A Competitive Multiperiod Supply Chain Network Model with Freight Carriers and Green Technology Investment Option

Sara Saberi

Jose M. Cruz (UConn), Joseph Sarkis (WPI), Anna Nagurney (UMass)

Foisie School of Business
Worcester Polytechnic Institute
Worcester, MA 01609-2280

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Motivation

- Increase environmental awareness and improve ecological footprint
- Walmart plan for CO2 reduction with its suppliers
- 2015 Siemens investment for emission reduction for future saving
- Carbon footprinting (Wiedmann and Minx (2008))
- IBM heat map to illustrate the degree of carbon impact
- We consider the environmental impact of production, inventory, transportation, and consumption of products in the supply chain network, and the tradeoff between the initial investment in technology and its ecological footprint effect

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Background

- Modeling supply chain decision making and management from operational, tactical, and strategic business perspectives (Brandenburg et al. (2014); Ding et al. (2016); Fahimnia et al. (2015); Ouardighi et al. (2016); Zhu and He (2017))
- Environmental decision making in supply chain management processes and associated optimization from a number of dimensions (Nagurney et al. (2007); Cruz (2008); Frota Neto et al. (2008))
- Utilizing regulatory policies related to internalizing externalities such as including emission taxes (Cruz and Liu (2011); Dhavale and Sarkis (2015); Diabat and Simchi-Levi (2009); Zakeri et al. (2015))
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- One important systemic tool that can provide useful insights, and has not seen much work in the environmental supply chain arena is network equilibrium models.

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Contributions

- Explicitly model competition among manufacturing firms, retail stores, and freight carriers in terms products and inventory quantities, product shipping costs, and energy rating levels using initial technology investments
- Integrating oligopolistic competition among manufacturers, retail stores, and freight carriers and environmentally sensitive demand functions with nonlinear cost functions
- Explicit integration of environmental preferences of retailers and manufacturers in selecting their manufacturers and carriers
- Consumer awareness of green technology and foot print outcomes in spatial price equilibrium conditions

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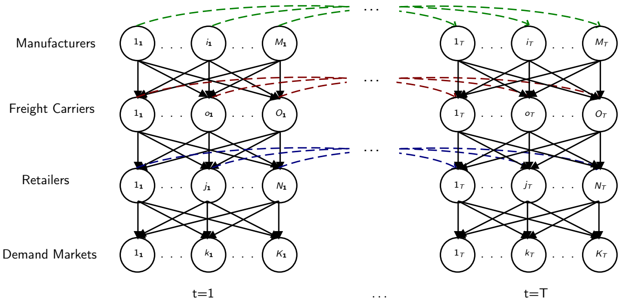
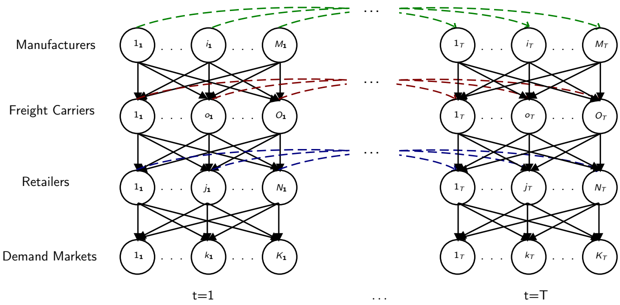


Figure: The supply chain network with freight carriers



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Notation	Definition
δ_{mi}	Energy rating of manufacturer i.
δ_{co}	Energy rating of carrier o.
δ_{rj}	Energy rating of retailer j.
δ_{max}	Maximum possible level of energy rating.

$$\text{Maximize} \quad \sum_{t=1}^{T} \frac{1}{(1+r)^t} \left\{ \sum_{j=1}^{N} p_{ijt}^{1*} q_{ijt}^{1} - PC_{it}(S_t, \delta_{mi}) - \sum_{j=1}^{N} TC_{ijt}(q_{ijt}^{1}, \delta_{mi}) - WC_{it}(I_{it}, \delta_{mi}) - \sum_{j=1}^{N} \sum_{o=1}^{O} R_{ijot}(p_t^{2*}, \delta_{co}) p_{ijot}^{2*} \right\} - TSI_i(\delta_{mi}) \quad (1)$$

subject to:

$$S_{i1} - I_{i1} \geq \sum_{j=1}^{N} q_{ij1}^{1} \quad (2)$$

$$I_{i(t-1)} + S_{it} - I_{it} \geq \sum_{j=1}^{N} q_{ijt}^{1}, \qquad \forall t = 2, \ldots, T \quad (3)$$

$$q_{ijt}^{1} = \sum_{o=1}^{O} R_{ijot}(p_t^{2}, \delta_{co}), \qquad \forall j, t \quad (4)$$

$$\delta_{mi} \leq \delta_{co}, \qquad \forall o \quad (5)$$

and the nonnegativity constraints: $q_{ijt}^{1} \geq 0$, $S_{it} \geq 0$, $I_{it} \geq 0$, $0 \leq \delta_{mi} \leq \delta_{max}$, $\forall j, t$.

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The optimality conditions for manufacturer i; $i = 1, \ldots, M$ simultaneously can be expressed as the following variational inequality, determine $(q^{1*}, S^*, I^*, \delta^{m*}, \mu^{1*}, \theta^*, \eta^{1*}) \in \mathcal{K}^1$ satisfying:

$$\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{N}\Big[\frac{1}{(1+r)^t}\Big(-p^{1*}_{ijt} + \frac{\partial TC_{ijt}(q^{1*}_{ijt}, \delta^*_{mi})}{\partial q^1_{ijt}}\Big) + \mu^*_{it} + \theta^*_{ijt}\Big] \times [q^1_{ijt} - q^{1*}_{ijt}]$$

$$+\sum_{i=1}^{M}\Big[\frac{\partial TSI_i(\delta^*_{mi})}{\partial \delta_{mi}} + \sum_{t=1}^{T}\frac{1}{(1+r)^t}\Big[\frac{\partial PC_{it}(S^*_t, \delta^*_{mi})}{\partial \delta_{mi}} + \frac{\partial WC_{it}(I^*_{it}, \delta^*_{mi})}{\partial \delta_{mi}} + \sum_{j=1}^{N}\frac{\partial TC_{ijt}(q^{1*}_{ijt}, \delta^*_{mi})}{\partial \delta_{mi}}\Big] + \sum_{o=1}^{O}\eta^*_{io}\Big] \times [\delta_{mi} - \delta^*_{mi}]$$

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$$+\sum_{i=1}^{M}\Big[\frac{1}{(1+r)^T}\Big(\frac{\partial WC_{iT}(I^*_{iT}, \delta^*_{mi})}{\partial I_{iT}}\Big) + \mu^*_{iT}\Big] \times [I_{iT} - I^*_{iT}]$$

$$+\sum_{i=1}^{M}\Big[S^*_{i1} - I^*_{i1} - \sum_{j=1}^{N} q^{1*}_{ij1}\Big] \times [\mu_{i1} - \mu^*_{i1}] + \sum_{t=2}^{T}\sum_{i=1}^{M}\Big[I^*_{i(t-1)} + S^*_{it} - I^*_{it} - \sum_{j=1}^{N} q^{1*}_{ijt}\Big] \times [\mu_{it} - \mu^*_{it}]$$

$$+\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{N}\Big[q^{1*}_{ijt} - \sum_{o=1}^{O} R_{ijot}(p^{2*}_t, \delta^*_{co})\Big] \times [\theta_{ijt} - \theta^*_{ijt}] + \sum_{i=1}^{M}\sum_{o=1}^{O}[\delta^*_{co} - \delta^*_{mi}] \times [\eta_{io} - \eta^*_{io}] \geq 0,$$

$$\forall (q^1, S, I, \delta_m, \mu, \theta, \eta^1) \in \mathcal{K}^1 \quad (6)$$

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$$+\sum_{i=1}^{M}\left[\frac{\partial TSI_i(\delta_{mi}^*)}{\partial \delta_{mi}} + \sum_{t=1}^{T}\frac{1}{(1+r)^t}[\frac{\partial PC_{it}(S_t^*, \delta_{mi}^*)}{\partial \delta_{mi}} + \frac{\partial WC_{it}(I_{it}^*, \delta_{mi}^*)}{\partial \delta_{mi}} + \sum_{j=1}^{N}\frac{\partial TC_{ijt}(q_{ijt}^{1*}, \delta_{mi}^*)}{\partial \delta_{mi}}] + \sum_{o=1}^{O}\eta_{io}^*\right] \times [\delta_{mi} - \delta_{mi}^*]$$

$$+\sum_{t=1}^{T}\sum_{i=1}^{M}[\frac{1}{(1+r)^t}(\frac{\partial PC_{it}(S_t^*, \delta_{mi}^*)}{\partial S_{it}}) - \mu_{it}^*] \times [S_{it} - S_{it}^*] + \sum_{t=1}^{T-1}\sum_{i=1}^{M}[\frac{1}{(1+r)^t}(\frac{\partial WC_{it}(I_{it}^*, \delta_{mi}^*)}{\partial I_{it}}) + \mu_{it}^* - \mu_{i(t+1)}^*] \times [I_{it} - I_{it}^*]$$

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$$+\sum_{i=1}^{M}[S_{i1}^* - I_{i1}^* - \sum_{j=1}^{N} q_{ij1}^{1*}] \times [\mu_{i1} - \mu_{i1}^*] + \sum_{t=2}^{T}\sum_{i=1}^{M}[I_{i(t-1)}^* + S_{it}^* - I_{it}^* - \sum_{j=1}^{N} q_{ijt}^{1*}] \times [\mu_{it} - \mu_{it}^*]$$

$$+\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{N}[q_{ijt}^{1*} - \sum_{o=1}^{O} R_{ijot}(p_t^{2*}, \delta_{co}^*)] \times [\theta_{ijt} - \theta_{ijt}^*] + \sum_{i=1}^{M}\sum_{o=1}^{O}[\delta_{co}^* - \delta_{mi}^*] \times [\eta_{io} - \eta_{io}^*] \geq 0,$$

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subject to:

$$\sum_{j=1}^{N} R_{ijo1}(p_1^2, \delta_{co}) - B_{io1} \geq \sum_{j=1}^{N} q_{ijo1}^2 \tag{8}$$

$$B_{io(t-1)} + \sum_{j=1}^{N} R_{ijot}(p_t^2, \delta_{co}) - B_{iot} \geq \sum_{j=1}^{N} q_{ijot}^2, \qquad \forall t = 2, \ldots, T \tag{9}$$

$$\sum_{t=1}^{T} \sum_{o=1}^{O} q_{ijot}^2 = \sum_{t=1}^{T} q_{ijt}^1, \qquad \forall i, j \tag{10}$$

and

$$p_{ijot}^2 \geq 0, \; B_{iot} \geq 0, \; q_{ijot}^2 \geq 0, \; 0 \leq \delta_{co} \leq \delta_{max}, \qquad \forall i, j, t.$$

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Generalized Nash equilibrium problem (GNEP)

$$\sum_{t=1}^{T}\sum_{o=1}^{O} q^2_{ijot} = \sum_{t=1}^{T} q^1_{ijt}, \qquad \forall i, j \tag{10}$$

We modify it to two inequality constraints as:

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$$\sum_{t=1}^{T}\sum_{o=1}^{O} q_{ijot}^{2} = \sum_{t=1}^{T} q_{ijt}^{1}, \qquad \forall i,j \tag{10}$$

We modify it to two inequality constraints as:

$$\sum_{t=1}^{T}\sum_{o=1}^{O} q_{ijot}^{2} \geq \sum_{t=1}^{T} q_{ijt}^{1}, \qquad \forall i,j \tag{11}$$

and

$$\sum_{t=1}^{T}\sum_{o=1}^{O} q_{ijot}^{2} \leq \sum_{t=1}^{T} q_{ijt}^{1}, \qquad \forall i,j \tag{12}$$

$$\text{Maximize} \quad \sum_{t=1}^{T} \frac{1}{(1+r)^t} \Big\{ p_{jt}^{3*} \sum_{k=1}^{K} q_{jkt}^{3} - IC_{jt}(Z_{jt}, \delta_{rj}) - HC_{jt}(Y_t, \delta_{rj})$$

$$- \sum_{k=1}^{K} TC_{jkt}(q_{jkt}^{3}, \delta_{rj}) - \sum_{i=1}^{M} p_{ijt}^{1*} q_{ijt}^{1} \Big\} - TSI_j(\delta_{rj}) \quad (13)$$

subject to:

$$\sum_{t=1}^{T} Y_{jt} = \sum_{t=1}^{T} \sum_{i=1}^{M} \sum_{o=1}^{O} q_{ijot}^{2} \quad (14)$$

$$\sum_{t=1}^{T} Y_{jt} = \sum_{t=1}^{T} \sum_{i=1}^{M} q_{ijt}^{1} \quad (15)$$

$$Y_{j1} - Z_{j1} \geq \sum_{k=1}^{K} q_{jk1}^{3} \quad (16)$$

$$Z_{j(t-1)} + Y_{jt} - Z_{jt} \geq \sum_{k=1}^{K} q_{jkt}^{3}, \qquad \forall t = 2, \ldots, T \quad (17)$$

$$\delta_{rj} \leq \delta_{mi}, \qquad \forall i \quad (18)$$

and $q_{jkt}^3 \geq 0, Y_{jt} \geq 0, Z_{jt} \geq 0,\ 0 \leq \delta_{rj} \leq \delta_{max}\ \forall k, t.$

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$$\sum_{t=1}^{T} Y_{jt} = \sum_{t=1}^{T} \sum_{i=1}^{M} q_{ijt}^{1} \quad (15)$$

$$Y_{j1} - Z_{j1} \geq \sum_{k=1}^{K} q_{jk1}^{3} \quad (16)$$

$$Z_{j(t-1)} + Y_{jt} - Z_{jt} \geq \sum_{k=1}^{K} q_{jkt}^{3}, \qquad \forall t = 2, \ldots, T \quad (17)$$

$$\delta_{rj} \leq \delta_{mi}, \qquad \forall i \quad (18)$$

and $q_{jkt}^3 \geq 0, Y_{jt} \geq 0, Z_{jt} \geq 0, \; 0 \leq \delta_{rj} \leq \delta_{max} \; \forall k, t.$

$$\text{Maximize} \quad \sum_{t=1}^{T} \frac{1}{(1+r)^t} \left\{ p_{jt}^{3*} \sum_{k=1}^{K} q_{jkt}^{3} - IC_{jt}(Z_{jt}, \delta_{rj}) - HC_{jt}(Y_t, \delta_{rj}) - \sum_{k=1}^{K} TC_{jkt}(q_{jkt}^{3}, \delta_{rj}) - \sum_{i=1}^{M} p_{ijt}^{1*} q_{ijt}^{1} \right\} - TSI_j(\delta_{rj}) \quad (13)$$

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$$\frac{1}{(1+r)^t}[p_{jt}^{3*} + SC_{jkt}(q_{jkt}^{3*})] \begin{cases} = \frac{1}{(1+r)^t} p_{kjt}^{4*}, & \text{if} \quad q_{jkt}^{3*} > 0, \\ \geq \frac{1}{(1+r)^t} p_{kjt}^{4*}, & \text{if} \quad q_{jkt}^{3*} = 0 \end{cases} \tag{19}$$

and

$$D_{kjt}(p^{4*}, \delta_{rj}^{*}) \begin{cases} = q_{jkt}^{3*}, & \text{if} \quad p_{kjt}^{4*} > 0, \\ \leq q_{jkt}^{3*}, & \text{if} \quad p_{kjt}^{4*} = 0. \end{cases} \tag{20}$$

Conditions (19) and (20) must hold simultaneously for all demand markets. These conditions correspond to the well-known spatial price equilibrium conditions (cf. Nagurney (1999); Takayama and Judge (1964)).

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Definition 1: The Equilibrium State of the Multiperiod Supply Chain Network with Freight Carriers

The equilibrium state is one where:

- all manufacturers have achieved optimality for their production levels, the retailers' order levels, inventory levels, and the energy rating level
- all carriers have achieved optimality for the prices of shipment, the amount of deliveries, and the backlog sizes as well as their energy rating level
- all retailers have achieved optimality for the order quantities from manufacturers, the inventory levels, and the sales volume to demand markets besides the level of energy rating
- the equilibrium conditions for all demand markets hold

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the multiperiod supply chain - freight carrier model are equivalent to the solution of the variational inequality problem given by: determine $(q^{1*}, q^{2*}, q^{3*}, S^*, I^*, \delta_m^*, p^{2*}, B^*, \delta_c^*, Y^*, Z^*, \delta_r^*, p^{4*}, \mu^{1*}, \mu^{2*}, \mu^{3*}, \theta^*, \eta^{1*}, \eta^{2*}, \nu^{1*}, \nu^{2*}, \gamma^*) \in \mathcal{K}$*, satisfying*

$$\langle F(X^*), X - X^* \rangle \geq 0, \qquad \forall X \in \mathcal{K} \tag{21}$$

where
$X \equiv (q^1, q^2, q^3, S, I, \delta_m, p^2, B, \delta_c, Y, Z, \delta_r, p^4, \mu^1, \mu^2, \mu^3, \theta, \eta^1, \eta^2, \nu^1, \nu^2, \gamma)$

$$F(X) \equiv (F_{q^1_{ijt}}, F_{q^2_{ijot}}, F_{q^3_{jkt}}, F_{S_{it}}, F_{I_{it}}, F_{\delta_{mi}}, F_{p^2_{ijot}}, F_{B_{iot}}, F_{\delta_{co}}, F_{Y_{jt}}, F_{Z_{jt}}, F_{\delta_{rj}}, F_{p^4_{jkt}}, F_{\mu^1_{it}}, F_{\mu^2_{iot}}, F_{\mu^3_{jt}}, F_{\theta_{ijt}}, F_{\eta^1}, F_{\eta^2}, F_{\nu^1}, F_{\nu^2}, F_{\gamma})$$

. The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

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$$F_{\mu^1_{it}}, F_{\mu^2_{iot}}, F_{\mu^3_{jt}}, F_{\theta_{ijt}}, F_{\eta^1}, F_{\eta^2}, F_{\nu^1}, F_{\nu^2}, F_{\gamma})$$

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The Modified Projection Method

Step 0: Initialization

Start with $X^0 \in \mathcal{K}$, as a feasible initial point, and let $\tau = 1$. Set ω such that $0 < \omega < \frac{1}{L}$, where L is the Lipschitz constant for function $F(X)$.

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \omega F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{22}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \omega F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{23}$$

Example 1

Two manufacturers, $M = 2$; two retailers, $N = 2$; two carriers, $O = 2$; and two demand markets, $K = 2$; competing over five planning periods, $T = 5$.

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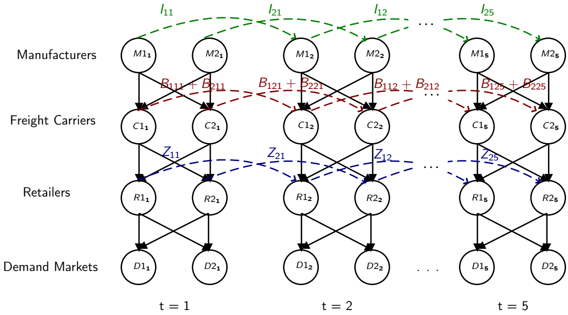

Figure: Example 1 Supply Chain Network

The energy rating, δ, can be zero and should not be more than 1, ($\delta_{max} = 1$)

Example 1

The cost functions are:

$$PC_{it}(S_{it},\delta_{mi}) = \alpha^{it} S_{1t} + 0.05(S_{it})^2 - \delta_{mi} S_{it}, \qquad i=1,2,\ t=1,\dots,5.$$

$$\alpha^{1t} = [2, 2.5, 3, 3.5, 4], \qquad \alpha^{2t} = [3, 4, 4.5, 5, 5.5].$$

$$WC_{it}(I_{it},\delta_{mi}) = 1.05 I_{it} + 0.002(I_{it})^2 - \delta_{mi} I_{it} + 10, \qquad i=1,2;\ t=1,\dots,5.$$

$$TC_{ijt}(q_{ijt},\delta_{mi}) = 1.5 q_{ijt} + 0.8(q_{ijt})^2 - \delta_{mi} q_{ijt}, \qquad i=1,2; j=1,2;\ t=1,\dots,5.$$

$$HC_{jt}(Y_{jt},\delta_{rj}) = 3Y_{jt} + 0.05(Y_{jt})^2 - \delta_{rj} Y_{jt}, \qquad j=1,2;\ t=1,\dots,5.$$

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$$R_{ijot}(p^2_t,\delta_{co}) = 20 - 1.5 p^2_{ijot} + 0.5 \sum_{c\neq o} p^2_{ijct} + 3\delta_{co}, \qquad i=1,2; j=1,2; o=1,2;\ t=1,\dots,5.$$

$$CC_{ijot}(q^2_{ijot},\delta_{co}) = 1.1 q^2_{ijot} + 0.003 q^2_{ijot} - \delta_{co} q^2_{ijot}, \qquad i=1,2; j=1,2; o=1,2; t=1,\dots,5.$$

$$AC_{iot}(B_{iot},\delta_{co}) = B_{iot} + 0.001(B_{iot})^2 - \delta_{co} B_{iot}, \qquad i=1,2;\ o=1,2;\ t=1,\dots,5.$$

The investment cost functions for manufacturers, retailers, and carriers are defined, respectively, as:

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The demand functions for customers within demand market 1 are defined to be less sensitive to future product prices, while customers within demand market 2 are defined to be more sensitive to future product prices.

$$D_{1j1}(p^4, \delta_{rj}) = 130 - 1.3p^4_{1j1} + 2\delta_{rj}, \quad D_{1j2}(p^4, \delta_{rj}) = 110 - 1.1p^4_{1j2} + 2\delta_{rj},$$
$$D_{1j3}(p^4, \delta_{rj}) = 80 - 0.9p^4_{1j3} + 2\delta_{rj}, \quad D_{1j4}(p^4, \delta_{rj}) = 50 - 0.7p^4_{1j4} + 2\delta_{rj},$$
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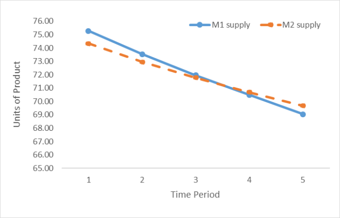
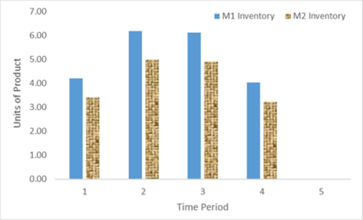
$$D_{2j3}(p^4, \delta_{rj}) = 150 - 1.2p^4_{2j3} + 2\delta_{rj}, \quad D_{2j4}(p^4, \delta_{rj}) = 180 - 1.7p^4_{2j4} + 2\delta_{rj},$$
$$D_{2j5}(p^4, \delta_{rj}) = 200 - 2p^4_{2j5} + 2\delta_{rj}, \quad j = 1, 2.$$



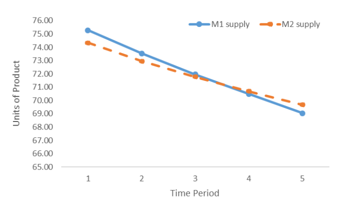
(a) Supplies at manufacturers



(b) Manufacturers' inventories

Figure: Manufacturers' supply and carriers' shipment and inventory

Example 1: Equilibrium Solution



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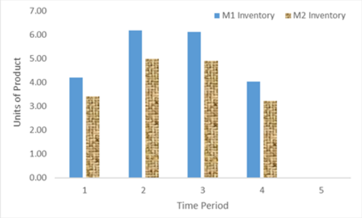
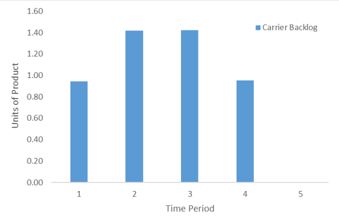
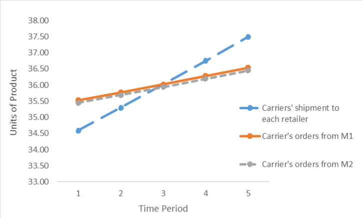


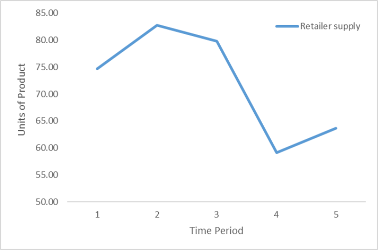
(b) Manufacturers' inventories



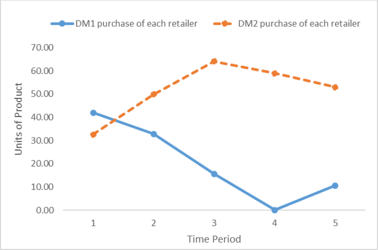
(c) Carriers' service backlogs



(d) Carriers' orders and shipment services from manufacturers



(a) Retailers' supply

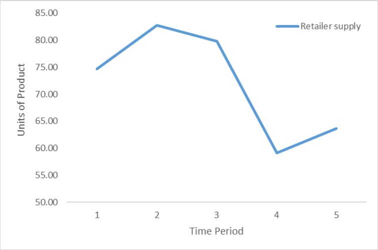


(b) Customers' purchase

Figure: Retailers' and customers' product flow

Energy rating level

$\delta_m = 1,\quad \delta_c = 1,\quad \delta_r = 0$



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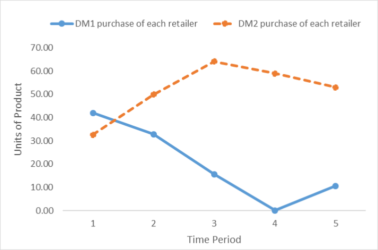

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Figure: Retailers' and customers' product flow

Energy rating level

$\delta_m = 1,\quad \delta_c = 1,\quad \delta_r = 0$

Baseline is Example 1, but the time periods have been extended $(T = 10)$.

The demand functions for periods 6 to 10 are:

$$D_{1j6}(p^4, \delta_{rj}) = 40 - 0.4p^4_{1j6} + 2\delta_{rj}, \quad D_{1j7}(p^4, \delta_{rj}) = 40 - 0.4p^4_{1j7} + 2\delta_{rj},$$
$$D_{1j8}(p^4, \delta_{rj}) = 40 - 0.4p^4_{1j8} + 2\delta_{rj}, \quad D_{1j9}(p^4, \delta_{rj}) = 40 - 0.4p^4_{1j9} + 2\delta_{rj},$$
$$D_{1j10}(p^4, \delta_{rj}) = 40 - 0.4p^4_{1j10} + 2\delta_{rj}, \quad j = 1, 2.$$

$$D_{2j6}(p^4, \delta_{rj}) = 200 - 2p^4_{2j6} + 2\delta_{rj}, \quad D_{2j7}(p^4, \delta_{rj}) = 160 - 1.7p^4_{2j7} + 2\delta_{rj},$$
$$D_{2j8}(p^4, \delta_{rj}) = 130 - 1.5p^4_{2j8} + 2\delta_{rj}, \quad D_{2j9}(p^4, \delta_{rj}) = 130 - p^4_{2j9} + 2\delta_{rj},$$
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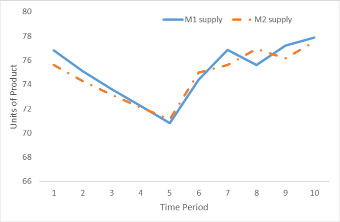
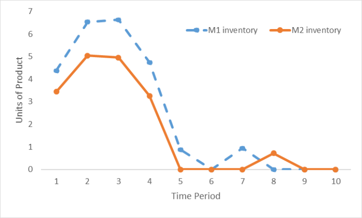
Energy rating level

$\delta_m = 1$, $\delta_c = 1$, $\delta_r = 1$

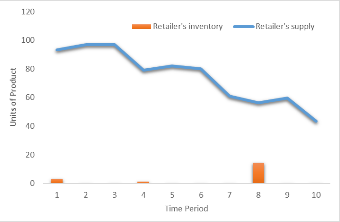
Example 2: Equilibrium Solution



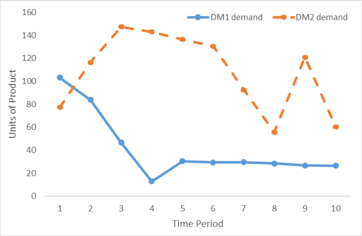
(a) Manufacturers' supply



(b) Manufacturers' inventory level



(c) Retailers' supply and inventory level



(d) Demand markets purchase

Example 3

Follows the same network structure as Example 1 but with varying cost functions for all network parties in order to focus on constraints

$$\delta_{mi} \le \delta_{co}, \qquad \forall o \qquad (5)$$

$$\delta_{rj} \le \delta_{mi}, \qquad \forall i \qquad (18)$$

Here, we vary the coefficient of δ in cost functions

$$TSI_i^1 = 500 + 360(\delta_{mi})^2, \qquad i = 1, 2.$$

$$TSI_o^2 = 500 + 360(\delta_{co})^2, \qquad o = 1, 2.$$

$$TSI_j^3 = 500 + 360(\delta_{rj})^2, \qquad j = 1, 2.$$

from 360 to 560 by increment of 20 and analyze the companies' capability in acquiring green technology.

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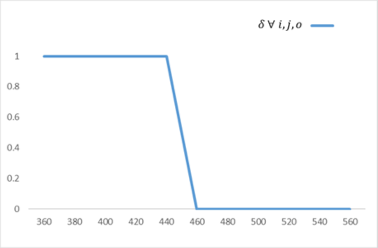
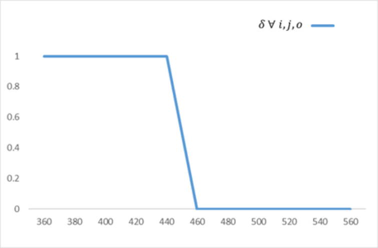
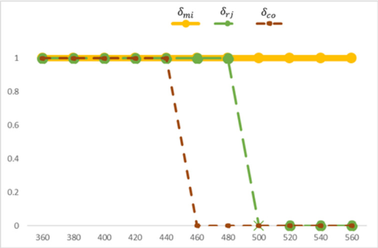


(e) In an obliged network

Figure: Energy rating of all entities for different investment levels



(a) In an obliged network



(b) In an uncommitted network

Figure: Energy rating of all entities for different investment levels

- Global warming is a huge issue for the world
- Governments can bring down the barrier of entry for green energy by taking steps to subsidize the green technology adoption and protect the posterity of our planet
- Time and the cost of investment affect firms' decisions, profitability, competitive advantage, and their environmental impact
- Our work fills the gap by capturing both Bertrand and Cournot competition for production and inventory flow and the prices of shipments in a multitiered multiperiod competitive supply chain-freight carrier network, along with the energy rating level for each entity as strategic variables.

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The Equilibrium Prices

The price that manufacturer $i; i = 1, \ldots, M$ charges retailer $j; j = 1, \ldots, N$ at time period $t; t = 1, \ldots, T$:

$$p_{ijt}^{1*} = (1+r)^t(\mu_{it}^* + \theta_{ijt}^*) + \frac{\partial TC_{ijt}(q_{ijt}^{1*}, \delta_{mi}^*)}{\partial q_{ijt}^1}, \quad (24)$$

The prices of products at the retailers:

$$p_{jt}^{3*} = (1+r)^t \mu_{jt}^* + \frac{\partial TC_{jkt}(q_{jkt}^{3*}, \delta_{rj}^*)}{\partial q_{jkt}}, \quad (25)$$

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The feasible set underlying the variational inequality problem is not compact. However, by imposing a rather weak condition, we can guarantee the existence of a solution pattern. Let

$$\mathcal{K}_b = \{(q^1, q^2, q^3, S, I, \delta_m, p^2, B, \delta_c, Y, Z, \delta_r, p^4, \mu^1, \mu^2, \mu^3, \theta, \eta^1, \eta^2, \nu^1, \nu^2, \gamma) | 0 \leqslant q^1 \leqslant b_1;$$
$$0 \leqslant q^2 \leqslant b_2; 0 \leqslant q^3 \leqslant b_3; 0 \leqslant S \leqslant b_4; 0 \leqslant I \leqslant b_5; 0 \leqslant \delta_m \leqslant \delta^b_{max}; 0 \leqslant p^2 \leqslant b_6; 0 \leqslant B \leqslant b_7;$$
$$0 \leqslant \delta_c \leqslant \delta^b_{max}; 0 \leqslant Y \leqslant b_8; 0 \leqslant Z \leqslant b_9; 0 \leqslant \delta_r \leqslant \delta^b_{max}; 0 \leqslant p^4 \leqslant b_{10}; 0 \leqslant \mu^1 \leqslant b_{11}; 0 \leqslant \mu^2 \leqslant b_{12};$$
$$0 \leqslant \mu^3 \leqslant b_{13}; -b_{14} \leqslant \theta \leqslant b_{15}; 0 \leqslant \eta^1 \leqslant b_{16}; 0 \leqslant \eta^2 \leqslant b_{17}; 0 \leqslant \nu^1 \leqslant b_{18}; -b_{19} \leqslant \nu^2 \leqslant b_{20},$$
$$-b_{21} \leqslant \gamma \leqslant b_{22}\} \quad (26)$$

Hence, the following variational inequality admits at least one solution $X^b \in \mathcal{K}_b$ since $\mathcal{K}_b$ is compact and F is continuous.

$$\langle F(X^b), X - X^b \rangle \geq 0, \qquad \forall X^b \in \mathcal{K}_b. \quad (27)$$

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