Multicommodity International Agricultural Trade Network Equilibrium: Competition for Limited Production and Transportation Capacity Under Disaster Scenarios with Implications for Food Security

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This presentation is dedicated to all Ukrainian farmers who have strived for global food security, even as the full-scale invasion continues.

Outline of Presentation

- **Background and Motivation**
- **Literature Review and Our Contributions**
- **The Multicommodity International Agricultural Trade Network Equilibrium Model with Production and Transportation Capacities**
- **The Algorithm**
- **Numerical Examples**
- **Insights and Summary**

Background and Motivation

International Agricultural Trade

International agricultural trade provides us with essential agri-food commodities throughout the year, ensuring our food security and simultaneously benefiting the farmers.

- **Our planet and people are being faced with immense challenges brought about by disasters, including man-made ones**.
- **Disasters can have an immense impact on nations, regions, businesses, organizations, and individuals. A multiplicity of disasters can also impact food security**.

Disasters and Food Security

- **Climate change and COVID-19 impacted the affordability and accessibility of agri-food products around the globe.**
- **With the added disruptions of Russia's full-scale invasion of Ukraine, around 47 million people are estimated to have been added to the more than 276 million who were already facing food insecurity.**

Disasters and Food Security

- **The FAO of the United Nations reports that between 20 and 30 percent of the Ukrainian land previously used for cultivating winter crops will probably remain unsown due to the ongoing war.**
- **In addition, there have been immense challenges in getting the exports of agricultural products out of Ukraine with the blockade of the Black Sea in wartime and various bottlenecks.**

Disasters and Food Security

- **With the major earthquake and aftershock striking Turkey and Syria in February 2023, the FAO is assessing their effects on the agricultural sector in both countries.**
- **In Turkey, the early estimate is approximately a 1% decrease in its GDP due to the damages to the agricultural sector.**
- **More than 4 million people in northern parts of Syria have already been identified as being food insecure.**

Disasters and Food Security

- **The Horn of Africa is now experiencing its worst drought recorded in modern history. The ongoing dry spell has caused food insecurity for 21 million people in the region.**
- **From July to December 2022, the number of children facing severe food insecurity in the Horn of Africa region doubled from 10 million to more than 20 million.**
- **The high food prices caused by COVID-19, climate change, and the shortage of grains due to the ongoing war in Ukraine have further complicated the disastrous situation in the region.**

Literature Review and Our Contributions

Literature Review

• The intellectual foundations of our work lie in the contributions of Samuelson (1952) and Takayama and Judge (1964, 1971) to spatial price equilibrium (SPE) modeling with notice of the following works that utilize variational inequality theory: Florian and Los (1982), Dafermos and Nagurney (1984), Nagurney and Aronson (1989), Nagurney (1989, 1999).

• SPE models have had wide application to the trade of different agricultural products (see, e.g., Thompson (1989), Bishop, Pratt, and Novakovic (1994), Ruijs et al. (2001), Barraza De La Cruz, Pizzolato, and Barraza de La Cruz (2010)).

• They have also gathered attention in the context of the quantification of the impacts of various policies such as quotas (e.g., Nagurney, Li, and Nagurney (2014), Nagurney (2022b), Nagurney, Salarpour, and Dong (2022)), tariffs, including tariff-rate quotas (see, for example, Nagurney, Besik, and Dong (2019)) and ad valorem tariffs (see Nagurney, Nicholson, and Bishop (1996)), as well as non-tariff measures in the form of sanitary and phytosanitary measures (Lopez, Rau, and Woltjer (2019) and Nagurney and Besedina (2023)).

Literature Review

• Although bounds have previously been imposed on commodity flows in SPE models in order to model either quotas or capacity limits in terms of transportation, such constraints were commodity-specific. **We, in contrast, have bounds across commodities in terms of production in each country as well as bounds across commodities on each route of transportation.**

• **The production bounds implicitly capture the available land for planting, which may have been reduced due to war and/or climate-related disasters. The transportation bounds, in turn, allow us to model the limits on the total volume of commodities that can be transported on different routes, which can correspond to different modes of transport.**

• **In addition, we include exchange rates in our model.** Exchange rates are essential parameters in international trade and associated decision-making. The inclusion of exchange rates in spatial price equilibrium models is very limited (see Devadoss and Sabala (2020), Nagurney et al. (2023), and Nagurney and Besedina (2023)).

Our first joint paper on agricultural trade with a focus on Ukraine in wartime was published in the *Journal of Global Optimization*.

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Exchange rates and multicommodity international trade: insights from spatial price equilibrium modeling with policy instruments via variational inequalities

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Abstract
In this paper, we construct a multicommodity international trade spatial price equilibrium model of special relevance to agriculture in which exchange rates are included along with policy instruments in the form of tariffs, subsidies as well as quotas. The model allows for multiple trade routes between country origin nodes and country destination nodes and these trade routes can include different modes of transportation and transport through distinct countries. We capture the impacts of exchange rates through the definition of effective path costs and identify the governing multicommodity international trade spatial price equilibrium conditions, which are then formulated as a variational inequality problem in product path flows. Existence results are established and a computational procedure presented. The illustrative numerical examples and a case study are inspired by the impacts of the war against Ukraine on agricultural trade flows and product prices. The modeling and algorithmic framework allows for the quantification of the impacts of exchange rates and various trade policies, as well as the addition or deletion of supply markets, demand markets and/or routes, on supply and demand market prices in local currencies, and on the volume of product trade flows with implications for food security.

Keywords Exchange rates · Spatial price equilibrium · International trade · Networks · Variational inequalities · Agriculture

1 Introduction

Exchange rates represent the value (price) of one currency relative to another currency. They are important economic parameters in international trade, with changes in the exchange rate affecting the decision-making of individuals, businesses, and governments. A separate

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Our Contributions

- **We use the theory of variational inequalities to construct a multicommodity international agricultural trade model, which contains novel features of capacities on the production outputs and on the transportation flows of agricultural commodities.**
- **Our model includes exchange rates, multiple agricultural commodities, multiple possible routes between country supply and demand market pairs, and expanded network equilibrium conditions to include the production and transportation bounds.**
- **The network equilibrium model allows for supply price, demand price, and unit transportation cost functions to depend on the commodity flow variables, and these functions can be nonlinear and asymmetric.**
- **The capacity constraints, along with the generality of the underlying functions, enable the modeling of competition for production and transportation capacity among the commodities.**

The Multicommodity International Agricultural Trade Model

The Multicommodity International Agricultural Trade Model

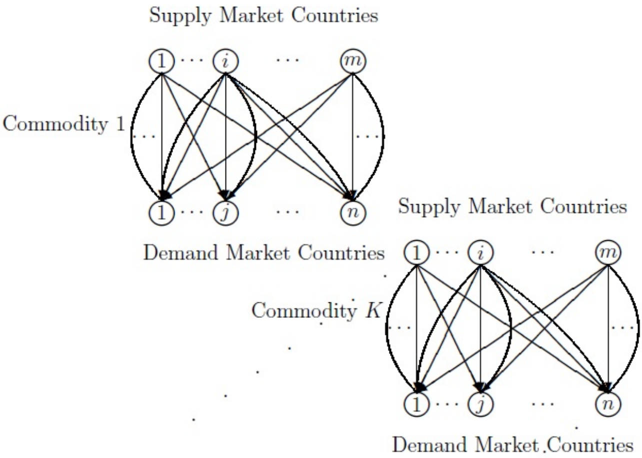


Figure: The Multicommodity International Trade Network

Variables

• Let Q_{ij}^{kl} denote the amount of commodity k produced at country supply market i and shipped on route l to country demand market j. The commodity flows are grouped into the vector $Q \in R_{+}^{KLmn}$. Associated with each pair of country supply and demand markets (i, j) is an exchange rate e_{ij} for $i = 1, \ldots, m$; $j = 1, \ldots, n$.

• Let s_i^k denote the supply of commodity k produced at country supply market i. All the commodity supplies are grouped into the vector $s \in R_{+}^{Km}$.

• The demand for commodity k at country demand market j is denoted by d_j^k, and all the demands are gathered into the vector $d \in R_{+}^{Kn}$.

A trade route can entail transportation via different modes, such as rail, truck, air, or water (sea, river, etc.).

Constraints

The conservation of flow equations are:

$$s_i^k = \sum_{j=1}^{n} \sum_{l=1}^{L} Q_{ij}^{kl}, \quad k = 1, \ldots, K; i = 1, \ldots, m, \tag{1}$$

$$d_j^k = \sum_{i=1}^{m} \sum_{l=1}^{L} Q_{ij}^{kl}, \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{2}$$

Also, all the commodity shipments must be nonnegative; that is:

$$Q_{ij}^{kl} \geq 0, \quad k = 1, \ldots, K; l = 1, \ldots, L; i = 1, \ldots, m; j = 1, \ldots, n. \tag{3}$$

Transportation Capacity Constraints

Let $\bar{Q}^l_{ij}$ denote the transportation capacity of route l between country supply market i and country demand market j, for all l, i, j. Typically, the units of flow for the agricultural commodities are in tons.

Hence, the following transportation capacity constraints must be satisfied:

$$\sum_{k=1}^{K} Q^{kl}_{ij} \leq \bar{Q}^l_{ij}, \quad l = 1, \ldots, L; i = 1, \ldots, m; j = 1, \ldots, n. \tag{4}$$

Expressions in (4) allow us to capture competition among the commodities for transportation services along particular routes.

Production Capacity Constraints

Let $\bar{S}_i$ denote the production capacity of country supply market i across all the commodities. The below production capacity constraints must be met:

$$\sum_{k=1}^{K} s_i^k \leq \bar{S}_i, \quad i = 1, \ldots, m. \tag{5a}$$

Due to the conservation of flow equations (1), constraints (5a) can take the form:

$$\sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{j=1}^{n} Q_{ij}^{kl} \leq \bar{S}_i, \quad i = 1, \ldots, m. \tag{5b}$$

According to (5a) or (5b), a country supply market i cannot violate its aggregate production capacity.

Lagrange Multipliers

We introduce Lagrange multipliers:

$$\lambda_{ij}^l, \quad l = 1, \ldots, L; i = 1, \ldots, m; j = 1, \ldots, n;$$

and:

$$\mu_i, \quad i = 1, \ldots, m;$$

associated with the capacity constraints in (4) and (5b), respectively, and we group these Lagrange multipliers into the vectors $\lambda \in R_+^{Lmn}$ and $\mu \in R_+^m$.

Supply Price Functions

The country supply price functions π_i^k, for all k, i, are:

$$\pi_i^k = \pi_i^k(s), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{6a}$$

Due to the conservation of flow equations (1), we may construct country supply price functions $\tilde{\pi}_i^k$, for all k, i, such that:

$$\tilde{\pi}_i^k(Q) \equiv \pi_i^k(s), \quad k = 1, \ldots, K; i = 1, \ldots, m. \tag{6b}$$

Demand Price Functions

The demand price of a commodity k in country j, ρ_j^k, in turn, can depend on the entire vector of demands of the commodities in all countries:

$$\rho_j^k = \rho_j^k(d), \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{7a}$$

Similarly, due to (2), we may construct new country demand price functions $\tilde{\rho}_j^k$, for all k, j, such that:

$$\tilde{\rho}_j^k(Q) \equiv \rho_j^k(d), \quad k = 1, \ldots, K; j = 1, \ldots, n. \tag{7b}$$

Transportation Cost Functions

The unit transportation cost associated with transporting commodity k from country i to country j on transportation route l is denoted by c_{ij}^{kl} and is as follows:

$$c_{ij}^{kl} = c_{ij}^{kl}(Q), \quad k = 1, \ldots, K; l = 1, \ldots, L; i = 1, \ldots, m; j = 1, \ldots, n. \tag{8}$$

The generality of the above transportation cost functions, where the unit transportation cost can depend on the vector of commodity shipments between all pairs of country supply and demand markets, allows one to further capture competition for transportation services among commodities.

Equilibrium Conditions

Definition 1: The Multicommodity International Agricultural Trade Equilibrium Conditions

A multicommodity shipment and Lagrange multiplier pattern $(Q^*, \lambda^*, \mu^*) \in \mathcal{K}^1$, *where* $\mathcal{K}^1 \equiv \{(Q, \lambda, \mu) | (Q, \lambda, \mu) \in R_+^{KLmn+Lmn+m}\}$ *is a multicommodity international agricultural trade network equilibrium with exchange rates, under limited production and transportation capacities, if the following conditions hold: For all commodities* k; $k = 1, \ldots, K$; *for all routes* l; $l = 1, \ldots, L$, *and for all country supply and demand market pairs:* (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$(\tilde{\pi}_i^k(Q^*) + c_{ij}^{kl}(Q^*))e_{ij} + \lambda_{ij}^{l*} + \mu_i^* \begin{cases} = \tilde{\rho}_j^k(Q^*), & \text{if } Q_{ij}^{kl*} > 0, \\ \geq \tilde{\rho}_j^k(Q^*), & \text{if } Q_{ij}^{kl*} = 0, \end{cases} \tag{9}$$

Equilibrium Conditions

Definition 1: The Multicommodity International Agricultural Trade Equilibrium Conditions

and for all routes l; $l = 1, \ldots, L$, and all country market pairs (i, j); $i = 1, \ldots, m$; $j = 1, \ldots, n$:

$$\lambda_{ij}^{l*} \begin{cases} \geq 0, & \text{if } \sum_{k=1}^{K} Q_{ij}^{kl*} = \bar{Q}_{ij}^{l}, \\ = 0, & \text{if } \sum_{k=1}^{K} Q_{ij}^{kl*} < \bar{Q}_{ij}^{l}, \end{cases} \tag{10}$$

and for all country supply markets i; $i = 1, \ldots, m$:

$$\mu_i^* \begin{cases} \geq 0, & \text{if } \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{j=1}^{n} Q_{ij}^{kl*} = \bar{S}_i, \\ = 0, & \text{if } \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{j=1}^{n} Q_{ij}^{kl*} < \bar{S}_i. \end{cases} \tag{11}$$

Variational Inequality Formulation

Theorem 1: Variational Inequality Formulation of the Multicommodity International Agricultural Trade Equilibrium Conditions

A multicommodity shipment and Lagrange multipliers pattern $(Q^*, \lambda^*, \mu^*) \in \mathcal{K}^1$ is a multicommodity international agricultural trade network equilibrium with exchange rates, under limited production and transportation capacities, according to Definition 1, if and only if it satisfies the variational inequality:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[(\tilde{\pi}_i^k(Q^*) + c_{ij}^{kl}(Q^*))e_{ij} + \lambda_{ij}^{l*} + \mu_i^* - \tilde{\rho}_j^k(Q^*)\right] \times (Q_{ij}^{kl} - Q_{ij}^{kl*})$$

$$+\sum_{l=1}^{L}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\bar{Q}_{ij}^{l} - \sum_{k=1}^{K} Q_{ij}^{kl*}\right] \times (\lambda_{ij}^{l} - \lambda_{ij}^{l*})$$

$$+\sum_{i=1}^{m}\left[\bar{S}_i - \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl*}\right] \times (\mu_i - \mu_i^*) \geq 0, \quad \forall (Q, \lambda, \mu) \in \mathcal{K}^1. \tag{12}$$

Variational Inequality Formulation

Standard Form

Variational inequality (12) is now put into standard form (cf. Nagurney (1999)), VI$(F,\mathcal{K})$, where one seeks to determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{13}$$

with F being a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, where $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Specifically, we define $X \equiv (Q, \lambda, \mu)$, $\mathcal{K} \equiv \mathcal{K}^1$, and $\mathcal{N} \equiv KLmn + Lmn + m$. Additionally, $F(X) \equiv (F_1(X), F_2(X), F_3(X))$ where $F_1(X)$ consists of the elements: $\left[(\tilde{\pi}_i^k(Q) + c_{ij}^{kl}(Q))e_{ij}) + \lambda_{ij}^l + \mu_i - \tilde{\rho}_j^k(Q)\right], \forall k, l, i, j$, and the components of $F_2(X)$ are: $\left[\bar{Q}_{ij}^l - \sum_{k=1}^K Q_{ij}^{kl}\right], \forall l, i, j$, and $F_3(X)$ is comprised of the elements: $\left[\bar{S}_i - \sum_{k=1}^K \sum_{l=1}^L \sum_{j=1}^n Q_{ij}^{kl}\right], \forall i$.

Clearly, variational inequality (12) can be put into standard form (13).

The Algorithm

The Algorithm

Modified Projection Method (Korpelevich (1977))

Step 0: Initialization
Initialize with $X^0 \in \mathcal{K}$. Set the iteration counter $\tau = 1$ and let β be a scalar such that $0 < \beta \leq \frac{1}{\eta}$, where η is the Lipschitz constant.

Step 1: Computation
Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \beta F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{14}$$

Step 2: Adaptation
Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \beta F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{15}$$

Step 3: Convergence Verification
If $|X^\tau - X^{\tau-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$ and go to Step 1.

The Algorithm

Explicit Formulae at Iteration τ in Step 1

The closed-form expressions for the multicommodity shipments for (14) for the solution of variational inequality (12) are:

$$\bar{Q}_{ij}^{kl\tau} = \max\{0, Q_{ij}^{kl\tau-1} + \beta(\tilde{\rho}_j^k(Q^{\tau-1}) - (\tilde{\pi}_i^k(Q^{\tau-1}) + c_{ij}^{kl}(Q^{\tau-1}))e_{ij} - \lambda_{ij}^{l\tau-1} - \mu_i^{\tau-1})\},$$
$$\forall k, l, i, j. \quad (16)$$

The closed-form expressions for the transportation capacity Lagrange multipliers for (14) for variational inequality (12) are:

$$\bar{\lambda}_{ij}^{l\tau} = \max\{0, \lambda_{ij}^{l\tau-1} + \beta(\sum_{k=1}^{K} Q_{ij}^{kl\tau-1} - \bar{Q}_{ij}^{l})\}, \quad \forall l, i, j. \quad (17)$$

The closed-form expressions for the production capacity Lagrange multipliers for (14) for variational inequality (12) are:

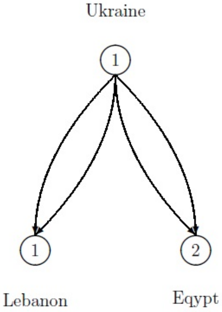
$$\bar{\mu}_i^{\tau} = \max\{0, \mu_i^{\tau-1} + \beta(\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{j=1}^{n} Q_{ij}^{kl\tau-1} - \bar{S}_i)\}, \quad \forall i. \quad (18)$$

The explicit formulae for the variables in (15) in Step 2 easily follow.

Numerical Examples

Numerical Examples

The examples focus on commodity flows from Ukraine to Lebanon and Egypt. There are two routes from Ukraine to each of the demand country markets, with the first route representing the export through a Black Sea port in Ukraine and the second route denoting the transportation of grains through the western borders of Ukraine to Romania, and then from a Romanian port on the Black Sea.



Example 1: Pre-War Scenario

This example considers the pre-war scenario when almost all of the grains in Ukraine were exported through their Black Sea ports. There are two commodities: wheat and corn, denoted by $k = 1, 2$, respectively.

The exchange rates are derived from early January 2022, before the full-scale invasion. The exchange rates are:

$$e_{11} = 55.0581, \quad e_{12} = .5714,$$

$$USD/UAH = 27.4619, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 15.7300.$$

The supply price functions for wheat and corn per ton in Ukrainian hryvnia are:

$$\pi_1^1(s) = .000136s_1^1 + .000068s_1^2 + 7,001.60,$$

$$\pi_1^2(s) = .000073s_1^1 + .000142s_1^2 + 6,728.20.$$

Example 1: Pre-War Scenario

The unit transportation cost functions for wheat and corn per ton in Ukrainian hryvnia are:

$$c_{11}^{11} = .000556Q_{11}^{11} + 2,046.80, \quad c_{11}^{12} = .007512Q_{11}^{12} + 10,984.60,$$

$$c_{12}^{11} = .000185Q_{12}^{11} + 2,046.80, \quad c_{12}^{12} = .007312Q_{12}^{12} + 10,984.60,$$

$$c_{11}^{21} = .005566Q_{11}^{21} + 2,046.80, \quad c_{11}^{22} = .006812Q_{11}^{22} + 10,984.60,$$

$$c_{12}^{21} = .001259Q_{12}^{21} + 2,046.80, \quad c_{12}^{22} = .007012Q_{12}^{22} + 10,984.60.$$

The demand price functions for wheat and corn in local currencies are:

$$\rho_1^1(d) = -.15d_1^1 + 602,344.00, \quad \rho_1^2(d) = -.68d_1^2 + 574,560.00,$$

$$\rho_2^1(d) = -.000475d_2^1 + 6,290.00, \quad \rho_2^2(d) = -.000758d_2^2 + 5,980.00.$$

Example 1: Pre-War Scenario

The supply capacity, in tons, in Ukraine is:

$$\bar{S}_1 = 5,000,000.00.$$

The transportation capacities, in tons, over routes are:

$$\bar{Q}^1_{11} = 5,000,000.00, \quad \bar{Q}^2_{11} = 500,000.00,$$

$$\bar{Q}^1_{12} = 5,000,000.00, \quad \bar{Q}^2_{12} = 500,000.00.$$

These capacities are derived based on the fact that Ukraine can, at most, export around 10% of its grains without its Black Sea ports.

The modified projection method yields the following equilibrium commodity shipment pattern:

$$Q^{11*}_{11} = 477,085.5938, \quad Q^{11*}_{12} = 1,605,672.5000, \quad Q^{12*}_{11} = 0.0000, \quad Q^{12*}_{12} = 0.0000,$$

$$Q^{21*}_{11} = 79,128.0781, \quad Q^{21*}_{12} = 560,130.3750, \quad Q^{22*}_{11} = 0.0000, \quad Q^{22*}_{12} = 0.0000.$$

This commodity flow pattern is quite close to Ukraine's actual wheat and corn exports to Lebanon and Egypt in 2021 and the projected amounts in 2022, with the assumption that the invasion would have never occurred. Lebanon, on the average, imports more than 70% of its wheat and about 20% of its corn from Ukraine, while these percentages for Egypt are, on the average, 25%, and 5%, for wheat and corn, respectively. Ukraine's wheat exports to Lebanon were at 520,000 tons in 2021, and an even greater amount of exports was expected for 2022.

The equilibrium commodity supplies are:

$$s_1^{1*} = 2,082,758.1250, \quad s_1^{2*} = 639,258.4375.$$

The equilibrium commodity demands are:

$$d_1^{1*} = 477,085.5938, \quad d_1^{2*} = 79,128.0781,$$

$$d_2^{1*} = 1,605,672.5000, \quad d_2^{2*} = 560,130.3750.$$

Example 1: Pre-War Scenario

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 7,328.3252 = \$266.8542, \quad \pi_1^2(s^*) = 6,971.0166 = \$253.8432.$$

Pre-war, Ukrainian farmers could earn close to $270 per ton for wheat and corn, which is very close to the reported supply prices in this example.

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 530,781.1875 = \$351.0457, \quad \rho_1^2(d^*) = 520,752.9063 = \$344.4132,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 5,527.3057 = \$351.3862, \quad \rho_2^2(d^*) = 5,555.4214 = \$353.1736.$$

We observe that the resultant demand prices in Lebanon and Egypt resemble the prices reported pre-war.

All the Lagrange multipliers were equal to: 0.0000, since the production and transportation capacities exceeded the corresponding flows. **Note that only the maritime routes have positive commodity flows.**

Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

We now consider the following disaster scenario. It is the early period after the full-scale invasion but before the Black Sea Grain Initiative. During this period, the Black Sea routes were mined and blockaded. Hence, the capacity of these maritime routes was greatly reduced to essentially zero.

Example 2 has identical data to that in Example 1, except the maritime route links are no longer available. We retain the same superscripts and subscripts as in Example 1 but note that for each pair of supply and demand country market pairs, there is only route 2 available for transporting the wheat and corn to Lebanon and Egypt.

The modified projection method yields the following equilibrium commodity shipment pattern:

$$Q_{11}^{12*} = 216,433.1406, \quad Q_{12}^{12*} = 500,000.0000,$$

$$Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

Now, with the cheaper maritime routes blockaded and no longer functional, the more expensive alternative routes are in use. After the start of the war, the transportation cost of grains inside Ukraine reached an unprecedented level of around $200.

The alternative routes are used for the export of wheat but not for corn. Lebanon and Egypt rely heavily on wheat as their main source of nutrition; however, corn is mostly used to feed animals.

Given that Egypt has a population of about twenty times that of Lebanon, its wheat import is such that the full capacity of the alternative route is used. On the other hand, the commodity flow of wheat to Lebanon does not even reach the low capacity of the alternative route, which is due to the high cost of transportation.

Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

The equilibrium commodity supplies are:

$$s_1^{1*} = 716,432.1875, \quad s_1^{2*} = 0.0000.$$

The equilibrium commodity demands are:

$$d_1^{1*} = 216,433.1406, \quad d_1^{2*} = 0.0000,$$

$$d_2^{1*} = 500,000.0000, \quad d_2^{2*} = 0.0000.$$

All Lagrange multipliers are equal to 0.0000 except that $\lambda_{12}^{2*} = 468.4277$.

Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 7,099.0347 = \$258.5048, \quad \pi_1^2(s^*) = 6,780.4995 = \$246.9056.$$

Note that the supply prices are lower than in Example 1. Ukrainian farmers are essentially selling their wheat at lower prices to compensate for the higher cost of transportation after the start of the invasion. As a matter of fact, in the later months after the start of the war, the supply price for Ukrainian wheat went as low as less than $100. This example is very early, right after the start of the war; therefore, the supply prices are just starting to go down.

Example 2: Early Period Post-Full-Scale Invasion of February 24, 2022

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 569,879.0000 = \$376.9041, \quad \rho_1^2(d^*) = 6,052.5005 = \$384.7743,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 574,560.0000 = \$380, \quad \rho_2^2(d^*) = 5,980.0000 = \$380.1652.$$

We observe that the demand prices in both demand country markets are now higher than in Example 1. In later months, close to the establishment of the Black Sea Grain Initiative, demand prices in Lebanon and Egypt reached high levels, even around \$500; here, however, the markets are just starting to react to the war in terms of higher prices and the associated supply and transportation challenges.

Example 3: Black Sea Grain Initiative in Place

In this example, we consider the scenario that the Black Sea Grain Initiative is in place (beginning in August). The exchange rates are derived from late August. The exchange rates are:

$$e_{11} = 41.3469, \quad e_{12} = .5236,$$

$$USD/UAH = 36.5686, \quad USD/LBP = 1,512.0000, \quad USD/EGP = 19.1500.$$

The supply price functions for wheat and corn per ton in Ukrainian hryvnia are:

$$\pi_1^1(s) = .000136 s_1^1 + .000068 s_1^2 + 3,364.60,$$

$$\pi_1^2(s) = .000073 s_1^1 + .000142 s_1^2 + 4,022.50.$$

Given the damages to production inputs and available arable land in Ukraine and the war-induced Ukrainian farmers' low share of the earnings, the supply price functions are updated accordingly to account for these factors.

Example 3: Black Sea Grain Initiative in Place

The unit transportation cost functions for wheat and corn per ton in Ukrainian hryvnia are:

$$c^{11}_{11} = .000556 Q^{11}_{11} + 13,867.90, \quad c^{12}_{11} = .007512 Q^{12}_{11} + 15,591.10,$$

$$c^{11}_{12} = .000185 Q^{11}_{12} + 13,867.90, \quad c^{12}_{12} = .007312 Q^{12}_{12} + 15,591.10,$$

$$c^{21}_{11} = .005566 Q^{21}_{11} + 13,867.90, \quad c^{22}_{11} = .006812 Q^{22}_{11} + 15,591.10,$$

$$c^{21}_{12} = .001259 Q^{21}_{12} + 13,867.90, \quad c^{22}_{12} = .007012 Q^{22}_{12} + 15,591.10.$$

The transportation cost functions are updated from the previous examples to highlight the war-induced unprecedented high transportation costs.

In this example, the maritime routes have the original capacity, as in Example 1 (although there are still slowdowns in processing, etc.). Still, due to mining and the destruction of agricultural land in Ukraine during the war, the supply capacity is now reduced, with $\bar{S}_1 = 1,000,000.00$.

Example 3: Black Sea Grain Initiative in Place

The demand price functions for wheat and corn per ton in local currencies are:

$$\rho_1^1(d) = -.15d_1^1 + 796,162.50, \quad \rho_1^2(d) = -.68d_1^2 + 718,256.40,$$

$$\rho_2^1(d) = -.000475d_2^1 + 10,000.60, \quad \rho_2^2(d) = -.000758d_2^2 + 9,900.50.$$

Both Lebanon and Egypt faced a severe food security crisis during this period. Both countries depend on the flow of Ukrainian grains to meet their populations' nutrition and caloric demands. The war and reduced supply of Ukrainian grains induced high demand prices in these demand country markets. These changes in prices are reflected in the updated demand price functions.

The modified projection method yields the following equilibrium commodity shipment pattern:

$$Q_{11}^{11*} = 477,651.1563, \quad Q_{12}^{11} = 552,348.4375, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 0.0000, \quad Q_{12}^{21*} = 0.0000, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Example 3: Black Sea Grain Initiative in Place

Once again, only efficient maritime routes are used for the transport of grains, and alternative routes are not utilized. This highlights the importance of the Black Sea Grain Initiative in facilitating the transport of grains from Ukraine, even with limited supply capacity.

The wheat commodity flows are improved compared to in Example 2, especially in the case of Lebanon, which has food security implications for the food crisis in both demand country markets.

Furthermore, as in Example 2, no corn is produced, which is, again, due to the high dependency of our network's demand country markets, Lebanon and Egypt, on Ukrainian wheat and Ukraine's war-induced limited supply capacity.

Example 3: Black Sea Grain Initiative in Place

The equilibrium commodity supplies are:

$$s_1^{1*} = 999,999.6250, \quad s_1^{2*} = 0.0000.$$

The equilibrium commodity demands are:

$$d_1^{1*} = 447,651.1563, \quad d_1^{2*} = 0.0000,$$

$$d_2^{1*} = 552,348.4375, \quad d_2^{2*} = 0.0000.$$

The computed equilibrium Lagrange multipliers are all equal to 0.0000 except that $\mu_1^{1*} = 591.6817$ since, essentially, the supply output of commodities is at the capacity $\bar{S}_1^1 = 1,000,000.00$.

Example 3: Black Sea Grain Initiative in Place

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 3,500.6001 = \$95.7269, \quad \pi_1^2(s^*) = 4,095.5000 = \$111.9949.$$

Observe that the price share of Ukrainian farmers is less than $100, even with the establishment of the Black Sea Grain Initiative and the facilitation of the transport of grains from Ukrainian Black Sea ports. This could be traced back to transportation costs remaining high even after the Initiative. In other words, due to the war and its associated risks, the transportation costs remain high, even through the transportation corridor provided by the Initiative.

Example 3: Black Sea Grain Initiative in Place

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 729,014.8125 = \$482.1526, \quad \rho_1^2(d^*) = 718,256.3750 = \$475.0372,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 9,738.2344 = \$508.5239, \quad \rho_2^2(d^*) = 9,900.5000 = \$516.9973.$$

Even though the transportation capacity limitations are lifted, one can see that, again, because of the high transportation costs and limited supply capacity, the demand prices remain at such high levels.

Example 4 explores the impact of transportation costs reverting to the pre-February 24, 2022 level. The rest of the data remains as in Example 2. Hence, this scenario considers changes in supply price and demand functions from those in Example 1, different exchange rates than those in Example 1, plus a reduction in production capacity, due to mining, etc., in wartime. This example helps to reveal the importance of transportation and a reduction in associated costs on the equilibrium pattern.

The modified projection method now yields the following equilibrium commodity shipment pattern:

$$Q_{11}^{11*} = 935,264.3750, \quad Q_{12}^{11*} = 552,348.4375, \quad Q_{11}^{12*} = 0.0000, \quad Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 64,735.4296, \quad Q_{12}^{21*} = 0.0000, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Example 4: Black Sea Grain Initiative in Place, Transportation Costs as Pre-Invasion, Supply Production Capacity as in Example 2

Again, only the efficient maritime routes representing transportation through Ukrainian Black Sea ports are in use.

Note that Lebanon and Egypt are essentially competing for the limited supply capacity of Ukrainian grains, as both countries are stricken by a food crisis. In this case, Lebanon is appropriating almost all of this limited production capacity, importing all of its wheat demand from Ukraine, while Egypt is shifting towards importing a small amount of Ukrainian corn.

As mentioned in Example 1, Lebanon imports, on the average, more than 70% of its wheat demand from Ukraine, while this percentage for Egypt is at around 25%. Accordingly, Lebanon is much more dependent on Ukrainian wheat than Egypt, and the above commodity flow pattern implies this higher dependency.

Example 4: Black Sea Grain Initiative in Place, Transportation Costs as Pre-Invasion, Supply Production Capacity as in Example 2

The equilibrium commodity supplies are:

$$s_1^{1*} = 935,264.3750, \quad s_1^{2*} = 64,735.4297.$$

The equilibrium commodity supplies are:

$$s_1^{1*} = 935,264.3750, \quad s_1^{2*} = 64,735.4297.$$

The equilibrium commodity demands are:

$$d_1^{1*} = 935,264.3750, \quad d_1^{2*} = 64,735.4297,$$

$$d_2^{1*} = 552,348.4375, \quad d_2^{2*} = 0.0000.$$

The computed equilibrium Lagrange multipliers are all equal to 0.0000 except that $\mu_1^{1*} = 405,189.5000$ since the supply output of commodities is at the capacity $\bar{S}_1^1 = 1,000,000.00$.

Example 4: Black Sea Grain Initiative in Place, Transportation Costs as Pre-Invasion, Supply Production Capacity as in Example 2

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 3,496.1982 = \$95.6065, \quad \pi_1^2(s^*) = 4,099.9668 = \$112.1171.$$

Note that the supply prices remain at the low levels observed in Example 3 without significant improvement. It could result from the war-induced low supply prices and full storage in Ukraine, at least in the short term. However, it could also highlight the significant impact of the damages to arable lands and production inputs in Ukraine due to the war, which has, in turn, resulted in a low production capacity.

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 655,872.8750 = \$433.7783, \quad \rho_1^2(d^*) = 674,236.3125 = \$445.9234,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 10,000.6000 = \$522.2245, \quad \rho_2^2(d^*) = 9,900.5000 = \$516.9973.$$

Lebanon sees an improvement in the prices of grains due to appropriating almost all of Ukraine's limited production capacity. At the same time, the demand prices in Egypt remain at the same high level as in Example 3. In other words, Lebanon makes an improvement on its food security crisis by winning the competition over Ukrainian grains when Egypt is left to deal with its food security concerns as severe as before. Thus, the importance of the Ukrainian grain harvest, and its capacity, for food security in MENA countries is further highlighted.

Example 5: Same Data as That for Example 4 but with Supply Capacity as in Example 1

Example 4 reveals not only the importance of investing in transportation routes but also the importance of having sufficient capacity for the production of agricultural products with $\mu_1^* = 405,189.5000$. **In Example 5, hence, we retain the data as in Example 4, but now we assume that the available supply capacity is as in Example 1. This example helps to illustrate the importance of having all the original land that Ukraine farmed pre-war made again available for critical agricultural commodities.** Hence, in this example $\bar{S}_1 = 5,000,000.00$.

The computed equilibrium commodity shipment pattern is now:

$$Q_{11}^{11*} = 3,122,624.0000, \; Q_{12}^{11} = 1,153,227.8750, \; Q_{11}^{12*} = 0.0000, \; Q_{12}^{12*} = 0.0000,$$

$$Q_{11}^{21*} = 487,816.0000, \quad Q_{12}^{21*} = 236,331.5313, \quad Q_{11}^{22*} = 0.0000, \quad Q_{12}^{22*} = 0.0000.$$

Example 5: Same Data as That for Example 4 but with Supply Capacity as in Example 1

Observe that, with the updated supply and demand price functions, transportation cost functions, and exchange rates relevant to the post-Black Sea Grain Initiative period but with recovered production and transportation capacities as in the pre-war period, all commodity flows on the maritime routes are now positive and increased, with the less efficient alternative routes, again, not utilized.

In this example, contrary to the pre-war case, Lebanon is doing better in terms of competition for Ukrainian grains, appropriating more of the production capacity of Ukraine for the nutritional and caloric needs of its population. The severity of the food crisis in Lebanon and its higher dependence on Ukrainian grains could be the reason for this shift in commodity shipments.

Additionally, the increase in flows could be related to months of little to no grains being shipped to the demand country markets.

Example 5: Same Data as That for Example 4 but with Supply Capacity as in Example 1

The equilibrium commodity supplies are:

$$s_1^{1*} = 4,275,852.0000, \quad s_1^{2*} = 724,147.50000.$$

The equilibrium commodity demands are:

$$d_1^{1*} = 3,122,624.0000, \quad d_1^{2*} = 487,816.0000,$$
$$d_2^{1*} = 1,153,227.8750, \quad d_2^{2*} = 236,331.5313.$$

The computed equilibrium Lagrange multipliers are all equal to 0.000 except that $\mu_1^{1*} = 6,171.1826$.

The incurred supply prices in Ukraine in hryvnia at the equilibrium are:

$$\pi_1^1(s^*) = 3,995.3579 = \$109.2565, \quad \pi_1^2(s^*) = 4,437.4663 = \$121.3463.$$

With the increase in commodity flows, the supply prices are now higher than those in Example 3 but still quite lower than the pre-war prices. Accordingly, farmers are earning more, but still quite less than pre-war.

Example 5: Same Data as That for Example 4 but with Supply Capacity as in Example 1

The incurred demand prices at the equilibrium in Lebanon in Lebanese pounds are:

$$\rho_1^1(d^*) = 327,768.8750 = \$216.7783, \quad \rho_1^2(d^*) = 386,541.5000 = \$255.6491,$$

whereas the corresponding demand prices in Egypt in Egyptian pounds are:

$$\rho_2^1(d^*) = 9,452.8164 = \$493.6196, \quad \rho_2^2(d^*) = 9,721.3604 = \$507.6428.$$

Note that, with the recovery of the production capacity, all demand prices are now lower than in Examples 3 and 4. The lower demand prices could translate into improvements in terms of food security issues in Lebanon and Egypt. However, observe that this improvement in the case of Lebanon is much more significant, as the country in this example appropriates a much higher commodity shipment than Egypt.

Sensitivity on Exchange Rates

Sensitivity on Exchange Rates

Here, we conduct sensitivity analysis on the exchange rates, and we report the equilibrium supplies of the commodities of wheat and corn and also their demand market prices in the countries of Lebanon and Egypt.

We use Example 5 as a baseline. In the first figure, we report the multicommodity supplies when $e_{12} = .5236$, but e_{11} varies with $e_{11} = 41.3469$ and then e_{11}=45, 50, and 55. The corresponding demand market prices at the equilibrium for these exchange rates are then reported in the second figure.

We then, again, using Example 5 as a baseline, keep $e_{11} = 41.3469$ as in Example 5, but vary e_{12} with $e_{12} = .5236$ and then $e_{12} = .6236$, .7236, and, finally, .8236. The computed equilibrium commodity supplies at these exchange rates are reported in the third figure, and the equilibrium commodity demand prices in the fourth figure.

Sensitivity on Exchange Rates

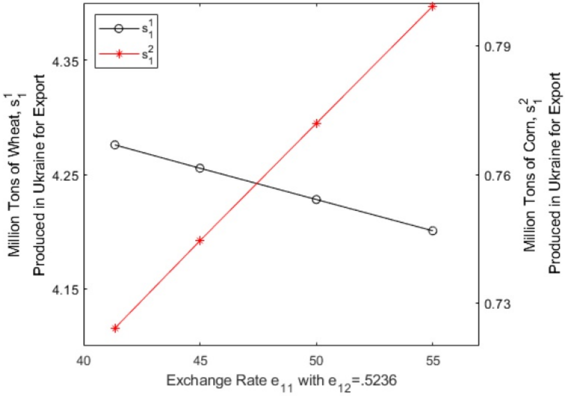


Figure: Sensitivity Analysis for Exchange Rates with $e_{12} = .5236$ but with e_{11} Varying: Impact on Commodity Supplies

Sensitivity on Exchange Rates

• Note that, as seen in the above figure, with the depreciation of the Lebanese pound with respect to Ukrainian hryvnia, that is, higher rates of e_{11}, while keeping e_{12} fixed, the production of wheat in Ukraine decreases, and the supply of Ukrainian corn increases with a sharper slope. In other words, with less demand from Lebanon because of the depreciation of LBP, Ukraine meets the demand for wheat in Egypt and shifts to produce more corn to satisfy the demand for corn in Egypt.

• It should be noted that, generally, the supply and demand of wheat are more price inelastic than corn; furthermore, when there is a global deficit, resulting in food security concerns, the trade volumes of wheat are even less sensitive to exchange rate fluctuations.

Sensitivity on Exchange Rates

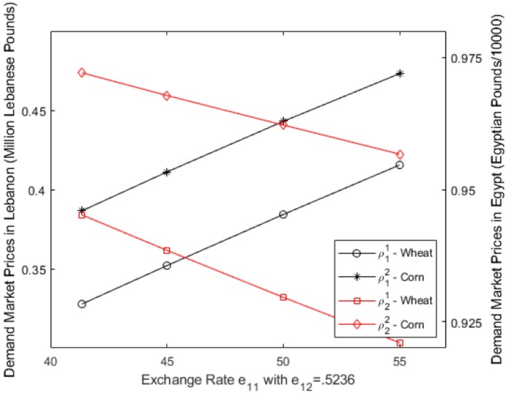


Figure: Sensitivity Analysis for Exchange Rates with $e_{12} = .5236$ but with e_{11} Varying: Impact on Commodity Demand Market Prices

Sensitivity on Exchange Rates

• Looking at the above figure, one can see that as LBP depreciates and the demand prices of both commodities increase in Lebanon, the demand prices go down in Egypt since Lebanon cannot buy as much grain as before, and Egypt, which is essentially competing with Lebanon for Ukrainian grain, is now appropriating more of the commodity shipments, driving its demand prices down.

• Also, observe that the decrease in the demand price of wheat in Egypt is sharper than that of corn, which is in line with the importance of wheat as a staple in the country's nutritional and caloric needs of its citizenry. In other words, Egypt will import more wheat than corn, that is, will import a higher commodity flow of wheat than corn, which translates into a sharper decrease in the demand price of wheat.

Sensitivity on Exchange Rates

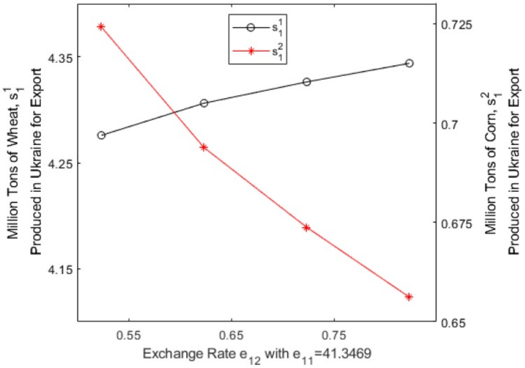


Figure: Sensitivity Analysis for Exchange Rates with $e_{11} = 41.3469$ but with e_{12} Varying: Impact on Commodity Supplies

Sensitivity on Exchange Rates

• **We observe that, as shown in the figure, with the depreciation of the Egyptian pound, with the Lebanese pound fixed, the supply of Ukrainian wheat increases, and the production of Ukrainian corn decreases, both at a decreasing rate.**

• **Essentially, with the depreciation of EGP, Egypt cannot afford as much Ukrainian grain as before, and Lebanon, which has a much higher demand for Ukrainian wheat than corn, imports more wheat. However, with the value of EGP going lower, and more wheat commodity flow appropriated by Lebanon, the country's demand for wheat is satisfied; it slowly shifts toward buying more corn, as such, causing a decreasing rate of decrease in the production of Ukrainian corn and a decreasing rate of increase in the production of Ukrainian wheat.**

• **Again, it must be noted that wheat has a lower price elasticity compared to corn, and corn plantings could have more variations based on the market conditions.**

Sensitivity on Exchange Rates

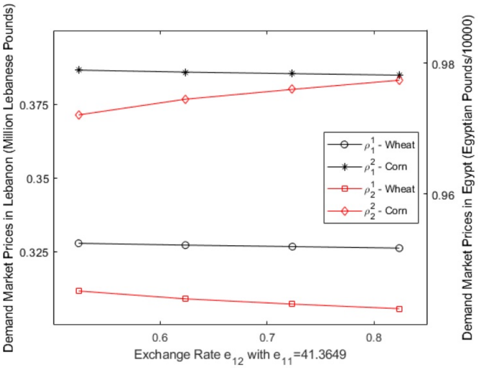


Figure: Sensitivity Analysis for Exchange Rates with $e_{11} = 41.3469$ but with e_{12} Varying: Impact on Commodity Demand Market Prices

Sensitivity on Exchange Rates

• **With the value of EGP going lower, the demand prices in Lebanon decrease slightly since Lebanon appropriates more commodity flow. At the same time, naturally, the demand price of corn in Egypt goes down, as Egypt cannot afford the previous level of commodity shipment.**

• **The demand price of wheat in Egypt is surprisingly decreasing, albeit at a decreasing rate. The reason for this decrease in the demand price is that Egypt, facing the depreciation of EGP, and given the higher priority of wheat in the country's caloric demand than corn, refrains from importing corn. Hence, at first, Egypt raises its wheat commodity flow by giving up its corn imports, lowering the demand price, although the value of its currency is depreciating. However, the rate of the decrease in the demand price is decreasing as the possible increase in wheat imports is limited, and the depreciation in the value of EGP finally catches up and increases the demand price of wheat.**

• **Furthermore, in practice, Egypt's ability to substitute Ukrainian wheat is higher than corn, as the country is far away from alternative sources of corn, e.g., the US and South America. Accordingly, Egypt, in general, is more prone to an increase in the prices of corn than those of wheat.**

Insights and Summary

Insights

• **We see that there is essentially no efficient alternative to the maritime transportation of grains from Ukrainian Black Sea ports, highlighting the importance of extending the Black Sea Grain Initiative during wartime to keep the transportation capacity sufficient not to disrupt the food security of demand country markets.**

• **The results confirm how the war has driven the earnings of the Ukrainian farmers, that is, the supply prices, to unprecedented low levels, possibly requiring the Ukrainian government's and global support of Ukrainian farmers for future harvest seasons, given the importance of Ukrainian grain to global food security, especially in the MENA region.**

• **The numerical results show the priority of wheat over corn in all scenarios in the demand markets of Lebanon and Egypt, as two countries representative of the MENA region.**

Insights

• **We find that Lebanon and Egypt compete over the war-induced limited production capacity at war-induced high prices to meet their populations' nutritional and caloric demands.**

• **The results demonstrate how the war-induced reduced production capacity in Ukraine intensifies this competition for meeting the fundamental need for food security of the Lebanese and Egyptian people.**

• **The economic instability of the demand country markets, in the form of the depreciation of their currencies, lowers their share of the Ukrainian wheat supply, causing food security concerns in these countries.**

• **The solutions to the numerical examples show the shift in the percentage of the limited production capacity in Ukraine utilized for producing each of the commodities of wheat and corn as the currency of each of the demand country markets depreciates.**

Summary

• We constructed a general multicommodity international agricultural trade network equilibrium model with production and transportation capacities.

• Our model enables the quantification of the impacts of decreases in agricultural commodity production and transportation capacities, due to disasters, as well as the magnitude of exchange rates, on the equilibrium commodity production, shipment, demand, and price patterns.

• Using the methodological framework of variational inequality theory, we provide alternative formulations of the governing equilibrium conditions, which are new. We also provide a deeper interpretation of the Lagrange multipliers associated with the production/supply and transportation constraints.

Summary

• **The proposed algorithm has nice features for implementation and resolves one of our derived variational inequalities into closed-form expressions for the commodity shipments and the Lagrange multipliers at each iteration.**

• **A series of numerical examples inspired by Russia's war on Ukraine and its impact on food security in MENA countries are solved to illustrate the multicommodity international agricultural trade network equilibrium model.**

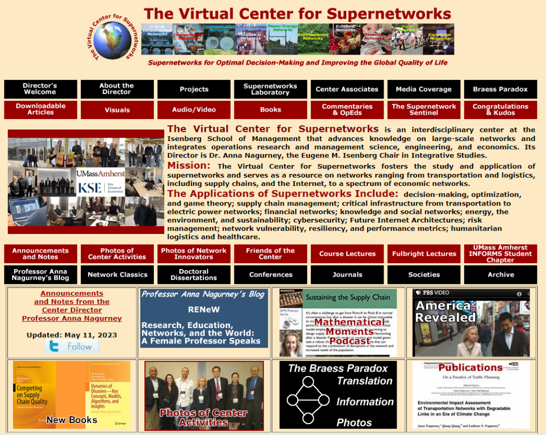
• **Different production and transportation capacities from several relevant periods; that is, pre-war, early after the start of the war, before the Black Sea Grain Initiative, and after the Initiative is in place, are used to highlight the changes in the equilibrium supply, commodity shipment, and demand patterns and the supply market and demand market prices.**

Summary

• **The numerical results have implications for the Ukrainian government. Also, knowledge of the changes in commodity shipment patterns and prices helps the governments of the demand country markets, in our case, Lebanon and Egypt, manage the food security of their people.**

• **Additionally, sensitivity analysis on exchange rates reveals how different exchange rates affect the supply and demand prices of the two commodities of wheat and corn at the equilibrium.**

Thank You Very Much!



More information on our work can be found on the Supernetwork Center site: https://supernet.isenberg.umass.edu/