A Knowledge Collaboration Network Model Across Disciplines

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Outline

- Introduction
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Introduction I

- According to Beckmann's (1993, 1994, 1995) models, which assumed face-to-face communication, the likelihood of two researchers collaborating decreases as the physical distance between them increases.
- Gibbons et al. (1994) and Hudson (1996) emphasized that the emergence of critical technologies greatly impacts researchers' output.
- Rosenblat and Mobius (2004), in turn, found that the number of co-authored papers in economics alone has increased thirty percent since the rise of the Internet.
- We also believe researchers will have to collaborate interdisciplinarily to solve problems more effectively.

Introduction II

- Ziman (1994) pointed out that "greater interdisciplinarity" and "more networking and collaboration" have been listed among the top characteristics that describe modern research.
- Thagard (1997) further pointed out that, given today's highly specialized disciplines, it is very difficult for a single researcher or a group of researchers in a single field to tackle a problem that requires cross-disciplinary knowledge.
- Wagner et al. (2002) analyzed international collaboration in science and technology. According to them, "The increasing ubiquity of information and communication technologies means that scientists can share information in real time. This had led to the growth of distributed research in a range of activities, some of which are called 'co-laboratories' or 'virtual laboratories'..."

Introduction III

- Börner, Maru, and Goldstone (2004) have indicated, there are now a great number of bibliometric studies that address the trends of co-authorship and interdisciplinary research in the literature of collaboration networks.

Our Relevant Work

- **Network Economics: A Variational Inequality Approach, Advances in Computational Economics**, vol. 1, Kluwer Academic Publishers, Boston, Massachusetts, 1993; second revised edition, vol. 10, Dordrecht, The Netherlands, 1999.
- Management of knowledge intensive systems as supernetworks: Modeling, analysis computations, and applications (A. Nagurney and J. Dong) (2005), *Mathematical and Computer Modelling, 42, 397-417.*
- Supernetworks: An introduction to the concept and its applications with a specific focus on knowledge supernetworks (A. Nagurney and T. Wakolbinger) (2005), *International Journal of Knowledge, Culture and Change Management, 4, 1523-1530.*

Contributions of This Research

We propose a theoretical framework for the modeling and analysis of knowledge collaboration networks that generalizes existing frameworks noted above in several ways:

- Research collaborators can be in the same or in different disciplines.
- The collaboration time is determined by the total distance which captures distance between disciplines as well as the communication distance.
- Instead of working with one another exclusively in a face-to-face mode, researchers can now also collaborate via different modes.

Assumptions and Definitions

- Researchers only collaborate in pairs.
- Two factors affecting collaboration time (both measured in time scale):
  1. Communication distance
  2. Virtual distance

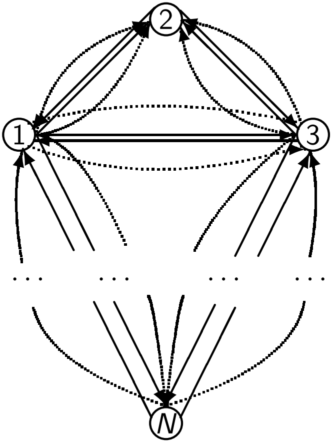

Figure: Structure of the Knowledge Collaboration Network

Knowledge Collaboration Network Model

$$\text{Maximize} \quad \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{O} u_{ijk}(x_{ijk}, x_{jik}) \tag{1}$$

subject to:

$$\sum_{j=1}^{N}\sum_{k=1}^{O} t_{ijk}x_{ijk} \leq T_i, \qquad i = 1, \ldots, N, \tag{2}$$

$$x \in R_{+}^{N\times N\times O}. \tag{3}$$

Optimality Conditions

$$\frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} + \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}} \begin{cases} = t_{ijk}\lambda^*_i, & \text{if} \quad x^*_{ijk} > 0, \\ \leq t_{ijk}\lambda^*_i, & \text{if} \quad x^*_{ijk} = 0, \end{cases} \tag{4}$$

$$T_i - \sum_{j=1}^{N}\sum_{k=1}^{O} t_{ijk}x^*_{ijk} \begin{cases} = 0, & \text{if} \quad \lambda^*_i > 0, \\ \geq 0, & \text{if} \quad \lambda^*_i = 0. \end{cases} \tag{5}$$

Theorem 1: Optimality Opportunity Costs

If in the knowledge collaboration network model presented in (1) – (3), the utility functions u_{ijk}, $\forall i, j, k$, are continuously differentiable and strictly monotonically increasing, then the optimal opportunity costs λ_i^*, $\forall i$, are positive.

Theorem 2: Varational Inequality Formulation

A solution to the knowledge network collaboration model is an optimal solution if and only if it satisfies the variational inequality problem: determine $(\lambda^*, x^*) \in \mathcal{K}$, where $\mathcal{K} \equiv \{(\lambda, x) \mid (\lambda, x) \in R_+^{N+N\times N\times O}\}$, such that

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{O}(t_{ijk}\lambda_i^* - \frac{\partial u_{ijk}(x_{ijk}^*, x_{jik}^*)}{\partial x_{ijk}} - \frac{\partial u_{jik}(x_{jik}^*, x_{ijk}^*)}{\partial x_{ijk}}) \times (x_{ijk} - x_{ijk}^*)$$

$$+\sum_{i=1}^{N}(T_i - \sum_{j=1}^{N}\sum_{k=1}^{O} t_{ijk}x_{ijk}^*) \times (\lambda_i - \lambda_i^*) \geq 0, \quad \forall (\lambda, x) \in \mathcal{K}. \qquad (6)$$

Assumptions of Symmetric Utility Function

every researcher has a utility function characterized by the property that a pair of collaborators shares the utility (which may also be interpreted as the research credit associated with co-authorship) evenly, that is, $u_{ijk}(x_{ijk}, x_{jik})$ is equal to $u_{jik}(x_{jik}, x_{ijk})$.

The following conditions then can be obtained:

$$\text{if } x^*_{ijk} > 0, \text{ then } \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} = \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}} = \frac{1}{2} t_{ijk} \lambda^*_i; \quad (7)$$

$$\text{if } x^*_{jik} > 0, \text{ then } \frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{jik}} = \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{jik}} = \frac{1}{2} t_{jik} \lambda^*_j. \quad (8)$$

Collaboration Using the Same Communication Mode

Case i: researcher i works independently;

Case ii: researcher i works with j who shares the same discipline with i;

Case iii: researcher i works with j who is not in the same discipline with i.

We denote the t_{ijk}s in the above cases as: t^1_{iik}, t^2_{ijk}, and t^3_{ijk}, respectively. According to the definition of t_{ijk}, we have that $t^3_{ijk} > t^2_{ijk} > t^1_{iik} = 1$. We also let $u^1_{iik}(x_{iik}, x_{iik})$, $u^2_{ijk}(x_{ijk}, x_{jik})$, and $u^3_{ijk}(x_{ijk}, x_{jik})$ denote researcher i's utility functions corresponding, respectively, to the above three cases. We conclude:

$$\frac{\partial u^3_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} > \frac{\partial u^2_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} > \frac{\partial u^1_{iik}(x^*_{iik}, x^*_{iik})}{\partial x_{iik}}. \tag{9}$$

and

$$\frac{\frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}}}{\frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{jik}}} = \frac{\lambda^*_i}{\lambda^*_j}. \tag{10}$$

Collaboration Across the Different Communication Modes

Assuming $t_{ijf} > t_{ijh}$,

Case iv: $x^*_{ijf} > 0$ and $x^*_{ijh} > 0$. We have that

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}} = \frac{t_{ijf}}{t_{ijh}}. \quad (11)$$

Case v: $x^*_{ijf} > 0$ and $x^*_{ijh} = 0$. We have that

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}} \geq \frac{t_{ijf}}{t_{ijh}}. \quad (12)$$

Case vi: $x^*_{ijf} = 0$ and $x^*_{ijh} > 0$. Similarly, we have that

$$\frac{\frac{\partial u_{ijf}(x^*_{ijf}, x^*_{jif})}{\partial x_{ijf}}}{\frac{\partial u_{ijh}(x^*_{ijh}, x^*_{jih})}{\partial x_{ijh}}} \leq \frac{t_{ijf}}{t_{ijh}}. \quad (13)$$

Qualitative Properties I

From the standard theory of variational inequality theory, optimization problem (1), subject to (2) and (3), under the assumption that the utility functions are concave and continuously differentiable, can also be formulated as the variational inequality problem: determine $x^* \in \mathcal{K}^2$, where $\mathcal{K}^2 \equiv \{x | x \in R_+^{N \times N \times O} \text{ and satisfies } (2), (3)\}$ such that

$$-\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{O}\left(\frac{\partial u_{ijk}(x^*_{ijk}, x^*_{jik})}{\partial x_{ijk}} + \frac{\partial u_{jik}(x^*_{jik}, x^*_{ijk})}{\partial x_{ijk}}\right) \times (x_{ijk} - x^*_{ijk}) \geq 0, \quad \forall x \in \mathcal{K}^2. \tag{14}$$

Theorem 3: Existence

Existence of a solution $x^* \in \mathcal{K}^2$ to variational inequality (14) is guaranteed.

Qualitative Properties II

Theorem 4: Uniqueness

Assume that the utility functions u_{ijk}, $\forall i, j, k$, are strictly concave functions. Then the optimal effort (time) allocation pattern satisfying variational inequality (14) is unique.

Computational Procedure - Modified Projection Method

Step 0: Initialization: Set $X^0 \in \mathcal{K}$. Let $\mathcal{T} = 1$ and set α so that $0 < \alpha \leq \frac{1}{L}$, where L is the Lipschitz continuity constant.
Step 1: Computation: Compute $\bar{X}^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (\bar{X}^{\mathcal{T}} + \alpha F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1})^T, X - \bar{X}^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (15)$$

Step 2: Adaptation: Compute $X^{\mathcal{T}}$ by solving the variational inequality subproblem:

$$\langle (X^{\mathcal{T}} + \alpha F(\bar{X}^{\mathcal{T}}) - X^{\mathcal{T}-1})^T, X - X^{\mathcal{T}} \rangle \geq 0, \quad \forall X \in \mathcal{K}. \quad (16)$$

Step 3: Convergence Verification: If $\max |X_l^{\mathcal{T}} - X_l^{\mathcal{T}-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Example 1

The utilities are directly proportional to the efforts spent and we consider Cobb-Douglas production-type functions of the form:
$u_{ijk}(x_{ijk}, x_{jik}) = b_{ijk}(x_{ijk})^{1/2}(x_{jik})^{1/2}, \quad \forall i, j, k$. We assume that $b_{ijk} = b_{jik}$, which is adopted from Beckmann's notation. Time budget is 100 for each researcher.

Collaborators i, j	**Mode** k	$\mathbf{r_{1ij}}$	$\mathbf{r_{2ijk}}$	$\mathbf{r_{ijk}}$	$\mathbf{a_{ijk}}$	$\mathbf{t_{ijk}}$	$\mathbf{b_{ijk}}$
1, 2	1	0	0	0	5	1	2
1, 3	1	3	0	3	5	16	5
1, 4	1	5	0	5	5	26	10
2, 3	1	3	0	3	5	16	5
2, 4	1	5	0	5	5	26	10
3, 4	1	5	0	5	5	26	9
1, 2	2	0	1	1	5	6	1
1, 3	2	3	1	4	5	21	4
1, 4	2	5	1	6	5	31	8
2, 3	2	3	1	4	5	21	6
2, 4	2	5	1	6	5	31	8
3, 4	2	5	1	6	5	31	12

Results from Example 1

The optimal efforts were given by: $x^*_{121} = x^*_{211} = 100.00$, $x^*_{331} = x^*_{332} = 50.00$, and $x^*_{441} = x^*_{442} = 50$, with all other $x_{ijk}s = 0.00$; the optimal opportunity costs are: $\lambda^*_1 = \lambda^*_2 = 2.00$ and $\lambda^*_3 = \lambda^*_4 = 1.00$.

Example 2

The data are the same except that the time budget for all researchers is increased to 120.
The optimal efforts were given by: $x^*_{121} = x^*_{211} = 120.00$, $x^*_{331} = x^*_{332} = 60.00$, and $x^*_{441} = x^*_{442} = 60$, with all other $x_{ijk}s = 0.00$; the optimal opportunity costs are: $\lambda^*_1 = \lambda^*_2 = 2.00$ and $\lambda^*_3 = \lambda^*_4 = 1.00$.

Conclusions

- A knowledge collaboration network model was developed that allows for multiple communication modes between researchers and handles also different disciplines.
- The behavior of the researchers was discussed and the system-optimization model constructed which yields the optimal effort/time allocation of the researchers, given their time budgets, so that the maximal utility for the knowledge collaboration network is attained.
- With an additional assumption of symmetric utility functions for the researchers/collaborators, we obtained explicit qualitative results regarding collaborative behaviors.
- Numerical example and computational procedure were discussed.


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