Networks, Performancement Assessment, and Vulnerability Analysis

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Outline

- Background and Motivation
- Why Behavior Matters and Paradoxes
- Methodologies for Formulation, Analysis, and Computations
- Which Nodes and Links Really Matter?
- Network Performance and Vulnerability
- From the Internet to Financial Networks
- Mergers and Acquisitions and Network Synergies

Background and Motivation

We Are in a New Era of Decision-Making Characterized by:

- *complex interactions* among decision-makers in organizations;
- *alternative and, at times, conflicting criteria* used in decision-making;
- *constraints on resources*: human, financial, natural, time, etc.;
- *global reach* of many decisions;
- *high impact* of many decisions;
- *increasing risk and uncertainty*;
- the *importance of dynamics* and realizing a timely response to evolving events.

The Era of Supernetworks

Supernetworks are *Networks of Networks*, and their prevalence in the world around us is illustrated by:

• *multimodal transportation networks*;

•*complex supply chain networks consisting of manufacturers, shippers and carriers, distributors, and retailers*;

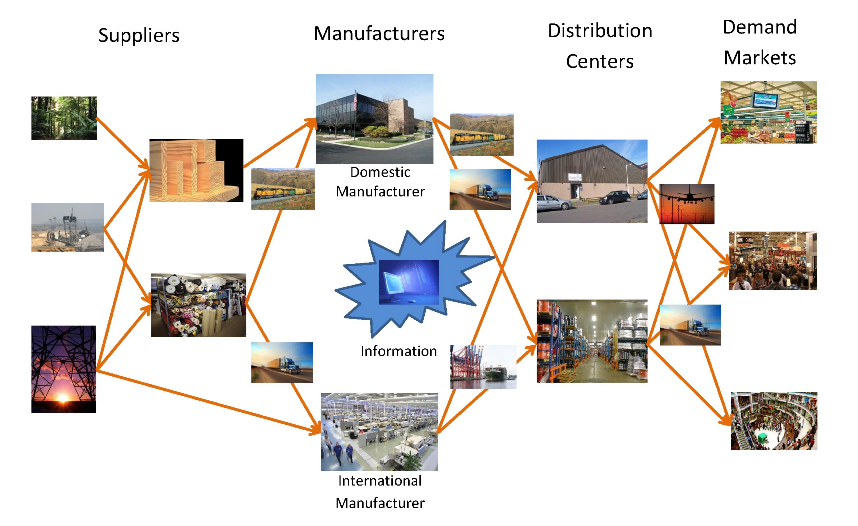
• *electric power generation and distribution networks*,

•*multitiered financial networks*, and

• *social network platforms such as Facebook and Twitter*, along with the Internet itself.

Multimodal Transportation

Complex Supply Chain Networks

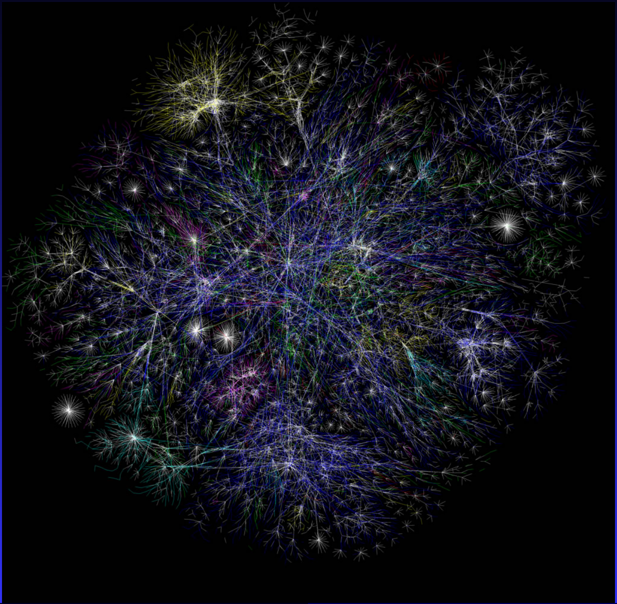


Electric Power Generation and Distribution Networks

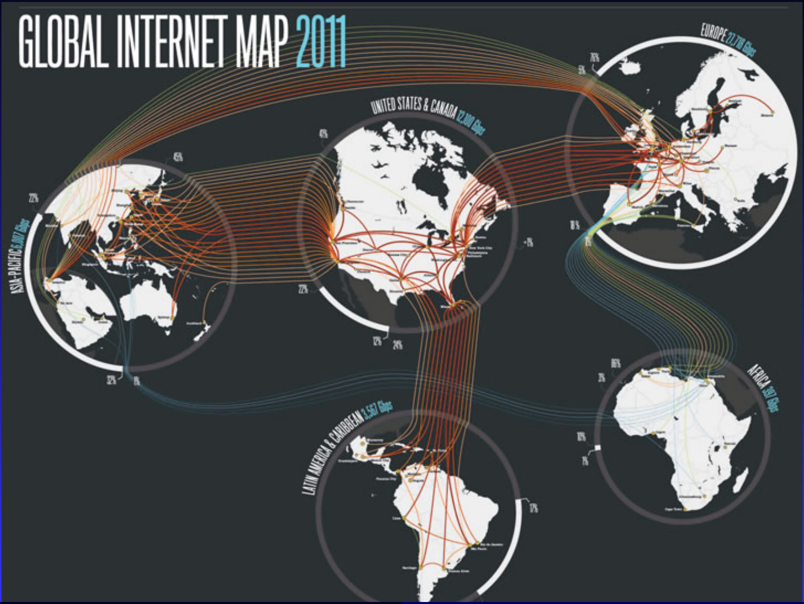
Financial Networks

Social Networks

Visual Image of the Internet (opte.org)



Internet Traffic Flows 2011



Some Characteristics of Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *interactions among networks themselves*, such as the Internet with financial networks, electric power networks, transportation and logistical networks;
- recognition of the fragility and vulnerability of network systems;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Network Problems and Methodologies

Networks consist of nodes, links, flows, and behavior associated with their operation, usage, and management.

Network problems are a distinct class of problems and they come in various forms and formulations, i.e., as optimization (linear or nonlinear) problems or as equilibrium problems and even dynamic network problems.

Methodologies that we have been using and extending to formulate, analyze, and compute solutions to large-scale network problems have included: network optimization, network theory, game theory, multicriteria decision-making, risk management, variational inequality theory, and projected dynamical systems theory.

Why Behavior Matters and Paradoxes

Network Models from Analysis to Design Must Capture the Behavior of Users

Behavior on Congested Networks

Flows are routed on individual cost-minimizing routes.

User-Optimized or Equilibrium

Decentralized	Selfish	U–O
vs. ⟺	vs. ⟺	vs.
Centralized	Unselfish	S–O

System-Optimized

Flows are routed so as to minimize the total cost.

Two fundamental principles of flow (traffic) behavior, due to Wardrop (1952), with terms coined by Dafermos and Sparrow (1969).

User-optimized (U-O) (network equilibrium) Problem – each user determines his/her cost minimizing route of travel between an origin/destination, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action (in the sense of Nash).

System-optimized (S-O) Problem – users are allocated among the routes so as to minimize the total cost in the system, where the total cost is equal to the sum over all the links of the link's user cost times its flow.

The U-O problems, under certain simplifying assumptions, possesses optimization reformulations. But now we can handle cost asymmetries, multiple modes of transport, and different classes of traffic (messages, vehicles, etc.), without such assumptions.

We Can State These Conditions Mathematically!

The U-O and S-O Conditions

Definition: U-O or Network Equilibrium – Fixed Demands

A path flow pattern x^*, *with nonnegative path flows and O/D pair demand satisfaction, is said to be U-O or in equilibrium, if the following condition holds for each O/D pair* $w \in W$ *and each path* $p \in P_w$*:*

$$C_p(x^*) \begin{cases} = \lambda_w, & \text{if} \quad x_p^* > 0, \\ \geq \lambda_w, & \text{if} \quad x_p^* = 0. \end{cases}$$

Definition: S-O Conditions

A path flow pattern x with nonnegative path flows and O/D pair demand satisfaction, is said to be S-O, if for each O/D pair $w \in W$ and each path $p \in P_w$:

$$\hat{C}'_p(x) \begin{cases} = \mu_w, & \text{if} \quad x_p > 0, \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p(x) = \sum_{a \in \mathcal{L}} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}$, and μ_w is a Lagrange multiplier.

The Braess Paradox Illustrates

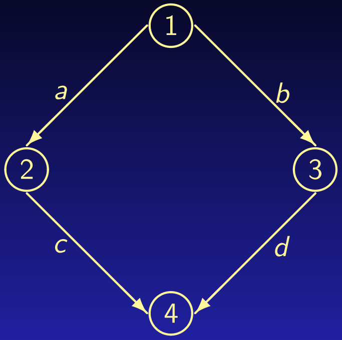
Why Behavior on Networks is Important

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6**, the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

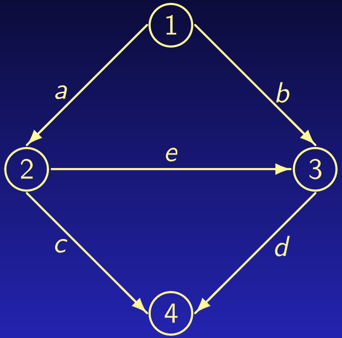
Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is
$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

The equilibrium path travel cost:
$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand of 6, would not be used.

The Braess paradox never occurs in S-O networks.

The 1968 Braess article has been translated from German to English and appears as:

"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) *Transportation Science* **39**, 446-450.

Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

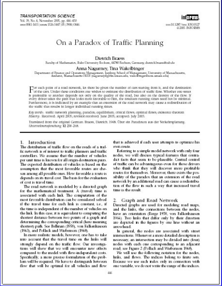
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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informs

On a Paradox of Traffic Planning

Dietrich Braess
Anna Nagurney, Tina Wakolbinger

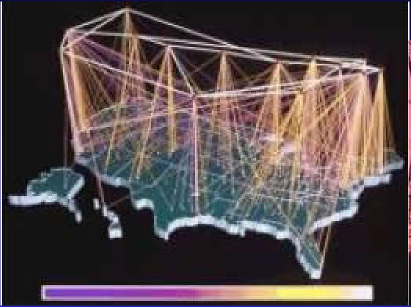
For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

Other Networks that Behave like Traffic Networks



The Internet, electric power networks, and even multitiered financial networks!

Methodologies for Formulation, Analysis, and Computations

The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized network problems.

Definition: The Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, *is to determine a vector* $X^* \in \mathcal{K}$, *such that:*

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N, $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N.

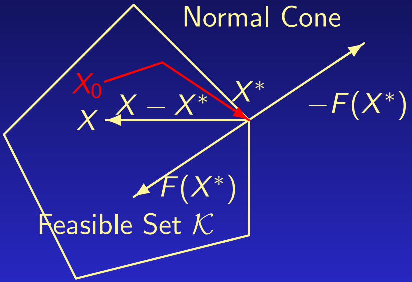
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of congested network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

To model the *dynamic behavior of supernetworks*, including transportation networks and supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

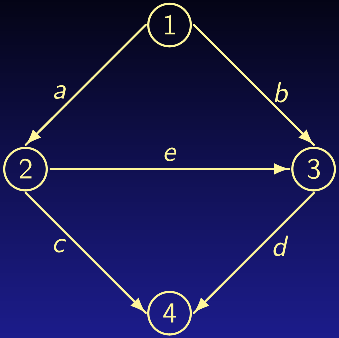
Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even *neuroscience* (Girard et al. (2008)).

Some of My Books



Recall the Braess network with the added link e.

What happens as the demand increases?

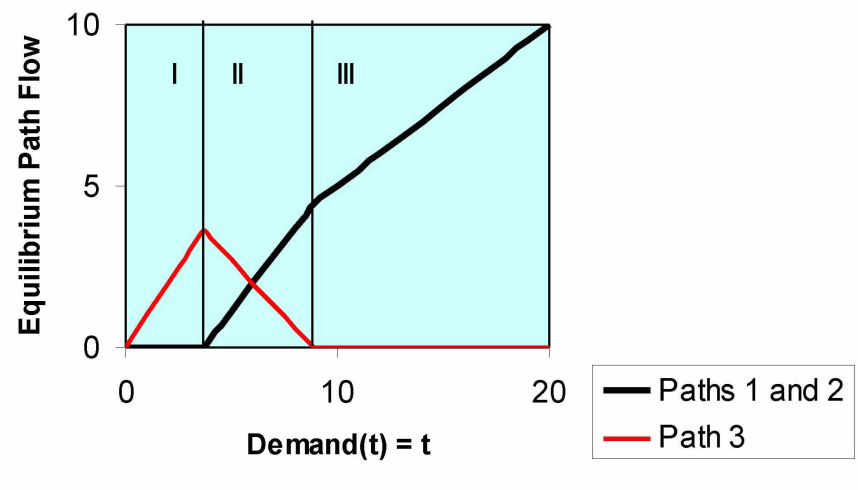
For Networks with Time-Dependent Demands

We Use Evolutionary Variational Inequalities

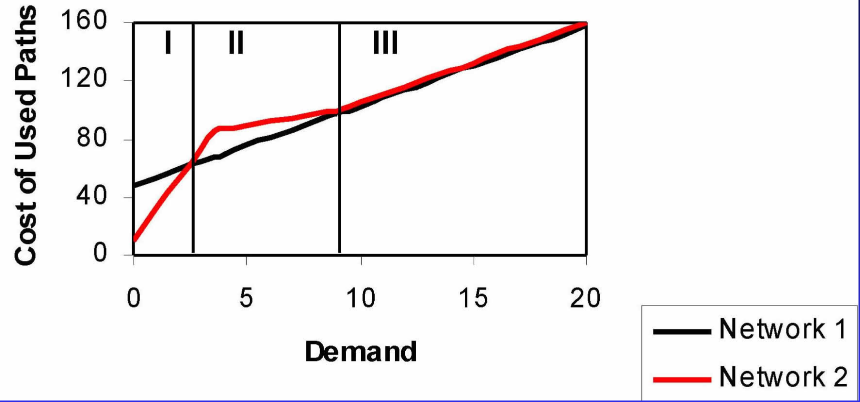
Radcliffe Institute for Advanced Study – Harvard University 2005-2006

Research with Professor David Parkes of Harvard University and Professor Patrizia Daniele of the University of Catania, Italy

The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* In a Model of the Internet (Nagurney, Daniele, and Parkes (2007)).



In Demand Regime I, Only the New Path is Used.
In Demand Regime II, the demand lies in the range [2.58, 8.89], and *the Addition of a New Link (Path) Makes Everyone Worse Off*!
In Demand Regime III, when the demand exceeds 8.89, *Only the Original Paths are Used!*



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!

Which Nodes and Links Really Matter?

Network Performance and Vulnerability

The analysis and the identification of the vulnerable components in networks have, recently, emerged as a major research theme.

However, in order to be able to evaluate the vulnerability and the reliability of a network, a measure that can quantifiably capture the performance of a network must be developed.

Recent disasters have vividly demonstrated the importance and vulnerability of our critical infrastructure systems

- The biggest blackout in North America, August 14, 2003;
- Two significant power outages in September 2003 – one in the UK and the other in Italy and Switzerland;
- The Indonesian tsunami (and earthquake), December 26, 2004;
- Hurricane Katrina, August 23, 2005;
- The Minneapolis I35 Bridge collapse, August 1, 2007;
- The Mediterranean cable destruction, January 30, 2008;
- The Sichuan earthquake on May 12, 2008;
- The Haiti earthquake that struck on January 12, 2010 and the Chilean one on February 27, 2010;
- The triple disaster in Japan on March 11, 2011.

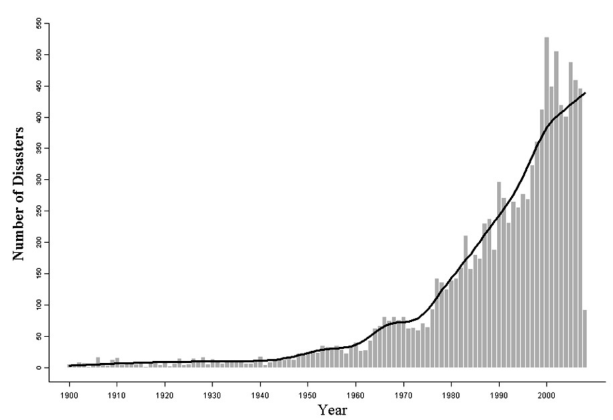
Hurricane Katrina in 2005

Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

The Haitian and Chilean Earthquakes

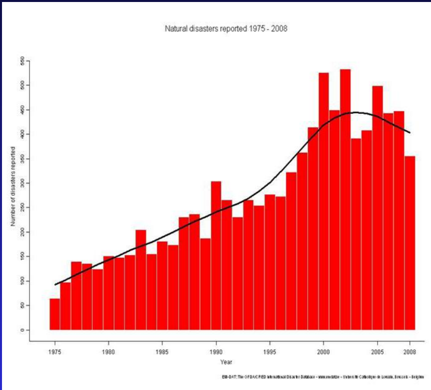
The Triple Disaster in Japan on March 11, 2011

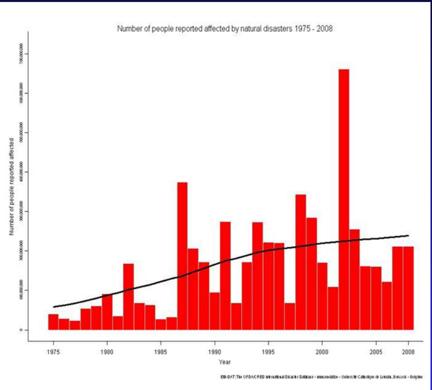
Disasters have brought an unprecedented impact on human lives in the 21st century and the number of disasters is growing. From January to October 2005, an estimated 97,490 people were killed in disasters globally; 88,117 of them because of natural disasters.



Frequency of disasters [Source: Emergency Events Database (2008)]

Natural Disasters (1975–2008)





Network Performance and Vulnerability

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources and economies.

By being able to identify which are the most important nodes and links we can then invest appropriately to protect such network components.

From the Internet to Financial Networks

The Nagurney and Qiang Network Efficiency Measure

Nagurney and Qiang (2007a, b, c) proposed a network efficiency measure for networks with fixed demands, which captures demand and flow information under network equilibrium:

The network performance/efficiency measure, $\mathcal{E}(G, d)$, according to Nagurney and Qiang (2007a, b, c), for a given network topology G and fixed demand vector d, is defined as:

$$\mathcal{E}(G, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W},$$

where n_W is the number of O/D pairs in the network and λ_w is the equilibrium disutility for O/D pair w.

Network Efficiency Measure for Dynamic Networks - Continuous Time

The network efficiency for the network G with time-varying demand d for $t \in [0, T]$, denoted by $\mathcal{E}(G, d, T)$, is defined as follows:

$$\mathcal{E}(G, d, T) = \frac{\int_0^T [\sum_{w \in W} \frac{d_w(t)}{\lambda_w(t)}]/n_W \, dt}{T}.$$

Note that the above measure is the average network performance over time of the dynamic network.

Network Efficiency Measure for Dynamic Networks - Discrete Time

Let $d_w^1,\ d_w^2, ..., d_w^H$ denote demands for O/D pair w in H discrete time intervals, given, respectively, by: $[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, where $t_H \equiv T$. We assume that the demand is constant in each such time interval for each O/D pair. Moreover, we denote the corresponding minimal costs for each O/D pair w at the H different time intervals by: $\lambda_w^1,\ \lambda_w^2, ..., \lambda_w^H$. The demand vector d, in this special discrete case, is a vector in $R^{n_W \times H}$.

Dynamic Network Efficiency: Discrete Time Version

The network efficiency for the network (G, d) *over* H *discrete time intervals:*
$[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, *where* $t_H \equiv T$, *and with the respective constant demands:*
$d_w^1,\ d_w^2, ..., d_w^H$ *for all* $w \in W$ *is defined as follows:*

$$\mathcal{E}(G, d, t_H = T) = \frac{\sum_{i=1}^{H}[(\sum_{w \in W} \frac{d_w^i}{\lambda_w^i})(t_i - t_{i-1})/n_W]}{t_H}.$$

Special Case

Assume that $d_w(t) = d_w$, *for all O/D pairs* $w \in W$ *and for* $t \in [0, T]$. *Then, the dynamic network efficiency measure collapses to the Nagurney and Qiang (2007a, b, c) measure:*

$$\mathcal{E} = \frac{1}{n_W} \sum_{w \in W} \frac{d_w}{\lambda_w}.$$

Importance of a Network Component

The importance of network component g of network G with demand d over time horizon T is defined as follows:

$$I(g, d, T) = \frac{\mathcal{E}(G, d, T) - \mathcal{E}(G - g, d, T)}{\mathcal{E}(G, d, T)}$$

where $\mathcal{E}(G - g, d, T)$ is the dynamic network efficiency after component g is removed.

Importance of Nodes and Links in the Dynamic Braess Network Using the New Measure When $T = 10$

Link	Importance Value	Importance Ranking
a	0.2604	1
b	0.1784	2
c	0.1784	2
d	0.2604	1
e	-0.1341	3

Node	Importance Value	Importance Ranking
1	1.0000	1
2	0.2604	2
3	0.2604	2
4	1.0000	1

Link e is never used after $t = 8.89$ and in the range $t \in [2.58, 8.89]$, it increases the cost, so the fact that link e has a negative importance value makes sense; over time, its removal would, on the average, improve the network efficiency!

Some Applications of the N-Q Measure

The Sioux Falls Network

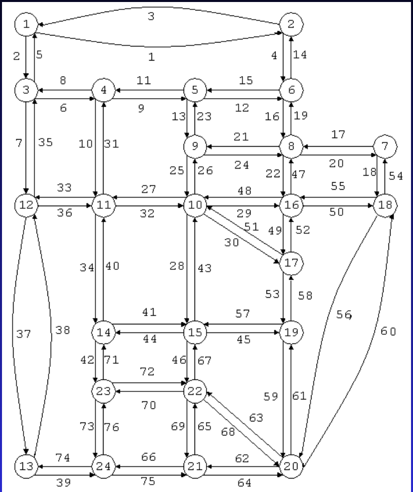


Figure 1: The Sioux Falls network with 24 nodes, 76 links, and 528 O/D pairs of nodes.

Importance of Links in the Sioux Falls Network

The computed network efficiency measure $\mathcal{E}$ for the Sioux Falls network is $\mathcal{E} = 47.6092$. Links 56, 60, 36, and 37 are the most important links, and hence special attention should be paid to protect these links accordingly, while the removal of links 10, 31, 4, and 14 would cause the least efficiency loss.

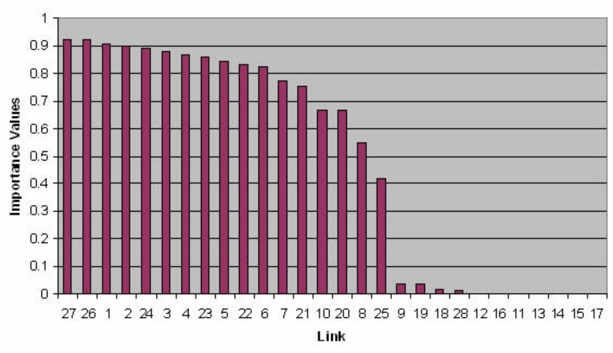


Figure 2: The Sioux Falls network link importance rankings

According to the European Environment Agency (2004), *since 1990, the annual number of extreme weather and climate related events has doubled, in comparison to the previous decade*. These events account for approximately 80% of all economic losses caused by catastrophic events. In the course of climate change, catastrophic events are projected to occur more frequently (see Schulz (2007)).

Schulz (2007) applied *N-Q network efficiency measure to a German highway system in order to identify the critical road elements* and found that this measure provided more reasonable results than the measure of Taylor and DEste (2007).

The N-Q measure can also be used to assess which links should be added to improve efficiency. *This measure was used for the evaluation of the proposed North Dublin (Ireland) Metro system* (October 2009 Issue of *ERCIM News*).

Financial Networks

The study of financial networks dates to the 1750s when Quesnay (1758), in his *Tableau Economique*, conceptualized the circular flow of financial funds in an economy as a network.

Copeland (1952) further explored the relationships among financial funds as a network and asked the question, *"Does money flow like water or electricity?"*

Financial Networks

The advances in information technology and globalization have further shaped today's financial world into a complex network, which is characterized by distinct sectors, the proliferation of new financial instruments, and with increasing international diversification of portfolios.

Financial Networks

Since today's financial networks may be highly interconnected and interdependent, any disruptions that occur in one part of the network may produce consequences in other parts of the network, which may not only be in the same region but many thousands of miles away in other countries.

As pointed out by Sheffi (2005) in his book, *The Resilient Enterprise*, one of the main characteristics of disruptions in networks is "the seemingly unrelated consequences and vulnerabilities stemming from global connectivity."

Financial Networks

In 2008 and 2009, the world reeled from the effects of the financial credit crisis; leading financial services and banks closed (including the investment bank Lehman Brothers), others merged, and the financial landscape was changed for forever.

The domino effect of the U.S. economic troubles rippled through overseas markets and pushed countries such as Iceland to the verge of bankruptcy.

Financial Networks

It is crucial for the decision-makers in financial systems (managers, executives, and regulators) to be able **to identify a financial network's vulnerable components** to protect the functionality of the network.

Financial networks, as extremely important infrastructure networks, have a great impact on the global economy, and their study has recently also attracted attention from researchers in the area of complex networks.

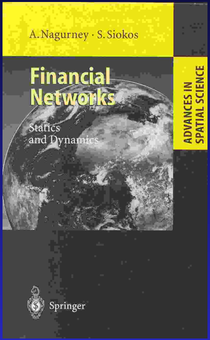
Financial Networks

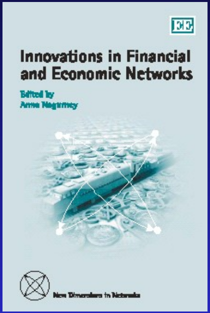
Several recent studies in finance, in turn, have analyzed the local consequences of catastrophes and the design of risk sharing/management mechanisms since the occurrence of disasters such as 9/11 and Hurricane Katrina (see, for example, Gilli and Këllezi (2006), Loubergé, Këllezi, and Gilli (1999), Doherty (1997), Niehaus (2002), and the references therein).

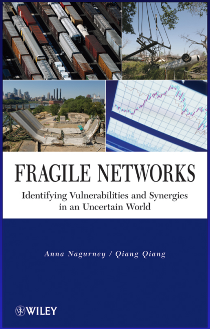
Nevertheless, there is very little literature that addresses the vulnerability of financial networks. Robinson, Woodard, and Varnado (1998) discussed, from the policy-making point of view, how to protect the critical infrastructure in the US, including financial networks.

To the best of our knowledge, however, there is no network performance measure to-date that has been applied to financial networks that captures both economic behavior as well as the underlying network/graph structure as well as the dynamic reallocation after disruptions.

Financial Networks



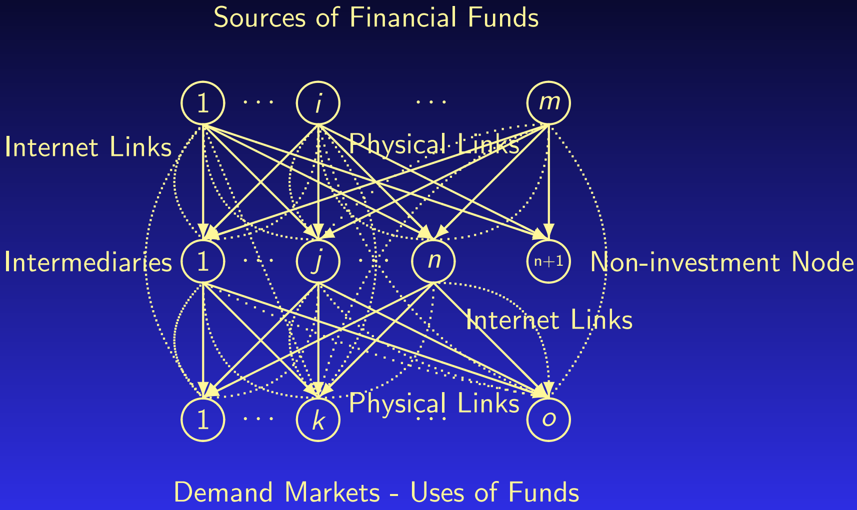




The Financial Network Model with Intermediation

This part of the presentation is based on the paper, "Identification of Critical Nodes and Links in Financial Networks with Intermediation and Electronic Transactions," Anna Nagurney and Qiang Qiang, in *Computational Methods in Financial Engineering*, E. J. Kontoghiorghes, B. Rustem, and P. Winker, Editors, Springer, Berlin, Germany (2008), pp 273-297.

The Financial Network Model with Intermediation



Examples of source agents include households and businesses.

The financial intermediaries, in turn, which can include banks, insurance companies, investment companies, etc., in addition to transacting with the source agents determine how to allocate the incoming financial resources among the distinct uses or financial products associated with the demand markets, which correspond to the nodes at the bottom tier of the financial network in the figure.

Both source agents and intermediaries maximize their net revenues while minimizing their risk.

Examples of demand markets are: the markets for real estate loans, household loans, business loans, etc.

Consumers at the demand markets react to the prices and their transaction costs.

The Financial Network Performance Measure

Definition: The Financial Network Performance Measure
The financial network performance measure, $\mathcal{E}$, for a given network topology G, and demand price functions $\rho_{3k}(d)$ $(k = 1, 2, \ldots, o)$, and available funds held by source agents S, is defined as follows:

$$\mathcal{E} = \frac{\sum_{k=1}^{o} \frac{d_k^*}{\rho_{3k}(d^*)}}{o},$$

where o is the number of demand markets in the financial network, and d_k^* and $\rho_{3k}(d^*)$ denote the equilibrium demand and the equilibrium price for demand market k, respectively.

The financial network performance measure $\mathcal{E}$ is actually the average demand to price ratio. It measures the overall (economic) functionality of the financial network. When the network topology G, the demand price functions, and the available funds held by source agents are given, a financial network is considered performing better if it can satisfy higher demands at lower prices.

Network Efficiency vs. Network Performance

Although in some networks as the Internet and certain transportation networks, the assumption of having a central planner to ensure the minimization of the total cost may, in some instances, be natural and reasonable, the same assumption faces difficulty when extended to the larger and more complex networks as in the case of financial networks, where the control by a "central planner" is not realistic.

The Importance of a Financial Network Component

The financial network performance is expected to deteriorate when a critical network component is eliminated from the network.

Such a component can include a link or a node or a subset of nodes and links depending on the financial network problem under investigation. Furthermore, the removal of a critical network component will cause severe damage than that of the damage caused by a trivial component.

The Importance of a Financial Network Component

The importance of a network component is defined as:

Definition: Importance of a Financial Network Component

The importance of a financial network component $g \in G$, $I(g)$, is measured by the relative financial network performance drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G) - \mathcal{E}(G - g)}{\mathcal{E}(G)}$$

where $G - g$ is the resulting financial network after component g is removed from network G.

Mergers and Acquisitions and Network Synergies

Mergers and Acquisitions and Network Synergies

A successful merger depends on the ability to measure the anticipated synergy of the proposed merger (cf. Chang (1988)) .

The rest of this presentation is based on the recent paper:

- Z. Liu and A. Nagurney (2011), "Risk Reduction and Cost Synergy in Mergers and Acquisitions via Supply Chain Network Integration," *Journal of Financial Decision Making* **7(2)**, 1-18.

Mergers and Acquisitions and Network Synergies

It is increasingly apparent and documented that *improving supply chain integration is key to improving the likelihood of post-merger success*!

This is understandable, since *up to 80% of a firm's costs are linked to operations* (Benitez and Gordon (2000)).

However, empirical studies demonstrate that *one out of two post-merger integration efforts fares poorly* (Gerds and Schewe (2009)).

In addition, in an empirical analysis of a global sample of over 45,000 data points of post-merger transactions in all significant sectors globally from services to manufacturing, *risk factors were identified to post-merger success* (see Gerds, Strottmann, and Jayaprakash (2010)).

Mergers and Acquisitions and Network Synergies

Risk in the context of supply chains may be associated with

- the production/procurement processes,
- the transportation/shipment of the goods,
- and/or the demand markets.

Such supply chain risks are directly reflected in firms' financial performances, and priced in the financial market.

Hendricks and Singhal (2010) estimated that the average stock price reaction to supply-demand mismatch announcements was approximately -6.8%. In addition, supply chain disruptions can cause firms' equity risks to increase by 13.50% on average after the disruption announcements (Hendricks and Singhal (2005)).

Illustrations of Supply Chain Risk

Mergers and Acquisitions and Network Synergies

We build upon the recent work in mergers and acquisitions of that focuses on horizontal network integration (cf. Nagurney (2009), Nagurney and Woolley (2010), and Nagurney, Woolley, and Qiang (2010)).

We develop the following significant extension: *we utilize a mean-variance (MV) approach in order to capture the risk associated with supply chain activities both prior to and post the merger/acquisition under investigation*. The MV approach to the measurement of risk dates to the work of the Nobel laureate Markowtiz (1952, 1959) and even today (cf. Schneeweis, Crowder, and Kazemi (2010)) remains a fundamental approach to minimizing volatility.

This new modeling framework allows one to capture quantitatively the risk associated *not only with the supply chain network activities but also with the merger/acquisition itself*.

The Pre- and Post-Merger Supply Chain Networks

All firms, both prior and post the merger, minimize both their expected total costs and the risk, as captured through the variance of the total costs, with a suitable weight assigned to the latter.

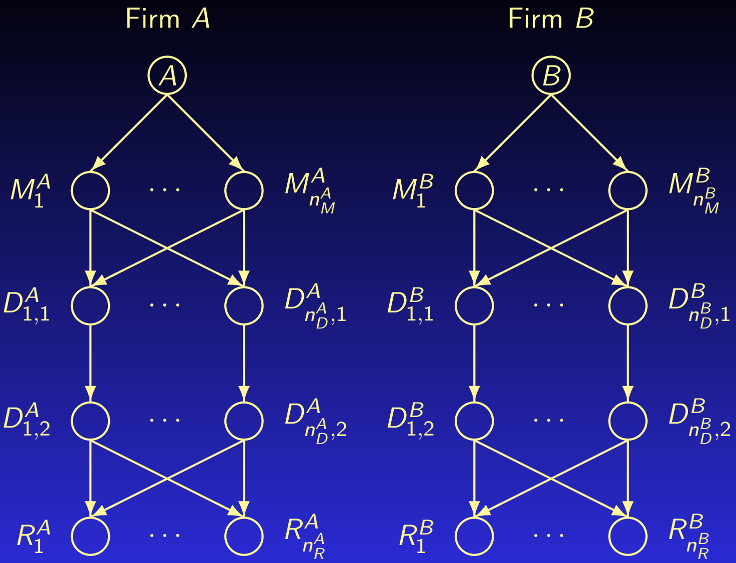


Figure 4: The Pre-Merger Supply Chain Network

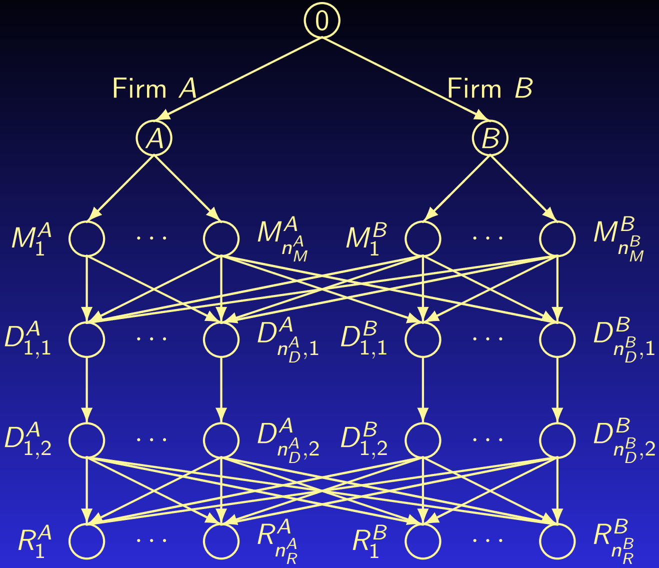


Figure 5: The Post-Merger Supply Chain Network

Three Synergy Measures for Mergers and Acquisitions

The measures to capture the gains, if any, are:

The Expected Total Cost Synergy

$$\mathcal{S}_{TC} \equiv \left[\frac{TC^0 - TC^1}{TC^0}\right] \times 100\%,$$

quantifies the expected total cost savings.

The Absolute Risk Synergy

$$\mathcal{S}_{TR} \equiv \left[\frac{TR^0 - TR^1}{TR^0}\right] \times 100\%,$$

represents the reduction of the absolute risk achieved through the merger.

The Relative Risk Synergy

$$\mathcal{S}_{CV} \equiv \left[\frac{CV^0 - CV^1}{CV^0}\right] \times 100\%,)$$

where CV^0 and CV^1 denote the coefficient of variation of the total cost for, respectively, the pre-merger and the post-merger networks, and are defined as follows:

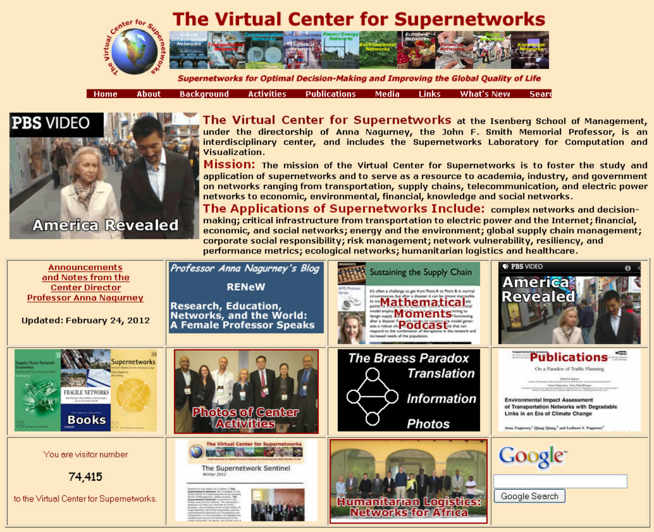
$$CV^0 \equiv \frac{\sqrt{TR^0}}{TC^0},$$

$$CV^1 \equiv \frac{\sqrt{TR^1}}{TC^1}.$$

Note that CV^0 and CV^1 represent the volatilities of the expected total costs of the pre- and post-merger networks, respectively.

This measure reflects the reduction of the relative risk through the merger.

THANK YOU!



For more information, see: http://supernet.isenberg.umass.edu

References - for Further Reading

Overview article on Financial Networks:
http://supernet.isenberg.umass.edu/articles/finhandbook.pdf

Link to Portfolio Optimization course in Executive Education at Harvard University:
http://supernet.isenberg.umass.edu/courses/Harvard-PortfolioOptimization.pdf

Link to Network Economics course at the World Bank:
http://supernet.isenberg.umass.edu/visuals/nagurney-worldbank-networkeconomics.pdf

Link to numerous articles on network modeling and applications, vulnerability and robustness analysis, as well as network synergy:
http://supernet.isenberg.umass.edu/dart.html

Link to books of interest:
http://supernet.isenberg.umass.edu/bookser.html