The Stella Dafermos Achievement Award in Transportation Science
A Tribute

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Professor Stella Dafermos

Stella was born on April 14, 1940 in Athens, Greece, and passed away in Providence, Rhode Island on April 5, 1990. She received her undergraduate degree in Civil Engineering from the National Technical University in Athens and PhD in 1968 in OR from Johns Hopkins.

EQUILIBRIUM MODELING, ANALYSIS AND COMPUTATION:
THE CONTRIBUTIONS OF STELLA DAFERMOS

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This memorial tribute to the late Stella Dafermos summarizes her contributions to equilibrium modeling, analysis, and computation, and provides a list of her published papers.

On April 5, 1990, with the death at 49 of Stella Dafermos, Professor of Applied Mathematics and Engineering at Brown University, Providence, Rhode Island, the operations research community lost one of its early woman doctorates. Throughout her quarter-century career she dedicated herself to developing rigorous mathematical formalisms for modeling, analyzing and making computations related to competitive equilibrium systems that spanned applications from congested urban transportation networks to multicommodity trade. Beginning with her 1968 doctoral dissertation, "Traffic Assignment and Resource Allocation in Transportation Networks," directed by F. T. Sparrow at Johns Hopkins University, which focused on the study of system-optimized and user-optimized transportation networks, she initiated a theme of advancement of methodologies by which the behavior of complex systems could be captured and studied.

EQUILIBRIUM MODELING

In her first paper, based on her thesis and published in 1969, she proposed convergent equilibration algorithms that simulated the adjustment behavior of travelers on a congested network operating in their own self-interest in choosing their routes. The governing equilibrium conditions, due to Wardrop, reflected that only the minimum cost routes connecting each origin/destination pair would be used. These algorithms took advantage of the problem structure and were later extended in papers published in 1971 and 1972 to network models that allowed for interaction among users of the system via the link cost functions. In her 1969 paper she also focused on stability issues, which began the theme of qualitative analysis of equilibrium patterns that was also to pervade her scholarly work.

These network equilibrium models, as well as the integrated models that allowed for both location and route choice, developed in her paper of 1976, were formulated as optimization problems, with the observation that the equilibrium conditions governing these problems were actually the Kuhn-Tucker conditions of an appropriately constructed optimization problem. Interestingly, parallel to these developments in nonlinear networks, the economics community was reformulating spatial price equilibrium problems, in which commodities are produced, consumed and traded, subject to transportation or transaction costs, as optimization problems. However, the assumption required for such a transformation—that of *symmetry*, in which cross-effects of various interactions had to be identical—precluded the realistic modeling of multiple modes of transportation and different classes of users of a given system, as well as multiple commodities. Moreover, the objective function thus constructed, although convenient and essential given the state-of-the-art at the time, was artificial. Furthermore, equilibrium problems, by their very nature, involve more than a single agent and objective function, with interactions among the agents, and, hence, the objective functions being the rule rather than the exception. To overcome such shortcomings, a methodological advance was needed.

In 1980 Stella made a fundamental discovery by observing that the equilibrium conditions of the traffic assignment problem actually had the structure of a variational inequality problem. Although the theory of variational inequalities had been introduced over a decade earlier for the study of partial differential equations, the emphasis there was on infinite-dimensional problems arising in mechanics, and its use as a powerful tool for equilibrium analysis in operations research was left unrealized. Stella's identification of network equilibrium conditions with a variational inequality problem opened up new horizons for mathematical modeling, analysis and the efficient computation of more general equilibrium systems than had heretofore been possible. With this

Subject classifications: Economics: solution of equilibrium problems. Networks/graphs: equilibrium modeling and computation. Transportation: formulation and solution of multimodal problems.

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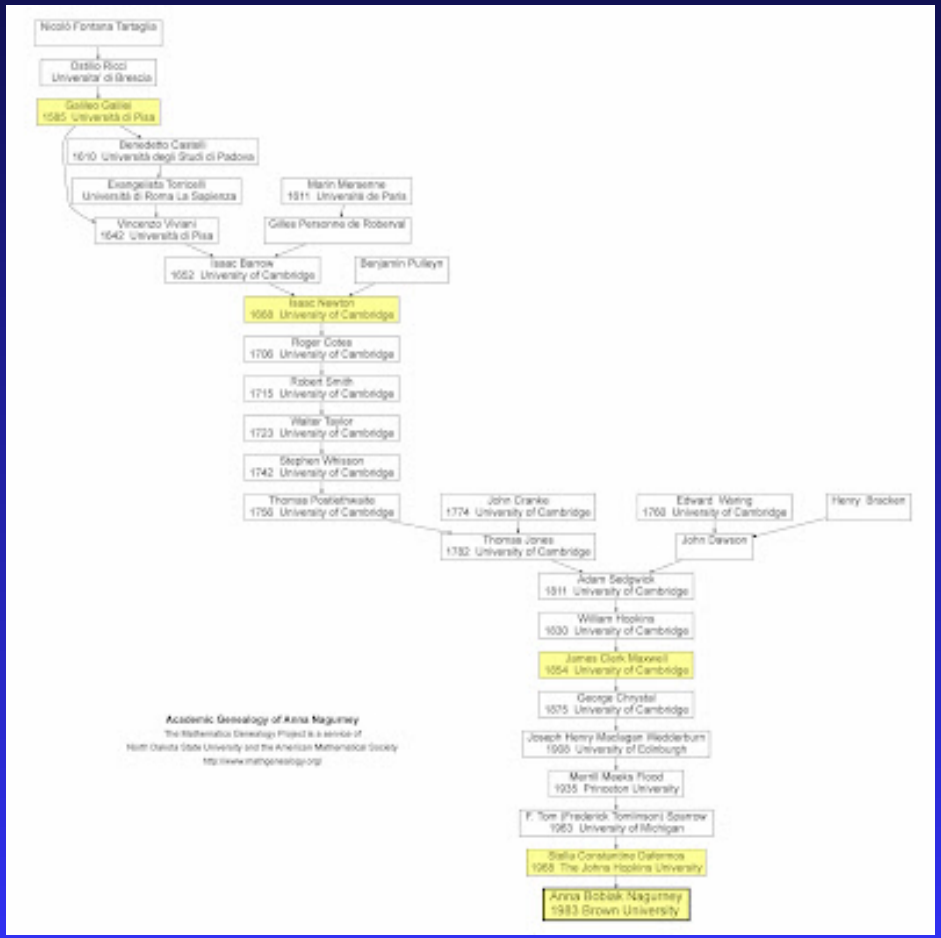
Her PhD dissertation, "Traffic Assignment and Resource Allocation in Transportation Networks," supervised by F.T. Sparrow, focused on the formulation, analysis, and solution of system-optimized and user-optimized transportation networks.

Professor Stella Dafermos

Stella Dafermos was the second female PhD in OR in the US.

Stella was the first female Full Professor at Brown University in Engineering and in Applied Mathematics, appointed in 1982.



Her Publications

PUBLISHED PAPERS OF STELLA DAFERMOS

This bibliography contains papers published in scholarly journals and books and a few easily obtainable reports.

1969. The Traffic Assignment Problem for a General Network (with F. T. Sparrow). *Journal of Research of the National Bureau of Standards* **73B**, 91–118.

1971. A Single Server Queue in Discrete Time (with M. Neuts). *Cahier du Centre de Recherche Opérationelle* **13**, 23–40.

1971. Optimal Resource Allocation and Toll Patterns in User Optimized Transportation Networks (with F. T. Sparrow). *Journal of Transport Economics and Policy* **V**, 1–17.

1971. An Extended Traffic Assignment Model With Applications to Two-Way Traffic. *Transportation Science* **5**, 366–389.

1972. The Traffic Assignment Problem for Multiclass-User Transportation Networks. *Transportation Science* **6**, 73–87.

1973. Toll Patterns for Multiclass-User Transportation Networks. *Transportation Science* **7**, 211–223.

1976. Integrated Equilibrium Flow Models for Transportation Planning. In *Lecture Notes in Economics and Mathematical Systems* **118**, M. Florian (ed.). Springer-Verlag, New York, 106–118.

1980. Continuum Modeling of Transportation Networks. *Transportation Research* **14B**, 295–301.

1980. Traffic Equilibrium and Variational Inequalities. *Transportation Science* **14**, 42–54.

1982. The General Multimodal Network Equilibrium Problem With Elastic Demand. *Networks* **12**, 57–72.

1982. Relaxation Algorithms for the General Asymmetric Traffic Equilibrium Problem. *Transportation Science* **16**, 231–240.

1983. Convergence of a Network Decomposition Algorithm for the Traffic Equilibrium Model. In *Proceedings of the Eighth International Symposium on Transportation and Traffic Theory*, M. Hurdle, E. Hauer and G. N. Stewart (eds.). University of Toronto Press, Toronto, Canada, 143–145.

1983. An Iterative Scheme for Variational Inequalities. *Mathematical Programming* **26**, 40–47.

1983. A Multicriteria Route-Mode Choice Traffic Equilibrium Model. *Bulletin of the Greek Mathematical Society* **24**, 13–32.

1984. Sensitivity Analysis for the Asymmetric Network Equilibrium Problem (with A. Nagurney). *Mathematical Programming* **28**, 174–184.

1984. Sensitivity Analysis for the General Spatial Equilibrium Problem (with A. Nagurney). *Operations Research* **32**, 1069–1086.

1984. On Some Traffic Equilibrium Theory Paradoxes (with A. Nagurney). *Transportation Research* **18B**, 101–110.

1984. Stability and Sensitivity Analysis for a Combined Network Equilibrium Model (with A. Nagurney). In *Proceedings of the Ninth International Symposium on Transportation and Traffic Theory*, J. Volmuller and R. Hamerslag (eds.). VNU Science Press, Utrecht, The Netherlands, 217–231.

1984. A Network Formulation of Market Equilibrium Problems and Variational Inequalities (with A. Nagurney). *Operations Research Letters* **3**, 247–250.

1985. Isomorphism Between Spatial Price Equilibrium and Traffic Network Equilibrium Models (with A. Nagurney). LCDS Report #85-17, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

1986. Isomorphic Multiclass Spatial Price and Multimodal Traffic Network Equilibrium Models. *Regional Science and Urban Economics* **16**, 197–209.

1986. Equilibria on Nonlinear Networks. LCDS Report #86-1, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

1986. Equilibrium Analysis of Competitive Economic Systems and Variational Inequalities (with S. C. McKelvey). LCDS Report #86-26, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

1987. Oligopolistic and Competitive Behavior of Spatially Separated Markets (with A. Nagurney). *Regional Science and Urban Economics* **17**, 245–254.

1987. Congested Transportation Networks and Variational Inequalities. In *Flow Control of Congested Networks* (NATO Series, Series F: Computer and System Sciences) **28**, A. Odoni, Bianco, and Szego (eds.). Springer-Verlag, New York.

1988. Sensitivity Analysis in Variational Inequalities. *Mathematics of Operations Research* **13**, 421–434.

1989. Supply and Demand Equilibration Algorithms for a Class of Market Equilibrium Problems (with A. Nagurney). *Transportation Science* **23**, 118–124.

1989. General Equilibrium and Variational Inequalities: Existence, Uniqueness, and Sensitivity (with L. Zhao). LCDS Report #89-2, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

1989. A General Market Equilibrium Problem and Partitionable Variational Inequalities (with S. C. McKelvey). LCDS Report #89-4, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

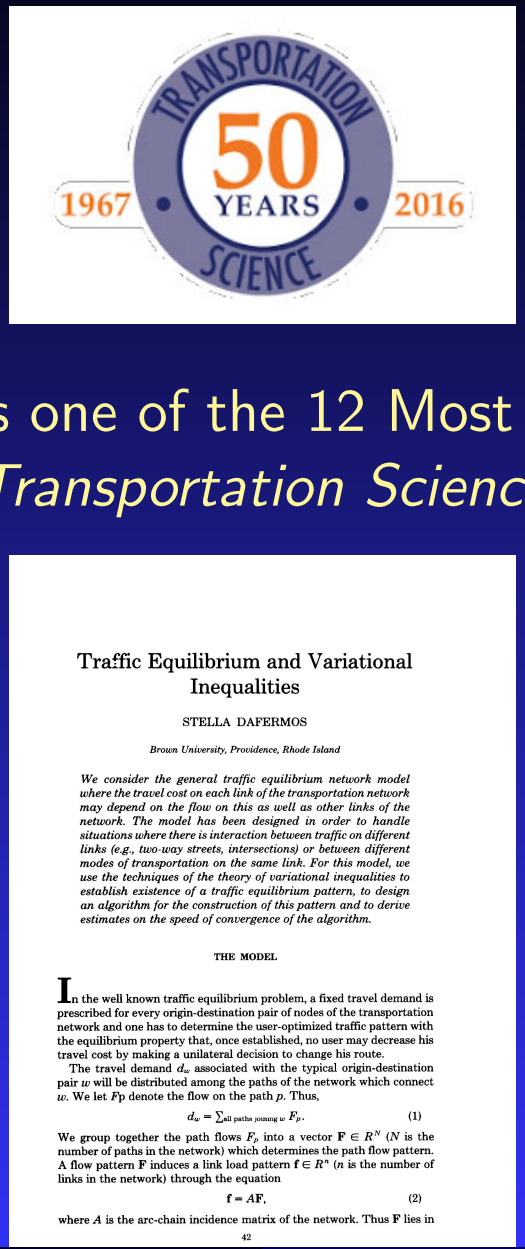
1990. General Economic Equilibrium and Variational Inequalities (with L. Zhao). LCDS Report #90-3, Division of Applied Mathematics, Brown University, Providence, Rhode Island.

1990. Exchange Price Equilibria and Variational Inequalities. *Mathematical Programming* **46**, 391–402.

Two of the working papers, with Lan Zhao and with Steve McKelvey, respectively, were published posthumously, in *Operations Research Letters* in 1991 and in *Journal of Optimization Theory and Applications* in 1992.

Impactful Research

Stella's 1980 paper was one of the 12 Most Impactful Papers in the 50 year history of *Transportation Science*.



Traffic Equilibrium and Variational Inequalities

STELLA DAFERMOS

Brown University, Providence, Rhode Island

We consider the general traffic equilibrium network model where the travel cost on each link of the transportation network may depend on the flow on this as well as other links of the network. The model has been designed in order to handle situations where there is interaction between traffic on different links (e.g., two-way streets, intersections) or between different modes of transportation on the same link. For this model, we use the techniques of the theory of variational inequalities to establish existence of a traffic equilibrium pattern, to design an algorithm for the construction of this pattern and to derive estimates on the speed of convergence of the algorithm.

THE MODEL

In the well known traffic equilibrium problem, a fixed travel demand is prescribed for every origin-destination pair of nodes of the transportation network and one has to determine the user-optimized traffic pattern with the equilibrium property that, once established, no user may decrease his travel cost by making a unilateral decision to change his route.

The travel demand d_w associated with the typical origin-destination pair w will be distributed among the paths of the network which connect w. We let F_p denote the flow on the path p. Thus,

$$d_w = \sum_{\text{all paths joining } w} F_p. \qquad (1)$$

We group together the path flows F_p into a vector $\mathbf{F} \in R^N$ (N is the number of paths in the network) which determines the path flow pattern. A flow pattern $\mathbf{F}$ induces a link load pattern $\mathbf{f} \in R^n$ (n is the number of links in the network) through the equation

$$\mathbf{f} = A\mathbf{F}, \qquad (2)$$

where A is the arc-chain incidence matrix of the network. Thus $\mathbf{F}$ lies in

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Stella as a Trailblazer

Stella's intellect, scholarship, attention to detail, scientific rigor, and creativity were remarkable and her legacy significant and sustained.

Her contributions to network equilibria and variational inequalities have influenced Operations Research, Engineering, Economics, and Regional Science and have helped to unveil numerous applications.

One wonders how much the world has missed because of her untimely passing.

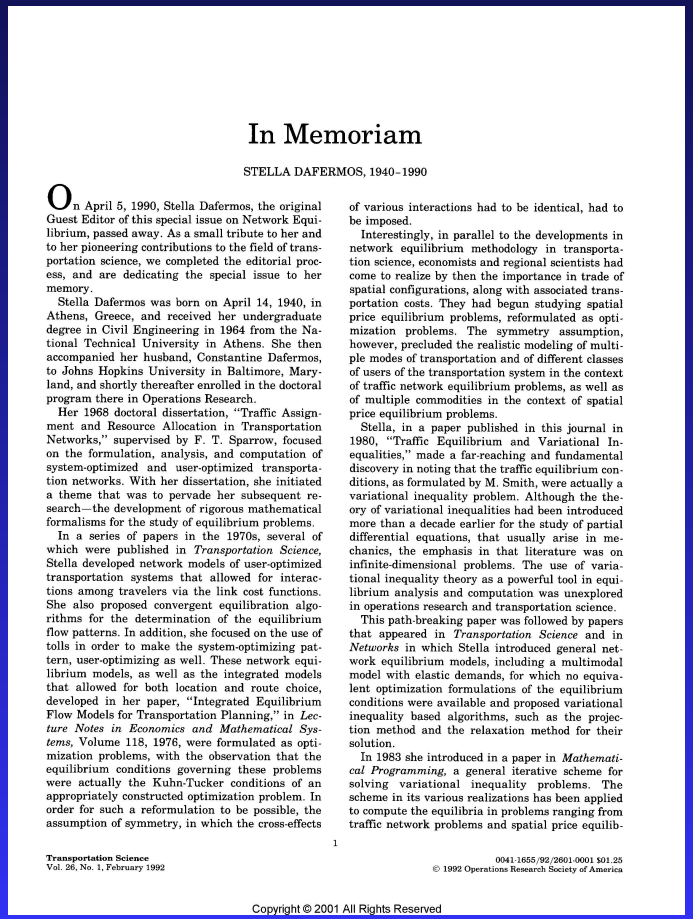
Professional Service and Visiting Appointments

Stella did not only contribute to the profession through her research. She was also an Associate Editor of *Transportation Science* and *Networks* and was a Council Member of the Transportation Science Section of ORSA.

She held a Visiting Professorship in the Department of Civil Engineering at MIT in 1984-1985 under NSF's VPW program. She also had a visiting appointment at the University of Wisconsin Madison and at the National Technical University of Athens.

Special Issue of *Transportation Science*

Stella Dafermos was guest editing a special issue on Network Equilibrium of *Transportation Science*, when she passed away, so Professor Amedeo Odoni and I completed the editorial process and we also published an *In Memoriam* to Stella.



In Memoriam

STELLA DAFERMOS, 1940–1990

On April 5, 1990, Stella Dafermos, the original Guest Editor of this special issue on Network Equilibrium, passed away. As a small tribute to her and to her pioneering contributions to the field of transportation science, we completed the editorial process, and are dedicating the special issue to her memory.

Stella Dafermos was born on April 14, 1940, in Athens, Greece, and received her undergraduate degree in Civil Engineering in 1964 from the National Technical University in Athens. She then accompanied her husband, Constantine Dafermos, to Johns Hopkins University in Baltimore, Maryland, and shortly thereafter enrolled in the doctoral program there in Operations Research.

Her 1968 doctoral dissertation, "Traffic Assignment and Resource Allocation in Transportation Networks," supervised by F. T. Sparrow, focused on the formulation, analysis, and computation of system-optimized and user-optimized transportation networks. With her dissertation, she initiated a theme that was to pervade her subsequent research—the development of rigorous mathematical formalisms for the study of equilibrium problems.

In a series of papers in the 1970s, several of which were published in *Transportation Science*, Stella developed network models of user-optimized transportation systems that allowed for interactions among travelers via the link cost functions. She also proposed convergent equilibration algorithms for the determination of the equilibrium flow patterns. In addition, she focused on the use of tolls in order to make the system-optimizing pattern, user-optimizing as well. These network equilibrium models, as well as the integrated models that allowed for both location and route choice, developed in her paper, "Integrated Equilibrium Flow Models for Transportation Planning," in *Lecture Notes in Economics and Mathematical Systems*, Volume 118, 1976, were formulated as optimization problems, with the observation that the equilibrium conditions governing these problems were actually the Kuhn-Tucker conditions of an appropriately constructed optimization problem. In order for such a reformulation to be possible, the assumption of symmetry, in which the cross-effects of various interactions had to be identical, had to be imposed.

Interestingly, in parallel to the developments in network equilibrium methodology in transportation science, economists and regional scientists had come to realize by then the importance in trade of spatial configurations, along with associated transportation costs. They had begun studying spatial price equilibrium problems, reformulated as optimization problems. The symmetry assumption, however, precluded the realistic modeling of multiple modes of transportation and of different classes of users of the transportation system in the context of traffic network equilibrium problems, as well as of multiple commodities in the context of spatial price equilibrium problems.

Stella, in a paper published in this journal in 1980, "Traffic Equilibrium and Variational Inequalities," made a far-reaching and fundamental discovery in noting that the traffic equilibrium conditions, as formulated by M. Smith, were actually a variational inequality problem. Although the theory of variational inequalities had been introduced more than a decade earlier for the study of partial differential equations, that usually arise in mechanics, the emphasis in that literature was on infinite-dimensional problems. The use of variational inequality theory as a powerful tool in equilibrium analysis and computation was unexplored in operations research and transportation science.

This path-breaking paper was followed by papers that appeared in *Transportation Science* and in *Networks* in which Stella introduced general network equilibrium models, including a multimodal model with elastic demands, for which no equivalent optimization formulations of the equilibrium conditions were available and proposed variational inequality based algorithms, such as the projection method and the relaxation method for their solution.

In 1983 she introduced in a paper in *Mathematical Programming*, a general iterative scheme for solving variational inequality problems. The scheme in its various realizations has been applied to compute the equilibria in problems ranging from traffic network problems and spatial price equilib-

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Conferencing with Stella

Stella very much enjoyed going to conferences and I had the pleasure of sharing hotel rooms with her at ORSA conferences and sometimes even serving as a tour guide with my husband when we traveled to conferences in The Netherlands and Japan (she helped us out when we conferenced in Greece).

Below is a photo I took of Stella and George Nemhauser in Tokyo at the Mathematical Programming Symposium, August 28 - September 2, 1988.

Comments from Stella's Closest Family on the TSL Award Named After Her

When I informed Stella's husband, Dr. Constantine Dafermos, the Alumni-Alumnae University Professor Applied Mathematics at Brown University, he responded as below.

"I am delighted, moved and also – on behalf of Stella – honored."

"I feel that establishing the prize is a major service to your field, as it will serve as a recognition of the contributions in the early days and as a reminder that women were among these contributors."

And her son, Dr. Mihalis Dafermos, Professor at Princeton University and the Lowndean Chair of Astronomy and Geometry at Cambridge University in the UK, on hearing the news, said: "That's very nice news! Many thanks for sharing this with me."

Thank You Very Much!

Many thanks to the President of the TSL Society, Professor Michael Hewitt, and to its Board, for honoring Stella Dafermos with the The Stella Dafermos Achievement Award in Transportation Science!

Let's continue to work towards the highest standards that she always set and to enjoy the wonderful community of TSL!