Reverse Supply Chain Management and Electronic Waste Recycling: A Multitiered Network Equilibrium Framework for E-Cycling

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Background

- 63 million PCs will be obsolete in 2003 in the U.S. (Appelbaum (2002)).

- About 10 million electric waste products are dumped per year in Japan. (Appelbaum (2002)).

- The shortage of landfills
- The problems of exporting hazardous materials to developing countries

Motivation

Recent legislation both in the United States as well as abroad has attention on recycling electronic wastes.

* Massachusetts in 2000 banned cathode-ray tubes from landfills.

* The Home Appliances Recycling Law in Japan (2001)

* Waste from Electrical and Electronic Equipment directive (WEEE) in EU (2008)

Motivation

- Electronic waste recycling differs from household recycling in two ways (Sodhi and Reimer (2001)).
  1. Electronic wastes contain hazardous materials.
  2. Electronic wastes contain precious materials.

↓

* Collecting and processing electronic wastes may be feasible or profitable.

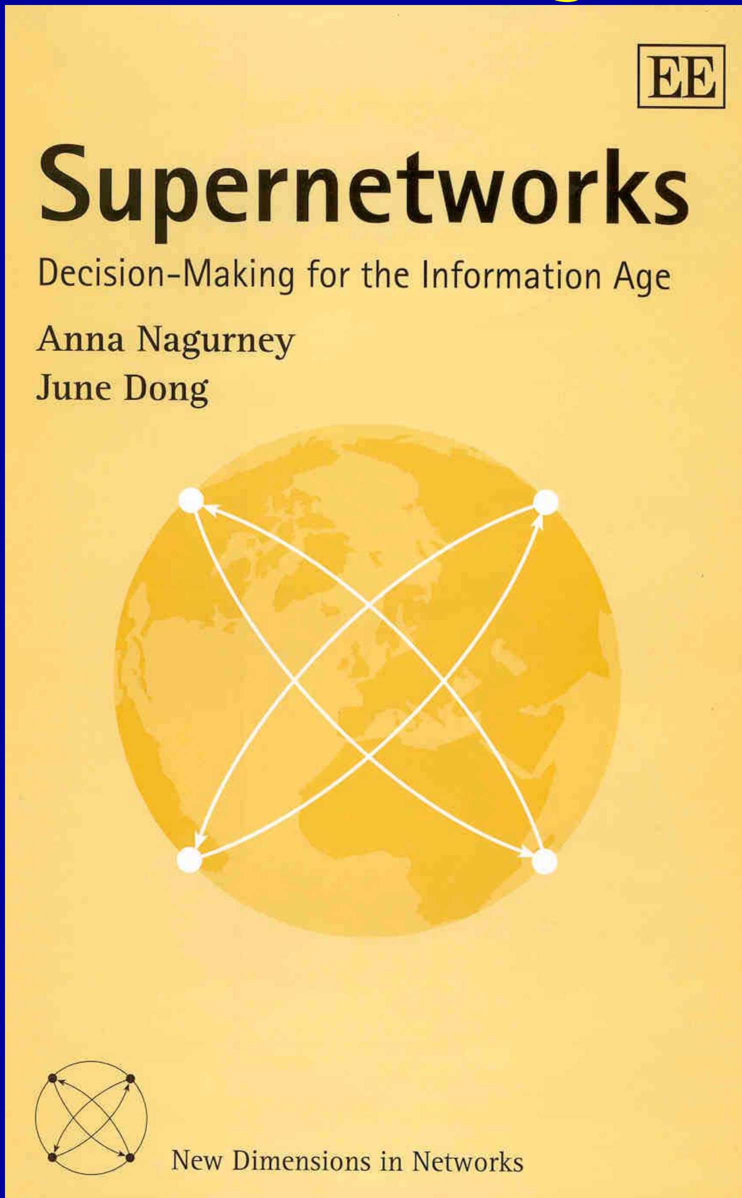
Motivation

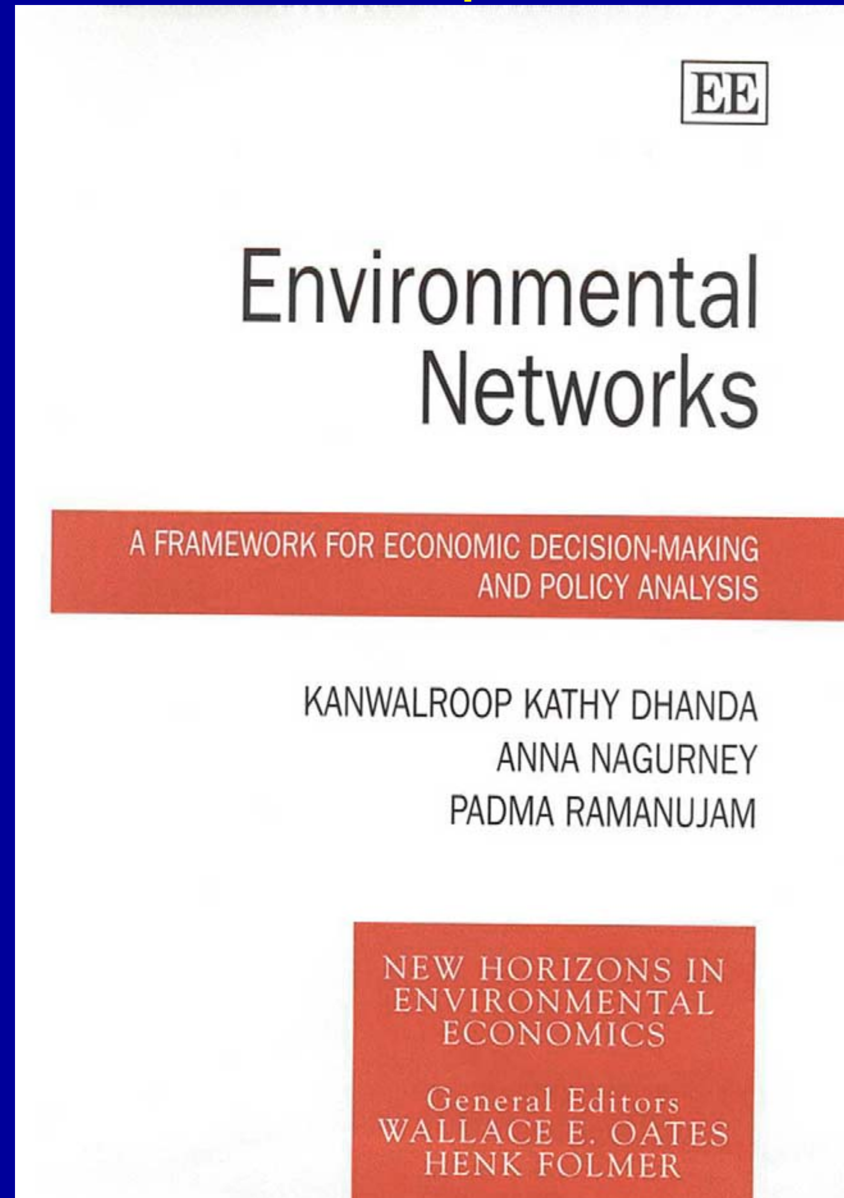
From supply chains to green logistics

Legislative and consumer pressure lead manufactures to apply cradle-to-grave product management (Bloemhof-Ruwaard and et.al. (1995)).

The need to manage E-cycling from the prospective of reverse supply chains.

Some Background for our Perspective





References

Nagurney, A., Dong, J., and Zhang, D. (2002) "A supply chain network equilibrium model." *Transportation Research E* **38**, 282-303.

Nagurney, A., Loo, J., Dong, J., and Zhang, D. (2002) "Supply chain networks and electronic commerce: a theoretical perspective." *Netnomics* **4**, 187-220.

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References

Many researchers have studied recycling issues; see,for example,

* Hoshino, T., Yura, K., and Hitomi, K. (1995)

- Maximize total profit and recycling rate
- Use linear programming

* Stuart, J.A., Ammons, J.C., and Turbini, L.J. (1999)

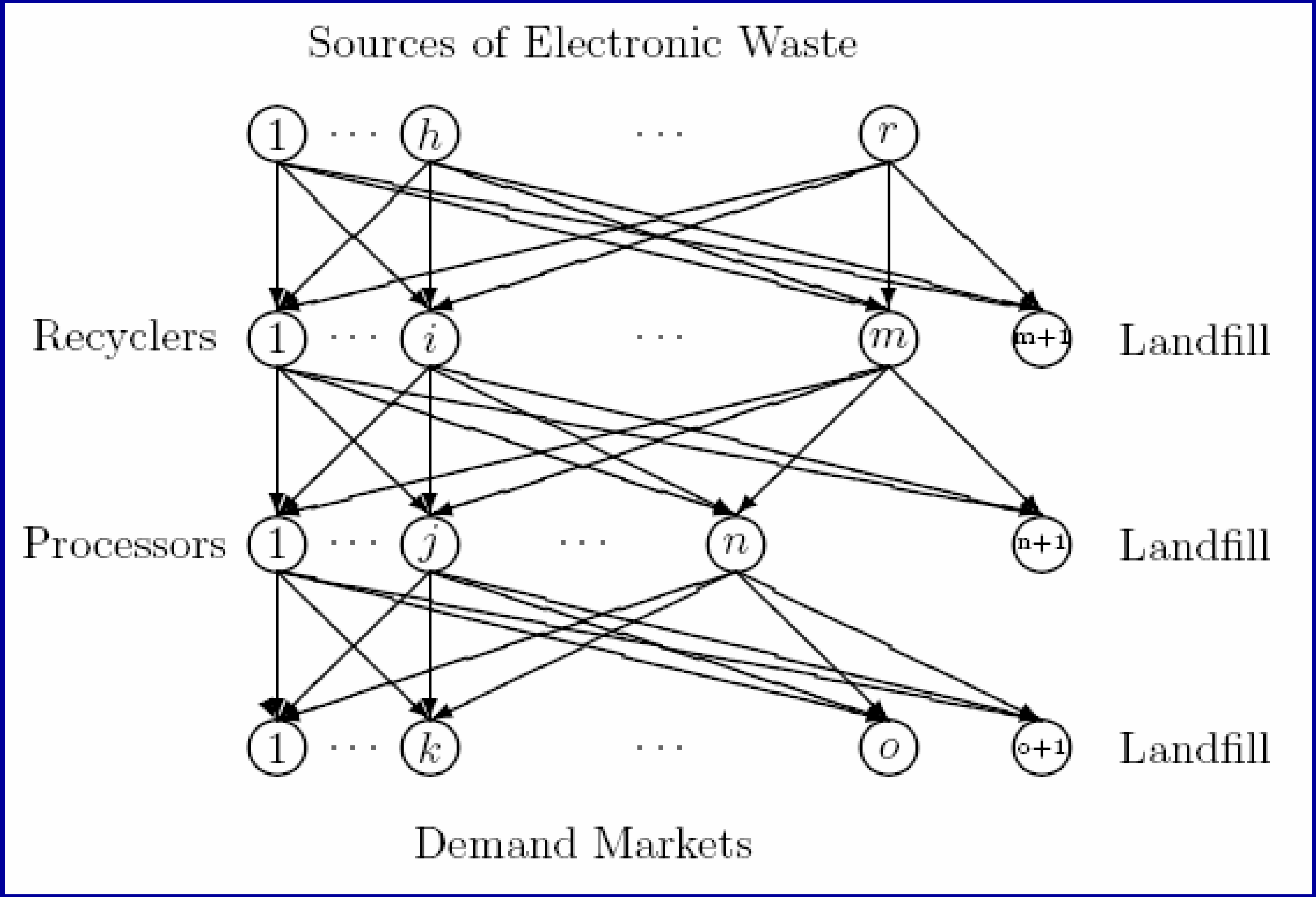
- Capture comprehensively measurable corporate environmental impact considerations for the product life cycle
- Use mixed integer programming

References

* Uzsoy, R., and Venkatachalam, G. (1998)
  - Consider recycling problems in terms of supply chains
  - Use linear programming

* Sodhi, M.S., and Reimer, B. (2001)
  - Optimization models for the sources, the recyclers, and the processors

The 4-Tiered E-Cycling Network



Assumptions of the Model

- The prices and material flows are endogenously determined.
- The sources generate a fixed amount of electronic waste.
- The sources minimize their costs.
- The recyclers and the processors maximize their profits, respectively.
- Cournot-Nash oligopoly market.

Sources' Behavior

$$\text{Minimize} \sum_{i=1}^{m} \rho_{1hi}^{*} q_{hi} + \bar{\rho}_{1h(m+1)} q_{h(m+1)} + \sum_{i=1}^{m+1} c_{hi}(q_{hi}) \tag{2}$$

subject to:

$$\sum_{i=1}^{m+1} q_{hi} = S^{h}, \tag{3}$$

$$q_{hi} \geq 0, \quad i = 1, \ldots, m+1. \tag{4}$$

Variational Inequality Formulation of the Sources

$$\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\frac{\partial c_{hi}(q_{hi}^*)}{\partial q_{hi}} + \rho_{1hi}^*\right] \times [q_{hi} - q_{hi}^*]$$

$$+\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q_{h(m+1)}^*)}{\partial q_{h(m+1)}} + \bar{\rho}_{1h(m+1)}\right] \times \left[q_{h(m+1)} - q_{h(m+1)}^*\right] \geq 0, \quad \forall Q^1 \in K,$$

where the feasible set K is defined as $K \equiv \{Q^1 \text{ such that (3) and (4) hold for all } h\}$.

Recyclers' Behavior

$$\text{Maximize} \sum_{j=1}^{n} \rho^*_{2ij} q_{ij} + \sum_{h=1}^{r} \rho^*_{1hi} q_{hi} - \bar{\rho}_{2i(n+1)} q_{i(n+1)} - \sum_{j=1}^{n+1} c_{ij}(q_{ij}) - \sum_{h=1}^{r} \hat{c}_{hi}(q_{hi}) - c_i(Q^2)$$

subject to:

$$\sum_{j=1}^{n+1} \alpha_{ij} q_{ij} \leq \sum_{h=1}^{r} q_{hi}$$

$$q_{hi} \geq 0, \quad h = 1, \ldots, r; \quad q_{ij} \geq 0, \quad j = 1, \ldots, n+1.$$

Processors' Behavior

$$\text{Maximize} \sum_{k=1}^{o} \rho^*_{3jk} q_{jk} - c_j(Q^3) - \bar{\rho}_{3j(o+1)} q_{j(o+1)} - \sum_{i=1}^{m} \rho^*_{2ij} q_{ij} - \sum_{k=1}^{o+1} c_{jk}(q_{jk}) - \sum_{i=1}^{m} \hat{c}_{ij}(q_{ij})$$

subject to:

$$\sum_{k=1}^{o+1} \beta_{jk} q_{jk} \leq \sum_{i=1}^{m} q_{ij}$$

$$q_{ij} \geq 0, \quad i = 1, \ldots, m; \quad q_{jk} \geq 0, \quad k = 1, \ldots, o+1.$$

The Demand Markets

For all processors j; $j = 1, \ldots, n$

$$\rho_{3jk}^* + \hat{c}_{jk}(q_{jk}^*) \begin{cases} = \rho_{4k}^*, & \text{if} \quad q_{jk}^* > 0 \\ \geq \rho_{4k}^*, & \text{if} \quad q_{jk}^* = 0, \end{cases}$$

$$d_k(\rho_4^*) \begin{cases} = \sum_{j=1}^{n} q_{jk}^*, & \text{if} \quad \rho_{4k}^* > 0 \\ \leq \sum_{j=1}^{n} q_{jk}^*, & \text{if} \quad \rho_{4k}^* = 0. \end{cases}$$

Variational Inequality Formulation

satisfying

$$
\begin{aligned}
&\sum_{h=1}^{r}\sum_{i=1}^{m}\left[\frac{\partial c_{hi}(q_{hi}^*)}{\partial q_{hi}}+\frac{\partial \hat{c}_{hi}(q_{hi}^*)}{\partial q_{hi}}-\gamma_i^*\right]\times\left[q_{hi}-q_{hi}^*\right]\\
&+\sum_{h=1}^{r}\left[\frac{\partial c_{h(m+1)}(q_{h(m+1)}^*)}{\partial q_{h(m+1)}}+\bar{\rho}_{1h(m+1)}\right]\times\left[q_{h(m+1)}-q_{h(m+1)}^*\right]\\
&+\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{ij}}+\frac{\partial c_{ij}(q_{ij}^*)}{\partial q_{ij}}+\frac{\partial \hat{c}_{ij}(q_{ij}^*)}{\partial q_{ij}}+\alpha_{ij}\gamma_i^*-\eta_j^*\right]\times\left[q_{ij}-q_{ij}^*\right]\\
&+\sum_{i=1}^{m}\left[\frac{\partial c_i(Q^{2*})}{\partial q_{i(n+1)}}+\frac{\partial c_{i(n+1)}(q_{i(n+1)}^*)}{\partial q_{i(n+1)}}+\bar{\rho}_{2i(n+1)}+\alpha_{i(n+1)}\gamma_i^*\right]\times\left[q_{i(n+1)}-q_{i(n+1)}^*\right]\\
&+\sum_{i=1}^{m}\left[\sum_{h=1}^{r}q_{hi}^*-\sum_{j=1}^{n+1}\alpha_{ij}q_{ij}^*\right]\times\left[\gamma_i-\gamma_i^*\right]\\
&+\sum_{j=1}^{n}\sum_{k=1}^{o}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{jk}}+\frac{\partial c_{jk}(q_{jk}^*)}{\partial q_{jk}}+\hat{c}_{jk}(q_{jk}^*)+\beta_{jk}\eta_j^*-\rho_{4k}^*\right]\times\left[q_{jk}-q_{jk}^*\right]\\
&+\sum_{j=1}^{m}\left[\frac{\partial c_j(Q^{3*})}{\partial q_{j(o+1)}}+\frac{\partial c_{j(o+1)}(q_{j(o+1)}^*)}{\partial q_{j(o+1)}}+\beta_{j(o+1)}\eta_j^*+\bar{\rho}_{3j(o+1)}\right]\times\left[q_{j(o+1)}-q_{j(o+1)}^*\right]\\
&+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}q_{ij}^*-\sum_{k=1}^{o+1}\beta_{jk}q_{jk}^*\right]\times\left[\eta_j-\eta_j^*\right]\\
&+\sum_{k=1}^{o}\left[\sum_{j=1}^{n}q_{jk}^*-d_k(\rho_4^*)\right]\times\left[\rho_{4k}-\rho_{4k}^*\right]\geq 0,\quad \forall(Q^{1*},Q^{2*},\gamma^*,Q^{3*},\eta^*,\rho_4^*)\in\mathcal{K},
\end{aligned}
$$

where $\mathcal{K}\equiv\{Q^1,Q^2,\gamma,Q^3,\eta,\rho_4)|(Q^1,Q^2,\gamma,Q^3,\eta,\rho_4)\geq 0,$ *and* Q^1 *satisfies* (3) *for all* $h\}$

Variational Inequality Formulation

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $X \equiv (Q^1, Q^2, \gamma, Q^3, \eta, \rho_4)$ and

$$F(X) \equiv (F_{hi}, F_{ij}, F_i, F_{jk}, F_{\hat{j}}, F_{\hat{k}})_{h=1,\ldots,r;i=1,\ldots,m+1;j=1,\ldots,n+1;k=1,\ldots,o+1;\hat{j}=1,\ldots,n;\hat{k}=1,\ldots,o},$$

Qualitative Properties

Bounded material flows and the imposition of a coercivity condition on demand functions or a boundedness condition → The existence of material flows and the demand prices is guaranteed.

Under the strict monotonicity of F(x) → The uniqueness is guaranteed.

The Algorithm

Modified projection method of Korpelevich (1977)

This algorithm requires only monotonicity of F(x) and Lipshitz continuity condition.

Step 0: Initialization

Step 1: Computation

Step 2: Adaptation

Step 3: Convergence Verification

Computation of the Top Tiers Material Flows

The top-tiered material flows are computed explicitly but not with specific formulae, since these flows must lie in the feasible set K.

$$\text{Minimize} \quad \sum_{i=1}^{m+1} \tilde{q}_{hi}^2 + \tilde{h}_{hi},$$

subject to:

$$\sum_{i=1}^{m+1} q_{hi} = S^h,$$

$$q_{hi} \geq 0, \quad i = 1, \ldots, m+1.$$

Computation of the Material Flows from the Recyclers to the Processors and the Landfill

In particular, compute, at iteration τ, the $\tilde{q}^{\tau}_{ij}$s according to:

$$\tilde{q}^{\tau}_{ij} = \max\{0, q^{\tau-1}_{ij} - \delta(\frac{\partial c_i(Q^{2\tau-1})}{\partial q_{ij}} + \frac{\partial c_{ij}(q^{\tau-1}_{ij})}{\partial q_{ij}} + \alpha_{ij}\gamma^{\tau-1}_i + \frac{\partial \hat{c}_{ij}(q^{\tau-1}_{ij})}{\partial q_{ij}} - \eta^{\tau-1}_j)\}, \quad \forall i, j,$$

and the $\tilde{q}^{\tau}_{i(n+1)}$s according to:

$$\tilde{q}^{\tau}_{i(n+1)} = \max\{0, q^{\tau-1}_{i(n+1)} - \delta(\frac{\partial c_i(Q^{2\tau-1})}{\partial q_{i(n+1)}} + \frac{\partial c_{i(n+1)}(q^{\tau-1}_{i(n+1)})}{\partial q_{i(n+1)}} + \alpha_{i(n+1)}\gamma^{\tau-1}_i + \bar{\rho}_{2i(n+1)})\}, \quad \forall i.$$

Computation of the Material Flows from Processors to the Demand Markets and the Landfill

Compute, at iteration τ, the $\tilde{q}^{\tau}_{jk}$s according to:

$$\tilde{q}^{\tau}_{jk} = \max\{0, q^{\tau-1}_{jk} - \delta(\frac{\partial c_j(Q^{3\tau-1})}{\partial q_{jk}} + \frac{\partial c_{jk}(q^{\tau-1}_{jk})}{\partial q_{jk}} + \beta_{jk}\eta^{\tau-1}_j + \hat{c}_{jk}(q^{\tau-1}_{jk}) - \rho^{\tau-1}_{4k})\}, \quad \forall j, k, \quad (36)$$

and the $\tilde{q}^{\tau}_{j(o+1)}$s according to:

$$\tilde{q}^{\tau}_{j(o+1)} = \max\{0, q^{\tau-1}_{j(o+1)} - \delta(\frac{\partial c_j(Q^{3\tau-1})}{\partial q_{j(o+1)}} + \frac{\partial c_{j(o+1)}(q^{\tau-1}_{j(o+1)})}{\partial q_{j(o+1)}} + \beta_{j(o+1)}\eta^{\tau-1}_j + \bar{\rho}_{3j(o+1)})\}, \quad \forall j, k. \quad (37)$$

Computation of the Prices

Computation of the Shadow Prices at the Recyclers

At iteration τ, compute the $\tilde{\gamma}_i^\tau$s according to:

$$\tilde{\gamma}_i^\tau = \max\{0, \gamma_i^{\tau-1} - \delta(\sum_{h=1}^{r} q_{hi}^{\tau-1} - \sum_{j=1}^{n+1} \alpha_{ij} q_{ij}^{\tau-1})\}, \quad \forall i.$$

Computation of the Shadow Prices at the Processors

Similarly, we can compute the $\tilde{\eta}_j^\tau$s according to:

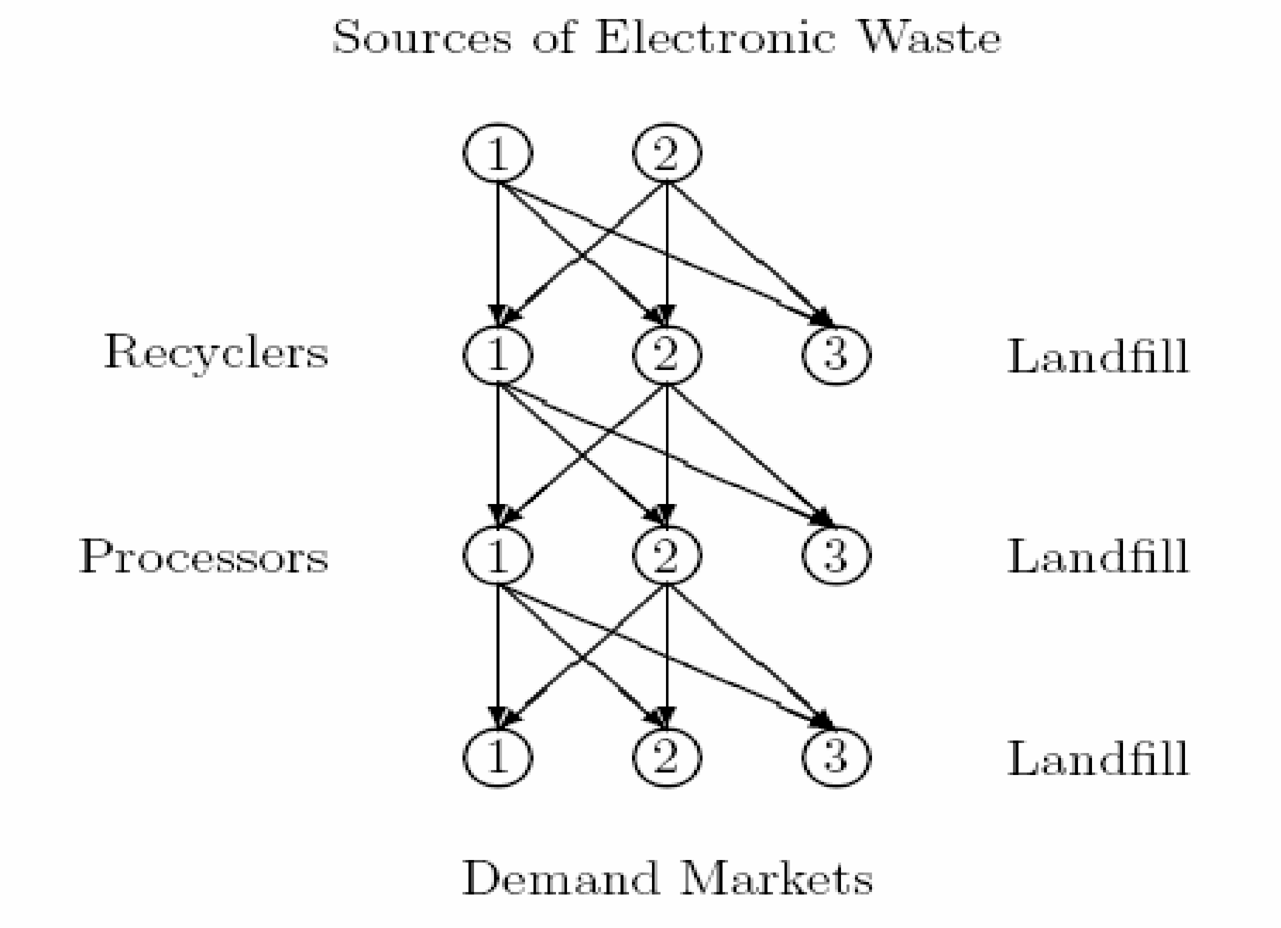
$$\tilde{\eta}_j^\tau = \max\{0, \eta_j^{\tau-1} - \delta(\sum_{i=1}^{m} q_{ij}^{\tau-1} - \sum_{k=1}^{o+1} \beta_{jk} q_{jk}^{\tau-1})\}, \quad \forall j.$$

Computation of the Demand Market Prices at the Demand Markets

Finally, we can compute at iteration τ the $\tilde{\rho}_{4k}^\tau$s according to:

$$\tilde{\rho}_{4k}^\tau = \max\{0, \rho_{4k}^{\tau-1} - \delta(\sum_{j=1}^{n} q_{jk}^{\tau-1} - d_k(\rho_4^{\tau-1}))\}, \quad \forall k.$$

E-Cycling Network for the Numerical Examples



Common Input Data for Set 1 Examples

Volumes of electronic waste	$S^1 = S^2 = 20$
Transaction cost functions between sources and recyclers (cf. (1))	$c_{hi} = .5q_{hi}^2 + 3.5, h = 1,2; i = 1,2$
Transaction cost functions between sources and the landfill (cf. (1))	$c_{h3} = .5q_{h3}^2 + 2, h = 1,2$
Second-tier landfill fixed prices	$\bar{\rho}_{1h3} = 1, h = 1,2$
Transaction cost functions between recyclers and processors (cf. (6))	$c_{ij} = .5q_{ij}^2 + 5, i = 1,2; j = 1,2$
Transaction cost functions between recyclers and the landfill (cf. (6))	$c_{i3} = .5q_{i3}^2 + 3, i = 1,2$
Third-tier landfill fixed prices	$\bar{\rho}_{2i3} = 1, i = 1,2$
Transaction cost functions between recyclers and sources (cf. (7))	$\hat{c}_{hi} = 1.5q_{hi}^2 + 3, h = 1,2; i = 1,2$
Recycling cost functions (cf. (8))	$c_i = \sum_{j=1}^{3} q_{ij}, i = 1,2$
Processing cost functions (cf. after (14))	$c_j = 2\sum_{k=1}^{3} q_{jk}, j = 1,2$
Transaction cost functions between the processors and the demand markets (cf. (13))	$c_{jk} = .5q_{jk}^2 + 1, j = 1,2; k = 1,2$
Transaction cost functions between processors and demand market pair (from the perspective of the consumers at the demand markets) (cf. (19))	$\hat{c}_{jk} = q_{jk} + 1, j = 1,2; k = 1,2$
Fourth-tier landfill fixed prices	$\bar{\rho}_{3j3} = 1, j = 1,2$
Demand functions (cf. after (19))	$d_1 = -2\rho_{41} - 1.5\rho_{42} + 1000$ $d_2 = -2\rho_{42} - 1.5\rho_{41} + 1000$

Equilibrium Solutions to Set 1 Examples (Baseline Set)

Set 1 Examples			
Equilibrium Solution	Example 1.1 $\alpha_{ij} = 1, \beta_{jk} = 1, \forall i, j, k$	Example 1.2 $\alpha_{ij} = .5, \beta_{jk} = 1, \forall i, j, k$	Example 1.3 $\alpha_{ij} = .5,\ \beta_{jk} = .25; \forall i, j, k$
Material Flows from Sources to Second Tier			
$q^*_{hi}, h = 1, 2; i = 1, 2$	10.00	10.00	9.53
$q^*_{h3}, h = 1, 2$	0.00	0.00	.95
Material Flows from Recyclers to Third Tier			
$q^*_{ij}, i = 1, 2; j = 1, 2$	10.00	20.00	19.06
$q^*_{i3}, i = 1, 2$	0,00	0.00	0.00
Material Flows from Processors to Bottom Tier			
$q^*_{jk}, j = 1, 2; k = 1, 2$	10.00	20.00	76.26
$q^*_{j3}, j = 1, 2$	0.00	0.00	0.00
Prices at the Recyclers			
$\gamma^*_i, i = 1, 2$	231.97	372.47	40.67
Prices at the Processors			
$\eta^*_j, j = 1, 2$	247.97	212.24	45.40
Prices at the Demand Markets			
$\rho^*_{4k}, k = 1, 2$	279.99	274.28	242.14

Common Input Data for Set 2 Examples

Volumes of electronic waste	$S^1 = S^2 = 20$
Transaction cost functions between sources and recyclers (cf. (1))	$c_{hi} = .5q_{hi}^2 + 3.5, h = 1,2; i = 1,2$
Transaction cost functions between sources and the landfill (cf. (1))	$c_{h3} = .5q_{h3}^2 + 2, h = 1,2$
Second-tier landfill fixed prices	$\bar{\rho}_{1h3} = 1, h = 1,2$
Transaction cost functions between recyclers and processors (cf. (6))	$c_{ij} = .5q_{ij}^2 + 5, i = 1,2; j = 1,2$
Transaction cost functions between recyclers and the landfill (cf. (6))	$c_{i3} = .5q_{i3}^2 + 3, i = 1,2$
Third-tier landfill fixed prices	$\bar{\rho}_{2i3} = 1, i = 1,2$
Transaction cost functions between recyclers and sources (cf. (7))	$\hat{c}_{hi} = 1.5q_{hi}^2 + 3, h = 1,2; i = 1,2$
Recycling cost functions (cf. (8))	$c_i = \sum_{j=1}^{3} q_{ij}, i = 1,2$
Processing cost functions (cf. after (14))	$c_j = 2\sum_{k=1}^{3} q_{jk}, j = 1,2$
Transaction cost functions between the processors and the demand markets (cf. (13))	$c_{jk} = .5q_{jk}^2 + 1, j = 1,2; k = 1,2$
Transaction cost functions between processors and demand market pair (from the perspective of the consumers at the demand markets) (cf. (19))	$\hat{c}_{jk} = q_{jk} + 1, j = 1,2; k = 1,2$
Fourth-tier landfill fixed prices	$\bar{\rho}_{3j3} = 1, j = 1,2$
Demand functions (cf. after (19))	$d_1 = -2\rho_{41} - 1.5\rho_{42} + 100$ $d_2 = -2\rho_{42} - 1.5\rho_{41} + 100$

Equilibrium Solutions to Set 2 Examples (with Demand Function Distinct from Set 1's Examples)

Set 2 Examples			
Equilibrium Solution	Example 2.1 $\alpha_{ij} = 1, \beta_{jk} = 1,$ $\forall i, j, k$	Example 2.2 $\alpha_{ij} = .5, \beta_{jk} = 1,$ $\forall i, j, k$	Example 2.3 $\alpha_{ij} = .5, \beta_{jk} = .25,$ $\forall i, j, k$
Material Flows from Sources to Second Tier			
$q^*_{hi}, h = 1, 2; i = 1, 2$ $q^8_{h3}, h = 1, 2$	3.50 13.00	2.75 14.50	2.75 14.50
Material Flows from Recyclers to Third Tier			
$q^*_{ij}, i = 1, 2; j = 1, 2$ $q^*_{i3}, i = 1, 2$	3.50 0.00	4.50 0.00	1.72 0.00
Material Flows from Processors to Bottom Tier			
$q^*_{jk}, j = 1, 2; k = 1, 2$ $q^*_{j3}, j = 1, 2$	3.50 0.00	4.50 0.00	6.90 0.00
Prices at the Recyclers			
$\gamma^*_i, i = 1, 2$	4.52	0.00	0.00
Prices at the Processors			
$\eta^*_j, j = 1, 2$	14.03	10.49	7.71
Prices at the Demand Markets			
$\rho^*_{4k}, k = 1, 2$	26.56	26.00	24.63

Input Data for Set 3 Examples

In Example 3.1, we used data identical to that of Example 2.1, except that the demand functions were

$$d_1 = -2\rho_{41} - 1.5\rho_{42} + 150, \quad d_2 = -2\rho_{42} - 1.5\rho_{41} + 100.$$

In Example 3.2, we used data identical to that of Example 3.1, except that the fixed unit prices associated with the sources using the landfill.

$$\bar{\rho}_{h3} = 10 \text{ for } h = 1, 2$$

Example 3.3 had data identical to that in Example 3.2, except that the values of the conversion rates were

$$\alpha_{ij}\text{s} = .5 \text{ for } i = 1, 2;\ j = 1, 2, 3 \qquad \beta_{jk}\text{s} = .25 \text{ for } j = 1, 2 \text{ and } k = 1, 2, 3.$$

Equilibrium Solutions to Set 3 Examples (with Demand Functions Distinct at each Market)

Set 3 Examples			
Equilibrium Solution	Example 3.1 $\alpha_{ij} = 1, \beta_{jk} = 1, \forall i, j, k$	Example 3.2 $\alpha_{ij} = 1, \beta_{jk} = 1, \forall i, j, k$	Example 3.3 $\alpha_{ij} = .5, \beta_{jk} = .25, \forall i, j, k$
Material Flows from Sources to Second Tier			
$q^*_{hi}, h = 1, 2, i = 1, 2$	5.36	5.87	4.25
$q^*_{h3}, h = 1, 2$	9.29	8.27	11.50
Material Flows from Recyclers to Third Tier			
$q^*_{ij}, i = 1, 2; j = 1, 2$	5.36	5.87	2.21
$q^*_{i3}, i = 1, 2$	0.00	0.00	0.00
Material Flows from Processors to Bottom Tier			
$q^*_{jk}, j = 1, 2; k = 1, 2$	10.73	11.75	$q^*_{j1} = 16.00; j = 1, 2$ $q^*_{j2} = 1.72, j = 1, 2$
$q^*_{j3}, j = 1, 2$	0.00	0.00	0.00
Prices at the Recyclers			
$\gamma^*_i, i = 1, 2$	15.64	9.71	0.00
Prices at the Processors			
$\eta^*_j, j = 1, 2$	27.00	21.58	8.20
Prices at the Demand Markets			
$\rho^*_{4k}, k = 1, 2$	4.11	5.87	9.22

Summary

- Developed a rigorous E-cycling mathematical model
  (the endogenous equilibrium prices and material shipments between tiers)

- Provided qualitative properties of the equilibrium electronic waste material flow and price pattern

- Provided numerical results

Thank You!

For more information on this paper and the

Virtual Center for Supernetworks and its activities see:

http://supernet.som.umass.edu