost efficient than acquiring additional virgin materials in this case
- Due to high transaction costs caused by necessary procurement and treatment of end-of-life products, manufacturers do not increase usage of recycled materials to meet demand.

Conclusions

- The model can be used by policy-makers and manufacturers to anticipate effects of new legislation and changes in economic conditions
- It is important to establish sound markets for recyclable materials and to provide manufacturers easy access to these markets
- Other economic networks can also benefit from the introduction of nonlinear constraints