A Stochastic Disaster Relief Game Theory Network Model

Anna Nagurney[1], Mojtaba Salarpour[1], June Dong[2], and Ladimer S. Nagurney[3]

[1] Department of Operations and Information Management
Isenberg School of Management
University of Massachusetts
Amherst, Massachusetts
[2] Department of Marketing & Management
School of Business, State University of New York, Oswego, New York
[3] Department of Electrical and Computer Engineering
University of Hartford, West Hartford, Connecticut

INFORMS Annual Meeting
November 7-13, 2020

This presentation is based on the paper:

Nagurney, A., Salarpour, M., Dong, J., Nagurney, L.S., 2020. A Stochastic Disaster Relief Game Theory Network Model. *SN Operations Research Forum*, 1(10), pp 1-33.

SN Operations Research Forum (2020) 1: 10
https://doi.org/10.1007/s43069-020-0010-0

Check for updates

ORIGINAL RESEARCH

A Stochastic Disaster Relief Game Theory Network Model

Anna Nagurney[1] · Mojtaba Salarpour[1] · June Dong[2] · Ladimer S. Nagurney[3]

Received: 26 December 2019 / Accepted: 20 March 2020 / Published online: 11 April 2020
© Springer Nature Switzerland AG 2020

Abstract

In this paper, we construct a novel game theory model for multiple humanitarian organizations engaged in disaster relief. Each organization is faced with a two-stage stochastic optimization problem associated with the purchase and storage of relief items pre-disaster, subject to a budget constraint, and, if need be, additional purchases and shipments post the disaster. The model integrates logistical and financial

Outline

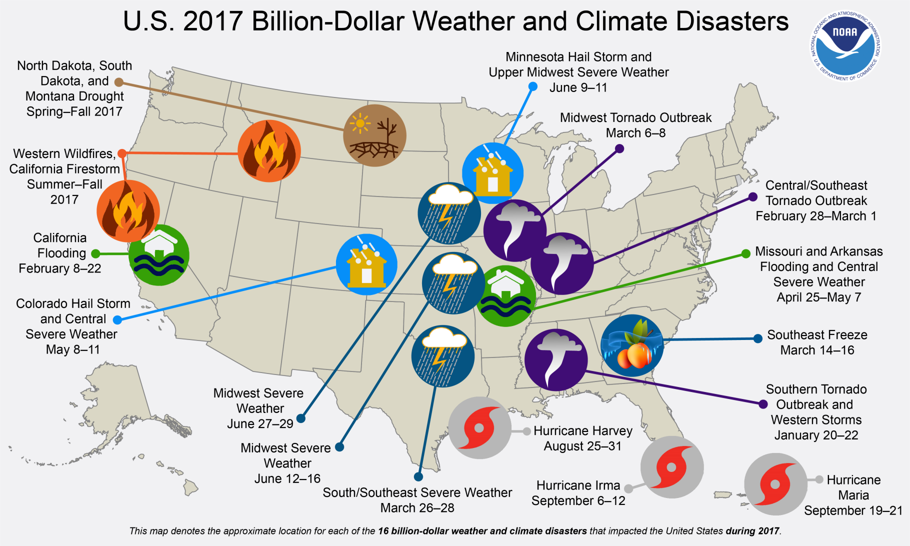
Introduction

- **Disasters**

  IFRC defines a disaster as "a sudden, calamitous event that seriously disrupts the functioning of a community or society and causes human, material, and economic or environmental losses that exceed the community's or society's ability to cope using its own resources. Though often caused by nature, disasters can have human origins."


Photographer: Chung Sung-Jun/Getty Images

Billion Dollar Disasters in the US in 2017



Introduction

- **Disaster Management has Four Phases**

  - Mitigation, preparedness, response, and recovery

  - Vital 72 hours!

  - The region's infrastructure may be compromised, or even destroyed.

  - Many natural disasters are highly **unpredictable** in terms of severity, timing, and location.

  - The main role of **Humanitarian Logistics**.

- **Humanitarian Organizations**

  - Governments alone cannot assume full responsibility for humanitarian operations.

  - National and international humanitarian organizations come to assist.

  - Lack of coordination among agencies may lead to the **duplication** of efforts, **confusion** at the "last mile", and issues of material convergence.

Introduction

- **Competition**

  - NGOs are nonprofit and dependent on **donations**.

  - **Competition** is natural in an environment in which Humanitarian Organizations (HOs) are competing for donor funding.

  - Donors respond to the **visibility** of HOs in disaster response in the **media**.

- **Uncertainty**

  - Some of the **population** may have perished in the disaster.

  - **Prices** of relief items post the disaster may increase due to competition and the demand.

  - Uncertainty due to the possibly compromised infrastructure, along with the **costs of freight service provision**.

  - Uncertainty surrounding the willingness of donors to give post a disaster and their **level of donations**.

- **The associated literature has been limited while the response phase of disaster management had been the phase researched the most intensively.:**

  - Toyasaki and Wakolbinger (2014) - Nagurney, Alvarez Flores, and Soylu (2016) - Coles, Zhang, and Zhuang (2018) - Nagurney et al. (2018) - Nagurney, Salarpour, and Daniele (2019).

  - Additional references on disaster management and game theory: see However, Muggy and Heier Stamm (2014) and the survey by Seaberg, Devine, and Zhuang (2017).

Optimization Under Uncertainty and Disaster Relief

- Background on the **two-stage scenario-based stochastic programming**: Dupacova (1996) - Barbarosoglu and Arda (2004) - The books by Birge and Louveaux (1997) and Shapiro, Dentcheva, and Ruszczynski (2009) and Derman et al. (1973)

  - Multicriteria optimization in humanitarian aid: Qiang and Nagurney (2012) constructed a supply chain network model for critical needs (food, medicines, etc.) in the case of disruptions. Gutjahr and Nolz (2016) presented a survey on multicriteria optimization in humanitarian aid.

  - He and Zhuang (2016) constructed a two-stage, dynamic model to assess the trade-off between pre-disaster preparedness and post-disaster relief.

Optimization Under Uncertainty and Disaster Relief

- Liu and Nagurney (2013) focused on competition in commercial supply chains and proposed a two-stage, game theory framework for supply chain networks with global outsourcing and quickresponse production under demand and cost uncertainty.
- Additional references on uncertainty in humanitarian logistics in disaster management: Liberatore et al. (2013) - Hoyos, Morales, and Akhavan-Tabatabaei (2015) - Rawls and Turnquist (2010), Mete and Zabinsky (2010), Falasca and Zobel (2011), Grass and Fischer (2016).

Our Contributions

- The topic of game theory for disaster relief under uncertainty remains essentially unexplored.
- Our model illustrates how organizations can optimize their decision-making, both in advance and, after a disaster, given the competitive environment.
- This paper builds on the work of Nagurney, Alvarez Flores, and Soylu (2016), Nagurney et al. (2018), and Nagurney, Salarpour, and Daniele (2019).
- **Two-stage stochastic optimization programming**
  Each decision-maker is faced with a two-stage stochastic optimization problem.
- **Demand uncertainty**
  Associated with the relief items - Can differ at the locations, post the disaster - Depends also on the disaster level at the location.

Our Contributions

- **Price uncertainty**
  Post the disaster - At the purchase locations (PLs) - The humanitarian organizations can purchase supplies from multiple locations prior to and post the disaster.
- **Uncertain logistical costs**
  Can be distinct for the different freight service providers (FSPs) since some may have suffered greater (or lesser) disruptions during the disaster.
- **Financial donations functions**
  Can differ for the HOs - Associated with the distinct demand points - Depend on the severity of the disaster.
- **Stochastic elements in the Generalized Nash Equilibrium model**
  Due to the common constraints corresponding to the lower bounds and the upper bounds on the relief item volumes at demand points, which are affected by the scale of the disaster.

Stochastic Disaster Relief Game Theory Network Model

- I humanitarian organizations, with a typical one denoted by i.
- J storage/distribution hubs, with a typical location denoted by j.
- K demand locations, with a typical location denoted by k.
- L freight service providers (FSPs) with a typical one denoted by l.
- H possible purchase locations, with a typical location denoted by h.
- Ω possible disaster scenarios, with a typical location denoted by ω.

Stochastic Disaster Relief Game Theory Network Model

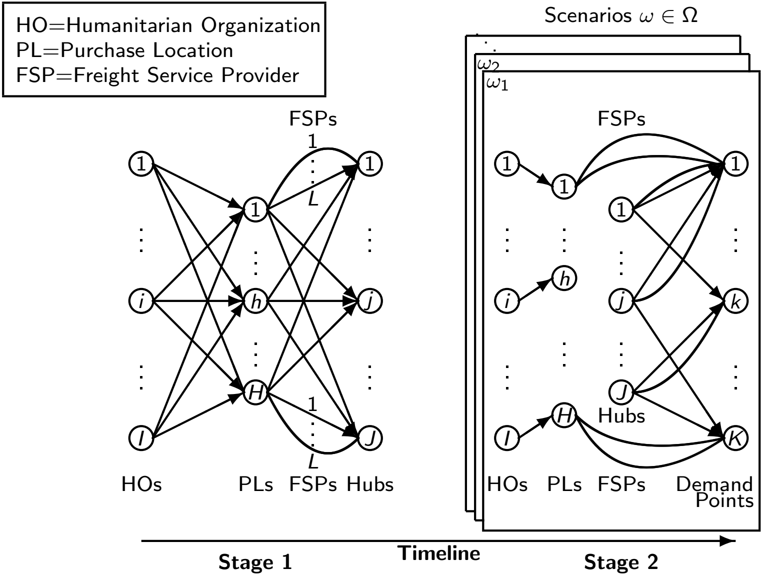


Figure: The Timeline of the Humanitarian Organizations.

The HOs' Optimization Problems

$q^{i1}_{hj,l}$: the amount of the disaster relief items purchased by HO i and carried by FSP l from PL h to Hub j.

$U_i(q)$: HO i's utility; $i = 1, \ldots, I$.

ρ_h: the price of the disaster relief item at purchasing location h before the disaster; $h = 1, \ldots, H$.

π_j: the price of storage per unit at storage/hub location j; $j = 1, \ldots, J$.

β_i: the weight imposed by HO i on the altruism component of his utility function in stage 2; $i = 1, \ldots, I$.

$c^{i1}_{hj,l}(q^1)$: the transportation cost encumbered by HO i to have its relief items delivered to hub j by freight service provider l from purchasing location h before the disaster.

B_i: HO i's budget in stage 1; $i = 1, \ldots, I$.

Stochastic Disaster Relief Game Theory Network Model

- In the first stage, the humanitarian organizations seek to determine their relief item volumes q^1.

- In the second stage, after the disaster scenario is revealed, each HO i determines the second stage purchase and direct shipment levels, the elements of its vector of strategies q^{i2}, and the shipments from the hubs to the demand points, the q^{i3}s.

- the humanitarian organizations need to determine their $q^{i1}_{hj,l}$s, $q^{i2,\omega}_{hk,l}$s, and $q^{i3,\omega}_{jk,l}$s in order to maximize the expected utilities, in equilibrium, across all scenarios.

The HOs' Optimization Problems

$$\text{Maximize} \quad E(U^i(q)) = -\sum_{h=1}^{H} \rho_h \sum_{j=1}^{J} \sum_{l=1}^{L} q_{hj,l}^{i1} - \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} c_{hj,l}^{i1}(q^1) - \sum_{j=1}^{J} \pi_j \sum_{h=1}^{H} \sum_{l=1}^{L} q_{hj,l}^{i1}$$

$$+E_{\Omega}\left[Q_i(q^1, q^2, q^3, \omega)\right] \tag{1}$$

subject to:

$$\sum_{h=1}^{H} \rho_h \sum_{j=1}^{J} \sum_{l=1}^{L} q_{hj,l}^{i1} + \sum_{h=1}^{H} \sum_{j=1}^{J} \sum_{l=1}^{L} c_{hj,l}^{i1}(q^1) + \sum_{j=1}^{J} \pi_j \sum_{h=1}^{H} \sum_{l=1}^{L} q_{hj,l}^{i1} \leq B_i, \tag{2}$$

$$q_{hj,l}^{i1} \geq 0, \quad h = 1, \ldots, H; j = 1, \ldots, J; l = 1, \ldots, L. \tag{3}$$

$Q_i(q^1, q^2, q^3, \omega)$ is the expected value of HO i's utility in Stage 2, over all scenarios. $Q_i(q^1, q^2, q^3, \omega)$ is the optimal value of the following problem.

Stochastic Disaster Relief Game Theory Network Model

The HOs' Optimization Problems

$q^{i2,\omega}_{hk,l}$: the amount of the disaster relief items purchased by HO i and carried by FSP l directly from PL h to demand point k when the disaster scenario ω happens.

$q^{i3,\omega}_{jk,l}$: the amount of the disaster relief items purchased by HO i and carried by FSP l from Hub j to demand point k when the disaster scenario ω occurs.

$c^{i2,\omega}_{hk,l}(q^{2,\omega})$: the transportation cost that HO i pays to have its relief items delivered to demand point k by freight service provider l from purchasing location h when the disaster scenario ω occurs.

$c^{i3,\omega}_{jk,l}(q^{3,\omega})$: the transportation cost encumbered by HO i to have his relief items delivered to demand point k by freight service provider l from hub j when the disaster scenario ω occurs.

$P^{\omega}_{ik}(q^{\omega})$: the donations received by HO i; $i = 1, \ldots, I$ due to visibility at location k; $k = 1, \ldots, K$, when disaster scenario $\omega \in \Omega$ strikes, with $E(P_i(q)) = \sum_{\omega \in \Omega} p_\omega \sum_{k=1}^{K} P^{\omega}_{ik}(q^{\omega})$.

The HOs' Optimization Problems

$\underline{d}_k$: base level of the lower bound on demand at demand point k; $k = 1, \ldots, K$.

$\overline{d}_k$: base level of the upper bound on demand at demand point k; $k = 1, \ldots, K$.

$\rho_{h,\omega}$: the price of disaster relief item at purchasing location h when the disaster scenario ω occurs; $h = 1, \ldots, H; \omega \in \Omega$.

$\underline{\gamma}_\omega$: coefficient reflecting the effect of disaster scenario ω on the lower bound demands; $\forall \omega \in \Omega$.

$\overline{\gamma}_\omega$: coefficient reflecting the effect of disaster scenario ω on the upper bound demands; $\forall \omega \in \Omega$.

p_ω: the probability of scenario ω; $\forall \omega \in \Omega$.

Stochastic Disaster Relief Game Theory Network Model

$$\text{Maximize} \quad -\sum_{h=1}^{H}\rho_{h,\omega}\sum_{k=1}^{K}\sum_{l=1}^{L}q_{hk,l}^{i2,\omega}-\sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}c_{hk,l}^{i2,\omega}(q^{2,\omega})-\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}c_{jk,l}^{i3,\omega}(q^{3,\omega})$$

$$+\beta_i(\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega}+\sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}q_{hk,l}^{i2,\omega})+\sum_{k=1}^{K}P_{ik}^{\omega}(q^{\omega}) \tag{4}$$

subject to:

$$\sum_{k=1}^{K}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega}\leq\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hj,l}^{i1}, \quad j=1,\ldots,J;\, h=1,\ldots,H;\, l=1,\ldots,L, \tag{5}$$

$$q_{jk,l}^{i3,\omega}\geq 0, \quad j=1,\ldots,J;\, k=1,\ldots,K;\, l=1,\ldots,L, \tag{6}$$

$$q_{hk,l}^{i2,\omega}\geq 0, \quad h=1,\ldots,H;\, k=1,\ldots,K;\, l=1,\ldots,L, \tag{7}$$

$$\underline{\gamma}_{\omega}\underline{d}_k\leq\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega}+\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hk,l}^{i2,\omega}, \quad k=1,\ldots,K;\, \forall\omega\in\Omega, \tag{8}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}q_{jk,l}^{i3,\omega}+\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L}q_{hk,l}^{i2,\omega}\leq\overline{\gamma}_{\omega}\overline{d}_k, \quad k=1,\ldots,K;\, \forall\omega\in\Omega. \tag{9}$$

Stochastic Disaster Relief Game Theory Network Model

Based on standard stochastic programming theory, we can reformulate HO i's two-stage optimization problem as the following maximization problem:

The HOs' Optimization Problems

$$\text{Maximize} \quad E(U^i(q)) = -\sum_{h=1}^{H}\rho_h\sum_{j=1}^{J}\sum_{l=1}^{L}q^{i1}_{hj,l} - \sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}c^{i1}_{hj,l}(q^1) - \sum_{j=1}^{J}\pi_j\sum_{h=1}^{H}\sum_{l=1}^{L}q^{i1}_{hj,l}$$

$$+\sum_{\omega\in\Omega}p_\omega\Bigg[-\sum_{h=1}^{H}\rho_{h,\omega}\sum_{k=1}^{K}\sum_{l=1}^{L}q^{i2,\omega}_{hk,l} - \sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}c^{i2,\omega}_{hk,l}(q^{2,\omega}) - \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}c^{i3,\omega}_{jk,l}(q^{3,\omega})$$

$$+\beta_i(\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}q^{i3,\omega}_{jk,l} + \sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}q^{i2,\omega}_{hk,l}) + \sum_{k=1}^{K}P^{\omega}_{ik}(q^\omega)\Bigg] \tag{10}$$

subject to:

$$\sum_{k=1}^{K}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega} \leq \sum_{h=1}^{H}\sum_{l=1}^{L} q_{hj,l}^{i1}, \quad j=1,\ldots,J; h=1,\ldots,H; l=1,\ldots,L; \forall\omega\in\Omega, \tag{11}$$

$$q_{jk,l}^{i3,\omega} \geq 0, \quad j=1,\ldots,J; k=1,\ldots,K; l=1,\ldots,L; \forall\omega\in\Omega, \tag{12}$$

$$q_{hk,l}^{i2,\omega} \geq 0, \quad h=1,\ldots,H; k=1,\ldots,K; l=1,\ldots,L; \forall\omega\in\Omega, \tag{13}$$

$$\sum_{h=1}^{H}\rho_h\sum_{j=1}^{J}\sum_{l=1}^{L} q_{hj,l}^{i1} + \sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L} c_{hj,l}^{i1}(q^1) + \sum_{j=1}^{J}\pi_j\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hj,l}^{i1} \leq B_i, \tag{14}$$

$$q_{hj,l}^{i1} \geq 0, \quad h=1,\ldots,H; j=1,\ldots,J; l=1,\ldots,L, \tag{15}$$

$$\underline{\gamma}_\omega \underline{d}_k \leq \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega} + \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega}, \quad k=1,\ldots,K; \forall\omega\in\Omega, \tag{16}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} q_{jk,l}^{i3,\omega} + \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L} q_{hk,l}^{i2,\omega} \leq \overline{\gamma}_\omega \overline{d}_k, \quad k=1,\ldots,K; \forall\omega\in\Omega. \tag{17}$$

Feasible set $\mathcal{K}_i$

We define the feasible set $\mathcal{K}_i$ corresponding to HO i as:

$$\mathcal{K}_i \equiv \{q^i \,\text{such that}\, (11) - (15) \,\text{hold}\}$$

and we let $\mathcal{K}^1 \equiv \prod_{i=1}^{I} \mathcal{K}_i$.

Feasible set $\mathcal{S}$

we define the feasible set of common constraints $\mathcal{S}$ as

$$\mathcal{S} \equiv \{q | (16) \,\text{and}\, (17) \,\text{hold}\}.$$

The feasible set

$$\mathcal{K}^2 \equiv \mathcal{K}^1 \cap \mathcal{S}.$$

Stochastic Disaster Relief Game Theory Network Model

Definition 1: Stochastic Generalized Nash Equilibrium for the Humanitarian Organizations

A relief item flow vector $q^* \in \mathcal{K}^2$ *is a Stochastic Generalized Nash Equilibrium if for each HO* i*;* $i = 1, \ldots, I$*:*

$$E(U^i(q^{i*}, \hat{q^{i*}})) \geq E(U^i(q^i, \hat{q^{i*}})), \quad \forall q^i \in K_i \cap \mathcal{S}, \tag{18}$$

where $\hat{q^{i*}} \equiv (q^{1*}, \ldots, q^{i-1*}, q^{i+1*}, \ldots, q^{I*})$.

- Not one of the HOs is willing to deviate from his current relief item flow pattern, given the relief flow item patterns of the other HOs.
- Each HO's utility depends not only on his own strategy but also on that of the others' strategies, and so do their feasible sets, since their feasible sets are linked because of the shared constraints. The latter condition makes the problem a **Generalized Nash Equilibrium** model.
- We know that the feasible sets $\mathcal{K}_i$ are convex for each i, as is the set $\mathcal{S}$, under the imposed assumptions.

Definition 2: Variational Equilibrium

A relief item flow vector q^* is a Variational Equilibrium of the above Stochastic Generalized Nash Equilibrium problem if $q^* \in \mathcal{K}^2$ is a solution to the following variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{q^i} E(U^i(q^*)), q^i - q^{i*} \rangle \geq 0, \quad \forall q \in \mathcal{K}^2, \tag{19}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space, where $\mathcal{N}$ here is equal to $IHJL + |\Omega|(IHKL + IJKL)$ and ∇ is the gradient.

Stochastic Disaster Relief Game Theory Network Model

Expanding variational inequality (19), we obtain: determine $q^* \in \mathcal{K}^2$ such that

$$\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[\rho_h+\sum_{r=1}^{H}\sum_{s=1}^{J}\sum_{t=1}^{L}\frac{\partial c^{i1}_{rs,t}(q^{1*})}{\partial q^{i1}_{hj,l}}+\pi_j\right]\times\left[q^{i1}_{hj,l}-q^{i1*}_{hj,l}\right]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\rho_{h,\omega}+\sum_{r=1}^{H}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c^{i2,\omega}_{ru,t}(q^{2,\omega*})}{\partial q^{i2,\omega}_{hk,l}}-\beta_i-\sum_{o=1}^{K}\frac{\partial P^{\omega}_{io}(q^{\omega*})}{\partial q^{i2,\omega}_{hk,l}}\right]\times\left[q^{i2,\omega}_{hk,l}-q^{i2,\omega*}_{hk,l}\right]$$

$$+\sum_{\omega\in\Omega}p_\omega\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\sum_{s=1}^{J}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c^{i3,\omega}_{su,t}(q^{3,\omega*})}{\partial q^{i3,\omega}_{jk,l}}-\beta_i-\sum_{o=1}^{K}\frac{\partial P^{\omega}_{io}(q^{\omega*})}{\partial q^{i3,\omega}_{jk,l}}\right]\times\left[q^{i3,\omega}_{jk,l}-q^{i3,\omega*}_{jk,l}\right]\geq 0,$$

$$\forall q\in\mathcal{K}^2. \tag{20}$$

Stochastic Disaster Relief Game Theory Network Model

We now put variational inequality (19) into standard form: determine a vector $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{21}$$

where F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$, $\mathcal{K}$ is a given closed, convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathcal{N}$-dimensional Euclidean space.

Clearly, VI (19) can be put into the above standard form. Specifically, we can define $X \equiv q$, $\mathcal{K} \equiv \mathcal{K}^2$, and $F(X) \equiv (F^1(X), F^2(X), F^3(X))$. The components of $F^1(X)$ correspond to the *IHJL* elements with a typical *ihjl* element as preceding the first multiplication sign in (20); the components of $F^2(X)$ correspond to the $|\Omega|$ *IHKL* elements with a typical such element as immediately preceding the second multiplication sign, and so on.

Existence of a solution to variational inequality (19) is guaranteed from the classical theory of variational inequalities (cf. Kinderlehrer and Stampacchia (1980)) since the feasible set $\mathcal{K}^2$ is compact and the function that enters the VI, $F(X)$, is continuous, under our imposed assumptions. Compactness follows because of the budget constraints and the lower and upper bounds on the demands at the demand points under all the scenarios.

- **Illustrative Example 1: One scenario and one HO**

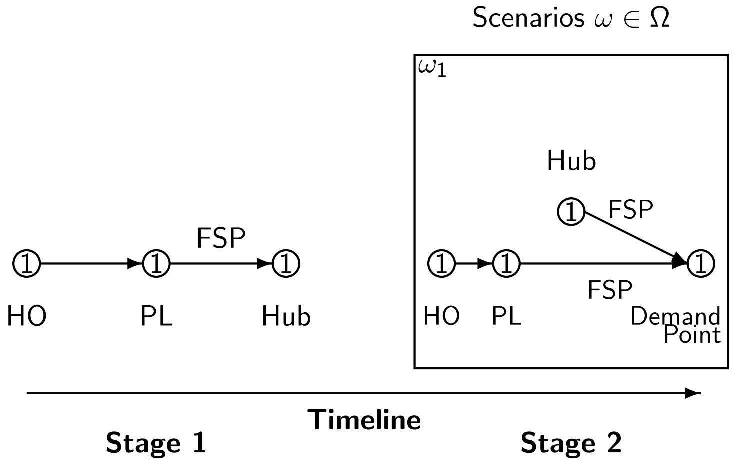


Figure: The Timeline of Illustrative Example 1

Illustrative Example 1

The purchasing prices of the relief items: $\rho_1 = 47, \rho_{1,1} = 100$.

The unit storage cost: $\pi_1 = 2$.

There is one scenario $\omega = \omega_1 = 1$ with probability $p_{\omega_1} = 1$.

The transportation costs:

$$c^{11}_{11,1}(q^1) = q^{11}_{11,1}, \quad c^{12,1}_{11,1}(q^{2,1}) = 10q^{12,1}_{11,1}, \quad c^{13,1}_{11,1}(q^{3,1}) = 5q^{13,1}_{11,1},$$

The budget for HO 1: $B_1 = 10,000$.

The altruism weight: $\beta_1 = 50$.

The lower and upper bounds on the demand point: $\underline{\gamma}_\omega \underline{d}_1 = 100, \quad \overline{\gamma}_\omega \overline{d}_1 = 300$.

The financial donation functions:

$$P^1_{11}(q^1) = 100\sqrt{2(q^{12,1}_{11,1} + q^{13,1}_{11,1})}$$

Illustrative Example 1

Results:

$$q_{11,1}^{11*} = 200.00, \quad q_{11,1}^{12,1*} = 0.00, \quad q_{11,1}^{13,1*} = 200.00.$$

We show that VI (20) holds. The solution lies in the feasible set $\mathcal{K}^2$. constructing the associated functions in (20) using the data for the example, we obtain:

$$[47 + 1 + 2] \times \left[q_{11,1}^{11} - 200\right] + [100 + 10 - 50 - 5] \times \left[q_{11,1}^{12,1} - 0\right]$$

$$+ [5 - 50 - 5] \times \left[q_{11,1}^{13,1} - 200\right]$$

$$= [50] \times \left[q_{11,1}^{11} - 200\right] + [55] \times \left[q_{11,1}^{12,1} - 0\right] + [-50] \times \left[q_{11,1}^{13,1} - 200\right]$$

$$= 50 \times q_{11,1}^{11} - 50 \times q_{11,1}^{13,1} + 55 q_{11,1}^{12,1} \geq 0,$$

The computed $P_{11}^1(q^{1*}) = 2000.00$ with expected donations $E(P_1(q^*))$ also equal to $2,000$. The humanitarian organization experiences an expected utility under this solution of: $E(U^1(q^*)) = 1,000.00$.

- **Illustrative Example 2: One scenario and two HOs**

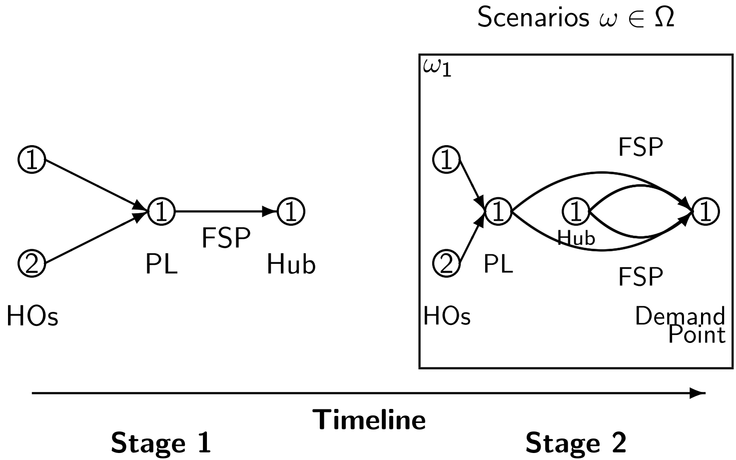


Figure: The Timeline of Illustrative Example 2

Illustrative Example 2

The purchasing prices of the relief items: $\rho_1 = 47, \rho_{1,1} = 100$.

The unit storage cost: $\pi_1 = 2$.

There is one scenario $\omega = \omega_1 = 1$ with probability $p_{\omega_1} = 1$.

The transportation costs:

$$c^{11}_{11,1}(q^1) = q^{11}_{11,1}, \quad c^{12,1}_{11,1}(q^{2,1}) = 10q^{12,1}_{11,1}, \quad c^{13,1}_{11,1}(q^{3,1}) = 5q^{13,1}_{11,1},$$

$$c^{21}_{11,1}(q^1) = q^{21}_{11,1}, \quad c^{22,1}_{11,1}(q^{2,1}) = 10q^{22,1}_{11,1}, \quad c^{23,1}_{11,1}(q^{3,1}) = 5q^{23,1}_{11,1}.$$

The budgets for HO 1 and 2: $B_1 = B_2 = 10,000$.

The altruism weight: $\beta_1 = 50$.

The lower and upper bounds on the demand point: $\underline{\gamma}_\omega \underline{d}_1 = 100, \quad \overline{\gamma}_\omega \overline{d}_1 = 300$.

The financial donation functions:

$$P^1_{11}(q^1) = 50\sqrt{2(q^{12,1}_{11,1} + q^{13,1}_{11,1}) - (q^{22,1}_{11,1} + q^{23,1}_{11,1})},$$

$$P^1_{21}(q^1) = 50\sqrt{2(q^{22,1}_{11,1} + q^{23,1}_{11,1}) - (q^{12,1}_{11,1} + q^{13,1}_{11,1})}$$

Illustrative Example 2

Results:

$$q^{21*}_{11,1} = q^{11*}_{11,1} = 100.00, \quad q^{22,1*}_{11,1} = q^{12,1*}_{11,1} = 0.00, \quad q^{23,1*}_{11,1} = q^{13,1*}_{11,1} = 100.00.$$

The expected utilities: $E(U^2(q^*)) = E(U^1(q^*)) = 0.00$.
The victims of the disaster, nevertheless, still have 200 relief item kits delivered (as in Example 1).

The HOs' computed financial donations: $P^1_{11}(q^{1*}) = 500.00$, $P^1_{21}(q^{1*}) = 500.00$.

With double the number of humanitarian organizations now involved in disaster relief, as compared to that in Example 1, the expected donations of the original HO 1 now drop.

The total volume of donations is now lower (\$1,000) than in Example 1 (\$2,000). Observe that these declines are also, due, in part, to the fact that the coefficient in the donation functions in Example 2 is 50 for each humanitarian organization, whereas it was 100 for the single one (HO 1) in Example 1.

Illustrative Example 3

- **Illustrative Example 3: Two scenarios** $\omega = \omega_1 = 1$ **and** $\omega = \omega_2 = 2$ **and two HOs**

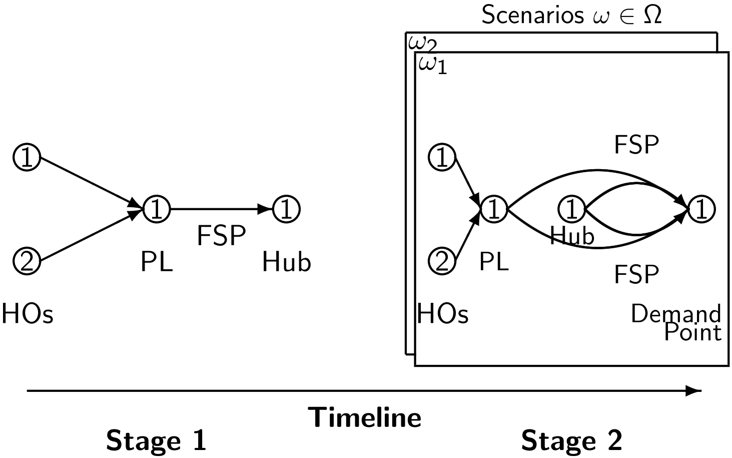


Figure: The Timeline of Illustrative Example 3

Illustrative Example 3

The purchasing prices of the relief items: $\rho_1 = 47, \rho_{1,1} = 100, \rho_{1,2} = 110$.

The unit storage cost: $\pi_1 = 2$.

The probabilities of two events, with the second event having a higher impact are:

$$p_1 = .4, \quad p_2 = .6.$$

The transportation costs:

$$c^{i1}_{11,1}(q^1) = q^{i1}_{11,1}, \quad i = 1, 2,$$

$$c^{i2,1}_{11,1}(q^{2,1}) = 10q^{i2,1}_{11,1}, \quad i = 1, 2, \qquad c^{i3,1}_{11,1}(q^{3,1}) = 5q^{i3,1}_{11,1}, \quad i = 1, 2,$$

$$c^{i2,2}_{11,1}(q^{2,2}) = 12q^{i2,2}_{11,1}, \quad i = 1, 2, \qquad c^{i3,2}_{11,1}(q^{3,2}) = 7q^{i3,2}_{11,1}, \quad i = 1, 2.$$

The budgets for HO 1 and 2: $B_1 = B_2 = 10,000$.

The altruism weight being: $\beta_1 = 50$.

The lower and upper bounds on the demand point:
$\underline{\gamma}_1 \underline{d}_1 = 100, \quad \overline{\gamma}_1 \overline{d}_1 = 300, \quad \underline{\gamma}_2 \underline{d}_1 = 200, \quad \overline{\gamma}_2 \overline{d}_1 = 500.$

Illustrative Example 3

The financial donation functions:

$$P^1_{11}(q^1) = 50\sqrt{2(q^{12,1}_{11,1} + q^{13,1}_{11,1}) - (q^{22,1}_{11,1} + q^{23,1}_{11,1})}, P^1_{21}(q^1) = 50\sqrt{2(q^{22,1}_{11,1} + q^{23,1}_{11,1}) - (q^{12,1}_{11,1} + q^{13,1}_{11,1})},$$

$$P^2_{11}(q^2) = 60\sqrt{2(q^{13,2}_{11,1} + q^{12,2}_{11,1}) - (q^{23,2}_{11,1} + q^{22,2}_{11,1})}, P^2_{21}(q^2) = 60\sqrt{2(q^{23,2}_{11,1} + q^{22,2}_{11,1}) - (q^{13,2}_{11,1} + q^{12,2}_{11,1})}.$$

Results:

$$q^{11*}_{11,1} = q^{21*}_{11,1} = 150.00,$$

$$q^{12,\omega*}_{11,1} = q^{22,\omega*}_{11,1} = 0.00, \quad \omega = 1, 2,$$

$$q^{13,\omega*}_{11,1} = q^{23,\omega*}_{11,1} = 150.00, \quad \omega = 1, 2.$$

$$P^1_{11}(q^{1*}) = 612.37, \quad P^1_{21}(q^{1*}) = 612.37,$$

$$P^2_{11}(q^{2*}) = 734.85, \quad P^2_{21}(q^{2*}) = 734.85,$$

$$E(P_1(q^*)) = 685.86, \quad E(P_2(q^*)) = 685.86.$$

Illustrative Example 3

In the case of two scenarios, with one being severe, the HOs in Example 3 stand to gain higher expected financial donations that in Example 2. However, their expected utilities are now negative: $E(U^1(q^*)) = E(U^2(q^*)) = -244.14$ since the humanitarian organizations are required to meet at least the lower bounds for the demand for relief items and now there is also the possibility of a more severe disaster scenario.

The Algorithm and Alternative Variational Inequality Formulation

The Modified Projection Method

Step 0: Initialization

Start with $X^0 \in \mathcal{K}$ (cf. (22)). Set $\tau := 1$ and select a, such that $0 < a \leq \frac{1}{L}$, where L is the Lipschitz continuity constant for $F(X)$.

Step 1: Construction and Computation

Compute $\overline{X}^{\tau-1}$ by solving the variational inequality subproblem:

$$\langle \overline{X}^{\tau-1} + (aF(X^{\tau-1}) - X^{\tau-1}), X - \overline{X}^{\tau-1} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{22}$$

Step 2: Adaptation

Compute X^{τ} by solving the variational inequality subproblem:

$$\langle X^{\tau} + (aF(\overline{X}^{\tau-1}) - X^{\tau-1}), X - X^{\tau} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{23}$$

Step 3: Convergence Verification

If $| X_l^{\tau} - X_l^{\tau-1} | \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else set $\tau := \tau + 1$, and go to step 1.

The Algorithm and Alternative Variational Inequality Formulation

We provide an alternative variational inequality formulation to that of (19), governing the Stochastic Generalized Nash Equilibrium, to which the algorithm, the modified projection method (cf. Korpelevich (1997) and Nagurney (1999)), is applied to compute solutions to numerical examples.

Necessary Lagrange multipliers:

η_i; $i = 1, \ldots, I$: the Lagrange multiplier associated with the budget constraint (14) for each HO i.

$\alpha_j^{i\omega}$; $i = 1, \ldots, I$; $j = 1, \ldots, J$; $\omega \in \Omega$: the Lagrange multiplier associated with HO i's Hub j inequality constraint (11) under scenario ω.

λ_k^ω: the Lagrange multiplier associated with the lower bound constraint (16) at demand point k and ω for $k = 1, \ldots, K$ and $\omega \in \Omega$.

μ_k^ω: the Lagrange multiplier associated with the upper bound constraint (17) at demand point k and scenario ω for $k = 1, \ldots, K$ and $\omega \in \Omega$.

We define the feasible set $\mathcal{K}^3$ as follows:

$$\mathcal{K}^3 \equiv \{(q, \eta, \alpha, \lambda, \mu) | q \in R_+^{IHJL+|\Omega|(IHKL+IJKL)}, \eta \in R_+^I, \alpha \in R_+^{|\Omega|IJ}, \text{and } \lambda \in R_+^{|\Omega|K}, \mu \in R_+^{|\Omega|K}\}.$$

The Algorithm and Alternative Variational Inequality Formulation

An alternative variational inequality to VI (19); equivalently, VI (20) is: determine $(q^*, \eta^*, \alpha^*, \lambda^*, \mu^*) \in \mathcal{K}^3$ such that

$$\sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}\left[(\rho_h + \sum_{r=1}^{H}\sum_{s=1}^{J}\sum_{t=1}^{L}\frac{\partial c^{i1}_{rs,t}(q^{1*})}{\partial q^{i1}_{hj,l}} + \pi_j)(1+\eta_i^*) - \sum_{\omega\in\Omega}\alpha_j^{i,\omega *}\right] \times \left[q^{i1}_{hj,l} - q^{i1*}_{hj,l}\right]$$

$$+\sum_{\omega\in\Omega} p_\omega \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\rho_{h,\omega} + \sum_{r=1}^{H}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c^{i2,\omega}_{ru,t}(q^{2,\omega *})}{\partial q^{i2,\omega}_{hk,l}} - \beta_i - \sum_{o=1}^{K}\frac{\partial P^{\omega}_{io}(q^{\omega *})}{\partial q^{i2,\omega}_{hk,l}} - \lambda_k^{\omega *} + \mu_k^{\omega *}\right]\times\left[q^{i2,\omega}_{hk,l} - q^{i2,\omega *}_{hk,l}\right]$$

$$+\sum_{\omega\in\Omega} p_\omega \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\left[\sum_{s=1}^{J}\sum_{u=1}^{K}\sum_{t=1}^{L}\frac{\partial c^{i3,\omega}_{su,t}(q^{3,\omega *})}{\partial q^{i3,\omega}_{jk,l}} - \beta_i - \sum_{o=1}^{K}\frac{\partial P^{\omega}_{io}(q^{\omega *})}{\partial q^{i3,\omega}_{jk,l}} + \alpha_j^{i,\omega *} - \lambda_k^{\omega *} + \mu_k^{\omega *}\right]\times\left[q^{i3,\omega}_{jk,l} - q^{i3,\omega *}_{jk,l}\right]$$

$$+\sum_{i=1}^{I}\left[B_i - \sum_{h=1}^{H}\rho_h\sum_{j=1}^{J}\sum_{l=1}^{L}q^{i1*}_{hj,l} - \sum_{h=1}^{H}\sum_{j=1}^{J}\sum_{l=1}^{L}c^{i1}_{hj,l}(q^{1*}) - \sum_{j=1}^{J}\pi_j\sum_{h=1}^{H}\sum_{l=1}^{L}q^{i1*}_{hj,l}\right]\times\left[\eta_i - \eta_i^*\right]$$

$$+\sum_{\omega\in\Omega}\sum_{i=1}^{I}\sum_{j=1}^{J}\left[\sum_{h=1}^{H}\sum_{l=1}^{L}q^{i1*}_{hj,l} - \sum_{k=1}^{K}\sum_{l=1}^{L}q^{i3,\omega *}_{jk,l}\right]\times\left[\alpha_j^{i,\omega} - \alpha_j^{i,\omega *}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}q^{i3,\omega *}_{jk,l} + \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L}q^{i2,\omega *}_{hk,l} - \underline{\gamma}_\omega \underline{d}_k\right]\times\left[\lambda_k^{\omega} - \lambda_k^{\omega *}\right]$$

$$+\sum_{\omega\in\Omega}\sum_{k=1}^{K}\left[\overline{\gamma}_\omega \overline{d}_k - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}q^{i3,\omega *}_{jk,l} - \sum_{i=1}^{I}\sum_{h=1}^{H}\sum_{l=1}^{L}q^{i2,\omega *}_{hk,l}\right]\times\left[\mu_k^{\omega} - \mu_k^{\omega *}\right] \geq 0,$$

$$\forall (q, \eta, \alpha, \lambda, \mu) \in \mathcal{K}^3. \quad (25)$$

Example 4

- This example has the same data as in Illustrative Example 3 but with modified transportation cost functions.
- The cost functions are nonlinear (rather than linear as in Example 3). This enables the better modeling of costs and time delays associated with congestion that can occur prior and post the disaster, but for different reasons.
- Modified transportation cost functions:

$$c^{i1}_{11,1}(q^1) = \frac{1}{2}(q^{i1}_{11,1})^2 + q^{i1}_{11,1}, \quad i = 1, 2,$$

$$c^{i2,1}_{11,1}(q^{2,1}) = \frac{1}{2}(q^{i2,1}_{11,1})^2 + 10q^{i2,1}_{11,1}, \quad i = 1, 2,$$

$$c^{i3,1}_{11,1}(q^{3,1}) = \frac{1}{2}(q^{i3,1}_{11,1})^2 + 5q^{i3,1}_{11,1}, \quad i = 1, 2,$$

$$c^{i2,2}_{11,1}(q^{2,2}) = \frac{1}{2}(q^{i2,2}_{11,1})^2 + 12q^{i2,2}_{11,1}, \quad i = 1, 2,$$

$$c^{i3,2}_{11,1}(q^{3,2}) = \frac{1}{2}(q^{i3,2}_{11,1})^2 + 7q^{i3,2}_{11,1}, \quad i = 1, 2.$$

Example 4

Results

- The computed product/shipment quantities:

$$q_{11,1}^{11*} = q_{11,1}^{21*} = 55.00,$$

$$q_{11,1}^{12,\omega*} = q_{11,1}^{22,\omega*} = 0.00, \quad \omega = 1,$$

$$q_{11,1}^{12,\omega*} = q_{11,1}^{22,\omega*} = 45.00, \quad \omega = 2,$$

$$q_{11,1}^{13,\omega*} = q_{11,1}^{23,\omega*} = 52.00, \quad \omega = 1,$$

$$q_{11,1}^{13,\omega*} = q_{11,1}^{23,\omega*} = 55.00, \quad \omega = 2.$$

- The Lagrange multipliers:

$$\eta_1^* = \eta_2^* = 0.00,$$

$$\lambda_1^{1*} = 0.00, \quad \lambda_1^{2*} = 111.00,$$

$$\mu_1^{1*} = 0.00, \quad \mu_1^{2*} = 0.00,$$

$$\alpha_1^{(1,1)*} = \alpha_1^{(2,1)*} = 0.00, \quad \alpha_1^{(1,2)*} = \alpha_1^{(2,2)*} = 105.00.$$

Results

- With the increase in transportation costs, we see a significant change in the humanitarian organizations' strategies as compared to Example 3.
- The humanitarian organizations change their strategies both quantitatively and qualitatively.
- Each HO now prepositions only 55 relief items in the Hub in Stage 1.
- In Stage 2, in response to the first scenario, no additional relief items are purchased and 52 items are shipped by each HO to the victims.
- In the case of the second scenario, which is associated with more severe damage, in Stage 2, each humanitarian organization purchases 45 additional relief items and these are transported to the victims along with each HO's 55 items that have been stored.
- In the case of scenario ω_2, the lower bound on the demand of 200 is precisely met, and, hence, the associated Lagrange multiplier λ_1^{2*} is positive.
- The equilibrium conditions hold with excellent accuracy.

Example 4

Results

- Financial Donations:

$$P_{11}^{1}(q^{1*}) = 360.56, \quad P_{21}^{1}(q^{1*}) = 360.56,$$

$$P_{11}^{2}(q^{2*}) = 600.00, \quad P_{21}^{2}(q^{2*}) = 600.00.$$

- Expected donations for HO 1 and HO 2:

$$E(P_1(q^*)) = 504.22, \quad E(P_2(q^*)) = 504.22.$$

- Faced with higher logistical costs, each HO, under each scenario, delivers fewer relief items that in Example 3.
- With fewer items delivered, donors respond accordingly, and the expected donations are significantly lower than in Example 3.

Example 5

- **Example 5: Two HOs, one FSP, one PL, one Hub, two Demand Points, and two Scenarios.**

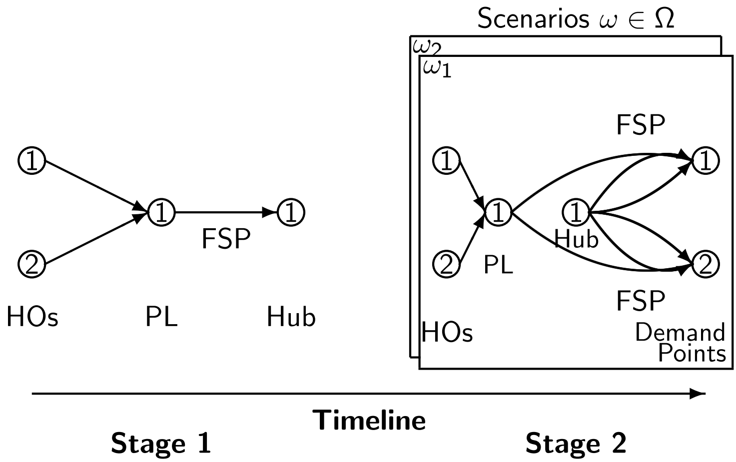


Figure: The Timeline of Example 5

Example 5

- Example 5 was constructed from Example 4 but now we added a new point of demand for the relief items.
- The data associated with the first demand point remained as in Example 4.
- We increased the budget of each HO from 10,000 to 20,000.
- The lower and upper bounds at Demand Point 2 under each scenario are the same as at Demand Point 1 in Example 4.
- The financial donation functions associated with the second demand point:

$$P^1_{12}(q^1) = 50\sqrt{2(q^{13,1}_{12,1} + q^{12,1}_{12,1}) - (q^{23,1}_{12,1} + q^{22,1}_{12,1})},$$

$$P^1_{22}(q^1) = 50\sqrt{2(q^{23,1}_{12,1} + q^{22,1}_{12,1}) - (q^{13,1}_{12,1} + q^{12,1}_{12,1})},$$

$$P^2_{12}(q^2) = 60\sqrt{2(q^{13,2}_{12,1} + q^{12,2}_{12,1}) - (q^{23,2}_{12,1} + q^{22,2}_{12,1})},$$

$$P^2_{22}(q^2) = 60\sqrt{2(q^{23,2}_{12,1} + q^{22,2}_{12,1}) - (q^{13,2}_{12,1} + q^{12,2}_{12,1})}.$$

- The additional cost functions associated with Demand Point 2:

$$c^{i2,1}_{12,1}(q^{2,1}) = 8q^{i2,1}_{11,1}, \quad i = 1, 2,$$

$$c^{i3,1}_{12,1}(q^{3,1}) = 4q^{i3,1}_{11,1}, \quad i = 1, 2,$$

$$c^{i2,2}_{12,1}(q^{2,2}) = 9q^{i2,2}_{11,1}, \quad i = 1, 2,$$

Example 5

Results

- The computed product/shipment quantities:

$$q^{11*}_{11,1} = q^{21*}_{11,1} = 100.00,$$

$$q^{12,\omega*}_{1k,1} = q^{22,\omega*}_{1k,1} = 0.00, \quad \omega = 1; k = 1,2,$$

$$q^{12,\omega*}_{11,1} = q^{22,\omega*}_{11,1} = 49.75, \quad \omega = 2, \qquad q^{12,\omega*}_{12,1} = q^{22,\omega*}_{12,1} = 50.25, \quad \omega = 2,$$

$$q^{13,\omega*}_{1k,1} = q^{23,\omega*}_{1k,1} = 50.00, \quad \omega = 1; k = 1,2,$$

$$q^{13,\omega*}_{11,1} = q^{23,\omega*}_{11,1} = 50.25, \quad \omega = 2, \qquad q^{13,\omega*}_{12,1} = q^{23,\omega*}_{12,1} = 49.75. \quad \omega = 2.$$

- The Lagrange multipliers:

$$\eta^*_1 = \eta^*_2 = 0.00,$$

$$\lambda^{1*}_1 = 33.43, \quad \lambda^{2*}_1 = 115.75, \quad \lambda^{1*}_2 = 32.43, \quad \lambda^{2*}_2 = 113.25,$$

$$\mu^{1*}_1 = \mu^{2*}_1 = \mu^{1*}_2 = \mu^{2*}_2 = 0.00,$$

$$\alpha^{(1,1)*}_1 = \alpha^{(2,1)*}_1 = 35.50, \quad \alpha^{(1,2)*}_1 = \alpha^{(2,2)*}_1 = 114.50.$$

Example 5

Results

- In contrast to Example 4, the humanitarian organizations need to plan and be prepared to respond post the disaster to two demand points.
- In Stage 1, each of the humanitarian organizations stores 100 disaster relief items at the Hub to be able to respond to the needs of the disaster victims in Stage 2.
- In scenario ω_1, the lower bound on the demand, 100, for both demand points is met by using only the items in the hub and there is no need to purchase extra items post the disaster.
- Under scenario ω_2, each humanitarian organization purchases 100 additional relief items post the disaster and has them transported directly to the demand points along with 100 items from the hub to satisfy the lower bound on the demand of 200, at both demand points.
- All the lower bounds hold tightly under both scenarios, and, therefore, the associated Lagrange multipliers are positive.

Example 5

Results

- Financial Donations:

$$P^1_{11}(q^{1*}) = 353.55, \quad P^1_{21}(q^{1*}) = 353.55,$$

$$P^2_{11}(q^{2*}) = 600.00, \quad P^2_{21}(q^{2*}) = 600.00,$$

$$P^1_{12}(q^{1*}) = 353.55, \quad P^1_{22}(q^{1*}) = 353.55,$$

$$P^2_{12}(q^{2*}) = 600.00, \quad P^2_{22}(q^{2*}) = 600.00.$$

HO 1, ω_1: $P^1_{11}(q^{1*}) + P^1_{12}(q^{1*}) = 707.10,$

HO 2, ω_1: $P^1_{21}(q^{1*}) + P^1_{22}(q^{1*}) = 707.10,$

HO 1, ω_2: $P^2_{11}(q^{2*}) + P^2_{12}(q^{2*}) = 1200.00,$

HO 2, ω_2: $P^2_{21}(q^{2*}) + P^2_{22}(q^{2*}) = 1200.00.$

- Expected donations for HO 1 and HO 2:

$$E(P_1(q^*)) = 1002.84, \quad E(P_2(q^*)) = 1002.84.$$

- With two demand points, each HO can expect to receive financial donations almost double of the amount in Example 4.

Summary and Conclusions

- We constructed, for the first time, a Stochastic Generalized Nash Equilibrium model for disaster relief consisting of multiple humanitarian organizations, multiple purchase locations for the disaster relief items, multiple hubs for storage, and multiple freight service provision options, ultimately, to multiple points of demand.
- Each humanitarian organization solves a two-stage stochastic optimization problem.
- In the first stage, each HO seeks to determine the optimal purchase quantities for storage at multiple hubs, subject to a budget constraint, and, in the second stage, which handles multiple disaster scenarios with associated probabilities of occurrence, each HO must determine how much to deliver from the hubs to multiple points of demand, and how many additional relief items to purchase, if need be, for delivery.
- The humanitarian organizations compete for financial donations and consider multiple costs in their objective functions, along with a weighted altruism component, since they are nonprofits.

Summary and Conclusions

- In the second stage, the humanitarian organizations are subject to both lower and upper bounds on the demand for the relief items at the demand points.
- The model is formulated as a finite-dimensional variational inequality problem, utilizing the concept of a Variational Equilibrium and existence established.
- An alternative variational inequality formulation is given, which includes Lagrange multipliers associated with the various constraints and enables elegant computations in that, at each iteration of the proposed algorithm scheme, closed form expressions for the product purchase/storage/shipment variables, and the Lagrange multipliers are obtained and presented.
- Both illustrative examples are presented as well as examples that are computed using the proposed algorithm.

Summary and Conclusions

- The results in this paper open up new directions for research in disaster relief, through the synthesis of stochastic elements with game theory, in a unified theoretical and computational framework, along with policy interventions.
- This paper serves as a "proof of concept" and we can expect additional followup theoretical, computational, as well as empirical work.
- This study has taken a major step in investigating the competition among humanitarian organizations in pre- and post-disaster operations by considering uncertainty as a major factor.
- This problem can be extended to a multi-stage problem where we model the different stages of information emergence and detailed decisions to be made. Similarly, multi-disasters, such as earthquakes and tsunamis, can be examined in different stages, along with appropriate responses.
- Competition among others, such as freight service providers, can also be considered in future research. This would result in multistage, multitiered stochastic game theory constructs.

Thank You!

For more information: https://supernet.isenberg.umass.edu/