Game Theory Network Models for Disaster Relief and Blood Supply Chains

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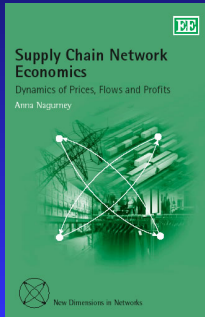Harvard University
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RADCLIFFE INSTITUTE
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Thanks to the Radcliffe Institute for Advanced Study for the opportunity to be a Summer Fellow. I remember fondly being a 2005-2006 Science Fellow at Radcliffe.

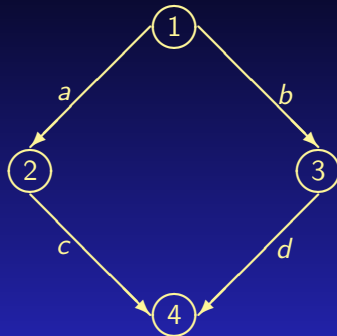While a Science Fellow I also wrote a book.

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6**, the equilibrium path flows are $x_{p_1}^* = x_{p_2}^* = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83$.

$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

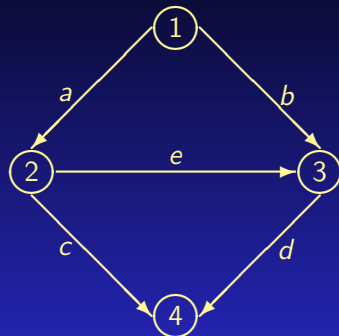Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is
$x_{p_1}^* = x_{p_2}^* = x_{p_3}^* = 2$.
The equilibrium path travel cost:
$C_{p_1} = C_{p_2} = C_{p_3} = 92$.

$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as:

On a Paradox of Traffic Planning,

Dietrich Braess, Anna Nagurney, and Tina Wakolbinger,
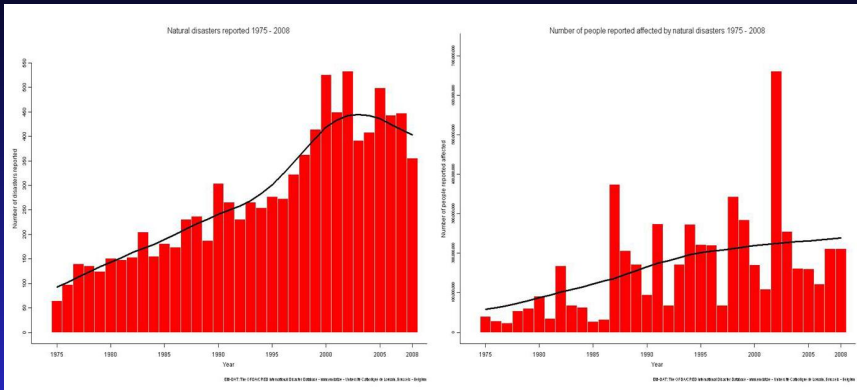*Transportation Science* 39 (2005), pp 446-450.

Network Models Are Also Very Useful in Disaster Relief

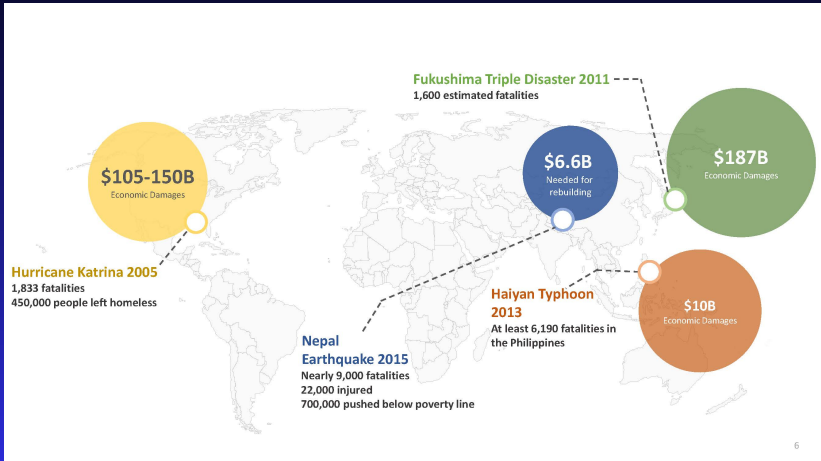# Also for Healthcare Supply Chains

Natural Disasters (1975–2008)

Disasters have a catastrophic effect on human lives and a region's or even a nation's resources. A total of 2.3 billion people were affected by natural disasters from 1995-2015 (UN Office of Disaster Risk (2015)).

Some Recent Disasters

Fukushima Triple Disaster 2011
1,600 estimated fatalities

$187B
Economic Damages

$105-150B
Economic Damages

$6.6B
Needed for rebuilding

Hurricane Katrina 2005
1,833 fatalities
450,000 people left homeless

Haiyan Typhoon 2013
At least 6,190 fatalities in the Philippines

$10B
Economic Damages

Nepal Earthquake 2015
Nearly 9,000 fatalities
22,000 injured
700,000 pushed below poverty line

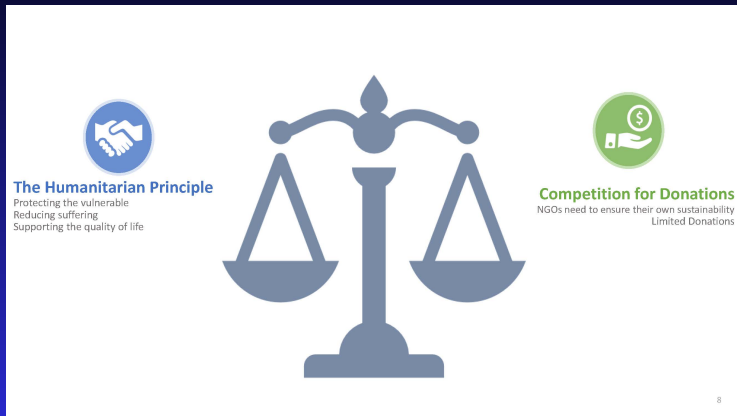Hurricane Katrina has been called an *"American tragedy,"* in which essential services failed completely.

Manhattan without power October 30, 2012 as a result of the devastation wrought by Superstorm Sandy.

Challenges Associated with Disaster Relief

• Timely delivery of relief items is challenged by damaged and destroyed infrastructure (transportation, telecommunications, hospitals, etc.).

• Shipments of the wrong supplies create congestion and materiel convergence (sometimes referred to as the second disaster).

• • Within three weeks following the 2010 earthquake in Haiti, 1,000 NGOs were operating in Haiti. News media attention of insufficient water supplies resulted in immense donations to the Dominican Red Cross to assist its island neighbor. Port-au-Price was saturated with both cargo and gifts-in-kind.

• • After the Fukushima disaster, there were too many blankets and items of clothing shipped and even broken bicycles.

• • After Katrina, even tuxedos were delivered to victims.

Better coordination among NGOs is needed.

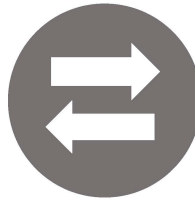# Challenges Associated with Disaster Relief - The NGO Balancing Act

There were 1.5 million registered NGOs in the US in 2012. $300 billion in donations given yearly to US charities.

Disasters

Will pose an ever increasing risk to the most vulnerable people on the planet.

NGOs

Will need to adapt their delivery mechanisms to an era of uncertainty and increased competition.

Therefore

there is a need to *develop appropriate analytical tools* that can assist NGOs, as well as governments in modeling the complex interactions in disaster relief to improve outcomes.

Although there have been quite a few optimization models developed for disaster relief there are very few game theory models.

Toyasaki and Wakolbinger (2014) constructed perhaps the first models of financial flows that captured the strategic interaction between donors and humanitarian organizations using game theory and also included earmarked donations.

We developed the first Generalized Nash Equilibrium (GNE) model for post-disaster humanitarian relief, which contains both a financial component and a supply chain component. The Generalized Nash Equilibrium problem is a generalization of the Nash Equilibrium problem (cf. Nash (1950, 1951)).

"A Generalized Nash Equilibrium Network Model for Post-Disaster Humanitarian Relief," Anna Nagurney, Emilio Alvarez Flores, and Ceren Soylu, *Transportation Research E* **95** (2016), pp 1-18.
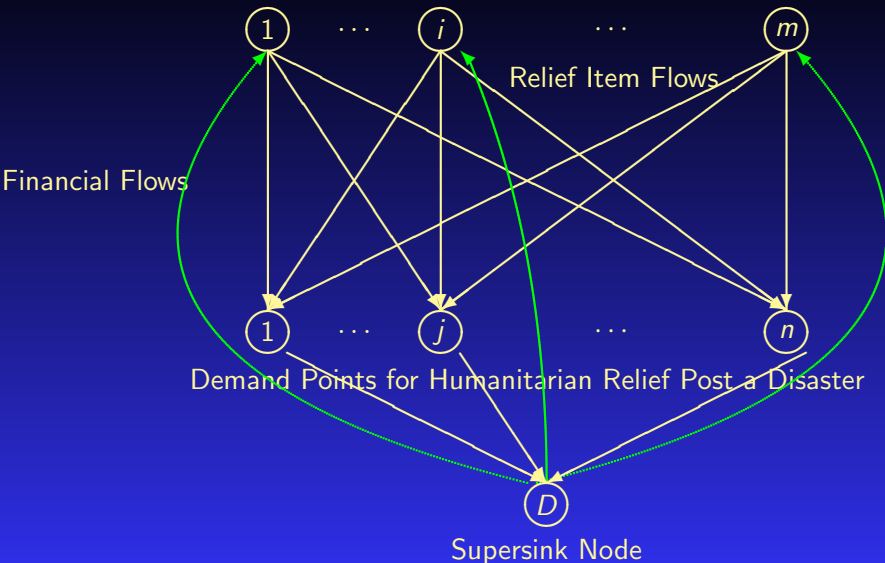
The Network Structure of the Model

Figure 1: The Network Structure of the Game Theory Model

The Game Theory Model

We assume that each NGO i has, at its disposal, an amount s_i of the relief item that it can allocate post-disaster. Hence, we have the following conservation of flow equation, which must hold for each i; $i = 1, \ldots, m$:

$$\sum_{j=1}^{n} q_{ij} \leq s_i. \tag{1}$$

In addition, we know that the product flows for each i; $i = 1, \ldots, m$, must be nonnegative, that is:

$$q_{ij} \geq 0, \quad j = 1, \ldots, n. \tag{2}$$

Each NGO i encumbers a cost, c_{ij}, associated with shipping the relief items to location j, denoted by c_{ij}, where we assume that

$$c_{ij} = c_{ij}(q_{ij}), \quad j = 1, \ldots n, \tag{3}$$

with these cost functions being strictly convex and continuously differentiable.

In addition, each NGO i; $i = 1, \ldots, m$, derives satisfaction or utility associated with providing the relief items to j; $j = 1, \ldots, n$, with its utility over all demand points given by $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$. Here γ_{ij} is a positive factor representing a measure of satisfaction/utility that NGO i acquires through its supply chain activities to demand point j.

Each NGO i; $i = 1, \ldots, m$, associates a positive weight ω_i with $\sum_{j=1}^{n} \gamma_{ij} q_{ij}$, which provides a monetization of, in effect, this component of the objective function.

The Game Theory Model

Finally, each NGO i; $i = 1, \ldots, m$, based on the media attention and the visibility of NGOs at location j; $j = 1, \ldots, n$, acquires funds from donors given by the expression

$$\beta_i \sum_{j=1}^{n} P_j(q), \tag{4}$$

where $P_j(q)$ represents the financial funds in donation dollars due to visibility of all NGOs at location j. Hence, β_i is a parameter that reflects the proportion of total donations collected for the disaster at demand point j that is received by NGO i.

Expression (4), therefore, represents the financial flow on the link joining node D with node NGO i.

The Game Theory Model

Each NGO i seeks to maximize its utility with the utility corresponding to the financial gains associated with the visibility through media of the relief item flow allocations, $\beta_i \sum_{j=1}^{n} P_j(q)$, plus the utility associated with the supply chain aspect of delivery of the relief items, $\omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij})$.

The optimization problem faced by NGO i; $i = 1, \ldots, m$, is, hence,

$$\text{Maximize} \quad \beta_i \sum_{j=1}^{n} P_j(q) + \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} - \sum_{j=1}^{n} c_{ij}(q_{ij}) \qquad (5)$$

subject to constraints (1) and (2).

The Game Theory Model

We also have that, at each demand point j; $j = 1, \ldots, n$:

$$\sum_{i=1}^{m} q_{ij} \geq \underline{d}_j, \tag{6}$$

and

$$\sum_{i=1}^{m} q_{ij} \leq \bar{d}_j, \tag{7}$$

where $\underline{d}_j$ denotes a lower bound for the amount of the relief items needed at demand point j and $\bar{d}_j$ denotes an upper bound on the amount of the relief items needed post the disaster at demand point j.

We assume that

$$\sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} \underline{d}_j, \tag{8}$$

so that the supply resources of the NGOs are sufficient to meet the minimum financial resource needs.

Each NGO i; $i = 1, \ldots, m$, seeks to determine its optimal vector of relief items or strategies, q_i^*, that maximizes objective function (5), subject to constraints (1), (2), and (6), (7).

The Game Theory Model

Theorem: Optimization Formulation of the Generalized Nash Equilibrium Model of Financial Flow of Funds

The above Generalized Nash Equilibrium problem, with each NGO's objective function (5) rewritten as:

$$\text{Minimize} \quad -\beta_i \sum_{j=1}^{n} P_j(q) - \omega_i \sum_{j=1}^{n} \gamma_{ij} q_{ij} + \sum_{j=1}^{n} c_{ij}(q_{ij}) \quad (9)$$

and subject to constraints (1) and (2), with common constraints (6) and (7), is equivalent to the solution of the following optimization problem:

$$\text{Minimize} \quad -\sum_{j=1}^{n} P_j(q) - \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\omega_i \gamma_{ij}}{\beta_i} q_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{1}{\beta_i} c_{ij}(q_{ij}) \quad (10)$$

subject to constraints: (1), (2), (6), and (7).

Hurricane Katrina Case Study

Making landfall in August of 2005, Katrina caused extensive damage to property and infrastructure, left 450,000 people homeless, and took 1,833 lives in Florida, Texas, Mississippi, Alabama, and Louisiana (Louisiana Geographic Information Center (2005)).

Given the hurricane's trajectory, most of the damage was concentrated in Louisiana and Mississippi. In fact, 63% of all insurance claims were in Louisiana, a trend that is also reflected in FEMA's post-hurricane damage assessment of the region (FEMA (2006)).

Hurricane Katrina Case Study

The total damage estimates range from $105 billion (Louisiana Geographic Information Center (2005)) to $150 billion (White (2015)), making Hurricane Katrina not only a far-reaching and costly disaster, but also a very challenging environment for providing humanitarian assistance.

We consider 3 NGOs: the Red Cross, the Salvation Army, and Others and 10 Parishes in Louisiana.
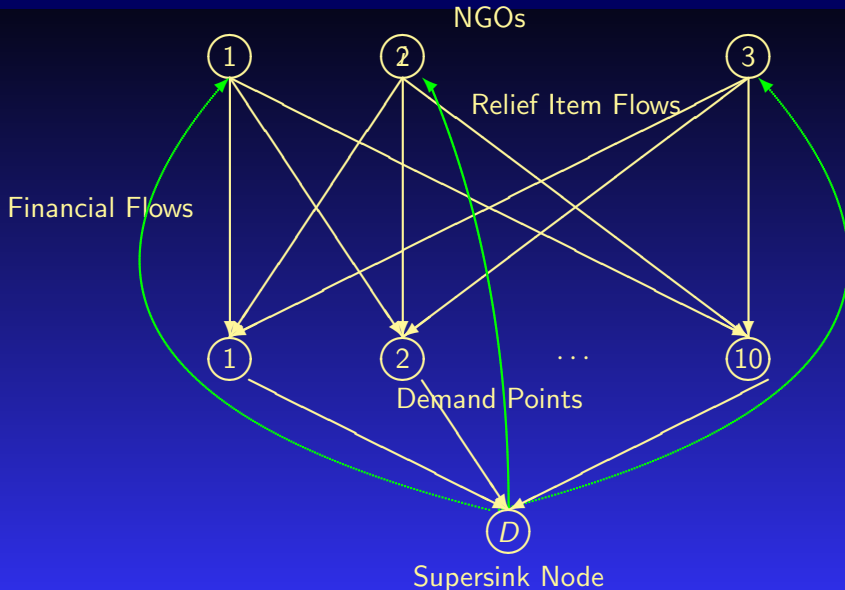
Figure 2: Hurricane Katrina Relief Network Structure

The structure of the P_j functions is as follows:

$$P_j(q) = k_j \sqrt{\sum_{i=1}^{m} q_{ij}}.$$

The weights are:

$$\omega_1 = \omega_2 = \omega_3 = 1,$$

with $\gamma_{ij} = 950$ for $i = 1, 2, 3$ and $j = 1, \ldots, 10$.

Hurricane Katrina Case Study

Hurricane Katrina Demand Point Parameters					
Parish	Node j	k_j	$\underline{d_j}$	$\bar{d_j}$	p_j (in %)
St. Charles	1	8	16.45	50.57	2.4
Terrebonne	2	16	752.26	883.82	6.7
Assumption	3	7	106.36	139.24	1.9
Jefferson	4	29	742.86	1,254.89	19.5
Lafourche	5	6	525.53	653.82	1.7
Orleans	6	42	1,303.99	1,906.80	55.9
Plaquemines	7	30	33.28	62.57	57.5
St. Barnard	8	42	133.61	212.43	78.4
St. James	9	9	127.53	166.39	1.2
St. John the Baptist	10	7	19.05	52.59	6.7

Table 1: Demand Point Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

We then estimated the cost of providing aid to the Parishes as a function of the total damage in the area and the supply chain efficiency of each NGO. We assume that these costs follow the structures observed by Van Wassenhove (2006) and randomly generate a number based on his research with a mean of $\hat{p} = .8$ and standard deviation of $s = \sqrt{\frac{.8(.2)}{3}}$.

We denote the corresponding coefficients by π_i. Thus, each NGO i; $i = 1, 2, 3$, incurs costs according the the following functional form:

$$c_{ij}(q_{ij}) = \left(\pi_i q_{ij} + \frac{1}{1 - p_j}\right)^2.$$

Data Parameters for NGOs Providing Aid					
NGO	i	π_i	γ_{ij}	β_i	s_i
Others	1	.82	950	.355	1,418
Red Cross	2	.83	950	.55	2,200
Salvation Army	3	.81	950	.095	382

Table 2: NGO Data for the Generalized Nash Equilibrium Problem for Hurricane Katrina

Hurricane Katrina Case Study

Generalized Nash Equilibrium Product Flows (in Millions of Aid Units)			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	17.48	28.89	4.192
Terrebonne	267.023	411.67	73.57
Assumption	49.02	77.26	12.97
Jefferson	263.69	406.68	72.45
Lafourche	186.39	287.96	51.18
Orleans	463.33	713.56	127.1
Plaquemines	21.89	36.54	4.23
St. Barnard	72.31	115.39	16.22
St. James	58.67	92.06	15.66
St. John the Baptist	18.2	29.99	4.40

Table 3: Flows to Demand Points under Generalized Nash Equilibrium

Hurricane Katrina Case Study

The total utility obtained through the above flows for the Generalized Nash Equilibrium for Hurricane Katrina is 9, 257, 899, with the Red Cross capturing 3,022,705, the Salvation Army 3,600,442.54, and Others 2,590,973.

In addition, we have that the Red Cross, the Salvation Army, and Others receive 2,200.24, 1418.01, and 382.31 million in donations, respectively.

The relief item flows meet at least the lower bound, even if doing so is very expensive due to the damages to the infrastructure in the region.

Furthermore, the above flow pattern behaves in a way that, after the minimum requirements are met, any additional supplies are allocated in the most efficient way. For example, only the minimum requirements are met in New Orleans Parish, while the upper bound is met for St. James Parish.

If we remove the shared constraints, we obtain a Nash Equilibrium solution, and we can compare the outcomes of the humanitarian relief efforts for Hurricane Katrina under the Generalized Nash Equilibrium concept and that under the Nash Equilibrium concept.

The Nash Equilibrium Solution

Nash Equilibrium Product Flows			
Demand Point	Others	Red Cross	Salvation Army
St. Charles	142.51	220.66	38.97
Terrebonne	142.50	220.68	38.93
Assumption	142.51	220.66	38.98
Jefferson	142.38	220.61	38.74
Lafourche	142.50	220.65	38.98
Orleans	141.21	219.59	37.498
Plaquemines	141.032	219.28	37.37
St. Barnard	138.34	216.66	34.59
St. James	142.51	220.65	38.58
St. John the Baptist	145.51	220.66	38.98

Table 4: Flows to Demand Points under Nash Equilibrium

The Nash Equilibrium Solution

Under the Nash Equilibrium, the NGOs obtain a higher utility than under the Generalized Nash Equilibrium. Specifically, of the total utility $10,346,005.44$, 2,804,650 units are received by the Red Cross, 5,198,685 by the Salvation Army, and 3,218,505 are captured by all other NGOs.

Under this product flow pattern, there are total donations of 3,760.73, of which 2,068.4 are donated to the Red Cross, 357.27 to the Salvation Army, and 1,355 to the other players.

It is clear that there is a large contrast between the flow patterns under the Generalized Nash and Nash Equilibria. For example, the Nash Equilibrium flow pattern results in about $500 million less in donations.

While this has strong implications about how collaboration between NGOs can be beneficial for their fundraising efforts, the differences in the general flow pattern highlights a much stronger point.

Under the Nash Equilibrium, NGOs successfully maximize their utility. Overall, the Nash Equilibrium solution leads to an increase of utility of roughly 21% when compared to the flow patterns under the Generalized Nash Equilibrium.

But they do so at the expense of those in need. In the Nash Equilibrium, each NGO chooses to supply relief items such that costs can be minimized. On the surface, this might be a good thing, but recall that, given the nature of disasters, it is usually more expensive to provide aid to demand points with the greatest needs.

Additional Insights

With this in mind, one can expect oversupply to the demand points with lower demand levels, and undersupply to the most affected under a purely competitive scheme. This behavior can be seen explicitly in the results summarized in the Tables.

For example, St. Charles Parish receives roughly 795% of its upper demand, while Orleans Parish only receives about 30.5% of its minimum requirements. That means that much of the 21% in 'increased' utility is in the form of waste.

In contrast, the flows under the Generalized Nash Equilibrium guarantee that minimum requirements will be met and that there will be no waste; that is to say, *as long as there is a coordinating authority that can enforce the upper and lower bound constraints, the humanitarian relief flow patterns under this bounded competition will be significantly better than under untethered competition.*

Extension of the Model

Figure 3: The Network Structure of the Extended Game Theory Model

The Extended Model

The extended model captures competition for logistic services, has more general benefit functions, as well as financial donation functions.

This work, "A Variational Equilibrium Network Framework for Humanitarian Organizations in Disaster Relief: Effective Product Delivery Under Competition for Financial Funds," 2017, is joint with Patrizia Daniele, Emilio Alvarez Flores, and Valeria Caruso and will be presented at the Dynamics of Disasters conference that I have co-organized with Fuad Aleskerov, Ilias S. Kotsireas, and Panos M. Pardalos. The conference will take place in Kalamata, Greece, July 5-9, 2017.

In the new model, we can no longer reformulate the Generalized Nash Equilibrium as an optimization problem but do so as a Variational Equilibrium.

Our case study is inspired by a disaster consisting of a series of tornados that hit western Massachusetts on June 1, 2011. The largest tornado was measured at EF3. It was the worst tornado outbreak in the area in a century (see Flynn (2011)). A wide swath from western to central MA of about 39 miles was impacted.

The tornado killed 4 persons, injured more than 200 persons, damaged or destroyed 1,500 homes, left over 350 people homeless in Springfield's MassMutual Center arena, left 50,000 customers without power, and brought down thousands of trees.

FEMA estimated that 1,435 residences were impacted with the following breakdowns: 319 destroyed, 593 sustaining major damage, 273 sustaining minor damage, and 250 otherwise affected. FEMA estimated that the primary impact was damage to buildings and equipment with a cost estimate of $24,782,299.

Total damage estimates from the storm exceeded $140 million, the majority from the destruction of homes and businesses.

Especially impacted were the city of Springfield and the towns of Monson and Brimfield. It has been estimated that, in the aftermath, the Red Cross served about 11,800 meals and the Salvation Army about 20,000 meals (cf. Western Massachusetts Regional Homeland Security Advisory Council (2012)).

We consider the American Red Cross and the Salvation Army as the NGOs. The demand points are: Springfield, Monson, and Brimfield.

We find in multiple examples comprising our case study of Massachusetts tornados that the NGOs garner greater financial funds through the Generalized Nash Equilibrium solution, rather than the Nash equilibrium one. Moreover, the needs of the victims are met under the Generalized Nash Equilibrium solution.

This further supports our contention that policy makers should impose realistic lower and upper bounds at demand points for relief item supplies. Not only do victims benefit but NGOs can as well financially in terms of donations.

While at Radcliffe I am also working on blood supply chains.

Blood Supply Chains

The American Red Cross is the major supplier of blood products to hospitals and medical centers satisfying about 45% of the demand for blood components nationally.

- The shelf life of platelets is 5 days and of red blood cells is 42.
- Over 39,000 donations are needed everyday in the US.
- Blood is a perishable product that cannot be manufactured but must be donated.
- As of February 1, 2016, the American Red Cross was facing an emergency need for blood and platelet donors. Severe winter weather in January forced the cancellation of more than 340 blood drives in 20 states, resulting in nearly 10,000 donations uncollected.
- There is increasing competition among blood service organizations for donors and, interestingly, overall, there has been a decrease in demand because of improved medical procedures, for example.
- Pressure to reduce costs is resulting in mergers and acquisitions in the blood services industry.

Blood Supply Chains

We (Nagurney, Masoumi, and Yu) developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of human blood.

Novel features of the model include:

- It captures *perishability of this life-saving product* through the use of arc multipliers;

- It contains *discarding costs* associated with waste/disposal;

- It handles *uncertainty* associated with demand points;

- It assesses *costs associated with shortages/surpluses at the demand points*, and

- It quantifies the *supply-side risk* associated with procurement.

We have begun research on a game theory model for blood donations (with Pritha Dutta) and are also working on capturing competition for blood services among hospitals and medical centers.

In addition, along with Amir Masoumi and Min Yu, we are constructing network models to assess possible synergies associated with mergers and acquisitions among blood service organizations, taking into account capacities and frequencies of various supply chaain network link activities.

Figure 4: Supply Chain Network Topology Pre-Merger

Figure 5: Supply Chain Network Topology Post-Merger

THANK YOU!

For more information, see:
http://supernet.isenberg.umass.edu