

# Multicommodity International Agricultural Trade Network Equilibrium: Competition for Limited Production and Transportation Capacity Under Disaster Scenarios with Implications for Food Security

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Innovative Applications of O.R.

Multicommodity international agricultural trade network equilibrium:  
Competition for limited production and transportation capacity under  
disaster scenarios with implications for food security

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ABSTRACT

The number of people affected by disasters, including man-made ones, is on the rise globally, with rising food insecurity being one of the most critical impacts. Disasters, both sudden-onset and slow-onset ones, can cause significant disruptions to the production and transportation of agricultural commodities. A model that can quantitatively assess the changes in agricultural commodity shipment volumes and their prices under disruptions caused by disaster scenarios is of major importance. In this paper, we utilize the theory of variational inequalities as the methodology to construct a multicommodity international agricultural trade network equilibrium model that can capture competition for limited production and transportation capacities of multiple agricultural commodities to capture competition. The model includes exchange rate and accounts for multiple routes and possibly distinct transportation modes and combinations. Theoretical results are given and an algorithm is proposed. A series of numerical examples, both alternative and algorithmically solved ones, illustrate the proposed model and the impact of the capacity of production and transportation capacities on food security in the Middle Eastern and North African (MENA) countries of Lebanon and Egypt. We also include sensitivity analysis results for exchange rates. The solutions reveal insight into the importance of the production and transportation capacities regarding food security, along with having multiple transportation routes that are cost-efficient as well as the importance of the magnitude of exchange rates.

# Acknowledgment and Dedication

**The authors acknowledge the partnership established between the University of Massachusetts Amherst and the Kyiv School of Economics, which supported this collaboration.**



**This presentation is dedicated to all Ukrainian farmers who have strived for global food security, even as the full-scale invasion continues.**



# International Agricultural Trade and Disasters

- International agricultural trade provides us with essential agri-food commodities throughout the year, ensuring our food security and simultaneously benefiting the farmers.
- Our planet and people are being faced with immense challenges brought about by disasters, including man-made ones.
- Disasters can have an immense impact on nations, regions, businesses, organizations, and individuals. A multiplicity of disasters can also impact food security.



# Disasters and Food Security

- Climate change and COVID-19 impacted the affordability and accessibility of agri-food products around the globe.
- With the added disruptions of Russia's full-scale invasion of Ukraine, around 47 million people are estimated to have been added to the more than 276 million who were already facing food insecurity.



# Disasters and Food Security

- With the major earthquake and aftershock striking Turkey and Syria in February 2023, the FAO is assessing their effects on the agricultural sector in both countries.
- In Turkey, the early estimate is approximately a 1% decrease in its GDP due to the damages to the agricultural sector.
- More than 4 million people in northern parts of Syria have already been identified as being food insecure.



# Disasters and Food Security

- The Horn of Africa is experiencing its worst drought recorded in modern history. The ongoing dry spell has caused food insecurity for 21 million people in the region.
- The high food prices caused by COVID-19, climate change, and the shortage of grains due to the ongoing war in Ukraine have further complicated the disastrous situation in the region.



# Literature Review

- The intellectual foundations of our work lie in the contributions of Samuelson (1952) and Takayama and Judge (1964, 1971) to spatial price equilibrium (SPE) modeling.
- SPE models have had wide application to the trade of different agricultural products (see, e.g., Ruijs et al. (2001), Barraza De La Cruz, Pizzolato, and Barraza de La Cruz (2010)).
- They have also gathered attention in the context of the quantification of the impacts of various policies such as quotas (e.g., Nagurney, Salarpour, and Dong (2022)), tariffs, including tariff-rate quotas (see, for example, Nagurney, Besik, and Dong (2019)) and ad valorem tariffs (see Nagurney, Nicholson, and Bishop (1996)), as well as non-tariff measures (Lopez, Rau, and Woltjer (2019) and Nagurney and Besedina (2023)).
- The inclusion of exchange rates in spatial price equilibrium models is very limited (see Devadoss and Sabala (2020), Nagurney et al. (2023), and Nagurney and Besedina (2023)).

# Our Contributions

- We use the theory of variational inequalities to construct a multicommodity international agricultural trade model, which contains capacities on agricultural commodities' production and transportation flows.
- Our model includes exchange rates, multiple agricultural commodities, multiple possible routes between country supply and demand market pairs.
- The network equilibrium model allows for supply price, demand price, and unit transportation cost functions to depend on the commodity flow variables, and these functions can be nonlinear and asymmetric.
- The capacity constraints, along with the generality of the underlying functions, enable the modeling of competition for production and transportation capacity among the commodities.

# The Multicommodity International Agricultural Trade Model

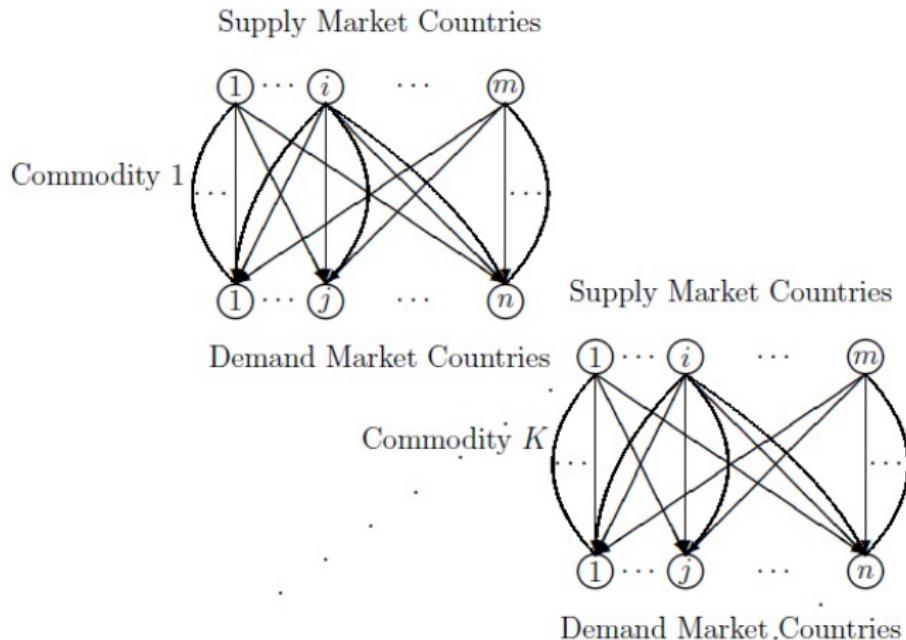


Figure: The Multicommodity International Trade Network

# Variables

- Let  $Q_{ij}^{kl}$  denote the amount of commodity  $k$  produced at country supply market  $i$  and shipped on route  $l$  to country demand market  $j$ . The commodity flows are grouped into the vector  $Q \in R_+^{KLmn}$ .
- Let  $s_i^k$  denote the supply of commodity  $k$  produced at country supply market  $i$ . All the commodity supplies are grouped into the vector  $s \in R_+^{Km}$ .
- The demand for commodity  $k$  at country demand market  $j$  is denoted by  $d_j^k$ , and all the demands are gathered into the vector  $d \in R_+^{Kn}$ .

**Associated with each pair of country supply and demand markets  $(i, j)$  is an exchange rate  $e_{ij}$  for  $i = 1, \dots, m; j = 1, \dots, n$ .**

**A trade route can entail transportation via different modes, such as rail, truck, air, or water (sea, river, etc.).**

# Constraints

The conservation of flow equations are:

$$s_i^k = \sum_{j=1}^n \sum_{l=1}^L Q_{ij}^{kl}, \quad k = 1, \dots, K; i = 1, \dots, m, \quad (1)$$

$$d_j^k = \sum_{i=1}^m \sum_{l=1}^L Q_{ij}^{kl}, \quad k = 1, \dots, K; j = 1, \dots, n. \quad (2)$$

Also, all the commodity shipments must be nonnegative; that is:

$$Q_{ij}^{kl} \geq 0, \quad k = 1, \dots, K; l = 1, \dots, L; i = 1, \dots, m; j = 1, \dots, n. \quad (3)$$

# Transportation Capacity Constraints

Let  $\bar{Q}_{ij}^l$  denote the transportation capacity of route  $l$  between country supply market  $i$  and country demand market  $j$ , for all  $l, i, j$ . Typically, the units of flow for the agricultural commodities are in tons.

Hence, the following transportation capacity constraints must be satisfied:

$$\sum_{k=1}^K Q_{ij}^{kl} \leq \bar{Q}_{ij}^l, \quad l = 1, \dots, L; i = 1, \dots, m; j = 1, \dots, n. \quad (4)$$

Expressions in (4) allow us to capture competition among the commodities for transportation services along particular routes.

# Production Capacity Constraints

Let  $\bar{S}_i$  denote the production capacity of country supply market  $i$  across all the commodities. The below production capacity constraints must be met:

$$\sum_{k=1}^K s_i^k \leq \bar{S}_i, \quad i = 1, \dots, m. \quad (5a)$$

Due to the conservation of flow equations (1), constraints (5a) can take the form:

$$\sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl} \leq \bar{S}_i, \quad i = 1, \dots, m. \quad (5b)$$

According to (5a) or (5b), a country supply market  $i$  cannot violate its aggregate production capacity.

# Lagrange Multipliers

We introduce Lagrange multipliers:

$$\lambda_{ij}^l, \quad l = 1, \dots, L; i = 1, \dots, m; j = 1, \dots, n;$$

and:

$$\mu_i, \quad i = 1, \dots, m;$$

associated with the capacity constraints in (4) and (5b), respectively, and we group these Lagrange multipliers into the vectors  $\lambda \in R_+^{Lmn}$  and  $\mu \in R_+^m$ .

# Functions

The country supply price functions  $\pi_i^k$ , for all  $k, i$ , are:

$$\pi_i^k = \pi_i^k(s) \equiv \tilde{\pi}_i^k(Q), \quad k = 1, \dots, K; i = 1, \dots, m. \quad (6)$$

The demand price functions  $\rho_j^k$ , for all  $k, j$ , are:

$$\rho_j^k = \rho_j^k(d) \equiv \tilde{\rho}_j^k(Q), \quad k = 1, \dots, K; j = 1, \dots, n. \quad (7)$$

And the unit transportation cost functions  $c_{ij}^{kl}$ , for all  $k, l, i, j$ , are as follows:

$$c_{ij}^{kl} = c_{ij}^{kl}(Q), \quad k = 1, \dots, K; l = 1, \dots, L; i = 1, \dots, m; j = 1, \dots, n. \quad (8)$$

The generality of the above transportation cost functions allows one to capture competition for transportation services among commodities.

# Equilibrium Conditions

## Definition 1: The Multicommodity International Agricultural Trade Equilibrium Conditions

*A multicommodity shipment and Lagrange multiplier pattern  $(Q^*, \lambda^*, \mu^*) \in \mathcal{K}^1$ , where  $\mathcal{K}^1 \equiv \{(Q, \lambda, \mu) | (Q, \lambda, \mu) \in R_+^{KLmn+Lmn+m}\}$  is a multicommodity international agricultural trade network equilibrium with exchange rates, under limited production and transportation capacities, if the following conditions hold: For all commodities  $k$ ;  $k = 1, \dots, K$ ; for all routes  $l$ ;  $l = 1, \dots, L$ , and for all country supply and demand market pairs:  $(i, j)$ ;  $i = 1, \dots, m$ ;  $j = 1, \dots, n$ :*

$$(\tilde{\pi}_i^k(Q^*) + c_{ij}^{kl}(Q^*))e_{ij} + \lambda_{ij}^{l*} + \mu_i^* \begin{cases} = \tilde{\rho}_j^k(Q^*), & \text{if } Q_{ij}^{kl*} > 0, \\ \geq \tilde{\rho}_j^k(Q^*), & \text{if } Q_{ij}^{kl*} = 0, \end{cases} \quad (9)$$

# Equilibrium Conditions

## Definition 1: The Multicommodity International Agricultural Trade Equilibrium Conditions

and for all routes  $l$ ;  $l = 1, \dots, L$ , and all country market pairs  $(i, j)$ ;  
 $i = 1, \dots, m$ ;  $j = 1, \dots, n$ :

$$\lambda_{ij}^{l*} \begin{cases} \geq 0, & \text{if } \sum_{k=1}^K Q_{ij}^{kl*} = \bar{Q}_{ij}^l, \\ = 0, & \text{if } \sum_{k=1}^K Q_{ij}^{kl*} < \bar{Q}_{ij}^l, \end{cases} \quad (10)$$

and for all country supply markets  $i$ ;  $i = 1, \dots, m$ :

$$\mu_i^* \begin{cases} \geq 0, & \text{if } \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl*} = \bar{S}_i, \\ = 0, & \text{if } \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl*} < \bar{S}_i. \end{cases} \quad (11)$$

# Variational Inequality Formulation

## Theorem 1: Variational Inequality Formulation of the Multicommodity International Agricultural Trade Equilibrium Conditions

A *multicommodity shipment and Lagrange multipliers pattern*  $(Q^*, \lambda^*, \mu^*) \in \mathcal{K}^1$  is a *multicommodity international agricultural trade network equilibrium with exchange rates, under limited production and transportation capacities, according to Definition 1*, if and only if it satisfies the variational inequality:

$$\begin{aligned} & \sum_{k=1}^K \sum_{l=1}^L \sum_{i=1}^m \sum_{j=1}^n \left[ (\tilde{\pi}_i^k(Q^*) + c_{ij}^{kl}(Q^*)) e_{ij} + \lambda_{ij}^{l*} + \mu_i^* - \tilde{\rho}_j^k(Q^*) \right] \times (Q_{ij}^{kl} - Q_{ij}^{kl*}) \\ & + \sum_{l=1}^L \sum_{i=1}^m \sum_{j=1}^n \left[ \bar{Q}_{ij}^l - \sum_{k=1}^K Q_{ij}^{kl*} \right] \times (\lambda_{ij}^l - \lambda_{ij}^{l*}) \\ & + \sum_{i=1}^m \left[ \bar{S}_i - \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl*} \right] \times (\mu_i - \mu_i^*) \geq 0, \quad \forall (Q, \lambda, \mu) \in \mathcal{K}^1. \end{aligned} \quad (12)$$

# Numerical Examples

The examples focus on commodity flows from Ukraine to Lebanon and Egypt. The Modified Projection Method (Korpelevich (1977)) is the algorithm used to solve these examples.

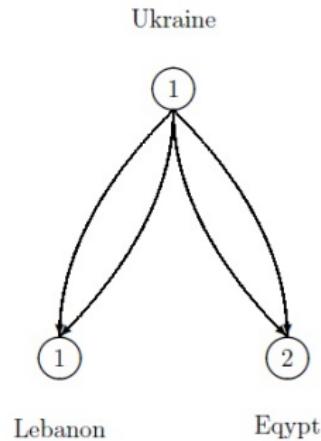


Figure: The International Trade Network for the Examples

## Example 1: Pre-War Scenario

This example considers the pre-war scenario when almost all of the grains in Ukraine were exported through their Black Sea ports. There are two commodities: wheat and corn, denoted by  $k = 1, 2$ , respectively.

The equilibrium commodity shipment pattern is as follows:

$$Q_{11}^{11*} = 477,085.5938, \quad Q_{12}^{11*} = 1,605,672.5000, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 79,128.0781, \quad Q_{12}^{21*} = 560,130.3750, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

All the Lagrange multipliers were equal to: 0.0000.

- Lebanon imports more than 70% of its wheat and about 20% of its corn from Ukraine, while these percentages for Egypt are 25%, and 5%. Ukraine's wheat exports to Lebanon were at 520,000 tons in 2021.
- Only the maritime routes have positive commodity flows.

## Example 1: Pre-War Scenario

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 7,328.3252 = \$266.8542, \quad \pi_1^2(s^*) = 6,971.0166 = \$253.8432.$$

- **Pre-war, Ukrainian farmers could earn close to \$270 per ton for wheat and corn.**

The equilibrium demand prices in Lebanon and Egypt are:

$$\rho_1^1(d^*) = 530,781.1875 = \$351.0457, \quad \rho_1^2(d^*) = 520,752.9063 = \$344.4132,$$

$$\rho_2^1(d^*) = 5,527.3057 = \$351.3862, \quad \rho_2^2(d^*) = 5,555.4214 = \$353.1736.$$

- **Before the war, the demand prices in Lebanon and Egypt were much closer to the global prices of wheat and corn.**

## Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

We now consider the early period after the full-scale invasion but before the Black Sea Grain Initiative. During this period, the Black Sea routes were mined and blockaded.

The equilibrium commodity shipment pattern is:

$$Q_{11}^{12*} = 216,433.1406, \quad Q_{12}^{12*} = 500,000.0000,$$

$$Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

All Lagrange multipliers are equal to 0.0000 except that  $\lambda_{12}^{2*} = 468.4277$ .

- **With the cheaper routes blockaded, the more expensive alternative routes are in use. After the start of the war, the transportation costs inside Ukraine reached around \$200.**
- **The alternative routes are used for the export of wheat but not for corn. Lebanon and Egypt rely heavily on wheat as their main source of nutrition.**

## Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

The incurred supply prices in Ukraine at the equilibrium are:

$$\pi_1^1(s^*) = 7,099.0347 = \$258.5048, \quad \pi_1^2(s^*) = 6,780.4995 = \$246.9056.$$

- **After the start of the war, the supply price for Ukrainian wheat went as low as less than \$100.**

The incurred demand prices at the equilibrium in Lebanon and Egypt are:

$$\rho_1^1(d^*) = 569,879.0000 = \$376.9041, \quad \rho_1^2(d^*) = 6,052.5005 = \$384.7743,$$

$$\rho_2^1(d^*) = 574,560.0000 = \$380, \quad \rho_2^2(d^*) = 5,980.0000 = \$380.1652.$$

- **In later months, demand prices in Lebanon and Egypt reached even around \$500; here, however, the markets are just starting to react.**

## Example 3: Black Sea Grain Initiative in Place

In this example, we consider the scenario in which the Black Sea Grain Initiative is in place (beginning in August).

The equilibrium commodity shipment pattern is:

$$Q_{11}^{11*} = 477,651.1563, \quad Q_{12}^{11} = 552,348.4375, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 0.0000, \quad Q_{12}^{21*} = 0.0000, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

The computed equilibrium Lagrange multipliers are all equal to 0.0000 except that  $\mu_1^{1*} = 591.6817$  since the supply output of commodities is at the capacity.

- Once again, only efficient maritime routes are used for the transport of grains.
- The wheat commodity flows are improved compared to Example 2, especially in the case of Lebanon. Furthermore, as in Example 2, no corn is produced.

## Example 3: Black Sea Grain Initiative in Place

The incurred supply prices in Ukraine at the equilibrium are:

$$\pi_1^1(s^*) = 3,500.6001 = \$95.7269, \quad \pi_1^2(s^*) = 4,095.5000 = \$111.9949.$$

- The price share of Ukrainian farmers is less than \$100, even with the establishment of the Black Sea Grain Initiative. This could be traced back to transportation costs remaining high even after the Initiative.

The incurred demand prices at the equilibrium in Lebanon and Egypt are:

$$\rho_1^1(d^*) = 729,014.8125 = \$482.1526, \quad \rho_1^2(d^*) = 718,256.3750 = \$475.0372,$$

$$\rho_2^1(d^*) = 9,738.2344 = \$508.5239, \quad \rho_2^2(d^*) = 9,900.5000 = \$516.9973.$$

- Even though the transportation capacity limitations are lifted, because of the high transportation costs and limited supply capacity, the demand prices remain at such high levels.

# Sensitivity on Exchange Rates

Here, we conduct sensitivity analysis on the exchange rates, and we report the equilibrium supplies of the commodities of wheat and corn.

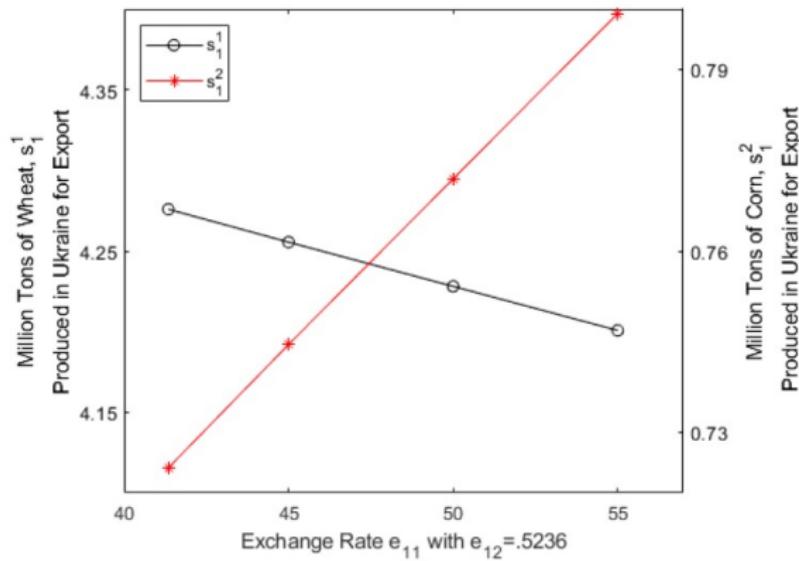


Figure:  $e_{12}$  fixed but with  $e_{11}$  Varying: Impact on Commodity Supplies

# Sensitivity on Exchange Rates

With the depreciation of the Lebanese pound, with the Egyptian pound fixed:

- The production of wheat in Ukraine decreases, and the supply of Ukrainian corn increases with a sharper slope.
- With less demand from Lebanon, Ukraine meets the demand for wheat in Egypt and shifts to produce more corn to satisfy the demand for corn in Egypt.
- Generally, the supply and demand of wheat are more price inelastic than corn; when there is a global deficit, the trade volumes of wheat are even less sensitive to exchange rate fluctuations.

# Sensitivity on Exchange Rates

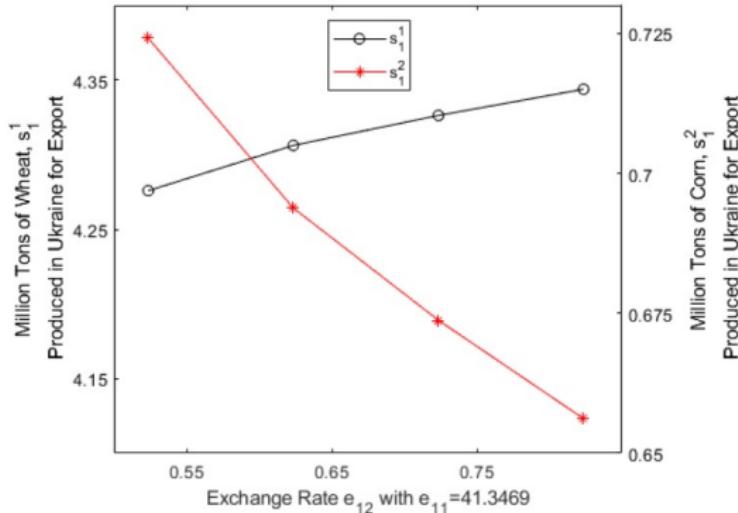


Figure:  $e_{11}$  fixed but with  $e_{12}$  Varying: Impact on Commodity Supplies

# Sensitivity on Exchange Rates

With the depreciation of the Egyptian pound, with the Lebanese pound fixed:

- The supply of Ukrainian wheat increases, and the production of Ukrainian corn decreases, both at a decreasing rate.
- Egypt cannot afford as much Ukrainian grain as before, and Lebanon, which has a much higher demand for Ukrainian wheat than corn, imports more wheat.
- Lebanon appropriates more wheat flow, its demand for wheat is satisfied, and slowly shifts toward buying more corn, causing a decreasing rate of decrease in the production of Ukrainian corn and a decreasing rate of increase in the production of Ukrainian wheat.

# Insights & Summary

- We see that there is essentially no efficient alternative to the maritime transportation of grains from Ukrainian Black Sea ports, highlighting the importance of the Black Sea Grain Initiative.
- The results confirm how the war has driven the earnings of the Ukrainian farmers to unprecedented low levels.
- The numerical results show the priority of wheat over corn in all scenarios in the demand markets of Lebanon and Egypt, as two countries representative of the MENA region.
- Lebanon and Egypt compete over the war-induced limited production capacity at war-induced high prices.
- The economic instability of the demand country markets, in the form of the depreciation of their currencies, lowers their share of the Ukrainian wheat supply.

# Thank You Very Much!



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