Financial Networks and Disruption Management

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Outline

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- The Variational Inequality Problem
- The Financial Network with Intermediation
- The Financial Network Performance Measure
- The Importance of a Financial Network Component
- Numerical Examples
- Summary and Conclusions

Background and Motivation

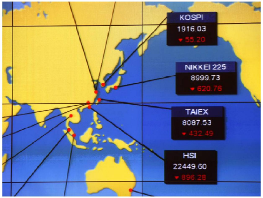
The study of financial networks dates to the 1750s when Quesnay (1758), in his *Tableau Economique*, **conceptualized the circular flow of financial funds in an economy as a network.**

Copeland (1952) further explored the relationships among financial funds as a network and asked the question, *"Does money flow like water or electricity?"*

Background and Motivation

The advances in information technology and globalization have further shaped today's financial world into a complex network, which is characterized by distinct sectors, the proliferation of new financial instruments, and with increasing international diversification of portfolios.

Financial Networks



Background and Motivation

Recently, financial networks have been studied using network models with multiple tiers of decision-makers, including intermediaries.

For a detailed literature review of financial networks, please refer to the paper by Nagurney (2008) (see also Fei (1960), Charnes and Cooper (1967), Thore (1969), Thore and Kydland (1972), Thore (1980), Christofides, Hewins, and Salkin (1979), Crum and Nye (1981), Mulvey (1987), Nagurney and Hughes (1992), Nagurney, Dong and Hughes (1992), Nagurney and Siokos (1997), Nagurney and Ke (2001, 2003), Boginski, Butenko, and Pardalos (2003), Geunes and Pardalos (2003), Nagurney and Cruz (2003a, 2003b), Nagurney, Wakolbinger, and Zhao (2006), and the references therein).

Since today's financial networks may be highly interconnected and interdependent, any disruptions that occur in one part of the network may produce consequences in other parts of the network, which may not only be in the same region but many thousands of miles away in other countries.

As pointed out by Sheffi (2005) in his book, *The Resilient Enterprise*, one of the main characteristics of disruptions in networks is **"the seemingly unrelated consequences and vulnerabilities stemming from global connectivity."**

Background and Motivation

For example, the unforgettable 1987 stock market crash was, in effect, a chain reaction throughout the world; it originated in Hong Kong, then propagated to Europe, and, finally, the United States.

Background and Motivation

The management at Merrill Lynch well understood the criticality of their operations in World Trade Center and established contingency plans.

Directly after the 9/11 terrorist attacks, management was able to switch their operations from the World Trade Center to the backup centers and the redundant trading floors near New York City. Therefore, the company managed to mitigate the losses for both its customers and itself (see Sheffi (2005)).

Background and Motivation

In 2008 and 2009, the world reeled from the effects of the financial credit crisis; leading financial services and banks closed (including the investment bank Lehman Brothers), others merged, and the financial landscape was changed for forever.

The domino effect of the U.S. economic troubles rippled through overseas markets and pushed countries such as Iceland to the verge of bankruptcy.

It is crucial for the decision-makers in financial systems (managers, executives, and regulators) to be able to to identify a financial network's vulnerable components to protect the functionality of the network.

Background and Motivation

The analysis and the identification of the vulnerable components in networks have, recently, emerged as a major research theme, especially in the study of what are commonly referred to as *complex* networks, or, collectively, as *network science* (see the survey by Newman (2003)).

However, in order to be able to evaluate the vulnerability and the reliability of a network, a measure that can quantifiably capture the performance of a network must be developed.

Recent disasters have vividly demonstrated the importance and vulnerability of our transportation and critical infrastructure systems

- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Mediterranean cable destruction, January 30, 2008;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011.

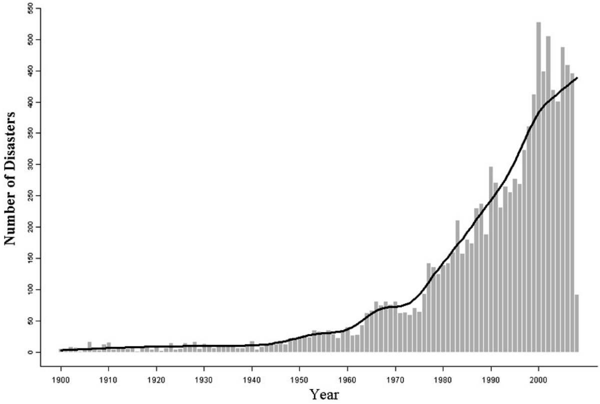
Hurricane Katrina in 2005

Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

The Haitian and Chilean Earthquakes

The Triple Disaster in Japan on March 11, 2011

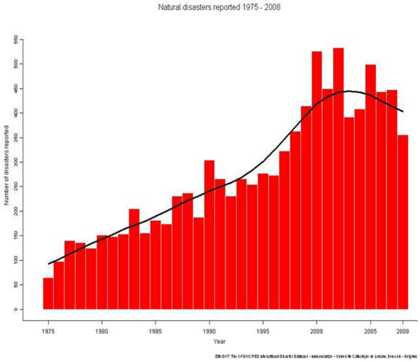
Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing. From January to October 2005, an estimated 97,490 people were killed in disasters globally; 88,117 of them because of natural disasters.

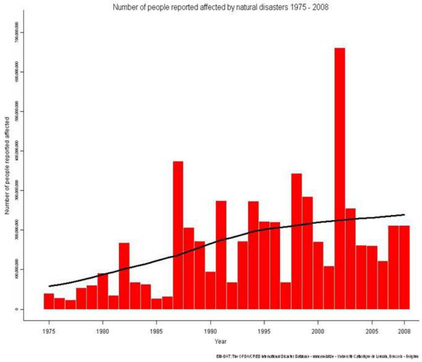


Frequency of disasters [Source: Emergency Events Database (2008)]

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources.

Natural Disasters (1975–2008)





Which Nodes and Links Really Matter?

Background and Motivation

Financial networks, as extremely important infrastructure networks, have a great impact on the global economy, and their study has recently also attracted attention from researchers in the area of complex networks.

Onnela, Kaski, and Kertész (2004) studied a financial network in which the nodes are stocks and the edges are the correlations among the prices of stocks (see also, Kim and Jeong (2005)).

Caldarelli et al. (2004) studied different financial networks, namely, board and director networks, and stock ownership networks and discovered that all these networks displayed scale-free properties (see also Boginski, Butenko, and Pardalos (2003)).

Background and Motivation

Several recent studies in finance, in turn, have analyzed the local consequences of catastrophes and the design of risk sharing/management mechanisms since the occurrence of disasters such as 9/11 and Hurricane Katrina (see, for example, Gilli and Këllezi (2006), Loubergé, Këllezi, and Gilli (1999), Doherty (1997), Niehaus (2002), and the references therein).

Background and Motivation

Nevertheless, there is very little literature that addresses the vulnerability of financial networks. Robinson, Woodard, and Varnado (1998) discussed, from the policy-making point of view, how to protect the critical infrastructure in the US, including financial networks.

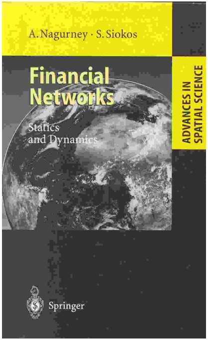
Odell and Phillips (2001) conducted an empirical study to analyze the impact of the 1906 San Francisco earthquake on bank loan rates in the financial network within San Francisco.

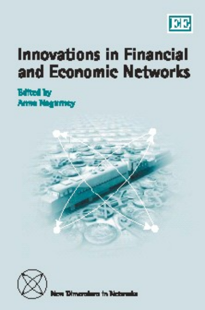
To the best of our knowledge, however, there is no network performance measure to-date that has been applied to financial networks that captures both economic behavior as well as the underlying network/graph structure.

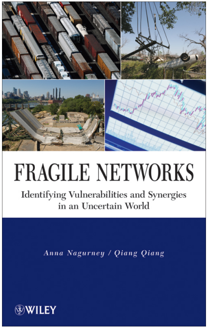
Background and Motivation

The only relevant network study is that by Jackson and Wolinsky (1996), which defines a value function for the network topology and proposes the network efficiency concept based on the value function from the point of view of network formation.

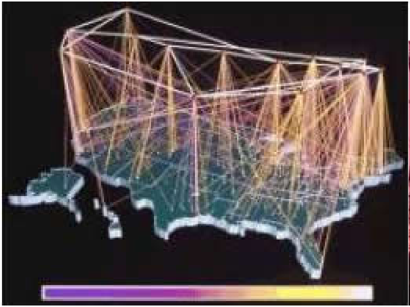
Financial Networks







Other Networks that are Related to Financial Networks



The Internet, electric power networks, and even transportation !

The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized network problems.

Definition: The Variational Inequality Problem
The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$*, is to determine a vector* $X^* \in \mathcal{K}$*, such that:*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N*,* $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N*.*

The Variational Inequality Problem

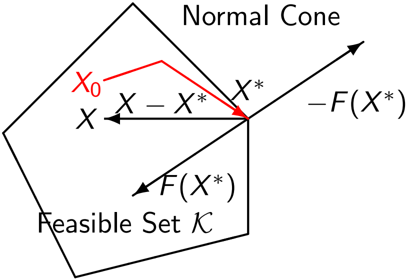
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of network problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

Dynamics

To model the *dynamic behavior of networks*, including transportation networks and supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

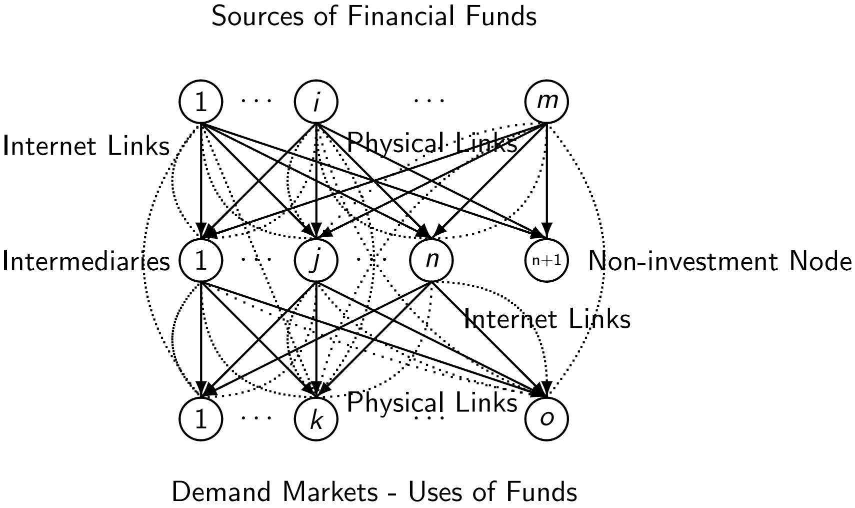
ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even **neuroscience** (Girard et al. (2008)).

The Financial Network Model with Intermediation

The rest of this presentation is based on the paper, "Identification of Critical Nodes and Links in Financial Networks with Intermediation and Electronic Transactions," Anna Nagurney and Qiang Qiang, in *Computational Methods in Financial Engineering*, E. J. Kontoghiorghes, B. Rustem, and P. Winker, Editors, Springer, Berlin, Germany (2008), pp 273-297.

The Financial Network Model with Intermediation



The Financial Network Model with Intermediation

The financial network consists of *m* sources of financial funds, *n* financial intermediaries, and *o* demand markets, as depicted in Figure 1. In the financial network model, the financial transactions are denoted by the links with the transactions representing electronic transactions delineated by hatched links. The majority of the notation for this model is given in Tables 1-3.

Table 1: Notation for the Financial Network Model

Notation	Definition
S	m-dimensional vector of the amounts of funds held by the source agents with component i denoted by S^i
q_i	$(2n+o)$-dimensional vector associated with source agent i; $i=1,\ldots,m$ with components: $\{q_{ijl}; j=1,\ldots,n; l=1,2; q_{ik}; k=1,\ldots,o\}$
q_j	$(2m+2o)$-dimensional vector associated with intermediary j; $j=1,\ldots,n$ with components: $\{q_{ijl}; i=1,\ldots,m; l=1,2; q_{jkl}; k=1,\ldots,o; l=1,2\}$
Q^1	$2mn$-dimensional vector of all the financial transactions/flows for all source agents/intermediaries/modes with component ijl denoted by q_{ijl}
Q^2	mo-dimensional vector of the electronic financial transactions/flows between the sources of funds and the demand markets with component ik denoted by q_{ik}
Q^3	$2no$-dimensional vector of all the financial transactions/flows for all intermediaries/demand markets/modes with component jkl denoted by q_{jkl}
g	n-dimensional vector of the total financial flows received by the intermediaries with component j denoted by g_j, with $g_j \equiv \sum_{i=1}^m \sum_{l=1}^2 q_{ijl}$
γ	n-dimensional vector of shadow prices associated with the intermediaries with component j denoted by γ_j
d	o-dimensional vector of market demands with component k denoted by d_k
$\rho_{3k}(d)$	the demand price (inverse demand) function at demand market k

The Financial Network Model with Intermediation

Table 2: Notation for the Financial Network Model

Notation	**Definition**
V^i	the $(2n+o) \times (2n+o)$ dimensional variance-covariance matrix associated with source agent i
V^j	the $(2m+2o) \times (2m+2o)$ dimensional variance-covariance matrix associated with intermediary j
$c_{ijl}(q_{ijl})$	the transaction cost incurred by source agent i in transacting with intermediary j using mode l with the marginal transaction cost denoted by $\frac{\partial c_{ijl}(q_{ijl})}{\partial q_{ijl}}$
$c_{ik}(q_{ik})$	the transaction cost incurred by source agent i in transacting with demand market k with marginal transaction cost denoted by $\frac{\partial c_{ik}(q_{ik})}{\partial q_{ik}}$
$c_{jkl}(q_{jkl})$	the transaction cost incurred by intermediary j in transacting with demand market k via mode l with marginal transaction cost denoted by $\frac{\partial c_{jkl}(q_{jkl})}{\partial q_{jkl}}$
$c_j(Q^1) \equiv c_j(g)$	conversion/handling cost of intermediary j with marginal handling cost with respect to g_j denoted by $\frac{\partial c_j}{\partial g_j}$ and the marginal handling cost with respect to q_{ijl} denoted by $\frac{\partial c_j(Q^1)}{\partial q_{ijl}}$
$\hat{c}_{ijl}(q_{ijl})$	the transaction cost incurred by intermediary j in transacting with source agent i via mode l with the marginal transaction cost denoted by $\frac{\partial \hat{c}_{ijl}(q_{ijl})}{\partial q_{ijl}}$

The Financial Network Model with Intermediation

Table 3: Notation for the Financial Network Model

Notation	**Definition**
$\hat{c}_{jkl}(Q^2, Q^3)$	the unit transaction cost associated with obtaining the product at demand market k from intermediary j via mode l
$\hat{c}_{ik}(Q^2, Q^3)$	the unit transaction cost associated with obtaining the product at demand market k from source agent i

The Financial Network Model with Intermediation

All vectors are assumed to be column vectors. The equilibrium solutions throughout this paper are denoted by *.

The m agents or sources of funds at the top tier of the financial network in Figure 1 seek to determine the optimal allocation of their financial resources transacted either physically or electronically with the intermediaries or electronically with the demand markets. Examples of source agents include: households and businesses.

The financial intermediaries, in turn, which can include banks, insurance companies, investment companies, etc., in addition to transacting with the source agents determine how to allocate the incoming financial resources among the distinct uses or financial products associated with the demand markets, which correspond to the nodes at the bottom tier of the financial network in Figure 1.

The Financial Network Model with Intermediation

Examples of demand markets are: the markets for real estate loans, household loans, business loans, etc. The transactions between the financial intermediaries and the demand markets can also take place physically or electronically via the Internet.

The Financial Network Model with Intermediation

We denote a typical source agent by i; a typical financial intermediary by j, and a typical demand market by k. The mode of transaction is denoted by l with $l = 1$ denoting the physical mode and with $l = 2$ denoting the electronic mode.

We now describe the behavior of the decision-makers with sources of funds. We then discuss the behavior of the financial intermediaries and, finally, the consumers at the demand markets. Subsequently, we state the financial network equilibrium conditions and derive the variational inequality formulation governing the equilibrium conditions.

The Behavior of the Source Agents

Since there is the possibility of non-investment allowed, the node $n+1$ in the second tier in Figure 1 represents the "sink" to which the uninvested portion of the financial funds flows from the particular source agent or source node.

We have the following conservation of flow equations:

$$\sum_{j=1}^{n}\sum_{l=1}^{2} q_{ijl} + \sum_{k=1}^{o} q_{ik} \leq S^i, \quad i = 1, \ldots, m, \tag{1}$$

that is, the amount of financial funds available at source agent i and given by S^i cannot exceed the amount transacted physically and electronically with the intermediaries plus the amount transacted electronically with the demand markets. Note that the "slack" associated with constraint (1) for a particular source agent i is given by $q_{i(n+1)}$ and corresponds to the uninvested amount of funds.

The Behavior of the Source Agents

Let ρ_{1ijl} denote the price charged by source agent i to intermediary j for a transaction via mode l and, let ρ_{1ik} denote the price charged by source agent i for the electronic transaction with demand market k.

The ρ_{1ijl} and ρ_{1ik} are endogenous variables and their equilibrium values ρ^*_{1ijl} and ρ^*_{1ik}; $i = 1, \ldots, m$; $j = 1, \ldots, n$; $l = 1, 2$, $k = 1, \ldots, o$ are determined once the complete financial network model is solved.

The Behavior of the Source Agents

Each source agent seeks to maximize his net revenue and to minimize his risk. The risk for source agent i is represented by the variance-covariance matrix V^i so that the optimization problem faced by source agent i can be expressed as:

$$\text{Maximize} \quad U^i(q_i) = \sum_{j=1}^{n} \sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} + \sum_{k=1}^{o} \rho^*_{1ik} q_{ik} - \sum_{j=1}^{n} \sum_{l=1}^{2} c_{ijl}(q_{ijl})$$

$$- \sum_{k=1}^{o} c_{ik}(q_{ik}) - q_i^T V^i q_i \qquad (2)$$

subject to:

$$\sum_{j=1}^{n} \sum_{l=1}^{2} q_{ijl} + \sum_{k=1}^{o} q_{ik} \leq S^i$$

$$q_{ijl} \geq 0, \quad \forall j, l,$$

$$q_{ik} \geq 0, \quad \forall k, \qquad q_{i(n+1)} \geq 0.$$

The Behavior of the Source Agents

The first four terms in the objective function (2) represent the net revenue of source agent i and the last term is the variance of the return of the portfolio, which represents the risk associated with the financial transactions.

The transaction cost functions for each source agent are continuously differentiable and convex, and the source agents compete in a noncooperative manner in the sense of Nash (1950, 1951).

The Behavior of the Source Agents

The optimality conditions for all decision-makers with source of funds coincide with the solution of the variational inequality: determine $(Q^{1*}, Q^{2*}) \in \mathcal{K}^0$ such that:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[2V^i_{z_{jl}} \cdot q^*_i + \frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}} - \rho^*_{1ijl}\right] \times \left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[2V^i_{z_{2n+k}} \cdot q^*_i + \frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}} - \rho^*_{1ik}\right] \times [q_{ik} - q^*_{ik}] \geq 0,$$

$$\forall (Q^1, Q^2) \in \mathcal{K}^0, \tag{3}$$

where $V^i_{z_{jl}}$ denotes the z_{jl}-th row of V^i and z_{jl} is defined as the indicator: $z_{jl} = (l-1)n + j$. Similarly, $V^i_{z_{2n+k}}$ denotes the z_{2n+k}-th row of V^i but with z_{2n+k} defined as the $2n+k$-th row, and $\mathcal{K}^0 \equiv \{(Q^1, Q^2) | (Q^1, Q^2) \in R^{2mn+mo}_+ \text{and (1) holds for all } i\}$.

The Behavior of the Financial Intermediaries

Let the endogenous variable ρ_{2jkl} denote the product price charged by intermediary j with ρ^*_{2jkl} denoting the equilibrium price, where $j = 1, \ldots, n$; $k = 1, \ldots, o$, and $l = 1, 2$. We assume that each financial intermediary also seeks to maximize his net revenue while minimizing his risk.

Note that a financial intermediary, by definition, may transact either with decision-makers in the top tier of the financial network as well as with consumers associated with the demand markets in the bottom tier.

The Behavior of the Financial Intermediaries

The financial intermediary is faced with the following optimization problem:

$$\text{Maximize} \quad U^j(q_j) = \sum_{k=1}^{o} \sum_{l=1}^{2} \rho^*_{2jkl} q_{jkl} - c_j(Q^1) - \sum_{i=1}^{m} \sum_{l=1}^{2} \hat{c}_{ijl}(q_{ijl})$$

$$- \sum_{k=1}^{o} \sum_{l=1}^{2} c_{jkl}(q_{jkl}) - \sum_{i=1}^{m} \sum_{l=1}^{2} \rho^*_{1ijl} q_{ijl} - q_j^T V^j q_j \tag{4}$$

subject to:

$$\sum_{k=1}^{o} \sum_{l=1}^{2} q_{jkl} \leq \sum_{i=1}^{m} \sum_{l=1}^{2} q_{ijl}, \tag{5}$$

$$q_{ijl} \geq 0, \quad \forall i, l,$$

$$q_{jkl} \geq 0, \quad \forall k, l.$$

The Behavior of the Financial Intermediaries

The first five terms in the objective function (4) denote the net revenue, whereas the last term is the variance of the return of the financial allocations, which represents the risk to each financial intermediary. Constraint (5) guarantees that an intermediary cannot reallocate more of its financial funds among the demand markets than it has available.

Let γ_j be the Lagrange multiplier associated with constraint (5) for intermediary j.

We assume that the cost functions are continuously differentiable and convex, and that the intermediaries compete in a noncooperative manner.

The optimality conditions for all intermediaries simultaneously can be expressed as: determine $(Q^{1*}, Q^{3*}, \gamma^*) \in R_+^{2mn+2no+n}$ such that:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[2V_{z_{il}}^{j}\cdot q_j^* + \frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \rho_{1ijl}^* + \frac{\partial \hat{c}_{ijl}(q_{ijl}^*)}{\partial q_{ijl}} - \gamma_j^*\right]\times\left[q_{ijl}-q_{ijl}^*\right]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[2V_{z_{kl}}^{j}\cdot q_j^* + \frac{\partial c_{jkl}(q_{jkl}^*)}{\partial q_{jkl}} - \rho_{2jkl}^* + \gamma_j^*\right]\times\left[q_{jkl}-q_{jkl}^*\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2}q_{ijl}^* - \sum_{k=1}^{o}\sum_{l=1}^{2}q_{jkl}^*\right]\times\left[\gamma_j-\gamma_j^*\right]\geq 0,$$

$$\forall (Q^1, Q^3, \gamma) \in R_+^{2mn+2no+n}, \qquad (6)$$

where $V_{z_{il}}^{j}$ denotes the z_{il}-th row of V^j and z_{il} is defined as the indicator: $z_{il} = (l-1)m + i$. Similarly, $V_{z_{kl}}^{j}$ denotes the z_{kl}-th row of V^j and z_{kl} is defined as the indicator: $z_{kl} = 2m + (l-1)o + k$.

The Consumers at the Demand Markets and the Equilibrium Conditions

We assume, as given, the inverse demand functions $\rho_{3k}(d)$; $k = 1, \ldots, o$, associated with the demand markets at the bottom tier of the financial network. The demand markets correspond to distinct financial products. Of course, if the demand functions are invertible, then one may obtain the price functions simply by inversion.

The Consumers at the Demand Markets and the Equilibrium Conditions

The following conservation of flow equations must hold:

$$d_k = \sum_{j=1}^{n} \sum_{l=1}^{2} q_{jkl} + \sum_{i=1}^{m} q_{ik}, \quad k = 1, \ldots, o. \tag{7}$$

Equations (7) state that the demand for the financial product at each demand market is equal to the financial transactions from the intermediaries to that demand market plus those from the source agents.

The Consumers at the Demand Markets and the Equilibrium Conditions

The equilibrium condition for the consumers at demand market k are as follows: for each intermediary j; $j = 1, \dots, n$ and mode of transaction l; $l = 1, 2$:

$$\rho^*_{2jkl} + \hat{c}_{jkl}(Q^{2*}, Q^{3*}) \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q^*_{jkl} > 0 \\ \geq \rho_{3k}(d^*), & \text{if} \quad q^*_{jkl} = 0. \end{cases} \tag{8}$$

In addition, we must have that, in equilibrium, for each source of funds i; $i = 1, \dots, m$:

$$\rho^*_{1ik} + \hat{c}_{ik}(Q^{2*}, Q^{3*}) \begin{cases} = \rho_{3k}(d^*), & \text{if} \quad q^*_{ik} > 0 \\ \geq \rho_{3k}(d^*), & \text{if} \quad q^*_{ik} = 0. \end{cases} \tag{9}$$

The Consumers at the Demand Markets and the Equilibrium Conditions

Condition (8) states that, in equilibrium, if consumers at demand market k purchase the product from intermediary j via mode l, then the price the consumers pay is exactly equal to the price charged by the intermediary plus the unit transaction cost via that mode.

However, if the sum of price charged by the intermediary and the unit transaction cost is greater than the price the consumers are willing to pay at the demand market, there will be no transaction between this intermediary/demand market pair via that mode. Condition (9) states the analogue but for the case of electronic transactions with the source agents.

The Consumers at the Demand Markets and the Equilibrium Conditions

In equilibrium, conditions (8) and (9) must hold for all demand markets. We can also express these equilibrium conditions using the following variational inequality: determine $(Q^{2*}, Q^{3*}, d^*) \in \mathcal{K}^1$, such that

$$\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[\rho^*_{2jkl} + \hat{c}_{jkl}(Q^{2*}, Q^{3*})\right] \times \left[q_{jkl} - q^*_{jkl}\right]$$

$$+ \sum_{i=1}^{m}\sum_{k=1}^{o}\left[\rho^*_{1ik} + \hat{c}_{ik}(Q^{2*}, Q^{3*})\right] \times [q_{ik} - q^*_{ik}]$$

$$- \sum_{k=1}^{o}\rho_{3k}(d^*) \times [d_k - d^*_k] \geq 0, \quad \forall (Q^2, Q^3, d) \in \mathcal{K}^1, \qquad (10)$$

where
$\mathcal{K}^1 \equiv \{(Q^2, Q^3, d) | (Q^2, Q^3, d) \in R^{2no+mo+o}_{+}$ and (7) holds.$\}$

The Equilibrium Conditions for Financial Network with Electronic Transactions

In equilibrium, the optimality conditions for all decision-makers with source of funds, the optimality conditions for all the intermediaries, and the equilibrium conditions for all the demand markets must be simultaneously satisfied so that no decision-maker has any incentive to alter his or her decision.

Definition 1: Financial Network Equilibrium with Intermediation and with Electronic Transactions
The equilibrium state of the financial network with intermediation is one where the financial flows between tiers coincide and the financial flows and prices satisfy the sum of conditions (3), (6), and (10).

Variational Inequality Formulation

We now define the feasible set:

$$\mathcal{K}^2 \equiv \{(Q^1, Q^2, Q^3, \gamma, d) | (Q^1, Q^2, Q^3, \gamma, d) \in R_+^{m+2mn+2no+n+o}$$

$$\text{and (1) and (7) hold}\}$$

and state the Theorem 1.

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the financial network model with intermediation are equivalent to the solution to the variational inequality problem given by: determine $(Q^{1^*}, Q^{2^*}, Q^{3^*}, \gamma^*, d^*) \in \mathcal{K}^2$ *such that:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{2}\left[2V^i_{z_{jl}}\cdot q^*_i + 2V^j_{z_{il}}\cdot q^*_j + \frac{\partial c_{ijl}(q^*_{ijl})}{\partial q_{ijl}} + \frac{\partial c_j(Q^{1*})}{\partial q_{ijl}} + \frac{\partial \hat{c}_{ijl}(q^*_{ijl})}{\partial q_{ijl}} - \gamma^*_j\right]$$

$$\times\left[q_{ijl} - q^*_{ijl}\right]$$

$$+\sum_{i=1}^{m}\sum_{k=1}^{o}\left[2V^i_{z_{2n+k}}\cdot q^*_i + \frac{\partial c_{ik}(q^*_{ik})}{\partial q_{ik}} + \hat{c}_{ik}(Q^{2*}, Q^{3*})\right]\times[q_{ik} - q^*_{ik}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{l=1}^{2}\left[2V^j_{z_{kl}}\cdot q^*_j + \frac{\partial c_{jkl}(q^*_{jkl})}{\partial q_{jkl}} + \hat{c}_{jkl}(Q^{2*}, Q^{3*}) + \gamma^*_j\right]\times\left[q_{jkl} - q^*_{jkl}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{l=1}^{2} q^*_{ijl} - \sum_{k=1}^{n}\sum_{l=1}^{2} q^*_{jkl}\right]\times\left[\gamma_j - \gamma^*_j\right] - \sum_{k=1}^{o}\rho_{3k}(d^*)\times[d_k - d^*_k] \geq 0,$$

$$\forall (Q^1, Q^2, Q^3, \gamma, d) \in \mathcal{K}^2. \quad (11)$$

Variational Inequality Formulation

The variables in the variational inequality problem (11) are: the financial flows from the source agents to the intermediaries, Q^1; the direct financial flows via electronic transaction from the source agents to the demand markets, Q^2; the financial flows from the intermediaries to the demand markets, Q^3; the shadow prices associated with handling the product by the intermediaries, γ, and the prices at demand markets ρ_3.

The solution to the variational inequality problem (11), $(Q^{0^*}, Q^{1^*}, Q^{2^*}, Q^{3^*}, \gamma^*, d^*)$, coincides with the equilibrium financial flow and price pattern according to Definition 1.

The Financial Network Performance Measure

Definition 2: The Financial Network Performance Measure
The financial network performance measure, $\mathcal{E}$, for a given network topology G, and demand price functions $\rho_{3k}(d)$ $(k = 1, 2, \ldots, o)$, and available funds held by source agents S, is defined as follows:

$$\mathcal{E} = \frac{\sum_{k=1}^{o} \frac{d_k^*}{\rho_{3k}(d^*)}}{o}, \tag{12}$$

where o is the number of demand markets in the financial network, and d_k^* and $\rho_{3k}(d^*)$ denote the equilibrium demand and the equilibrium price for demand market k, respectively.

The financial network performance measure $\mathcal{E}$ defined in (12) is actually the average demand to price ratio. It measures the overall (economic) functionality of the financial network. When the network topology G, the demand price functions, and the available funds held by source agents are given, a financial network is considered performing better if it can satisfy higher demands at lower prices.

The Financial Network Performance Measure

By referring to the equilibrium conditions (8) and (9), we assume that if there is a positive transaction between a source agent or an intermediary with a demand market at equilibrium, the price charged by the source agent or the intermediary plus the respective unit transaction costs is always positive.

Furthermore, we assume that if the equilibrium demand at a demand market is zero, the demand market price (i.e., the inverse demand function value) is positive. Hence, the demand market prices will always be positive and the above network performance measure is well-defined.

The Financial Network Performance Measure

We assume that all the demand markets are given the same weight when aggregating the demand to price ratio, which can be interpreted as all the demand markets are of equal strategic importance.

Of course, it is interesting to weigh demand markets differently by incorporating managerial or governmental factors into the measure. For example, we can give more preference to the markets with large demand quantity.

Network Efficiency vs. Network Performance

Although in some networks as the Internet and certain transportation networks, the assumption of having a central planner to ensure the minimization of the total cost may, in some instances, be natural and reasonable, the same assumption faces difficulty when extended to the larger and more complex networks as in the case of financial networks, where the control by a "central planner" is not realistic.

The Importance of a Financial Network Component

The financial network performance is expected to deteriorate when a critical network component is eliminated from the network.

Such a component can include a link or a node or a subset of nodes and links depending on the financial network problem under investigation. Furthermore, the removal of a critical network component will cause severe damage than that of the damage caused by a trivial component.

The importance of a network component is defined as:

Definition 3: Importance of a Financial Network Component

The importance of a financial network component $g \in G$, $I(g)$, *is measured by the relative financial network performance drop after* g *is removed from the network:*

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G-g)}{\mathcal{E}(G)} \tag{13}$$

where $G - g$ *is the resulting financial network after component* g *is removed from network* G.

The Importance of a Financial Network Component

It is worth pointing out that the above importance of the network components is well-defined even in a financial network with disconnected source agent/demand market pairs.

In our financial network performance measure, the elimination of a transaction link is treated by removing that link from the network while the removal of a node is managed by removing the transaction links entering or exiting that node.

In the case that the removal results in no transaction path connecting a source agent/demand market pair, we simply assign the demand for that source agent/demand market pair to an abstract transaction path with an associated cost of infinity. The above procedure(s) to handle disconnected agent/demand market pairs, will be illustrated in our numerical examples.

Numerical Examples

In order to further demonstrate the applicability of the financial network performance measure, two numerical financial network examples are presented.

For each example, our network performance measure is computed and the importance and the rankings of links and the nodes are also reported.

The examples consist of two source agents, two financial intermediaries, and two demand markets.

These examples have the financial network structure depicted in Figure 2.

For simplicity, we exclude the electronic transactions.

The Structure of the Network for the Numerical Examples

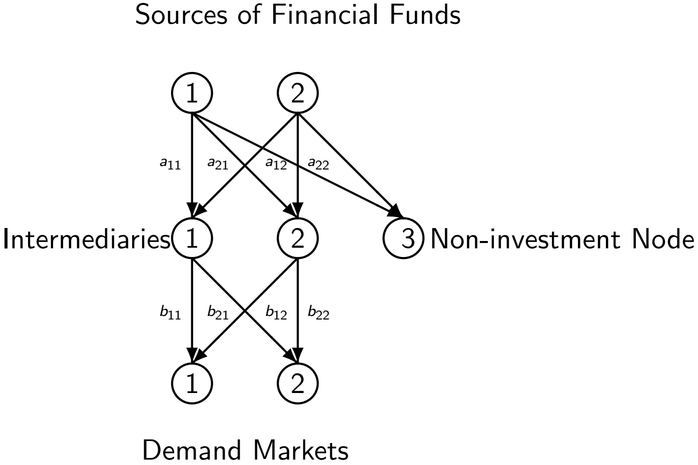


Figure 2: The Financial Network Structure of the Numerical Examples

Numerical Examples

The transaction links between the source agents and the intermediaries are denoted by a_{ij} where $i = 1, 2$; $j = 1, 2$.

The transaction links between the intermediaries and the demand markets are denoted by b_{jk} where $j = 1, 2$; $k = 1, 2$.

Since the non-investment portions of the funds do not participate in the actual transactions, we will not discuss the importance of the links and the nodes related to the non-investment funds.

The examples below were solved using the Euler method (see, Nagurney and Zhang (1996, 1997), Nagurney and Ke (2003), and Nagurney, Wakolbinger, and Zhao (2006)).

Example 1

The financial holdings for the two source agents in the first example are: $S^1 = 10$ and $S^2 = 10$.

The variance-covariance matrices V^i and V^j are identity matrices for all the source agents $i = 1, 2$.

We have suppressed the subscript l associated with the transaction cost functions since we have assumed a single (physical) mode of transaction being available.

Please refer to Tables 1-3 for a compact exposition of the notation.

Example 1

The transaction cost function of source agent 1 associated with his transaction with intermediary 1 is given by:

$$c_{11}(q_{11}) = 4q_{11}^2 + q_{11} + 1.$$

The other transaction cost functions of the source agents associated with the transactions with the intermediaries are given by:

$$c_{ij}(q_{ij}) = 2q_{ij}^2 + q_{ij} + 1, \quad \text{for } i = 1, 2; j = 1, 2$$

while i and j are not equal to 1 at the same time.

The transaction cost functions of the intermediaries associated with transacting with the sources agents are given by:

$$\hat{c}_{ij}(q_{ij}) = 3q_{ij}^2 + 2q_{ij} + 1, \quad \text{for } i = 1, 2; j = 1, 2.$$

The handling cost functions of the intermediaries are:

$$c_1(Q^1) = 0.5(q_{11} + q_{21})^2, \quad c_2(Q^1) = 0.5(q_{12} + q_{22})^2.$$

Example 1

We assumed that in the transactions between the intermediaries and the demand markets, the transaction costs perceived by the intermediaries are all equal to zero, that is,

$$c_{jk} = 0, \quad \text{for } j = 1, 2; k = 1, 2.$$

The transaction costs between the intermediaries and the consumers at the demand markets, in turn, are given by:

$$\hat{c}_{jk} = q_{jk} + 2, \quad \text{for } j = 1, 2; k = 1, 2.$$

The demand price functions at the demand markets are:

$$\rho_{3k}(d) = -2d_k + 100, \quad \text{for } k = 1, 2.$$

Example 1

The equilibrium financial flow pattern, the equilibrium demands, and the incurred equilibrium demand market prices are reported below.

For Q^{1*}, we have:

$$q_{11}^* = 3.27, \ \ q_{12}^* = 4.16, \ \ q_{21}^* = 4.36, \ \ q_{22}^* = 4.16.$$

For Q^{2*}, we have:

$$q_{11}^* = 3.81, \ \ q_{12}^* = 3.81, \ \ q_{21}^* = 4.16, \ \ q_{22}^* = 4.16.$$

Also, we have:

$$d_1^* = 7.97, \ \ d_2^* = 7.97,$$

$$\rho_{31}(d^*) = 84.06, \ \ \rho_{32}(d^*) = 84.06.$$

Example 1

The financial network performance measure (cf. (12)) is:

$$\mathcal{E} = \frac{\frac{7.97}{84.06} + \frac{7.97}{84.06}}{2} = 0.0949.$$

Example 1

The importance of the links and the nodes and their ranking are reported in Table 4 and 5, respectively.

Table 4: Importance and Ranking of the Links in Example 1

Link	Importance Value	Ranking
a_{11}	0.1574	3
a_{12}	0.2003	2
a_{21}	0.2226	1
a_{22}	0.2003	2
b_{11}	0.0304	5
b_{12}	0.0304	5
b_{21}	0.0359	4
b_{22}	0.0359	4

Numerical Examples

Table 5: Importance and Ranking of the Nodes in Example 1

Node	Importance Value	Ranking
Source Agent 1	0.4146	4
Source Agent 2	0.4238	3
Intermediary 1	0.4759	2
Intermediary 2	0.5159	1
Demand Market 1	0.0566	5
Demand Market 2	0.0566	5

Discussion

First note that, in Example 1, both source agents choose not to invest a portion of their financial funds.

Given the cost structure and the demand price functions in the network of Example 1, the transaction link between source agent 2 **and intermediary** 1 **is the most important link because it carries a large amount of financial flow, in equilibrium, and the removal of the link causes the highest performance drop assessed by the financial network performance measure.**

Similarly, because intermediary 2 **handles the largest amount of financial input from the source agents, it is ranked as the most important node in the above network.** On the other hand, since the transaction links between intermediary 1 to demand markets 1 and 2 carry the least amount of equilibrium financial flow, they are the least important links.

Example 2

In the second example, the parameters are identical to those in Example 1, except for the following changes.

The transaction cost function of source agent 1 associated with his transaction with intermediary 1 is changed to:

$$c_{11}(q_{11}) = 2q_{11}^2 + q_{11} + 1$$

and the financial holdings of the source agents are changed, respectively, to $S_1 = 6$ and $S_2 = 10$.

Example 2

The equilibrium financial flow pattern, the equilibrium demands, and the incurred equilibrium demand market prices are reported below.

For Q^{1*}, we have:

$$q_{11}^* = 3.00, \ q_{12}^* = 3.00, \ q_{21}^* = 4.48, \ q_{22}^* = 4.48.$$

For Q^{2*}, we have:

$$q_{11}^* = 3.74, \ q_{12}^* = 3.74, \ q_{21}^* = 3.74, \ q_{22}^* = 3.74.$$

Also, we have:

$$d_1^* = 7.48, \ d_2^* = 7.48,$$

$$\rho_{31}(d^*) = 85.04, \ \rho_{32}(d^*) = 85.04.$$

Example 2

The financial network performance measure (cf. (12)) is:

$$\mathcal{E} = \frac{\frac{7.48}{85.04} + \frac{7.48}{85.04}}{2} = 0.0880.$$

The importance of the links and the nodes and their ranking are reported in Table 6 and 7, respectively.

Example 2

Table 6: Importance and Ranking of the Links in Example 2

Link	Importance Value	Ranking
a_{11}	0.0917	2
a_{12}	0.0917	2
a_{21}	0.3071	1
a_{22}	0.3071	1
b_{11}	0.0211	3
b_{12}	0.0211	3
b_{21}	0.0211	3
b_{22}	0.0211	3

Example 2

Table 7: Importance and Ranking of the Nodes in Example 2

Node	Importance Value	Ranking
Source Agent 1	0.3687	3
Source Agent 2	0.6373	1
Intermediary 1	0.4348	2
Intermediary 2	0.4348	2
Demand Market 1	-0.0085	4
Demand Market 2	-0.0085	4

Discussion

In Example 2, the first source agent has no funds non-invested.

Given the cost structure and the demand price functions, since the transaction links between source agent 2 and intermediaries 1 and 2 carry the largest amount of equilibrium financial flow, they are ranked the most important.

In addition, since source agent 2 allocates the largest amount of financial flow in equilibrium, it is ranked as the most important node.

The negative importance value for demand markets 1 and 2 is due to the fact that the existence of each demand market brings extra flows on the transaction links and nodes and, therefore, increases the marginal transaction cost.

Discussion

The removal of one demand market has two effects:

• first, the contribution to the network performance of the removed demand market becomes zero;

• second, the marginal transaction cost on links/nodes decreases, which decreases the equilibrium prices and increases the demand at the other demand markets.

If the performance drop caused by the removal of the demand markets is overcompensated by the improvement of the demand-price ratio of the other demand markets, the removed demand market will have a negative importance value. It simply implies that the "negative externality" caused by the demand market has a larger impact than the performance drop due to its removal.

Summary and Conclusions

• We described a novel financial network performance measure, which is motivated by the research of Qiang and Nagurney (2008) and Nagurney and Qiang (2007a, b, c) in assessing the importance of network components in the case of disruptions in network systems ranging from transportation networks to such critical infrastructure networks as electric power generation and distribution networks.

• **The financial network measure examines the network performance by incorporating the economic behavior of the decision-makers, with the resultant equilibrium prices and transaction flows, coupled with the network topology.**

Summary and Conclusions

• The financial network performance measure, along with the network component importance definition, provide valuable methodological tools for evaluating the financial network vulnerability and reliability.

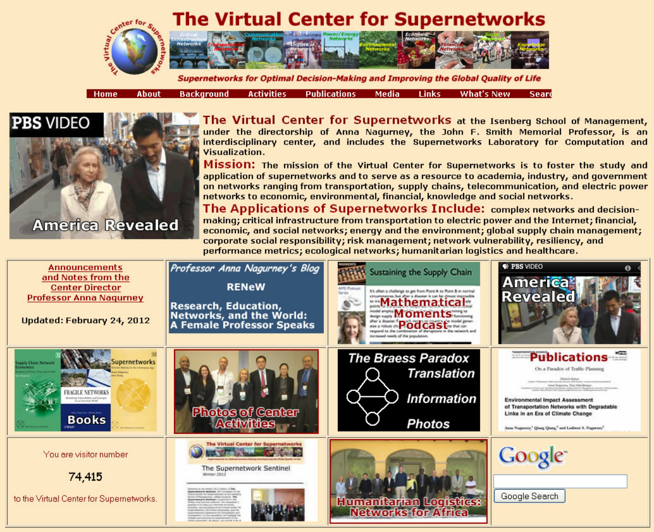
• **Furthermore, our measure is shown to be able to evaluate the importance of nodes and links in financial networks even when the source agent/demand market pairs become disconnected.**

Summary and Conclusions

We believe that our network performance measure is a good starting point to analyze the functionality of an economic network, in general, and a financial network, in particular.

Especially, in a network where agents compete in a noncooperative manner in the same tier and coordinate between different tiers without the intervention from the government and central planner, our proposed measure examines the network on a functional level other than in the traditional Pareto sense.

THANK YOU!



For more information, see: http://supernet.isenberg.umass.edu

References and Additional Readings

Beckmann, M. J., McGuire, B. C. and Winsten, B. C.: 1956, *Studies in the Economics of Transportation*, Yale University Press, New Haven, Connecticut.

Boginski, V., Butenko, S. and Pardalos, P. M.: 2003, On structural properties of the market graph, in *Innovations in Financial and Economic Networks*, A. Nagurney, Editor, Edward Elgar Publishing, Cheltenham, England, 29-42.

Buchanan, J. M. and Musgrave, R. A.: 1999, *Public Finance and Public Choice: Two Contrasting Visions of the State*, MIT Press, Boston, Massachusetts.

Caldarelli, G., Battiston, S., Garlaschelli, D. and Catanzaro, M.: 2004, Emergence of complexity in financial networks, *Lecture Notes in Physics*, **650**, 399-423.

Charnes, A. and Cooper, W. W.: 1967, Some network characterizations for mathematical programming and accounting approaches to planning and control, *The Accounting Review*, **42**, 24-52.

Chipman, J. S.: 1987, Compensation principle, in *The New Palgrave: A Dictionary of Economics, Vol. 1*, Eatwell, J., Milgate, M. and Newman, P., Editors, The Stockton Press, New York.

Christofides, N., Hewins, R. D. and Salkin, G. R.: 1979, Graph theoretic approaches to foreign exchange operations, *Journal of Financial and Quantitative Analysis*, **14**, 481-500.

Copeland, M. A.: 1952, *A Study of Moneyflows in the United States*, National Bureau of Economic Research, New York, New York.

Crum, R. L. and Nye, D. J.: 1981, A network model of insurance company cash flow management, *Mathematical Programming Study*, **15**, 86-101.

Doherty, N. A.: 1997, Financial innovation in the management of catastrophe risk, *Journal of Applied Corporate Finance*, **10**, 84-95.

Dafermos, S. C.: 1973, Toll patterns for multi-class user transportation networks, *Transportation Science*, **7**, 211-223.

Fei, J. C. H.: 1960, The study of the credit system by the method of the linear graph, *The Review of Economics and Statistic*, **42**, 417-428.

References and Additional Readings

Geunes, J. and Pardalos, P. M.: 2003, Network optimization in supply chain management and financial engineering: An annotated bibliography, *Networks*, **42**, 66-84.

Gilli, M. and Këllezi, E.: 2006, An application of extreme value theory for measuring financial risk, *Computational Economics*, **27**, 207-228.

Jackson, M. O. and Wolinsky, A.: 1996, A strategic model of social and economic networks, *Journal of Economic Theory*, **71**, 4474.

Kim, D. H. and Jeong, H.: 2005, Systematic analysis of group identification in stock markets, *Physical Review E*, **72**, Article No. 046133.

Kontoghiorghes, E. J., Rustem, B. and Siokos, S., Editors: 2002, *Computational Methods in Decision-Making, Economics and Finance, Optimization Models*, Kluwer Academic Publishers, Boston, Massachusetts.

Latora, V. and Marchiori, M.: 2001, Efficient behavior of small-world networks, *Physical Review Letters*, **87**, Article No. 198701.

Latora, V. and Marchiori, M.: 2002, Is the Boston subway a small-world network? *Physica A: Statistical Mechanics and its Applications*, **314**, 109-113.

Latora, V. and Marchiori, M.: 2003, Economic small-world behavior in weighted networks, *The European Physical Journal B*, **32**, 249-263.

Latora, V. and Marchiori, M.: 2004, How the science of complex networks can help developing strategies against terrorism, *Chaos, Solitons and Fractals*, **20**, 69-75.

Liu, Z. and Nagurney, A.: 2007, Financial networks with intermediation and transportation network equilibria: A supernetwork equivalence and computational management reinterpretation of the equilibrium conditions with computations, *Computational Management Science*, **4**, 243-281.

Loubergé, H., Këllezi, E. and Gilli, M.: 1999, Using catastrophe-linked securities to diversify insurance risk: A financial analysis of cat-bonds, *The Journal of Insurance Issues*, **2**, 125-146.

Mas-Colell, A., Whinston, M. and Green, J. R.: 1995, *Microeconomic Theory*, Oxford University Press, New York.

Mulvey, J. M.: 1987, Nonlinear networks in finance, *Advances in Mathematical Programming and Financial Planning*, **20**, 187-217.

Nagurney, A.: 2000, *Sustainable Transportation Networks*, Edward Elgar Publishers, Cheltenham, England.

References and Additional Readings

Nagurney, A.: 2008, Networks in finance, in the *Handbook on Information Technology and Finance*, D. Seese, C. Weinhardt, and F. Schlottmann, Editors, Springer, Berlin, Germany, 383-420.

Nagurney, A. and Cruz, J.: 2003a, International financial networks with electronic transactions, in *Innovations in Financial and Economic Networks*, A. Nagurney, Editor, Edward Elgar Publishing, Cheltenham, England, 136-168.

Nagurney, A. and Cruz, J.: 2003b, International financial networks with intermediation: modeling, analysis, and computations, *Computational Management Science*, **1**, 31-58.

Nagurney, A., Dong, J. and Hughes, M.: 1992, Formulation and computation of general financial equilibrium, *Optimization*, **26**, 339-354.

Nagurney, A. and Hughes, M.: 1992, Financial flow of funds networks, *Networks*, **22**, 145-161.

Nagurney, A. and Ke, K.: 2001, Financial networks with intermediation, *Quantitative Finance*, **1**, 309-317.

Nagurney, A. and Ke, K.: 2003, Financial networks with electronic transactions: modeling, analysis, and computations, *Quantitative Finance*, **3**, 71-87.

Nagurney, A. and Qiang, Q.: 2007a, A transportation network efficiency measure that captures flows, behavior, and costs with applications to network component importance identification and vulnerability, *Proceedings of the 18th Annual POMS Conference*, Dallas, Texas, May 4-7.

Nagurney, A. and Qiang, Q.: 2007b, A network efficiency measure with application to critical infrastructure networks, *Journal of Global Optimization*, in press.

Nagurney, A. and Qiang, Q.: 2007c, A network efficiency measure for congested networks, *Europhysics Letters*, 38005, p1-p5.

Nagurney, A. and Qiang, Q: 2007d, Robustness of transportation networks subject to degradable links, to appear in *Europhysics Letters*.

Nagurney, A. and Siokos, S.: 1997, *Financial Networks: Statics and Dynamics*, Springer-Verlag, Heidelberg, Germany.

Nagurney, A., Wakolbinger, T. and Zhao, L.: 2006, The evolution and emergence of integrated social and financial networks with electronic transactions: A dynamic supernetwork theory for the modeling, analysis, and computation of financial flows and relationship levels, *Computational Economics*, **27**, 353-393.

Nagurney, A. and Zhang, D.: 1996, *Projected Dynamical Systems and Variational Inequalities with Applications*, Kluwer Academic Publishers, Boston, Massachusetts.

References and Additional Readings

Nagurney, A. and Zhang, D.: 1997, Projected dynamical systems in the formulation, stability analysis, and computation of fixed demand traffic network equilibria, *Transportation Science*, **31**, 147-158.

Nash, J. F.: 1950, Equilibrium points in N-person games, *Proceedings of the National Academy of Sciences, USA*, **36**, 48-49.

Nash, J. F.: 1951, Noncooperative games, *Annals of Mathematics*, **54**, 286-298.

Newman, M.: 2003, The structure and function of complex networks, *SIAM Review*, **45**, 167-256.

Niehaus, G.: 2002, The allocation of catastrophe risk, *Journal of Banking and Finance*, **26**, 585-596.

Odell, K. and Phillips, R. J.: 2001, Testing the ties that bind: financial networks and the 1906 San Francisco earthquake, working paper, Department of Economics, Colorado State University, Fort Collins, Colorado.

Onnela, J. P., Kaski, K. and Kertész, J.: 2004, Clustering and information in correlation based financial networks, *The European Physical Journal B*, **38**, 353-362.

Qiang, Q. and Nagurney, A.: 2008, A unified network performance measure with importance identification and the ranking of network components, *Optimization Letters*, **2**, 127-142.

Quesnay, F.: 1758, *Tableau Economique*, reproduced in facsimile with an introduction by H. Higgs by the British Economic Society, 1895.

Robinson, C. P., Woodard, J. B. and Varnado, S. G.: 1998, Critical infrastructure: interlinked and vulnerable, *Issues in Science and Technology*, **15**, 61-67.

Roughgarden, T.: 2005, *Selfish Routing and the Price of Anarchy*, MIT Press, Cambridge, Massachusetts.

Rustem, B., and Howe, M.: 2002, *Algorithms for Worst-Case Design and Risk Management*, Princeton University Press, Princeton, New Jersey.

Sheffi, Y.: 2005, *The Resilient Enterprise: Overcoming Vulnerability for Competitive Advantage*, MIT Press, Cambridge, Massachusetts.

Samuelson, P. A.: 1983, *Foundations of Economic Analysis*, Enlarged Edition, Harvard University Press, Boston. Massachusetts.

Thore, S.: 1969, Credit networks, *Economica*, **36**, 42-57.

Thore, S., and Kydland, F.: 1972, Dynamic flow-of-funds networks, in *Applications of Management Science in Banking and Finance*, S. Eilon and T. R. Fowkes, Editors, Epping, England, 259-276.

Thore, S.: 1980 *Programming the Network of Financial Intermediation*, Universitetsforlaget, Oslo, Norway.