Financial Networks

Anna Nagurney

Visiting Professor
School of Business, Economics and Law
Gothenburg University
Gothenburg, Sweden
and
John F. Smith Memorial Professor
Director – Virtual Center for Supernetworks
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts 01003

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School of Business, Economics and Law
Gothenburg University, Sweden

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Outline

- Background and Motivation
- Brief Early History of the Science of Networks
- An Overview of Some Financial Network Models
- Networks in Mergers and Acquisitions
- The Pre- and Post-Merger Supply Chain Network Models with Cost and Risk
- Three Synergy Measures for Mergers and Acquisitions
- Numerical Examples
- First Set of Numerical Examples
- Second Set of Numerical Examples
- Managerial Insights and Conclusions

Background and Motivation

We Are in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- *alternative and, at times, conflicting criteria* used in decision-making;
- *constraints on resources*: human, financial, natural, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- *increasing risk and uncertainty*;
- the *importance of dynamics* and realizing a timely response to evolving events.

Characteristics of Networks Today

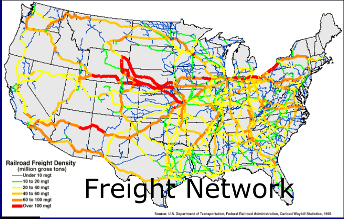
- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *interactions among networks themselves*, such as the Internet with electric power networks, financial networks, and transportation and logistical networks, as well as *social networks with financial networks and supply chains*;
- recognition of *the fragility and vulnerability of network systems*;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Networks consist of nodes, links, and flows and one must capture the underlying behavior and interactions of decision-makers and the induced costs, the relevant risks, and prices.

Network methodologies provide a spectrum of tools for financial problem formulation, analysis, and solution.

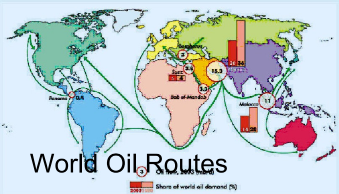
Components of Common Physical Networks

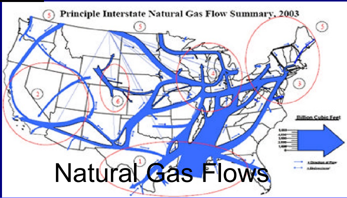
Network System	Nodes	Links	Flows
Transportation	Intersections, Homes, Workplaces, Airports, Railyards	Roads, Airline Routes, Railroad Track	Automobiles, Trains, and Planes,
Manufacturing and logistics	Workstations, Distribution Points	Processing, Shipment	Components, Finished Goods
Communication	Computers, Satellites, Telephone Exchanges	Fiber Optic Cables Radio Links	Voice, Data, Video
Energy	Pumping Stations, Plants	Pipelines, Transmission Lines	Water, Gas, Oil, Electricity



Network

Systems





Interdisciplinary Impact of Networks

Models and Algorithms

Networks

Economics and Finance

Interregional Trade

General Equilibrium

Industrial Organization

Portfolio Optimization

Flow of Funds Accounting

OR/MS and Engineering

Energy

Manufacturing

Telecommunications

Transportation

Supply Chains

Sociology

Social Networks

Organizational Theory

Computer Science

Routing Algorithms

Price of Anarchy

Biology

DNA Sequencing

Targeted Cancer Therapy

Brief History of the Science of Networks

Brief Early History of the Science of Networks

- 1736 - Euler - the earliest paper on graph theory - Konigsberg bridges problem.
- 1758 - Quesnay in his *Tableau Economique* introduced a network/graph to depict the circular flow of financial funds in an economy.
- 1781 - Monge, who had worked under Napoleon Bonaparte, publishes what is probably the first paper on transportation in minimizing cost.
- 1838 - Cournot states that a competitive price is determined by the intersection of supply and demand curves in the context of spatially separate markets in which transportation costs are included.
- 1841 - Kohl considered a two node, two route transportation network problem.
- 1845 - Kirchhoff wrote Laws of Closed Electric Circuits.

- 1920 - Pigou studied a transportation network system of two routes and noted that the decision-making behavior of the users on the network would result in different flow patterns.

- 1936 - Konig published the first book on graph theory.

- 1939, 1941, 1947 - Kantorovich, Hitchcock, and Koopmans considered the network flow problem associated with the classical minimum cost transportation problem and provided insights into the special network structure of these problems, which yielded special-purpose algorithms.

- 1948, 1951 - Dantzig published the simplex method for linear programming and adapted it for the classical transportation problem.

- 1951 - Enke showed that spatial price equilibrium problems can be solved using electronic circuits

• 1952 - Copeland in his book, *Studies of Moneyflows in the United States*, asked, Does money flow like water or electricity?

• 1952 - Samuelson gave a rigorous mathematical formulation of spatial price equilibrium and emphasized the network structure.

• 1952 - Markowitz unveils portfolio optimization theory.

• 1956 - Beckmann, McGuire, and Winsten in their book, *Studies in the Economics of Transportation*, provided a rigorous treatment of congested urban transportation systems under different behavioral mechanisms due to Wardrop (1952).

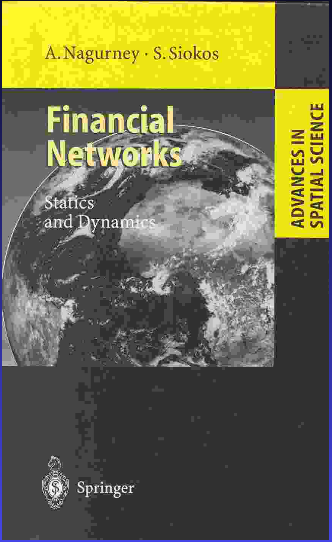
• 1962 - Ford and Fulkerson publish *Flows in Networks*.

• 1969 - Dafermos and Sparrow coined the terms user-optimization and system-optimization and develop algorithms for the computation of solutions that exploit the network structure of transportation problems.

An Overview of Some Financial Network Models

Financial Networks

Financial Networks



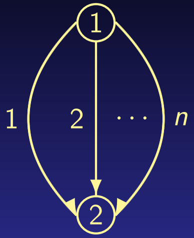


Figure 1: Network Structure of the Classical Markowitz (1952) Model

The structure of the classical portfolio optimization problem is that of a network.

Financial Network Models with Multiple Sectors

The need to expand upon the Markowitz and Sharpe frameworks in order to capture interactions among investors / sectors / decision-makers let to financial system network models.

Such models began with the work of Nagurney and Hughes (1992) in the estimation of financial flow of funds accounts.

The book by Nagurney and Siokos (1997) documents the evolution of financial networks models to that date.

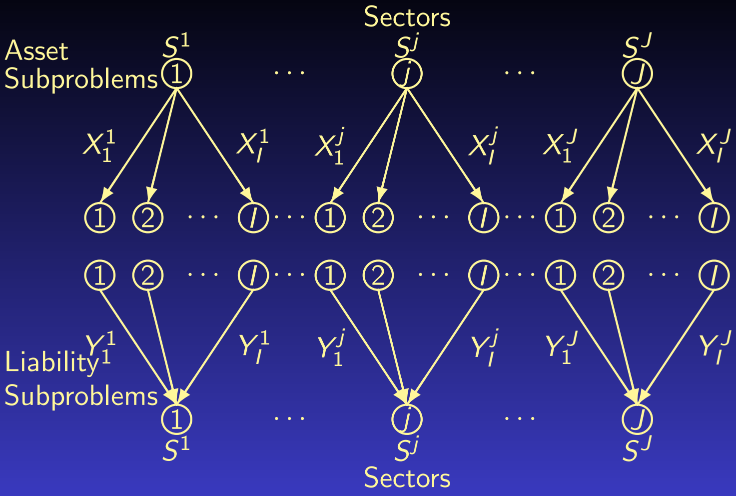


Figure 2: Network Structure of Multiple Financial Sectors' Portfolio Optimization Problems (Nagurney and Hughes (1992))

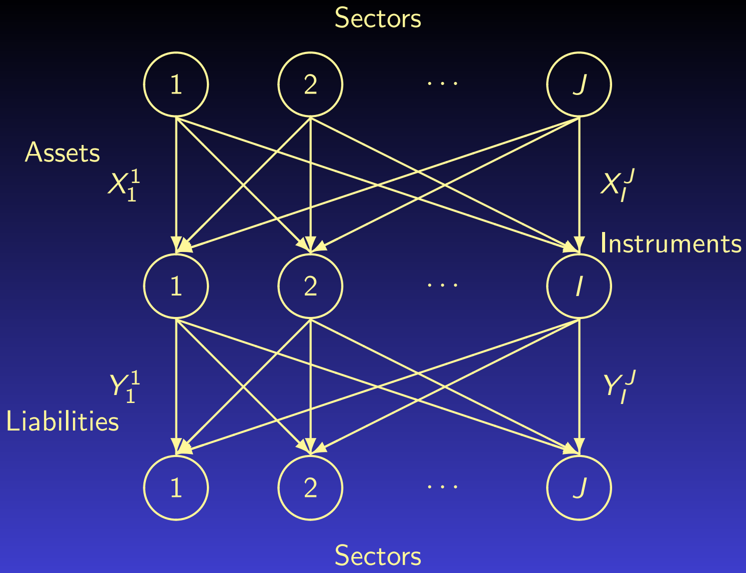
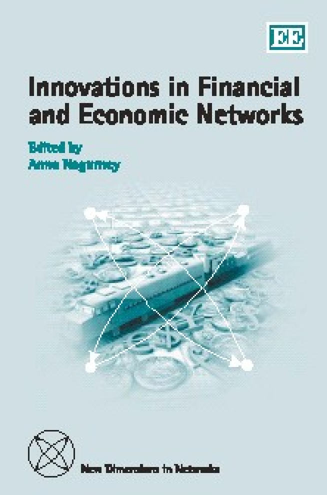


Figure 3: Network Structure at Equilibrium (Nagurney and Hughes (1992))

Financial Networks with Intermediation

There exist now general financial network models that capture the complexity of financial interactions on a macro level.



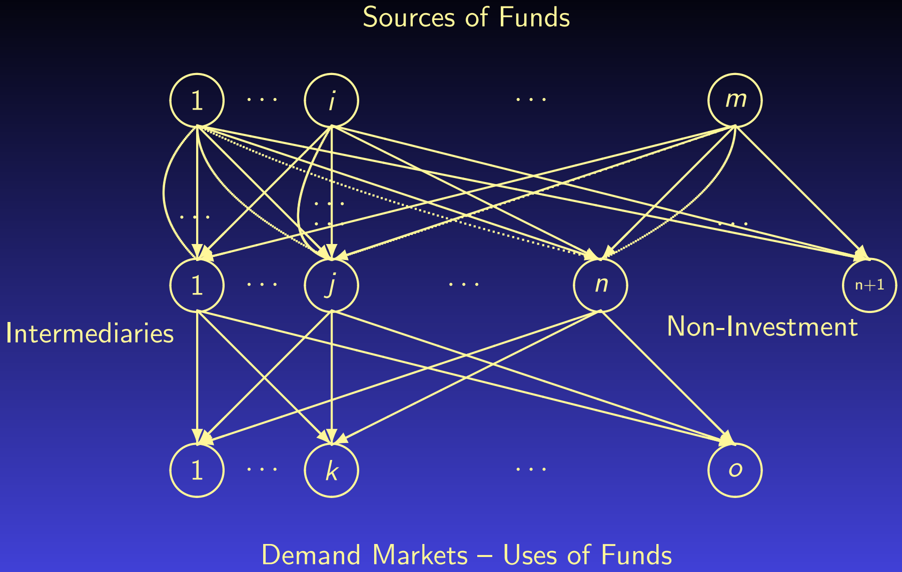


Figure 4: Network Structure of the Financial Economy with Intermediation (Nagurney and Ke (2001, 2003), *Quantitative Finance*)

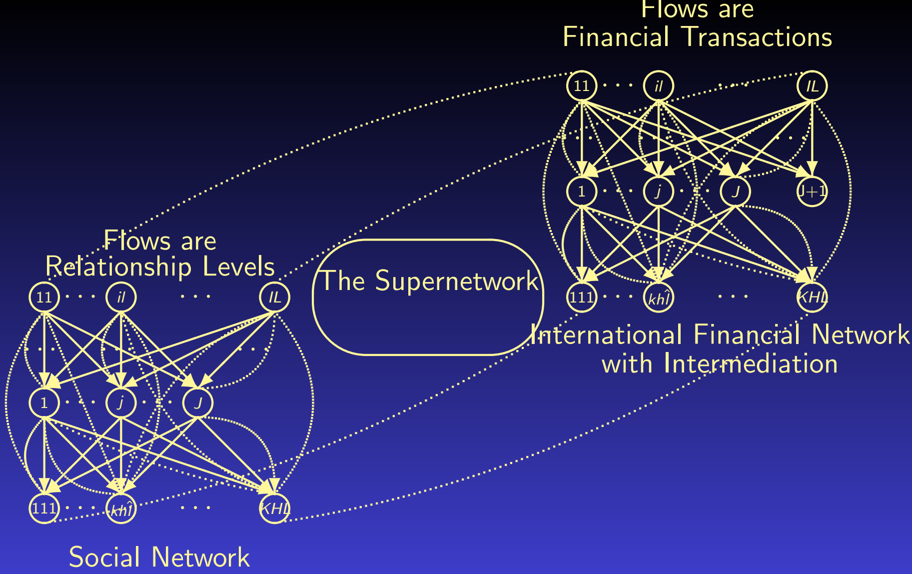
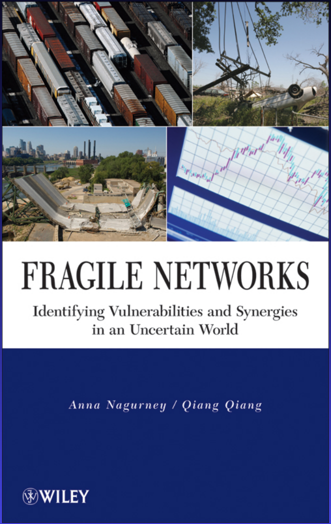


Figure 5: Multilevel Supernetwork Structure of the Integrated International Financial Network / Social Network System (Nagurney, Cruz, and Wakolbinger (2007), *Globalization and Regional Economic Modeling*)

Fragile Networks



We are living in a world of Fragile Networks.

Because today's financial networks may be *highly interconnected and interdependent*, any disruptions that occur in one part of the network may produce consequences in other parts of the network, which may not only be in the same region but miles away in other countries.

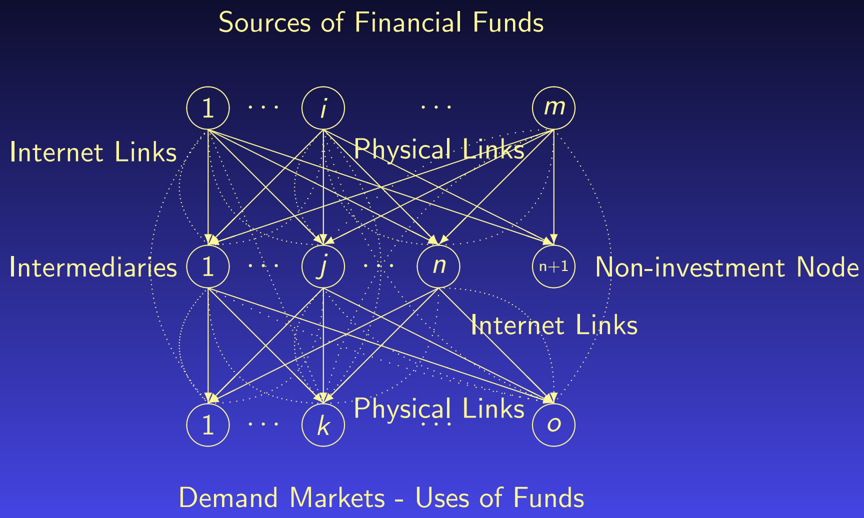
In 2008 and 2009, the world reeled from the effects of the financial credit crisis; leading financial services and banks closed (including the investment bank Lehman Brothers), others merged, and the financial landscape was changed for forever.

The domino effect of the U.S. economic troubles rippled through overseas markets and pushed countries such as Iceland to the verge of bankruptcy.

It is crucial for the decision-makers in financial systems (managers, executives, and regulators) to be able *to identify a financial network's vulnerable components* to protect the functionality of the network.

Our financial network performance measure (Nagurney and Qiang (2008)) and component importance indicator was published in the edited volume *Computational Methods in Financial Engineering*.

The Financial Network Model with Intermediation



The Financial Network Model with Intermediation

Sources of funds are, for example, households and business.

Financial intermediaries are banks, insurance compaies, etc. The demand markets correspond to financial products associated with consumers.

Source agents and financial intermediaries seek to maximize their net returns and to minimize their risk.

Consumers evaluate the financial products subject to transaction costs.

Definition: The Financial Network Performance Measure

The financial network performance measure, $\mathcal{E}$, for a given network topology G, and demand price functions $\rho_{3k}(d)$ $(k = 1, 2, \ldots, o)$, and available funds held by source agents S, is defined as follows:

$$\mathcal{E} = \frac{\sum_{k=1}^{o} \frac{d_k^*}{\rho_{3k}(d^*)}}{o},$$

where o is the number of demand markets in the financial network, and d_k^* and $\rho_{3k}(d^*)$ denote the equilibrium demand and the equilibrium price for demand market k, respectively.

The financial network performance measure $\mathcal{E}$ defined above is actually the average demand to price ratio. It measures the overall (economic) functionality of the financial network.

When the network topology G, the demand price functions, and the available funds held by source agents are given, a financial network is considered performing better if it can satisfy higher demands at lower prices.

Definition: Importance of a Financial Network Component

The importance of a financial network component $g \in G$, $I(g)$, is measured by the relative financial network performance drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G-g)}{\mathcal{E}(G)}$$

where $G - g$ is the resulting financial network after component g is removed from network G.

It is worth pointing out that the importance of the network components is well-defined even in a financial network with disconnected source agent/demand market pairs.

In our financial network performance measure, the elimination of a transaction link is treated by removing that link from the network while the removal of a node is managed by removing the transaction links entering or exiting that node.

In the case that the removal results in no transaction path connecting a source agent/demand market pair, we simply assign the demand for that source agent/demand market pair to an abstract transaction path with an associated cost of infinity.

Networks in Mergers and Acquisitions

We have also been using network methodologies to assess *not only financial performance and vulnerability* but also *synergies* associated with network integration as in mergers and acquisitions

Current Merger and Acquisition Activity

M&As totaled over $2 trillion in 2009, down 32% from full-year 2008 and down 53% from the record high in 2007, according to data from Thomson Reuters.

Mergers announced in October 2010 include Bain Capital / Gymboree, at $1.789 billion and Dynamex / Greenbriar Equity Group ($207 million).

Some of the most visible recent mergers have occurred in the airline industry with Delta and Northwest completing their merger in October 2008 and United and Continental closing on the formation of United Continental Holdings Oct. 1, 2010.

Global 2010 M&A activity is estimated to rise as much as 35% from 2009 figures (Sanford C. Bernstein research firm).

Successful mergers can add tremendous value; however, the failure rate is estimated to be between 74% and 83% (Devero (2004)).

It is worthwhile to develop tools to better predict the associated strategic gains, which include, among others, cost savings.

Mergers and Acquisitions and Network Synergies

A successful merger depends on the ability to measure the anticipated synergy of the proposed merger (cf. Chang (1988)) .

The rest of this presentation is based on the recent paper:

- Z. Liu and A. Nagurney (2011), “Risk Reduction and Cost Synergy in Mergers and Acquisitions via Supply Chain Network Integration,” *Journal of Financial Decision Making* **7(2)**, 1-18.

The economic and financial collapse of 2008 and 2009 due to the credit crisis in the U.S. with global ramifications, impacted dramatically the Mergers and Acquisitions (M&A) landscape.

According to *The Economist* (2009), 2007 broke all records in terms of M&A with *approximately 4.8 trillion dollars in M&A deals transacted*.

But in the year ending in August 2009, the value of such deals globally was *just below 1.5 trillion dollars*.

According to *The Economist* (2010), *emerging countries from Thailand to India and China have entered a period of dynamism* as developed countries continue to struggle with the recession with emerging-market companies pursuing growth through M&As with a focus on acquiring brands and distribution channels.

In addition, it is being reported that we can expect M&As in the healthcare, high tech, media, and energy sectors (cf. Zendrian (2010)).

It is increasingly apparent and documented that *improving supply chain integration is key to improving the likelihood of post-merger success*!

This is understandable, since *up to 80% of a firm's costs are linked to operations* (Benitez and Gordon (2000)).

However, empirical studies demonstrate that *one out of two post-merger integration efforts fares poorly* (Gerds and Schewe (2009)).

In addition, in an empirical analysis of a global sample of over 45,000 data points of post-merger transactions in all significant sectors globally from services to manufacturing, *risk factors were identified to post-merger success* (see Gerds, Strottmann, and Jayaprakash (2010)).

Risk in the context of supply chains may be associated with

- the production/procurement processes,
- the transportation/shipment of the goods,
- and/or the demand markets.

Such supply chain risks are directly reflected in firms' financial performances, and priced in the financial market.

Hendricks and Singhal (2010) estimated that the average stock price reaction to supply-demand mismatch announcements was approximately -6.8%. In addition, supply chain disruptions can cause firms' equity risks to increase by 13.50% on average after the disruption announcements (Hendricks and Singhal (2005)).

Illustrations of Supply Chain Risk

We build upon the recent work in mergers and acquisitions of that focuses on horizontal network integration (cf. Nagurney (2009), Nagurney and Woolley (2010), and Nagurney, Woolley, and Qiang (2010)).

We develop the following significant extension: *we utilize a mean-variance (MV) approach in order to capture the risk associated with supply chain activities both prior to and post the merger/acquisition under investigation*. The MV approach to the measurement of risk dates to the work of the Nobel laureate Markowtiz (1952, 1959) and even today (cf. Schneeweis, Crowder, and Kazemi (2010)) remains a fundamental approach to minimizing volatility.

This new modeling framework allows one to capture quantitatively the risk associated *not only with the supply chain network activities but also with the merger/acquisition itself, which has been identified as being critical in practice*.

For a comprehensive review of supply chain risk management models, please see Kleindorfer and Saad (2005), Tang (2006), Nagurney (2006), and Wu and Blackhurst (2009).

We believe that *is essential to study supply chain risk management from a holistic point of view, even in the context of mergers and acquisitions*, since *to capture the full complexity of the network may result in paradoxical behavior* (see Nagurney (2010)).

The Pre- and Post-Merger Supply Chain Network Models

All firms, both prior and post the merger, minimize both their expected total costs and the risk, as captured through the variance of the total costs, with a suitable weight assigned to the latter.

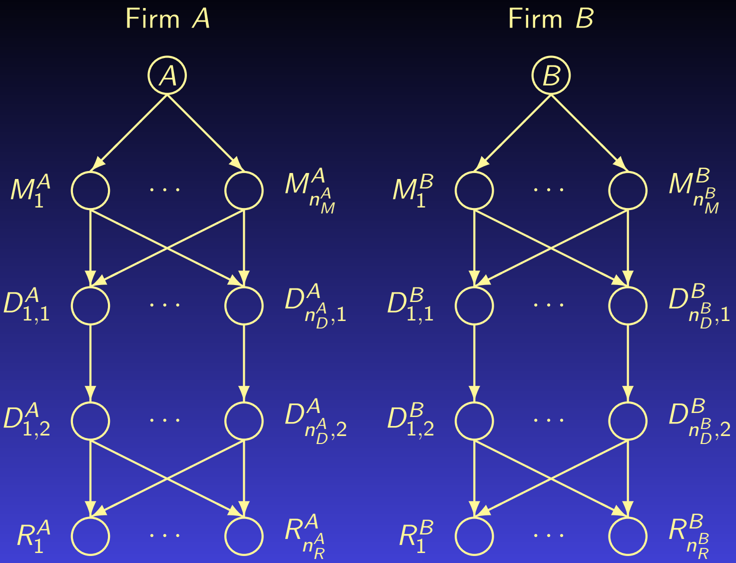


Figure 7: The Pre-Merger Supply Chain Network

The Pre-Merger Supply Chain Network Model with Risk

The links in Figure 7 are denoted by a, b, etc., and the total cost on link a by $\hat{c}_a$.

Let $d_{R^i_k}$ denote the fixed demand for the product at retailer R^i_k associated with firm i; $i = A, B$; $k = 1, \ldots, n^i_R$.

Let x_p denote the nonnegative flow of the product on path p connecting (origin) node i with a (destination) retail node of firm i; $i = A, B$.

The conservation of flow equations must hold for each firm i:

$$\sum_{p \in P^0_{R^i_k}} x_p = d_{R^i_k}, \quad i = A, B; k = 1, \ldots, n^i_R, \tag{1}$$

where $P^0_{R^i_k}$ denotes the set of paths joining node i with retail node R^i_k. Hence, the demand at each retail node must be satisfied by the product flows destined to that node.

The flow on link a is denoted by f_a.

The following conservation of flow equations must also hold:

$$f_a = \sum_{p \in P_i} x_p \delta_{ap}, \quad \forall a \in L_i; \ i = A, B, \tag{2}$$

where $\delta_{ap} = 1$ if link a is contained in path p and $\delta_{ap} = 0$, otherwise.

Here P_i denotes the set of *all* paths in firm i's network in Figure 7, that is, $P_i = \cup_{k=1,\ldots,n_R^i} P^0_{R_k^i}$.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P_i; \ i = A, B. \tag{3}$$

The total cost on link a, $\hat{c}_a$, takes the form:

$$\hat{c}_a = \hat{c}_a(f, \omega_a) = \omega_a \hat{h}_a f_a + h_a f_a, \quad \forall a \in L_i; \; i = A, B, \qquad (4)$$

where ω_a denotes the exogenous random variable affecting the total cost of link a.

We allow ω_a to follow any distribution, and permit the ω_as of different links to be correlated with one another. The ω_as can represent *factors of uncertainty, such as, for example, those associated with foreign exchange rates, the production disruption frequencies, and/or the energy and material prices*.

In (4), $\hat{h}_a f_a$, represents that part of the total cost that is subject to the variation of ω_a, whereas $h_a f_a$ denotes that part of the total cost that is independent of ω_a.

We assume that there are nonnegative capacities on the links with the capacity on link a denoted by u_a, $\forall a$.

The firms consider both costs and risks in their operations using a mean-variance framework and each seeks to minimize its expected total cost and the valuation of its risk. The optimization problem faced by firm i; $i = A, B$, can be expressed as:

$$\text{Minimize} \quad \sum_{a \in L_i} E(\hat{c}_a(f_a, \omega_a)) + \alpha_i V(\sum_{a \in L_i} \hat{c}_a(f_a, \omega_a)) \tag{5}$$

subject to: constraints (1) – (3) and

$$f_a \leq u_a, \quad \forall a \in L_i. \tag{6}$$

where the first term in the objective function (5) denotes the expected total cost; α_i denotes the risk aversion factor of firm i; and $V(\sum_{a \in L_i} \hat{c}_a(f_a, \omega_a))$ represents the variance of the total cost.

Note that we can substitute (4) into (5), to obtain the equivalent optimization problem:

$$\text{Minimize} \quad \sum_{a\in L_i} E(\omega_a)\hat{h}_a f_a + \sum_{a\in L_i} h_a f_a + \alpha_i V(\sum_{a\in L_i} \omega_a \hat{h}_a f_a) \qquad (7)$$

subject to: constraints (1) – (3) and

$$f_a \leq u_a, \quad \forall a \in L_i. \qquad (8)$$

We assume that the objective function in (7) is convex and that the individual terms are continuously differentiable. This optimization problem is a constrained, convex nonlinear programming problem. According to the standard theory of nonlinear programming (cf. Bazaraa, Sherali, and Shetty, 1993) if the feasible set of the problem represented by the constraints (1) – (3) and (6) is non-empty, then the optimal solution, denoted by $f^* \equiv \{f_a^*\}, a \in L_i$, exists.

We define $\mathcal{K}_i \equiv \{f | \exists x \geq 0, \text{ and } (1) - (3) \text{ and } (6) \text{ hold}\}$, where f is the vector of link flows and x the vector of path flows and we let β_a denote the Lagrange multiplier associated with constraint (6) for link a.

Theorem 1

The vector of link flows of firm i, $f^* \in \mathcal{K}_i$; i=A,B, is an optimal solution to problem (5), subject to (1) through (3) and (6), if and only if it satisfies the following variational inequality problem with the vector of optimal nonnegative Lagrange multipliers β^*:

$$\sum_{a \in L_i} \left[E(\omega_a)\hat{h}_a + h_a + \frac{\partial V(\sum_{a \in L_i} \omega_a \hat{h}_a f_a)}{\partial f_a} + \beta_a^* \right] \times [f_a - f_a^*] + \sum_{a \in L_i} [u_a - f_a^*]$$

$$\times [\beta_a - \beta_a^*] \geq 0, \quad \forall f \in \mathcal{K}_i, \quad \forall \beta_a \geq 0, \forall a \in L_i. \qquad (9)$$

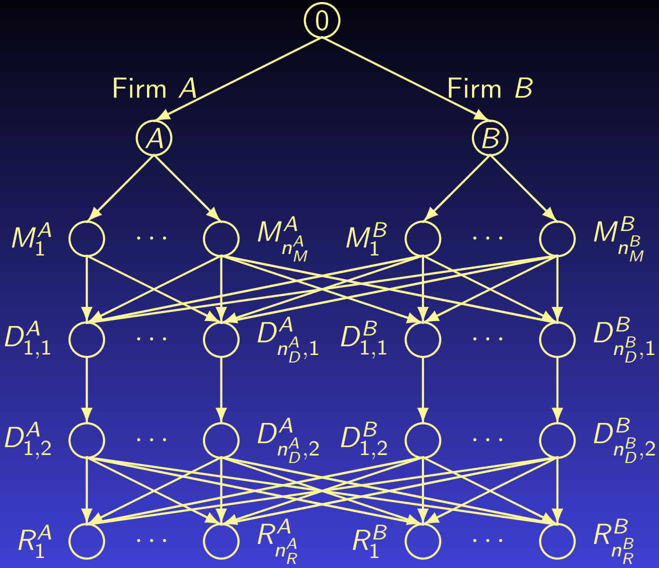


Figure 8: The Post-Merger Supply Chain Network

Let x_p denote the flow of the product on path p connecting node 0 with a retailer node.

Then the following conservation of flow equations must hold:

$$\sum_{p \in P^1_{R^i_k}} x_p = d_{R^i_k}, \quad i = A, B; k = 1, \ldots, n^i_R, \tag{12}$$

where $P^1_{R^i_k}$ denotes the set of paths joining node 0 with retail node R^i_k. The set $P^1 \equiv \cup_{i=A,B;k=1,\ldots,n^i_R} P^1_{R^i_k}$.

In addition, as before, we let f_a denote the flow of the product on link a, and we must have the following conservation of flow equations satisfied:

$$f_a = \sum_{p \in P^1} x_p \delta_{ap}, \quad \forall a \in L^1, \tag{13}$$

$$x_p \geq 0, \quad \forall p \in P^1. \tag{14}$$

In the case of *an acquisition*, we can expect the acquiring firm to impose its valuation of risk on the integrated network link activities, whereas in the case of *a merger*, the risk aversion factor may be obtained after some negotiations between the two firms that merge.

Hence, we assume that the risk aversion factor, post-merger (or acquisition), is denoted by α and recognize that, in the case of an acquisition, $\alpha = \alpha_i$, with i being an acquiring firm and, in the case of a merger, $\alpha = \frac{\alpha_A + \alpha_B}{2}$ being reasonable factors.

The optimization problem associated with the post-merger firm which minimizes the expected total cost and the total risk subject to the demand for the product being satisfied at the retailers, is, thus, given by:

$$\text{Minimize} \quad \sum_{a \in L^1} E(\hat{c}_a(f_a, \omega_a)) + \alpha V(\sum_{a \in L^1} \hat{c}_a(f_a, \omega_a)) \tag{15}$$

subject to: constraints (12) – (14) and

$$f_a \leq u_a, \quad \forall a \in L^1. \tag{16}$$

We can substitute (4) into (15) to obtain the equivalent optimization problem:

$$\text{Minimize} \quad \sum_{a \in L^1} E(\omega_a)(\hat{h}_a f_a + \hat{g}_a) + \sum_{a \in L^1} (h_a f_a + g_a) + \alpha V(\sum_{a \in L^1} \omega_a \hat{h}_a f_a) \tag{17}$$

subject to: constraints (12) – (14) and

$$f_a \leq u_a, \quad \forall a \in L^1. \tag{18}$$

The expected total cost associated with the merger, TC^1, which is defined as:

$$TC^1 \equiv \sum_{a \in L^1} E(c_a(f_a^*, \omega_a)), \tag{19}$$

and, the total risk associated with the merger, TR^1, which is defined as:

$$TR^1 \equiv V(\sum_{a \in L^1} (\hat{c}_a(f_a^*, \omega_a)), \tag{20}$$

with TC^0 and TR^0 denoting, respectively, the total minimal cost and variance prior to the merger.

Three Synergy Measures for Mergers and Acquisitions

The measures to capture the gains, if any, are:

The Expected Total Cost Synergy

$$\mathcal{S}_{TC} \equiv \left[\frac{TC^0 - TC^1}{TC^0}\right] \times 100\%, \tag{21}$$

quantifies the expected total cost savings.

The Absolute Risk Synergy

$$\mathcal{S}_{TR} \equiv \left[\frac{TR^0 - TR^1}{TR^0}\right] \times 100\%, \tag{22}$$

represents the reduction of the absolute risk achieved through the merger.

The Relative Risk Synergy

$$\mathcal{S}_{CV} \equiv \left[\frac{CV^0 - CV^1}{CV^0} \right] \times 100\%, \tag{23}$$

where CV^0 and CV^1 denote the coefficient of variation of the total cost for, respectively, the pre-merger and the post-merger networks, and are defined as follows:

$$CV^0 \equiv \frac{\sqrt{TR^0}}{TC^0}, \tag{24}$$

$$CV^1 \equiv \frac{\sqrt{TR^1}}{TC^1}. \tag{25}$$

Note that CV^0 and CV^1 represent the volatilities of the expected total costs of the pre- and post-merger networks, respectively.

This measure reflects the reduction of the relative risk through the merger.

Numerical Examples

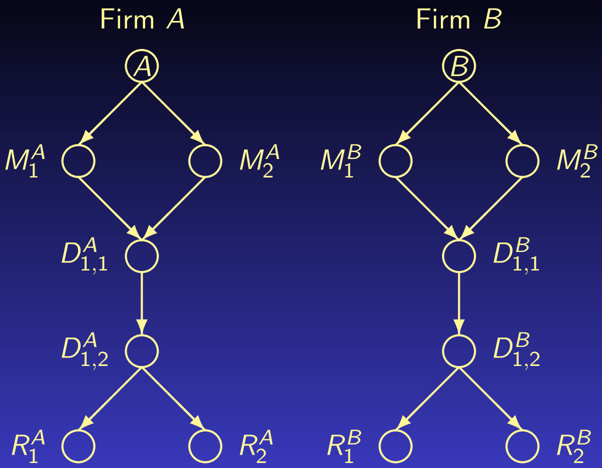


Figure 9: The Pre-Merger Supply Chain Network Topology for the Numerical Examples

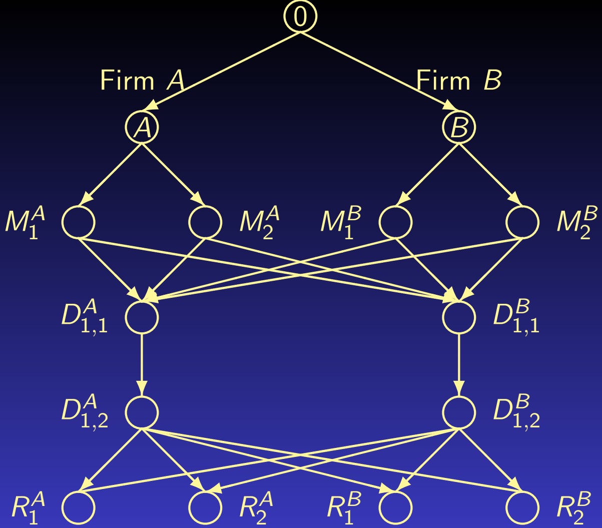


Figure 10: The Post-Merger Supply Chain Network Topology for the Numerical Examples

Both variational inequality problems (pre-merger and post-merger) can be solved using the modified projection method, which we embedded with the equilibration algorithm, for the solution of all the numerical examples. The code was implemented in Matlab.

The weights $\alpha_1 = \alpha_2 = 1$ and $\alpha = 1$.

For details, please refer to our paper.

First Set of Numerical Examples

Table 1: Definition of Links and Associated Total Cost Functions for the Numerical Examples

Link a	From Node	To Node	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$
1	A	M_1^A	$\omega_1 2f_1 + f_1$	$E(\omega_1) = 1$
2	A	M_2^A	$\omega_2 4f_2 + f_2$	$E(\omega_2) = 1$
3	M_1^A	$D_{1,1}^A$	$\omega_3 f_3 + f_3$	$E(\omega_3) = 1$
4	M_2^A	$D_{1,1}^A$	$\omega_4 f_4 + f_4$	$E(\omega_4) = 1$
5	$D_{1,1}^A$	$D_{1,2}^A$	$\omega_5 f_5 + f_5$	$E(\omega_5) = 1$
6	$D_{1,2}^A$	R_1^A	$\omega_6 f_6 + f_6$	$E(\omega_6) = 1$
7	$D_{1,2}^A$	R_2^A	$\omega_7 f_7 + f_7$	$E(\omega_7) = 1$
8	B	M_1^B	$\omega_8 2f_8 + f_8$	$E(\omega_8) = 1$
9	B	M_2^B	$\omega_9 4f_9 + f_9$	$E(\omega_9) = 1$
10	M_1^B	$D_{1,1}^B$	$\omega_{10} f_{10} + f_{10}$	$E(\omega_{10}) = 1$
11	M_2^B	$D_{1,1}^B$	$\omega_{11} f_{11} + f_{11}$	$E(\omega_{11}) = 1$
12	$D_{1,1}^B$	$D_{1,2}^B$	$\omega_{12} f_{12} + f_{12}$	$E(\omega_{12}) = 1$
13	$D_{1,2}^B$	R_1^B	$\omega_{13} f_{13} + f_{13}$	$E(\omega_{13}) = 1$
14	$D_{1,2}^B$	R_2^B	$\omega_{14} f_{14} + f_{14}$	$E(\omega_{14}) = 1$

Table 2: Definition of Links and Associated Total Cost Functions for the Numerical Examples

Link a	From Node	To Node	$\hat{c}_a(f_a, \omega_a)$	$E(\omega_a)$
15	M_1^A	$D_{1,1}^B$	$\omega_{15} f_{15} + f_{15}$	$E(\omega_{15}) = 1$
16	M_2^A	$D_{1,1}^B$	$\omega_{16} f_{16} + f_{16}$	$E(\omega_{16}) = 1$
17	M_1^B	$D_{1,1}^A$	$\omega_{17} f_{17} + f_{17}$	$E(\omega_{17}) = 1$
18	M_2^B	$D_{1,1}^A$	$\omega_{18} f_{18} + f_{18}$	$E(\omega_{18}) = 1$
19	$D_{1,2}^A$	R_1^B	$\omega_{19} f_{19} + f_{19}$	$E(\omega_{19}) = 1$
20	$D_{1,2}^A$	R_2^B	$\omega_{20} f_{20} + f_{20}$	$E(\omega_{20}) = 1$
21	$D_{1,2}^B$	R_1^A	$\omega_{21} f_{21} + f_{21}$	$E(\omega_{21}) = 1$
22	$D_{1,2}^B$	R_2^A	$\omega_{22} f_{22} + f_{22}$	$E(\omega_{22}) = 1$
23	0.00	A	0.00	$--$
24	0.00	B	0.00	$--$

In the first set of examples, since we assumed that the total cost and the total risk of the merger process are negligible, the total cost of the merger links (emanating from node 0) are assumed to be zero. The capacities on all the links in all the examples were set to: $u_a = 40, \forall a \in L_1$.

The demands at the retailers were: $d_{R_1^A} = 10$, $d_{R_2^A} = 10$, and $d_{R_1^B} = 10$, $d_{R_2^B} = 10$.

COV, the covariance matrix of the random cost factors, the ω_as, takes the form:

$$COV = \sigma^2 I, \tag{26}$$

where I is a 24×24 identity matrix, and σ represents the magnitude of the variance. In the simulation examples, we vary σ^2 from 0.01 to 0.1 to show how the costs and the risks of the pre-merger and post-merger networks change as the uncertainty increases.

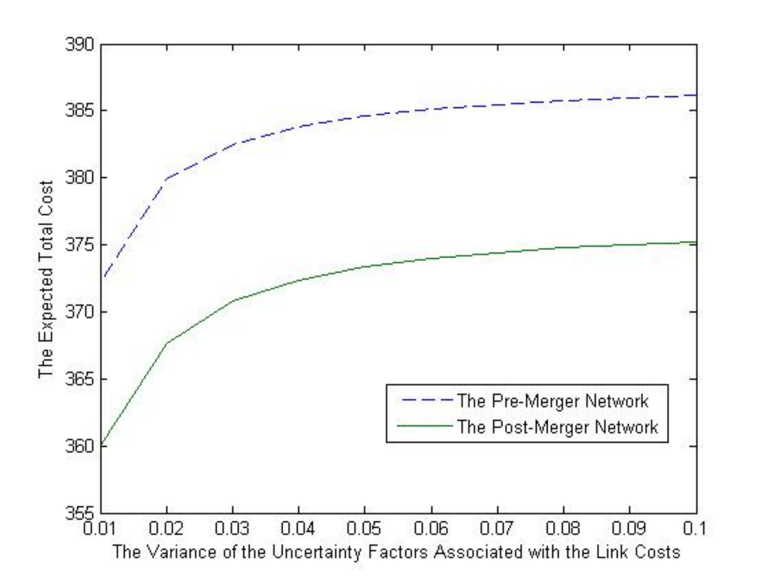


Figure 11: The Expected Total Costs of the Pre-Merger and the Post-Merger Networks

Figure 11 shows that, as the variance of the link cost uncertainty factors, σ^2, increases, the expected total costs of both the pre-merger and the post-merger networks will increase.

In addition, the expected total cost of the post-merger network is consistently lower than that of the pre-merger network.

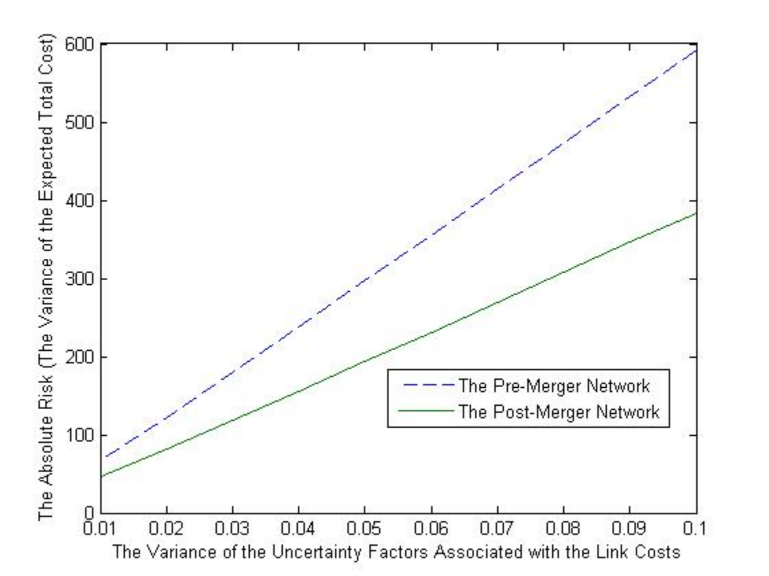


Figure 12: The Absolute Risks of the Pre-Merger and the Post-Merger Networks

Figure 12 shows that, as the variance of the link cost uncertainty factors, σ^2, increases, the total absolute risks of both networks represented by the variances of total costs both increase.

In addition, the total absolute risk of the post-merger network is always lower than that of the pre-merger network. Moreover, Figure 12 also shows that the total absolute risk of the post-merger network increases less quickly than that of the pre-merger network, which makes the gap between the total absolute risks of the two networks become larger as the link cost variance increases.

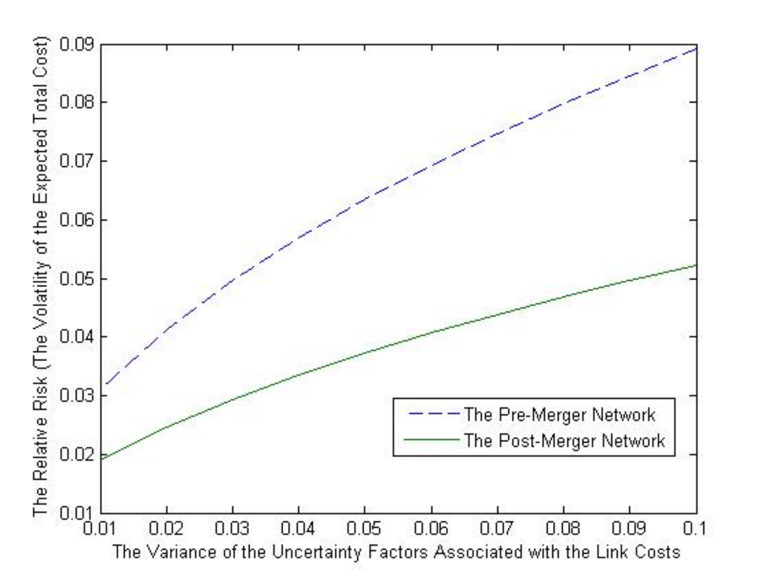


Figure 13: The Relative Risks of the Pre-Merger and the Post-Merger Networks

Figure 13 exhibits the trend of the relative risks of the two networks where the relative risks are represented by the volatilities (coefficient of variation) of the expected total costs.

Figure 13 shows that, as the variance of the link cost uncertainty factors, σ^2, increases the total cost volatilities of both networks increase.

Observe that the relative risk of the post-merger network is always lower than that of the pre-merger network. Moreover, the relative risk of the post-merger network increases less quickly than that of the pre-merger network which makes the gap between the relative risks of the two networks wider as the link cost uncertainty, σ^2, increases.

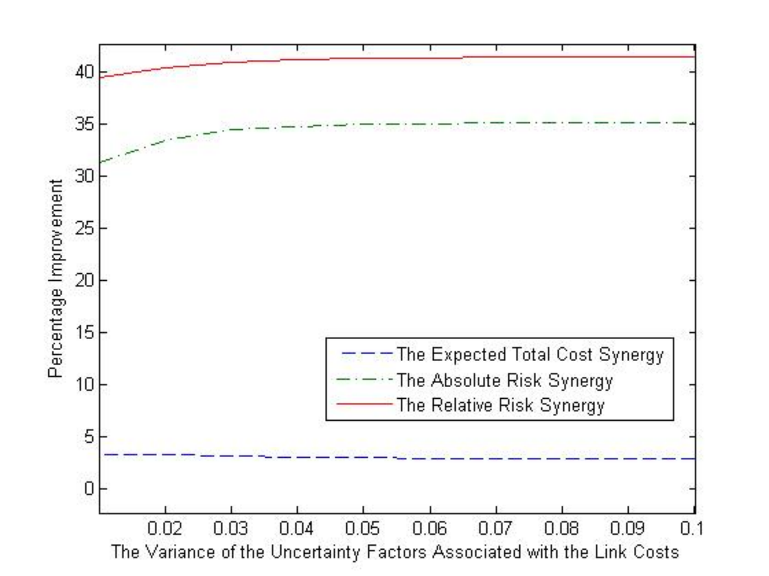


Figure 14: The Three Synergy Measures for Set 1

From Figure 14, we see that, in this example set, all the three measures are always positive which indicates that the merger of the two networks reduces both the expected total cost and the total risk when the cost and the risk of merger links are negligible.

In addition, the value of the expected total cost synergy, $\mathcal{S}_{TC}$, is relatively low, and is below 5% while the values of the two risk reduction synergy measures, $\mathcal{S}_{TR}$ and $\mathcal{S}_{VC}$, are both consistently higher than 30%.

Finally, we observe that, as the variance of the link cost uncertainty factors, σ^2, increases, the values of the two risk reduction synergy measures also increase while the value of the expected total cost synergy slightly decreases.

Second Set of Numerical Examples

In the second set of examples, we used the same networks and cost functions as in Set 1 except that we now assumed that the costs and risks of the merger links are not negligible.

In particular, we assumed that the total cost functions of the two merger links are as follows:

$$\hat{c}_{23}(f_{23}, \omega_{23}) = \omega_{23} f_{23}, \tag{27}$$

$$\hat{c}_{24}(f_{24}, \omega_{24}) = \omega_{24} f_{24}, \tag{28}$$

where $E(\omega_{23}) = 1$, $E(\omega_{24}) = 1$, and the variance of ω_{23} and ω_{24} are equal to $\hat{\sigma}^2$.

We varied $\hat{\sigma}^2$ from 0.0 to 0.8 to show how the three measures change as the risk incurred in the merger process increases. We now assumed that the variance of the uncertainty factors associated with the other links, σ^2, is fixed and is equal to 0.1.

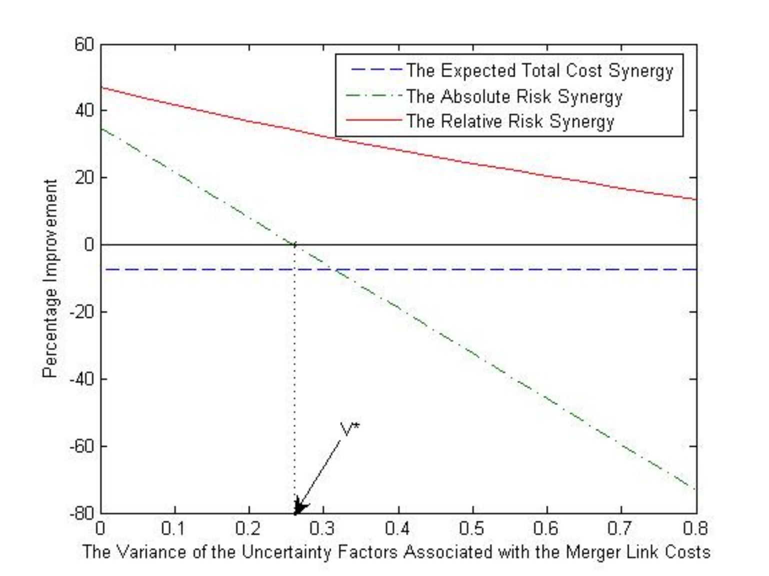


Figure 15: The Three Synergy Measures for Set 2

Figure 15 shows the values of the three synergy measures as the variances of the cost uncertainty factors of the merger links, $\hat{\sigma}^2$, increase.

We can see that the expected total cost synergy, $\mathcal{S}_{TC}$, is negative, which indicates that the merger of the two supply chain networks will increase the expected total cost. This is due to the fact that the cost incurred in the merger process offsets the potential savings through network integration.

Figure 15 shows that the absolute risk synergy, $\mathcal{S}_{TR}$, decreases as $\hat{\sigma}^2$ increases, and becomes negative when $\hat{\sigma}^2 > V^*$. This trend indicates that the reduction of absolute risk diminishes when the risk of the merger process increases, and the absolute risk actually starts to increase when $\hat{\sigma}^2 > V^*$.

The third measure, $\mathcal{S}_{CV}$, remains positive in Figure 15, which indicates that now the relative risk or the total cost volatility is always reduced by this supply chain network merger.

Figure 15 also shows that as $\hat{\sigma}^2$ increases, $\mathcal{S}_{CV}$ decreases and approaches zero. This trend implies that the reduction in the relative risk or the total cost volatility becomes smaller as the risk of the merger process becomes larger.

It is interesting that in this second set of examples the three synergy measures evaluate different aspects of potential gains through the merger. Synergy measure 1 shows that the merger of the two networks does not reduce the expected total cost.

However, the merger can still be beneficial to the firms' stakeholders since the total risk may be reduced through the merger. Moreover, if $\hat{\sigma}^2 < V^*$, both the absolute risk and the relative risk are reduced, and if $\hat{\sigma}^2 > V^*$, only the relative risk is reduced while the absolute risk will increase in the post-merger network.

Therefore, if the decision-maker's only concern is cost synergy this merger may not make sense.

Nevertheless, if the decision-maker also cares about risk he or she will need to carefully compare different risk measures in order to correctly evaluate the potential risk reduction through the merger.

Managerial Insights and Conclusions

- We focused on the potential cost synergy and risk reduction achievable through mergers/acquisitions via supply chain network integration.

- We developed a variational inequality modeling framework that considers the costs and the risks associated not only with the production, transportation, and storage activities in supply chain networks, but also with the merger/acquisition itself. The framework allows one to estimate the expected total cost and the total risk of the supply chain networks before and after the merger.

- We provided three synergy measures that can assist decision-makers in the evaluation of potential gains of M&As from different perspectives.

- We provided numerical results.

Our results provide interesting managerial insights for executives who are faced with M&A decisions.

• Our first set of examples showed that, if the expected total costs and the risks of the merger are negligible, both the total cost and the total risk would be reduced through the merger.

• In addition, the risk reduction achieved through the merger was more prominent when the uncertainty of link costs was higher.

• Our second set of examples showed that the cost and the risk of merger could have a significant impact on the total cost and the total risk of the post-merger firm, and should be carefully evaluated.

• Our examples also demonstrated that whether a merger makes sense economically may depend on the priority concerns of the decision-makers, and on the measures used to evaluate the gains.

• For instance, a merger that could not lower the expected total cost might still be able to reduce the total risk, and, hence, be considered beneficial to the firms' stakeholders.

I hope that you have enjoyed the overview of and some highlights of financial networks as well as a recent model developed to assess risks and costs associated with mergers and acquisitions through a supply chain network integration framework.

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu

References - for Further Reading

Overview article on Financial Networks:
http://supernet.isenberg.umass.edu/articles/finhandbook.pdf

Link to Portfolio Optimization course in Executive Education at Harvard University:
http://supernet.isenberg.umass.edu/courses/Harvard-PortfolioOptimization.pdf

Link to Network Economics course at the World Bank:
http://supernet.isenberg.umass.edu/visuals/nagurney-worldbank-networkeconomics.pdf

Link to numerous articles on network modeling and applications, vulnerability and robustness analysis, as well as network synergy:
http://supernet.isenberg.umass.edu/dart.html

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