Network Efficiency/Performance Measurement with Vulnerability and Robustness Analysis with Application to Critical Infrastructure

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- Algorithm
- A Weighted Supply Chain Performance Measure
- Numerical Examples

6. Identification of Critical Nodes and Links in Financial Networks
   - The Financial Network Model
   - The Financial Network Performance Measure
   - The Importance of a Financial Network Component

7. Conclusions and Future Research Plan

Trend of Increasing Number of Disasters

- Recent disasters have demonstrated the importance as well as the vulnerability of network systems.
- For example:
  - ◇ 9/11 Terrorist Attacks, September 11, 2001
  - ◇ The biggest blackout in North America, August 14, 2003
  - ◇ Two significant power outages during the month of September 2003 - one in England and one in Switzerland and Italy
  - ◇ Hurricane Katrina, August 23, 2005
  - ◇ Minneapolis Bridge Collapse, August 1, 2007
  - ◇ Sichuan Earthquake, May 12, 2008

Facts on the Impact of Critical Infrastructure Disruptions

- The 2008 Chinese winter storm caused $12 billion in damage to the nation's economy (BBC News, February 1, 2008). Due to the transport breakdowns, more than 180 million people were stranded on their way home during the Chinese holiday season. Moreover, millions of people suffered from power outages and substantial food price inflation because of the transportation delays in raw materials and crops (BBC News, January 31, 2008).

The U.S. Ailing Infrastructure I

- Over one-quarter of the nation's 590,750 bridges were rated structurally deficient or functionally obsolete. The degradation of transportation networks due to poor maintenance, natural disasters, deterioration over time, as well as unforeseen attacks now lead to estimates of $94 billion in the U.S. in terms of needed repairs for roads alone. Poor road conditions in the U.S. cost motorists $54 billion in repairs and operating costs annually (ASCE Survey (2005)).

The U.S. Ailing Infrastructure II

- The U.S. is experiencing a freight capacity crisis that threatens the strength and productivity of the U.S. economy. According to the American Road & Transportation Builders Association (see Jeanneret (2006)), nearly 75% of U.S. freight is carried in the U.S. on highways, and bottlenecks are causing truckers 243 million hours of delay annually with an estimated associated cost of $8 billion.
- The number of motor vehicles in the U.S. has risen by 157 million (or 212.16%) since 1960 while the population of licensed drivers grew by 109 million (or 125.28%) (U.S. Department of Transportation (2004)).

Transportation Network Capacity and Its Environmental Impact

- According to a U.S. EPA (2006) report, the transportation sector in 2003 accounted for 27% of the total greenhouse gas emissions in the U.S. and the increase in this sector was the largest of any in the period 1990 – 2003.
- A study claims that infrastructure capacity increases are directly linked to decreases in polluting emissions from motor vehicles. Using a traffic micro-simulation, it showed, for example, that upgrading narrow, winding roads or adding a lane to a congested motorway can yield decreases of up to 38% in CO_2 emissions, 67% in CO emissions and 75% in NO_x emissions, without generating substantially more car trips (Knudsen and Bang (2007)).

Figure: Examples from Alaska, Source: Smith and Levasseur

Figure: Roads are Damaged by Floods, Source: The Oregonian

- Algorithm
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6 Identification of Critical Nodes and Links in Financial Networks
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7 Conclusions and Future Research Plan

Network Centrality Measures

- Barrat et al. (2004, pp. 3748), "The identification of the most central nodes in the system is a major issue in network characterization."
- Centrality Measures for Non-weighted Networks
  - Degree, betweenness (node and edge), closeness (Freeman (1979), Girvan and Newman (2002))
  - Eigenvector centrality (Bonacich (1972))
  - Flow centrality (Freeman, Borgatti and White (1991))
  - Betweenness centrality using flow (Izquierdo and Hanneman (2006))
  - Random-work betweenness, Current-flow betweenness (Newman and Girvan (2004))
- Centrality Measures for Weighted Networks (Very Few)
  - Weighted betweenness centrality (Dall'Asta et al. (2006))
  - Network Efficiency Measure (Latora-Marchiori (2001))

Network Vulnerability Analysis in Complex Network Research

- Non-weighted Networks
  - Random attack vs. Target attack (Albert, Jeong and Barabási (2000))
  - Applications include power grids (cf. Albert, Albert, and Nakarado (2004), Chassin and Posse (2005), Holme et al. (2002), and Holmgren (2007)), the Internet (cf. Doyle et al. (2005), and Cohen et al. (2000a) and (2000b)), and worldwide air transportation networks (Guimerà et al. (2005))
- Weighted Networks
  - Worldwide air transportation networks (Dall'Asta et al. (2006))
  - MBTA and the Internet (Latora and Marchiori (2002, 2004))

Network Vulnerability Research in Transportation

- System surplus as a performance measure (Nicholson and Du (1997))
- Link importance indicators (Jenelius, Petersen, and Mattsson (2006))
- Vulnerability index and disruption index (Murray-Tuite and Mahmassani (2004))
- Game between travelers and an “evil” entity (Bell (2000))
- Other related research (Taylor, Sekhar and D’Este (2006), Dueñas-Osorio, Craig, and Goodno (2005))

Network Robustness Research

- Definition
  - *Robustness* is "the degree to which a system or component can function correctly in the presence of invalid inputs or stressful environmental conditions." (IEEE (1990))
  - "Robustness signifies that the system will retain its system structure (function) intact (remain unchanged or nearly unchanged) when exposed to perturbations." (Holmgren (2007))
  - Schillo et al. (2001) argued that robustness has to be studied "in relation to some definition of performance measure."
- Related network robustness studies in the complex network research: Callaway et al. (2000), Albert, Jeong, and Barabási (2000), Newman (2003), Albert, Albert, and Nakarado (2004), and Holme et al. (2002).
- Related network robustness studies in transportation research: Sakakibara, Kajitani, and Okada (2004) and Scott et al. (2006).

Our Research on Network Efficiency, Vulnerability, and Robustness I

- Nagurney, A., Qiang, Q., 2007a. A network efficiency measure for congested networks. *Europhysics Letters* 79, 38005, 1-5.
- Nagurney, A., Qiang, Q., 2007b. A transportation network efficiency measure that captures flows, behavior, and costs with applications to network component importance identification and vulnerability, In *Proceedings of the 18th Annual POMS Conference*, Dallas, Texas.
- Nagurney, A., Qiang, Q., 2007c. Robustness of transportation networks subject to degradable links *Europhysics Letters* 80, 68001, 1-6.
- Nagurney, A., Qiang, Q., 2008a. A network efficiency measure with application to critical infrastructure networks. *Journal of Global Optimization* 40, 261-275.

Our Research on Network Efficiency, Vulnerability, and Robustness II

- Nagurney, A., Qiang, Q., 2008b. Identification of critical nodes and links in financial networks with intermediation and electronic transactions. In **Computational Methods in Financial Engineering**, E. J. Kontoghiorghes, B. Rustem, and P. Winker, Editors, Springer, Berlin, Germany, pp. 273-297.
- Nagurney, A., Qiang, Q., 2008c. An efficiency measure for dynamic networks with application to the Internet and vulnerability analysis. *Netnomics 9*, 1-20.
- Nagurney, A., Qiang, Q., 2008d. A relative total cost index for the evaluation of transportation network robustness in the presence of degradable links and alternative travel behavior. *International Transactions in Operational Research* 16, 49-67.

Our Research on Network Efficiency, Vulnerability, and Robustness III

- Nagurney, A., Qiang, Q., Nagurney, L. S., 2008. Environmental impact assessment of transportation networks with degradable links in an era of climate change. To appear in the *International Journal of Sustainable Transportation*.
- Qiang, Q., Nagurney, A., Dong, G., 2009. Modeling of supply chain risk under disruptions with performance measurement and robustness analysis. In **Managing Supply Chain Risk and Vulnerability: Tools and Methods for Supply Chain Decision Makers**. Wu, T., Blackhurst, J., Editors, Springer, Berlin, Germany, in press.

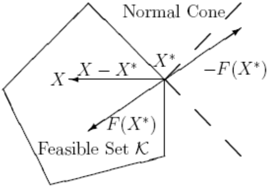
Variational Inequality Theory

Definition: Variational Inequality (VI)

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset \mathcal{R}^n$, such that

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K} \tag{1}$$

where F is a given continuous function from $\mathcal{K}$ to $\mathcal{R}^n$, $\mathcal{K}$ is a given closed convex set and $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional space.



Wardrop's Two Principles

Over half a century ago, Wardrop (1952) explicitly considered alternative possible behaviors of users of transportation networks, notably, urban transportation networks and stated two principles, which are commonly named after him:

- First Principle: The journey times of all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.
- Second Principle: The average journey time is minimal.

The first principle corresponds to the behavioral principle in which travelers seek to (unilaterally) determine their minimal costs of travel whereas the second principle corresponds to the behavioral principle in which the total cost in the network is minimal.

The Latora-Marchiori (L-M) Measure

Latora and Marchiori (2001) proposed a network performance/efficiency measure, $E(G)$, of a network G, defined as:

$$E(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}, \tag{2}$$

where n is the number of nodes in the network and d_{ij} is the shortest path length between node i and node j.

- According to the L-M measure, a network is efficient if nodes are close to each other.
- No information on network flows, the flow induced costs, and the behavior of users of the network.

The Zhu et al. Measure

Zhu et al. (2006) introduced another measure, which I denote as $\hat{E}(G)$, for a network with fixed demands:

$$\hat{E}(G) = \frac{\sum_{w \in W} \lambda_w d_w}{\sum_{w \in W} d_w} \tag{3}$$

- Assume network is connected.
- Not a suitable measure for network with elastic demands.

A Unified Network Efficiency/Performance Measure

Qiang and Nagurney (2008) (see also Nagurney and Qiang (2007a, b, e)) propose a unified network efficiency/performance measure: The network performance/efficiency measure, $\mathcal{E}(G, d)$, for a given network topology G and the equilibrium (or fixed) demand vector d, is defined as follows:

$$\mathcal{E} = \mathcal{E}(G, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_W}, \tag{4}$$

where recall that n_W is the number of O/D pairs in the network, and d_w and λ_w denote, for simplicity, the equilibrium (or fixed) demand and the equilibrium disutility for O/D pair w, respectively.

- The demand d_w is measured over a period of time, such as an hour, whereas λ_w is the minimum equilibrium travel cost (or time) associated with the O/D pair w.
- For general networks, the performance/efficiency measure $\mathcal{E}$ is the average demand to price ratio. When G and d are fixed, a network is more efficient if it can satisfy a higher demand at a lower price!

The Importance of a Network Component

The importance of a network component $g \in G$, $I(g)$, is measured by the relative network efficiency drop after g is removed from the network:

$$I(g) = \frac{\triangle \mathcal{E}}{\mathcal{E}} = \frac{\mathcal{E}(G,d) - \mathcal{E}(G-g,d)}{\mathcal{E}(G,d)} \tag{5}$$

where $G - g$ is the resulting network after component g is removed from network G.

The elimination of a link is treated by removing that link from the network while the removal of a node is managed by removing the links entering or exiting that node. In the case that the removal results in no path connecting an O/D pair, I simply assign the demand for that O/D pair (either fixed or elastic) to an abstract path with a cost of infinity.

Example 1

There are two O/D pairs: $w_1 = (1,2)$ and $w_2 = (1,3)$ with demands given, respectively, by $d_{w_1} = 100$ and $d_{w_2} = 20$. We have that path $p_1 = a$ and path $p_2 = b$. Assume that the link cost functions are given by: $c_a(f_a) = .01f_a + 19$ and $c_b(f_b) = .05f_b + 19$. Clearly, we must have that $x^*_{p_1} = 100$ and $x^*_{p_2} = 20$ so that $\lambda_{w_1} = \lambda_{w_2} = 20$. The proposed network measure $\mathcal{E} = 3.0000$ whereas the L-M measure $E = .0167$.

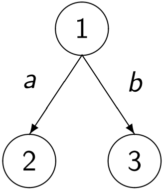


Figure: Example 1

Importance and Rankings of Nodes and Links in Example 1

Table: Importance Values and Ranking of Links in Example 1

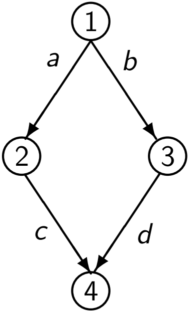
Link	Importance Value from the Proposed Measure	Importance Ranking from the Proposed Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	0.8333	1	0.5000	1
b	0.1667	2	0.5000	1

Table: Importance Values and Ranking of Nodes in Example 1

Node	Importance Value from the Proposed Measure	Importance Ranking from the Proposed Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.0000	1	1.0000	1
2	0.8333	2	0.5000	2
3	0.1667	3	0.5000	2

Example 2: An Application of the Proposed Measure to the Braess Paradox Network (Braess (1968))

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$. For a travel demand of 6, the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and The equilibrium path travel costs are $C_{p_1} = C_{p_2} = 83$.

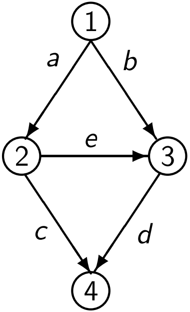


$c_a(f_a) = 10f_a \quad c_b(f_b) = f_b + 50$

$c_c(f_c) = f_c + 50 \quad c_d(f_d) = 10f_d$

Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$. The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, C_{p_3}=70. The new equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$. The equilibrium path travel costs are $C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

Importance and Rankings of the Braess Paradox Network

Table: Importance and Ranking of Links in the Braess Paradox Network

Link	Importance Value from the New Measure	Importance Ranking from the New Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
a	0.2069	1	0.1056	3
b	0.1794	2	0.2153	2
c	0.1794	2	0.2153	2
d	0.2069	1	0.1056	3
e	-0.1084	3	0.3616	1

Table: Importance and Ranking of Nodes in the Braess Paradox Network

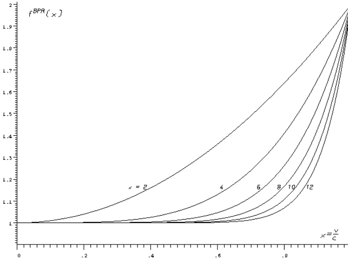
Node	Importance Value from the New Measure	Importance Ranking from the New Measure	Importance Value from the L-M Measure	Importance Ranking from the L-M Measure
1	1.0000	1	N/A	N/A
2	0.2069	2	0.7635	1
3	0.2069	2	0.7635	1
4	1.0000	1	N/A	N/A

Example 3: The Sioux-Falls Network (cf. LeBlanc, Morlok, and Pierskalla (1975))

The Bureau of Public Road (BPR) link travel cost function form is used, which is given by (see Bureau of Public Road (1964), and Sheffi (1985)):

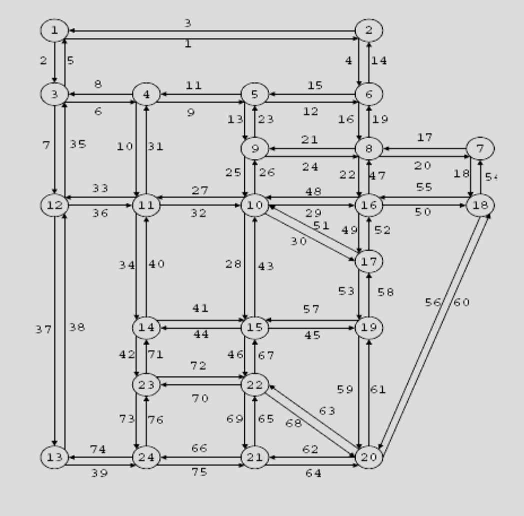
$$c_a(f_a) = t_a^0[1 + k(\frac{f_a}{u_a})^{\beta}], \quad \forall a \in L, \qquad (6)$$

where f_a is the flow on link a; u_a is the "practical" capacity on link a, which also has the interpretation of the level-of-service flow rate; t_a^0 is the free-flow travel time or cost on link a; k and β are the model parameters and both take on positive values.



Measure for Static Networks

The network topology is shown in below. There are 528 O/D pairs, 24 nodes, and 76 links in the Sioux-Falls network.



Algorithms

- The projection method (cf. Dafermos (1980) and Nagurney (1999)) with the embedded Dafermos and Sparrow (1969) equilibration algorithm (see also, e.g., Nagurney (1984)) and the column generation algorithm (cf. Leventhal, Nemhauser, and Trotter (1973)) were utilized to compute the equilibrium solutions.
- Based on the equilibrium solutions, the network efficiency was determined and the importance values and the importance rankings of the links were computed.

Measure for Static Networks

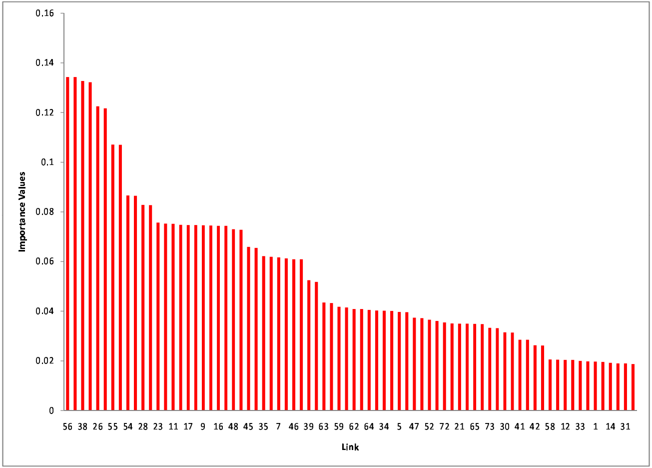


Figure: Link Importance Rankings in the Sioux-Falls Networks

Roughgarden (2005) on page 10 notes that, “A network like the Internet is volatile. Its traffic patterns can change quickly and dramatically ... The assumption of a static model is therefore particularly suspect in such networks.”

Dynamic Network Equilibrium

A route flow pattern $x^* \in \mathcal{K}$ is said to be a dynamic network equilibrium (according to the generalization of Wardrop's first principle), if, at each time t, only the minimum cost routes not at their capacities are used (that is, have positive flow) for each O/D pair unless the flow on a route is at its upper bound (in which case those routes' costs can be lower than those on the routes not at their capacities). The state can be expressed by the following equilibrium conditions which must hold for every O/D pair $w \in W$, every route $r \in P_w$, and a.e. on $[0, T]$:

$$C_r(x^*(t)) - \lambda_w^*(t) \begin{cases} \leq 0, & \text{if } \quad x_r^*(t) = \mu_r(t), \\ = 0, & \text{if } \quad 0 < x_r^*(t) < \mu_r(t), \\ \geq 0, & \text{if } \quad x_r^*(t) = 0. \end{cases} \tag{7}$$

Theorem of Nagurney, Parkes, and Daniele (2007)

$x^* \in \mathcal{K}$ is an equilibrium flow according to the definition of dynamic network equilibrium if and only if it satisfies the evolutionary variational inequality:

$$\int_0^T \langle C(x^*(t)), x(t) - x^*(t) \rangle dt \geq 0, \quad \forall x \in \mathcal{K}. \tag{8}$$

Network Efficiency Measure for Dynamic Networks - Continuous Time

The network efficiency for the network G with time-varying demand d for $t \in [0, T]$, denoted by $\mathcal{E}(G, d, T)$, is defined as follows:

$$\mathcal{E}(G, d, T) = \frac{\int_0^T [\sum_{w \in W} \frac{d_w(t)}{\lambda_w(t)}]/n_W \, dt}{T}. \tag{9}$$

Note that the above measure is the average network performance over time of the dynamic network.

Network Efficiency Measure for Dynamic Networks - Discrete Time

Let $d_w^1, d_w^2, ..., d_w^H$ denote demands for O/D pair w in H discrete time intervals, given, respectively, by: $[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, where $t_H \equiv T$. Assume that the demand is constant in each such time interval for each O/D pair. Denote the corresponding minimal costs for each O/D pair w at the H different time intervals by: $\lambda_w^1, \lambda_w^2, ..., \lambda_w^H$. The demand vector d, in this special discrete case, is a vector in $R^{n_W \times H}$.

Dynamic Network Efficiency: Discrete Time Version

The network efficiency for the network (G, d) over H discrete time intervals: $[t_0, t_1], (t_1, t_2], ..., (t_{H-1}, t_H]$, where $t_H \equiv T$, and with the respective constant demands: $d_w^1, d_w^2, ..., d_w^H$ for all $w \in W$ is defined as follows:

$$\mathcal{E}(G, d, t_H = T) = \frac{\sum_{i=1}^{H}[(\sum_{w \in W} \frac{d_w^i}{\lambda_w^i})(t_i - t_{i-1})/n_W]}{t_H}. \qquad (10)$$

Importance of a Network Component

The importance of network component g of network G with demand d over time horizon T is defined as follows:

$$I(g, d, T) = \frac{\mathcal{E}(G, d, T) - \mathcal{E}(G - g, d, T)}{\mathcal{E}(G, d, T)} \tag{11}$$

where $\mathcal{E}(G - g, d, T)$ is the dynamic network efficiency after component g is removed.

The Dynamic Braess Network Without Link e

Now construct time-dependent link costs, route costs, and demand for $t \in [0, T]$. It is important to emphasize that the case where time t is discrete, that is, $t = 0, 1, 2, \ldots, T$, is trivially included in the equilibrium conditions and also captured in the EVI formulation.

Consider, to start, the first network, consisting of links: a, b, c, d. Assume that the capacities $\mu_{r_1}(t) = \mu_{r_2}(t) = \infty$ for all $t \in [0, T]$. The link cost functions are assumed to be given and as follows for time $t \in [0, T]$:

$$c_a(f_a(t)) = 10 f_a(t), \quad c_b(f_b(t)) = f_b(t) + 50,$$

$$c_c(f_c(t)) = f_c(t) + 50, \quad c_d(f_d(t)) = 10 f_d(t).$$

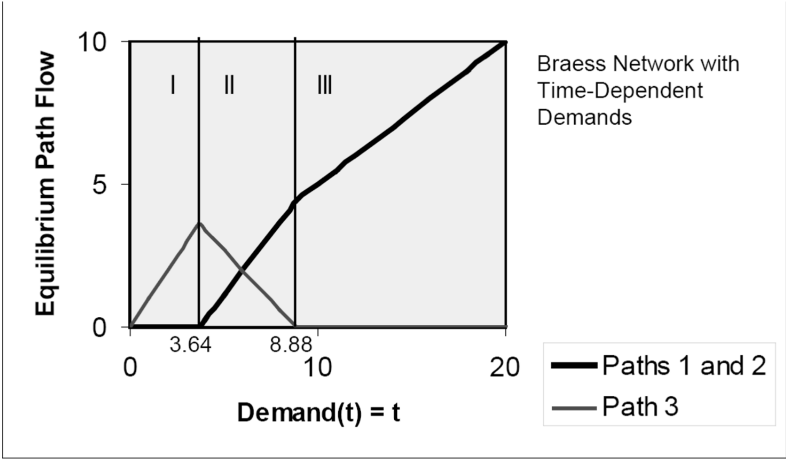
Assume a time-varying demand $d_w(t) = t$ for $t \in [0, T]$.

The Dynamic Braess Network - Solution

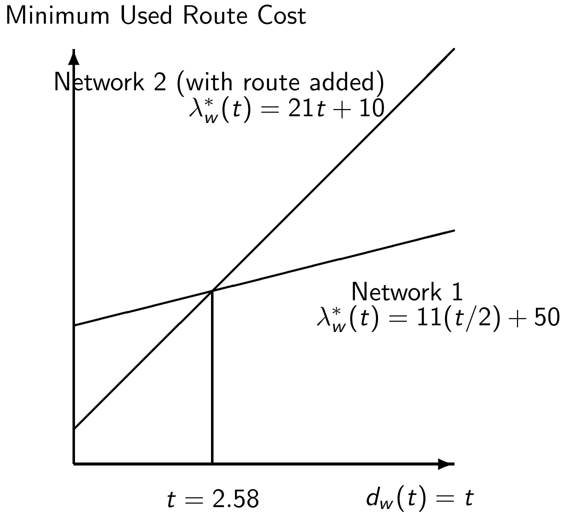
Solving the EVI, we have the equilibrium path flows are $x^*_{r_1}(t) = \frac{t}{2}$ and $x^*_{r_2}(t) = \frac{t}{2}$ for $t \in [0, T]$.

The equilibrium route costs for $t \in [0, T]$ are given by: $C_{r_1}(x^*_{r_1}(t)) = 5\frac{1}{2}t + 50 = C_{r_2}(x^*_{r_2}(t)) = 5\frac{1}{2}t + 50$, and, clearly, equilibrium conditions hold for $\in [0, T]$ a.e..

The Dynamic Braess Network Adding Link e



The Dynamic Braess Network



For demand in the range $2.58 < d_w(t) = t < 8.89$, the addition of the new route will result in everyone being worse off.

Importance of Nodes and Links in the Dynamic Braess Network Using the New Measure When $T = 10$

Link	Importance Value	Importance Ranking
a	0.2604	1
b	0.1784	2
c	0.1784	2
d	0.2604	1
e	-0.1341	3

Node	Importance Value	Importance Ranking
1	1.0000	1
2	0.2604	2
3	0.2604	2
4	1.0000	1

Link e is never used after $t = 8.89$ and in the range $t \in [2.58, 8.89]$, it increases the cost, so the fact that link e has a negative importance value makes sense; over time, its removal would, on the average, improve the network efficiency!

Transportation Network Robustness

The robustness measure $\mathcal{R}^\gamma$ for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative performance retained under a given uniform capacity retention ratio γ with $\gamma \in (0,1]$ so that the new capacities are given by γu. Its mathematical definition is given as:

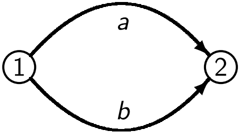
$$\mathcal{R}^\gamma = \mathcal{R}(G, c, d, \gamma, u) = \frac{\mathcal{E}^\gamma}{\mathcal{E}} \times 100\% \tag{12}$$

where $\mathcal{E}$ and $\mathcal{E}^\gamma$ are the network performance measures with the original capacities and the remaining capacities, respectively.

Example: A Simple Network

Consider a simple network as depicted in the figure below. There are two nodes: 1 and 2; two links: a and b; and a single O/D pair $w_1 = (1, 2)$. Therefore, there are two paths connecting the single O/D pair, which are denoted, respectively, by: $p_1 = a$ and $p_2 = b$. The demand is given by $d_{w_1} = 10$. Assume that in the BPR link cost functions that $k = 1$ and $\beta = 4$; and $t_a^0 = 10$ and $t_b^0 = 1$. Two sets of capacities:

- In Capacity Set A, $u_a = u_b = 50$. BPR link cost functions: $c_a(f_a) = 10(1 + (\frac{f_a}{50}))^4$ $c_b(f_b) = 10(1 + (\frac{f_b}{50}))^4$.
- In Capacity Set B, $u_a = 50$ and $u_b = 10$. BPR link cost functions: $c_a(f_a) = 10(1 + (\frac{f_a}{50}))^4$ $c_b(f_b) = 10(1 + (\frac{f_b}{10}))^4$.



Robustness Based on the Performance Measure

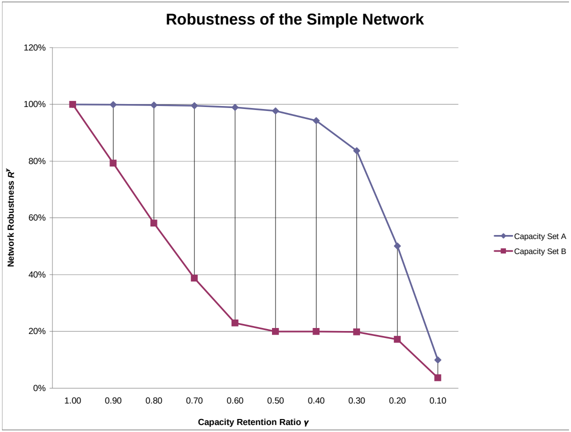


Figure: Robustness vs. Capacity Retention Ratio

An Application to the Sioux Falls Network

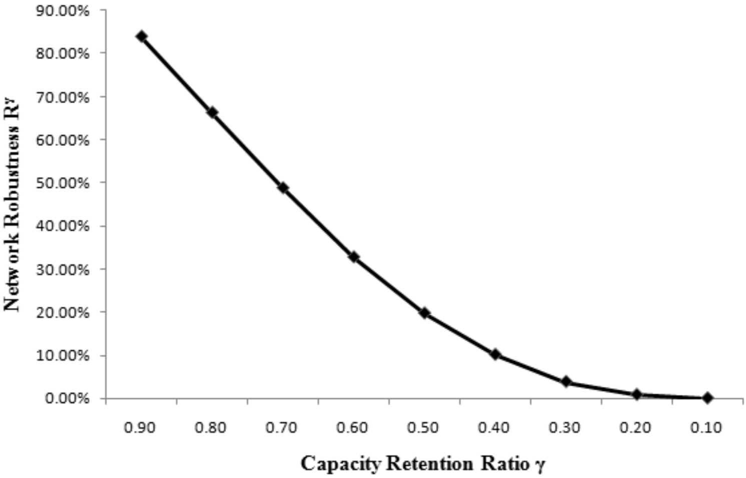


Figure: Robustness vs. Capacity Retention Ratio for the Sioux Falls

An Application to the Anaheim, California Network

There are 461 nodes, 914 links, and 1,406 O/D pairs in the Anaheim network.

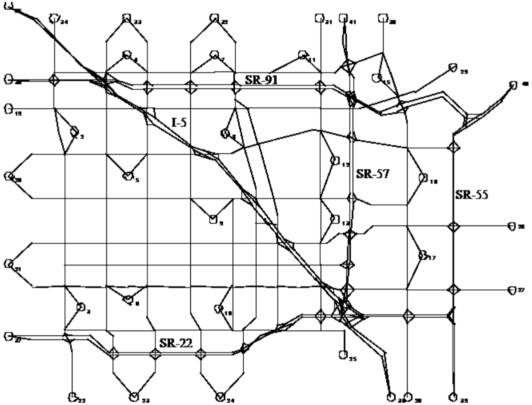


Figure: The Anaheim Network

Robustness Based on the Performance Measure

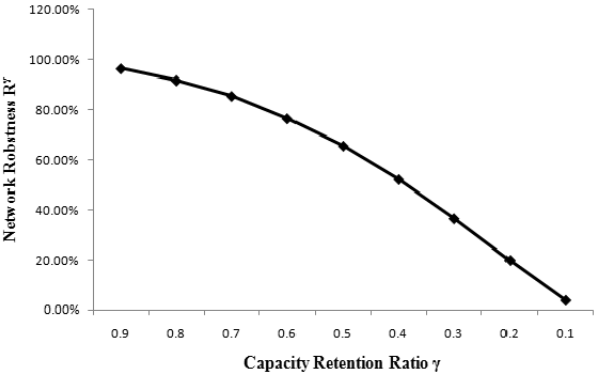


Figure: Robustness vs. Capacity Retention Ratio for the Anaheim Network

Theoretical Results I

Theorem

Consider a network consisting of two nodes 1 and 2, which are connected by a single link a and with a single O/D pair $w_1 = (1, 2)$. Assume that the user link cost function associated with link a is of the BPR form. Then the network robustness given by the expression is given by the explicit formula:

$$\mathcal{R}^{\gamma} = \frac{\gamma^{\beta}[u_a^{\beta} + kd_{w_1}^{\beta}]}{[\gamma^{\beta} u_a^{\beta} + kd_{w_1}^{\beta}]} \times 100\%, \tag{13}$$

where d_{w_1} is the given demand for O/D pair $w_1 = (1, 2)$. Moreover, the network robustness $\mathcal{R}$ is bounded from below by $\gamma^{\beta} \times 100\%$.

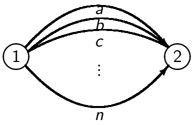
Theoretical Results II

Theorem

Consider a network consisting of two nodes 1 and 2 as in the figure below, which are connected by a set of parallel links. Assume that the associated BPR link cost functions have $\beta = 1$. Furthermore, let's assume that there are positive flows on all the links at both the original and partially degraded capacity levels. Then the network robustness given by the expression is given by the explicit formula:

$$\mathcal{R}^{\gamma} = \frac{\gamma U + k\gamma d_{w_1}}{\gamma U + k d_{w_1}} \times 100\%, \tag{14}$$

where d_{w_1} is the given demand for O/D pair $w_1 = (1, 2)$ and $U \equiv u_a + u_b + \cdots + u_n$. Moreover, the network robustness $\mathcal{R}^{\gamma}$ is bounded from below by $\gamma \times 100\%$.



Definitions

- The index is based on the two behavioral solution concepts, namely, the total cost evaluated under the U-O flow pattern, denoted by TC_{U-O}, and the S-O flow pattern, denoted by TC_{S-O}, respectively.
- The relative total cost index for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative total cost increase under a given uniform capacity retention ratio γ ($\gamma \in (0, 1]$) so that the new capacities are given by γu. Let c denote the vector of BPR user link cost functions and let d denote the vector of O/D pair travel demands.

BPR Total Link Cost Function

$$\hat{c}_a = \hat{c}_a(f_a) = c_a(f_a) \times f_a = t_a^0[1 + k(\frac{f_a}{u_a})^{\beta}] \times f_a, \ \forall a \in L \qquad (15)$$

Definition of $\mathcal{I}_{U-O}$

$$\mathcal{I}^{\gamma}_{U-O} = \mathcal{I}_{U-O}(G, c, d, \gamma, u) = \frac{TC^{\gamma}_{U-O} - TC_{U-O}}{TC_{U-O}} \times 100\%, \tag{16}$$

where TC_{U-O} and TC^{γ}_{U-O} are the total network costs evaluated under the U-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

Definition of $\mathcal{I}_{S-O}$

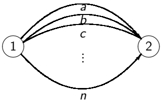
$$\mathcal{I}^{\gamma}_{S-O} = \mathcal{I}_{S-O}(G, c, d, \gamma, u) = \frac{TC^{\gamma}_{S-O} - TC_{S-O}}{TC_{S-O}} \times 100\%, \tag{17}$$

where TC_{S-O} and TC^{γ}_{S-O} are the total network costs evaluated at the S-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

Theoretical Results I

Theorem

Consider a network consisting of two nodes 1 and 2 as in the figure below, which are connected by n parallel links. If the free-flow term, t_a^0, $\forall a \in L$, is the same for all links $a \in L$ in the BPR link cost function, the S-O flow pattern coincides with the U-O flow pattern and, therefore, $\mathcal{I}_{U-O} = \mathcal{I}_{S-O}$.



Theorem

The upper bound for $\mathcal{I}^{\gamma}_{S-O}$ for a transportation network with BPR link cost functions is $\frac{1-\gamma^{\beta}}{\gamma^{\beta}} \times 100\%$.

Theoretical Results II

Theorem

Consider a network consisting of two nodes 1 and 2 as in the previous figure, which are connected by n parallel links. Assume that the associated BPR link cost functions have $\beta = 1$. Furthermore, assume that there are positive flows on all the links at both the original and the partially degraded capacity levels given, respectively, by u and γu. Then the relative total cost index under the U-O flow pattern is given by the explicit formula:

$$\mathcal{I}^{\gamma}_{U-O} = \left(\frac{\gamma U + kd_w}{\gamma U + k\gamma d_w} - 1\right) \times 100\%, \tag{18}$$

where d_w is the given demand for O/D pair $w = (1, 2)$ and $U \equiv u_a + u_b + \cdots + u_n$. Moreover, the network robustness $\mathcal{I}^{\gamma}_{U-O}$ is bounded from above by $\frac{1-\gamma}{\gamma} \times 100\%$.

The Sioux Falls Network

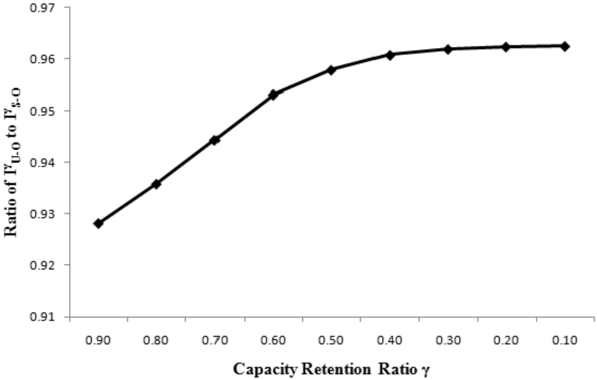


Figure: Ratio of I^{γ}_{U-O} to I^{γ}_{S-O} for the Sioux Falls Network under the Capacity Retention Ratio γ

The Anaheim Network

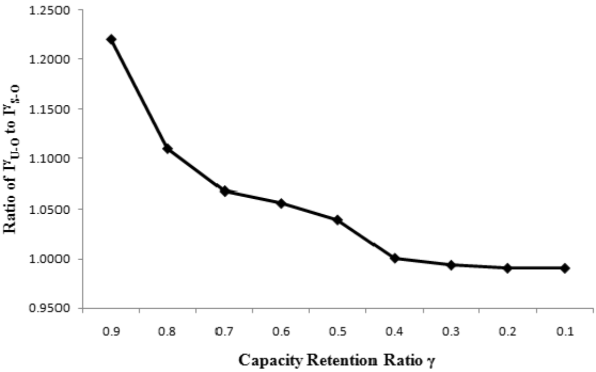


Figure: Ratio of I^{γ}_{U-O} to I^{γ}_{S-O} for the Anaheim Network under Capacity Retention Ratio γ

Extensions

- The link and node importance identification approach introduced previously does not apply directly to environmental impact assessment.
- An approach to consider both U-O and S-O behaviors.
- An approach to capture the impact of alternative behaviors on the environment as the transportation network is subject to link capacity degradations.

Emission Functions on Transportation Networks

CO Link Emission Function (Yin and Lawphongpanich (2006))

$$e_a(f_a) = 0.2038 \times c_a(f_a) \times e^{0.7962 \times (\frac{l_a}{c_a(f_a)})}, \tag{19}$$

where l_a denotes the length of link a and c_a corresponds to the travel time (in minutes) to traverse link a. The length l_a is measured in kilometers for each link $a \in L$ and the emissions are in grams per hour.

Total Emissions on a Link

The expression for total emissions on a link a, denoted by $\hat{e}_a(f_a)$, is given by:

$$\hat{e}_a(f_a) = e_a(f_a) \times f_a. \tag{20}$$

The Total Emissions of CO, TE, Generated on a Network

$$\mathrm{TE} = \sum_{a \in L} \hat{e}_a(f_a). \tag{21}$$

The Environmental Impact Assessment Index I

The environmental impact assessment index for a transportation network G with the vector of demands d, the vector of user link cost functions c, and the vector of link capacities u is defined as the relative total emission increase under a given uniform capacity retention ratio γ ($\gamma \in (0, 1]$) so that the new capacities are given by γu. Let c denote the vector of BPR user link cost functions and let d denote the vector of O/D pair travel demands.

The Environmental Impact Assessment Index II

Environmental Impact Assessment Index under the U-O Flow Pattern

$$\mathrm{EI}^{\gamma}_{U-O} = \mathrm{EI}_{U-O}(G, c, d, \gamma, u) = \frac{\mathrm{TE}^{\gamma}_{U-O} - \mathrm{TE}_{U-O}}{\mathrm{TE}_{U-O}}, \qquad (22)$$

where TE_{U-O} and $\mathrm{TE}^{\gamma}_{U-O}$ are the total emissions generated under the U-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

Environmental Impact Assessment Index under the S-O Flow Pattern

$$\mathrm{EI}^{\gamma}_{S-O} = \mathrm{EI}_{S-O}(G, c, d, \gamma, u) = \frac{\mathrm{TE}^{\gamma}_{S-O} - \mathrm{TE}_{S-O}}{\mathrm{TE}_{S-O}}, \qquad (23)$$

where TE_{S-O} and $\mathrm{TE}^{\gamma}_{S-O}$ are the total emissions generated at the S-O flow pattern with the original capacities and the remaining capacities (i.e., γu), respectively.

Environmental Importance Identification for Links

$$\mathrm{I}^{l}_{U-O} = \frac{\mathrm{TE}_{U-O}(G-l) - \mathrm{TE}_{U-O}}{\mathrm{TE}_{U-O}}, \tag{24}$$

$$\mathrm{I}^{l}_{S-O} = \frac{\mathrm{TE}_{S-O}(G-l) - \mathrm{TE}_{S-O}}{\mathrm{TE}_{S-O}}, \tag{25}$$

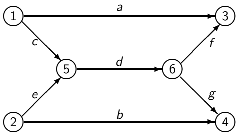
where I^{l}_{U-O} denotes the importance indicator for link l assuming U-O behavior and I^{l}_{S-O} denotes the analogue under S-O behavior; $\mathrm{TE}_{U-O}(G-l)$ denotes the total emissions generated under U-O behavior if link l is removed from the network and $\mathrm{TE}_{S-O}(G-l)$ denotes the same but under S-O behavior.

Example (Data from Yin and Lawphongpanich (2006))

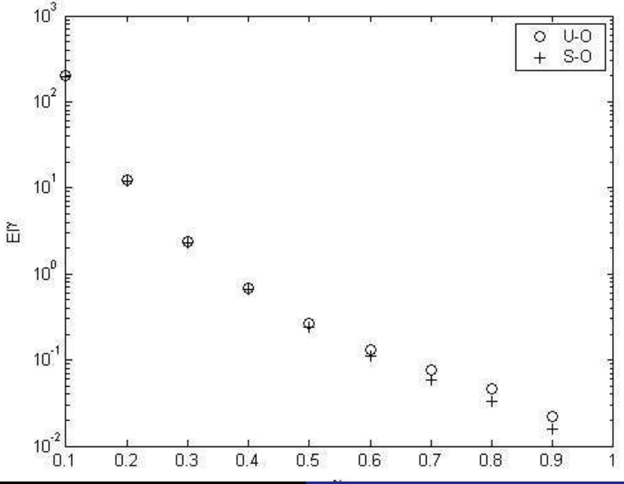
The network topology is in the figure on the right. There are two O/D pairs in the network: $w_1 = (1,3)$ and $w_2 = (2,4)$ with demands of $d_{w_1} = 3000$ vehicles per hour and $d_{w_2} = 3000$ vehicles per hour. The user link cost functions, which here correspond to travel time in minutes, are as follows:

$c_a(f_a) = 8(1 + .15(\frac{f_a}{2000})^4)$, $c_b(f_b) = 9(1 + .15(\frac{f_b}{2000})^4)$, $c_c(f_c) = 2(1 + .15(\frac{f_c}{2000})^4)$, $c_d(f_d) = 6(1 + .15(\frac{f_d}{4000})^4)$, $c_e(f_e) = 3(1 + .15(\frac{f_e}{2000})^4)$, $c_f(f_f) = 3(1 + .15(\frac{f_f}{2500})^4)$, $c_g(f_g) = 4(1 + .15(\frac{f_g}{2500})^4)$.

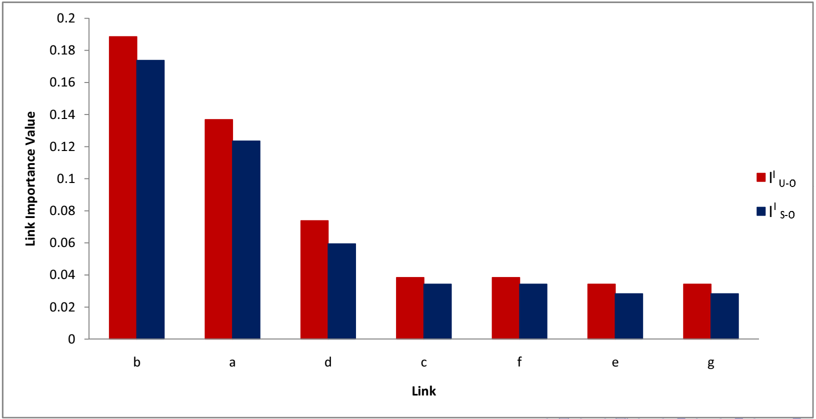
The lengths of the links, in kilometers, in turn, which are needed to compute the environmental emissions, are given by: $l_a = 8.0$, $l_b = 9.0$, $l_c = 2.0$, $l_d = 6.0$, $l_e = 3.0$, $l_f = 3.0$, $l_g = 4.0$.



Environmental Impact Indices under U-O and S-O Behaviors



Link Importance Values and Rankings Under U-O and S-O Behavior

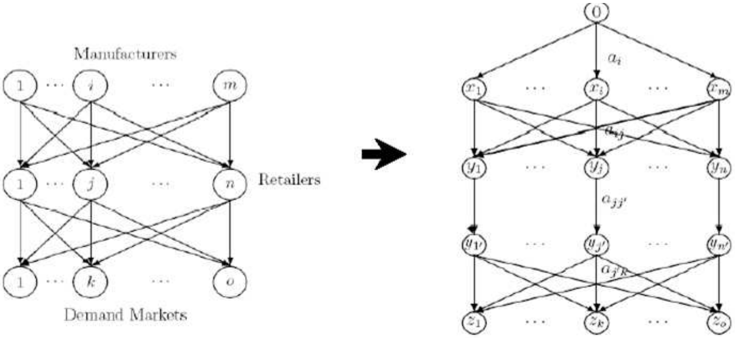


- Algorithm
- A Weighted Supply Chain Performance Measure
- Numerical Examples

6 Identification of Critical Nodes and Links in Financial Networks
- The Financial Network Model
- The Financial Network Performance Measure
- The Importance of a Financial Network Component

7 Conclusions and Future Research Plan

The Equivalence of Decentralized Supply Chains and Transportation Networks



Nagurney, *Transportation Research E* (2006).

Motivation III

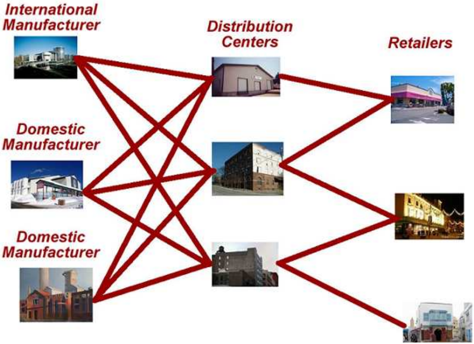
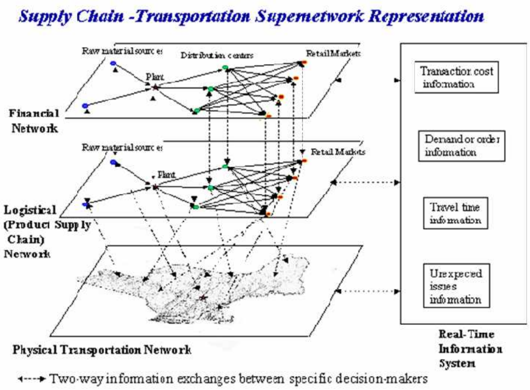


Figure: Depiction of a Global Supply Chain Network

Motivation III

Supply chain networks depend on infrastructure networks for their effective and efficient operations from: manufacturing and logistical networks, to transportation networks, to electric power networks, financial networks, and telecommunication networks, most, notably, the Internet.



Motivation III

The economic and financial troubles of the automobile companies in the United States among the "Big Three" are creating a domino effect throughout the supply chain and the vast network of auto supplier firms. For example, GM alone has approximately 2,000 suppliers, whereas Ford has about 1,600 suppliers, and Chrysler about 900 suppliers. Although Ford is in better shape in terms of the cash the company has, it shares most of the same big parts suppliers, so a disruption in the supply chain that a bankruptcy would invariably cause would hurt Ford too, and even halt production temporarily.

Motivation III

As summarized by Sheffi (2005), one of the main characteristics of disruptions in supply networks is "the seemingly unrelated consequences and vulnerabilities stemming from global connectivity."

Indeed, supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a holistic, system-wide approach to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers.

- Hendricks and Singhal (2005) analyzed 800 instances of supply chain disruptions experienced by firms whose stocks are publicly traded. They found that the companies that suffered supply chain disruptions experienced share price returns 33 percent to 40 percent lower than the industry and the general market benchmarks. Furthermore, share price volatility was 13.5 percent higher in these companies in the year following a disruption than in the prior year.
- Snyder and Daskin (2005) examined supply chain disruptions in the context of facility location. The objective of their model was to select locations for warehouses and other facilities that minimize the transportation costs to customers and, at the same time, account for possible closures of facilities that would result in re-routing of the product. However, as commented in Snyder and Shen (2006), "Although these are multi-location models, they focus primarily on the local effects of disruptions.

- Most supply disruption studies have focused on a local point of view, in the form of a single-supplier problem (see, e. g., Gupta (1996) and Parlar (1997)) or a two-supplier problem (see, e. g., Parlar and Perry (1996)). Very few studies/papers have examined supply chain risk management in an environment with multiple decision-makers and in the case of uncertain demands (cf. Tomlin (2006)).
- Tang (2006a) discussed how to deploy strategies in order to enhance the robustness and the resilience of supply chains. Kleindorfer and Saad (2005) provided an overview of strategies for mitigating supply chain disruption risks, which were exemplified by a case study in a chemical product supply chain.
- Beamon (1998, 1999) reviewed the supply chain literature and suggested directions for research on supply chain performance measures, which should include criteria on efficient resource allocation, output maximization, and flexible adaptation to the environmental changes (see also, Lee and Whang (1999), Lambert and Pohlen (2001), and Lai, Ngai, and Cheng (2002)).

Contributions of This Research

- Developed a multi-tiered, multi transportation modal supply chain network with interactions among various decision-makers.
- The model captures the supply-side risks together with uncertain demand.
- The mean-variance approach is used to model individual's attitude towards risks.
- Developed a weighted measure to study the supply chain network performance.

Supply Chain Model

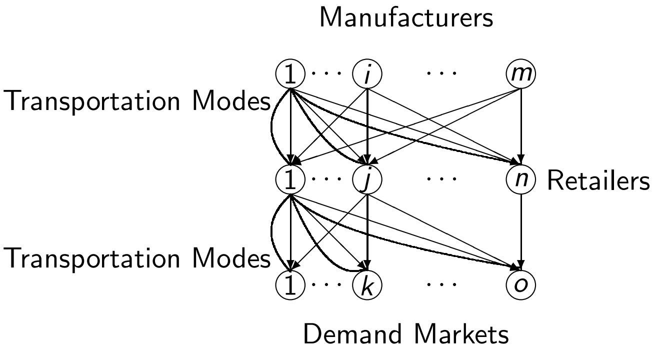


Figure: The Multitiered Network Structure of the Supply Chain

Assumptions

- Manufacturers and retailers are multicriteria decision-makers
- Manufacturers and retailers try to
  - Maximize profit
  - Minimize risk
  - Individual weight assigned to the risk level according to decision maker's attitude towards risk
- Nash Equilibrium

For each manufacturer i, there is a random parameter α_i that reflects the impact of disruption to his production cost function. The expected production cost function is given by:

$$\hat{F}_i(Q^1) \equiv \int f_i(Q^1, \alpha_i) d\mathcal{F}_i(\alpha_i), \quad i = 1, \ldots, m. \tag{26}$$

The variance of the above production cost function is denoted by $VF_i(Q^1)$ where $i = 1, \ldots, m$.
We assume that each manufacturer has g types of transportation modes available to ship the product to the retailers, the cost of which is also subject to disruption impacts. The expected transportation cost function is given by:

$$\hat{C}^u_{ij}(q^u_{ij}) \equiv \int_{\beta^u_{ij}} c^u_{ij}(q^u_{ij}, \beta^u_{ij}) d\mathcal{F}^u_{ij}(\beta^u_{ij}), \; i = 1, \ldots, m; \; j = 1, \ldots, n; \; u = 1, \ldots, g. \tag{27}$$

We further denote the variance of the above transportation cost function as $VC^u_{ij}(Q^1)$ where $i = 1, \ldots, m$; $j = 1, \ldots, n$; $u = 1, \ldots, g$.

Manufacturer's Maximization Problem

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{u=1}^{g}\rho_{1ij}^{u*}q_{ij}^{u} - \hat{F}_i(Q^1) - \sum_{j=1}^{n}\sum_{u=1}^{g}\hat{C}_{ij}^{u}(q_{ij}^{u})$$

$$-\theta_i\left[\sum_{i=1}^{m}VF_i(Q^1) + \sum_{j=1}^{n}\sum_{u=1}^{g}VC_{ij}^{u}(q_{ij}^{u})\right] \tag{28}$$

Nonnegative weight θ_i is assigned to the variance of the cost functions for each manufacturer to reflect his attitude towards disruption risks.

We assume that for each manufacturer, the production cost function and the transaction cost function without disruptions are continuously differentiable and convex. Hence, the optimality conditions for all manufacturers simultaneously (cf. Bazaraa, Sherali, and Shetty (1993) and Nagurney (1999)) can be expressed as the following VI:

The Optimal Conditions for All Manufacturers

Determine $Q^{1*} \in R_+^{mng}$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^{u*}}$$

$$+ \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^{u*}}) - \rho_{1ij}^{u*}] \times [q_{ij}^u - q_{ij}^{u*}] \geq 0, \quad \forall Q^1 \in R_+^{mng}. \qquad (29)$$

A random risk/disruption related random parameter η_j is associated with the handling cost of retailer j. The expected handling cost is:

$$\hat{C}_j^1(Q^1, Q^2) \equiv \int c_j(Q^1, Q^2, \eta_j) d\mathcal{F}_j(\eta_j), \;\; j = 1, \ldots, n \qquad (30)$$

The variance of the handling cost function is denoted by $VC_j^1(Q^1, Q^2)$ where $j = 1, \ldots, n$.

Retailer's Maximization Problem

The objective function for distributor j; $j = 1, \ldots, n$ can be expressed as follows:

$$\text{Maximize} \;\; \sum_{k=1}^{o} \sum_{v=1}^{h} \rho_{2jk}^{v*} q_{jk}^{v} - \hat{C}_j^1(Q^1, Q^2) - \sum_{i=1}^{m} \sum_{u=1}^{g} \rho_{1ij}^{u*} q_{ij}^{u} - \varpi_j VC_j^1(Q^1, Q^2) \qquad (31)$$

subject to:

$$\sum_{k=1}^{o} \sum_{v=1}^{h} q_{jk}^{v} \leq \sum_{i=1}^{m} \sum_{u=1}^{g} q_{ij}^{u} \qquad (32)$$

We assume that, for each retailer, the handling cost without disruptions is continuously differentiable and convex.

The Optimal Conditions for All Retailers

Determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R_+^{mng+noh+n}$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}\left[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \rho_{1ij}^{u*} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*\right]$$

$$\times \left[q_{ij}^u - q_{ij}^{u*}\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}[-\rho_{2jk}^{v*} + \gamma_j^* + \frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right] \times \left[\gamma_j - \gamma_j^*\right] \geq 0, \ \forall (Q^1, Q^2, \gamma) \in R_+^{mng+noh+n} \tag{33}$$

The Market Stochastic Economic Equilibrium Conditions

For any retailer with associated demand market k; $k = 1, \ldots, o$:

$$\hat{d}_k(\rho_3^*) \begin{cases} \leq \sum_{j=1}^{o} \sum_{v=1}^{h} q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^* = 0, \\ = \sum_{j=1}^{o} \sum_{v=1}^{h} q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^* > 0, \end{cases}$$

$$\rho_{2jk}^{v*} + c_{jk}^{v}(Q^{2*}) \begin{cases} \geq \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*} = 0, \\ = \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*} > 0. \end{cases}$$

The above market equilibrium conditions are equivalent to the following VI problem, after taking the expected value and summing over all retailers/demand markets k:

Equivalent VI Problem

Determine $(Q^{2*}, \rho_3^*) \in R_+^{noh+o}$ satisfying:

$$\sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*]$$

$$+\sum_{k=1}^{o}\sum_{j=1}^{n}\sum_{v=1}^{h}(\rho_{2jk}^{v*}+c_{jk}^{v}(Q^{2*})-\rho_{3k}^*)\times[q_{jk}^{v}-q_{jk}^{v*}] \geq 0, \ \forall \rho_3 \in R_+^o, \ \forall Q^2 \in R_+^{noh}, \tag{34}$$

where ρ_3 is the o-dimensional vector with components: $\rho_{31}, \ldots, \rho_{3o}$ and Q^2 is the noh-dimensional vector.

Remark:

We are interested in the cases where the expected demands are positive, that is, $\hat{d}_k(\rho_3) > 0$, $\forall \rho_3 \in R^o_+$ for $k = 1, \ldots, o$. Furthermore, we assume that the unit transaction costs: $c^v_{jk}(Q^2) > 0$, $\forall j, k$, $\forall Q^2 \neq 0$.
Under the above assumptions, we can show that $\rho^*_{3k} > 0$ and $\hat{d}_k(\rho^*_3) = \sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h q^{v*}_{jk}$, $\forall k$.

Definition: Supply Chain Network Equilibrium with Uncertainty and Expected Demands

The equilibrium state of the supply chain network with disruption risks and expected demands is one where the flows of the product between the tiers of the decision-makers coincide and the flows and prices satisfy the sum of conditions (29), (33), and (34).

Theorem: VI Formulation of the Supply Chain Network Equilibrium with Uncertainty and Expected Demands

Determine $(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in R_+^{mng+noh+n+o}$ *satisfying:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u})$$

$$+\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*] \times [q_{ij}^u - q_{ij}^{u*}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\gamma_j^* + c_{jk}^v(Q^{2*}) - \rho_{3k}^*] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right] \times \left[\gamma_j - \gamma_j^*\right] + \sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}. \tag{35}$$

where $\mathcal{K} \equiv \{(Q^1, Q^2, \gamma, \rho_3) | (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}\}$.

Algorithm–The Modified Projection Method (Korpelevich(1977))

Step 0: Initialization

Start with an $x^0 \in \mathcal{K}$. Set $k := 1$ and select ρ, such that $0 < \rho < \frac{1}{L}$, where L is the Lipschitz constant for function F in (35).

Step 1: Construction and Computation

Compute $\hat{x}^{k-1}$ by solving the VI subproblem:

$$\langle (\hat{x}^{k-1} + (\rho F(x^{k-1}) - x^{k-1}))^T, x - \hat{x}^{k-1} \rangle \geq 0, \quad \forall x \in \mathcal{K}. \tag{36}$$

Step 2: Adaptation

Compute x^k by solving the VI problem:

$$\langle (x^{k-1} + (\rho F(\hat{x}^{k-1}) - \hat{x}^{k-1}))^T, x - x^k \rangle \geq 0, \quad \forall x \in \mathcal{K}. \tag{37}$$

Step 3: Convergence Verification

If $|x^k - x^{k-1}| \leq \epsilon$, for $\epsilon > 0$, a prespecified tolerence, then stop; otherwise, set $k := k + 1$ and go to Step 1.

A Supply Chain Network Performance Measure

The supply chain network performance measure, $\mathcal{E}^{SCN}$, for a given supply chain, and expected demands: $\hat{d}_k$; $k = 1, 2, \ldots, o$, is defined as follows:

$$\mathcal{E}^{SCN} \equiv \frac{\sum_{k=1}^{o} \frac{\hat{d}_k}{\rho_{3k}}}{o}, \tag{38}$$

where o is the number of demand markets in the supply chain network, and $\hat{d}_k$ and ρ_{3k} denote, respectively, the expected equilibrium demand and the equilibrium price at demand market k.

Assume that all the random parameters take on a given threshold probability value; say, for example, 95%. Moreover, assume that all the cumulative distribution functions for random parameters have inverse functions. Hence, we have that: $\alpha_i = \mathcal{F}_i^{-1}(.95)$, for $i = 1, \ldots, m$; $\beta_{ij}^u = \mathcal{F}_{ij}^{u^{-1}}(.95)$, for $i = 1, \ldots, m$; $j = 1, \ldots, n$, and so on.

Supply Chain Robustness Measurement

Let $\mathcal{E}_w$ denote the supply chain performance measure with random parameters fixed at a certain level as described above. Then, the supply chain network robustness measure, $\mathcal{R}$, is given by the following:

$$\mathcal{R}^{SCN} = \mathcal{E}^0_{SCN} - \mathcal{E}_w, \tag{39}$$

where $\mathcal{E}^0_{SCN}$ gauges the supply chain performance based on the supply chain model, but with weights related to risks being zero.

$\mathcal{E}^0$ examines the "base" supply chain performance while $\mathcal{E}_w$ assesses the supply chain performance measure at some prespecified uncertainty level. If their difference is small, a supply chain maintains its functionality well and we consider the supply chain to be robust.

Note that different supply chains may have different requirements regarding the performance and robustness concepts introduced in the previous sections. For example, in the case of a supply chain of a toy product one may focus on how to satisfy demand in the most cost efficient way and not care too much about supply chain robustness. A medical/healthcare supply chain, on the other hand, may have a requirement that the supply chain be highly robust when faced with uncertain conditions. Hence, in order to be able to examine and to evaluate the different application-based supply chains from both perspectives, we now define a weighted supply chain performance measure as follows:

A Weighted Supply Chain Performance Measure

$$\hat{\mathcal{E}}^{SCN} = (1-\epsilon)\mathcal{E}^{0}_{SCN} + \epsilon(-\mathcal{R}^{SCN}), \tag{40}$$

where $\epsilon \in [0,1]$ is the weight that is placed on the supply chain robustness.

Supply Chain Example 1

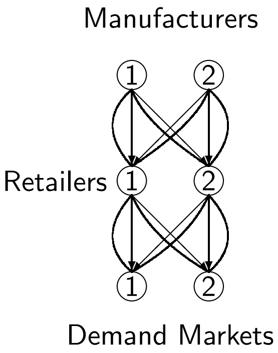


Figure: The Supply Chain Network for the Numerical Examples

For illustration purposes, we assumed that all the random parameters followed uniform distributions. The relevant parameters are as follows:
$\alpha_i \sim [0,2]$ for $i = 1,2$; $\beta_{ij}^u \sim [0,1]$ for $i = 1,2$; $j = 1,2$; $u = 1,2$;
$\eta_j \sim [0,3]$ for $j = 1,2$.
Demand functions are assumed followed a uniform distribution given by $[200 - 2\rho_{3k}, 600 - 2\rho_{3k}]$, for $k = 1,2$. Hence, the expected demand functions are:

$$\hat{d}_k(\rho_3) = 400 - 2\rho_{3k}, \quad \text{for } k = 1,2.$$

The production cost functions for the manufacturers are given by:

$$f_1(Q^1,\alpha_1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2\alpha_1(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u),$$

$$f_2(Q^1,\alpha_2) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2\alpha_2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u).$$

The expected production cost functions for the manufacturers are given by:

$$\hat{F}_1(Q^1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u),$$

$$\hat{F}_2(Q^1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u).$$

The variances of the production cost functions for the manufacturers are given by:
$VF_1(Q^1) = \frac{4}{3}(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2$;$VF_2(Q^1) = \frac{4}{3}(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2$.
The transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$c_{ij}^1(q_{ij}^1, \beta_{ij}^1) = .5(q_{ij}^1)^2 + 3.5\beta_{ij}^1 q_{ij}^1, \text{ for } i = 1,2; j = 1,2,$$

$$c_{ij}^2(q_{ij}^2, \beta_{ij}^2) = (q_{ij}^2)^2 + 5.5\beta_{ij}^2 q_{ij}^2, \text{ for } i = 1,2; j = 1,2.$$

The expected transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$\hat{C}^1_{ij}(q^1_{ij}) = .5(q^1_{ij})^2 + 1.75q^1_{ij}, \text{ for } i = 1,2; j = 1,2,$$

$$\hat{C}^2_{ij}(q^2_{ij}) = .5(q^2_{ij})^2 + 2.75q^2_{ij}, \text{ for } i = 1,2; j = 1,2.$$

The variances of the transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$VC^1_{ij}(q^1_{ij}) = 1.0208(q^1_{ij})^2, \text{ for } i = 1,2; j = 1,2,$$

$$VC^2_{ij}(q^2_{ij}) = 2.5208(q^2_{ij})^2, \text{ for } i = 1,2; j = 1,2.$$

The handling costs of the retailers, in turn, are given by:

$$c_j(Q^1, Q^2, \eta_j) = .5(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij})^2 + \eta_j(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij}), \text{ for } j = 1,2.$$

The expected handling costs of the retailers are given by:

$$\hat{C}_j^1(Q^1, Q^2) = .5(\sum_{i=1}^{2}\sum_{u=1}^{2} q_{ij}^u)^2 + 1.5(\sum_{i=1}^{2}\sum_{u=1}^{2} q_{ij}^u), \text{ for } j = 1, 2.$$

The variance of the handling costs of the retailers are given by:

$$VC_j(Q^1, Q^2) = \frac{3}{4}(\sum_{i=1}^{2}\sum_{u=1}^{2} q_{ij}^u)^2, \text{ for } j = 1, 2.$$

The unit transaction costs from the retailers to the demand markets are given by:

$$c_{jk}^1(Q^2) = .3q_{jk}^1, \quad \text{for } j = 1, 2; \; k = 1, 2,$$

$$c_{jk}^2(Q^2) = .6q_{jk}^2, \quad \text{for } j = 1, 2; \; k = 1, 2.$$

We assumed that the manufacturers and the retailers placed zero weights on the disruption risks to compute $\mathcal{E}^0$.

The equilibrium shipments between manufacturers and retailers are:

$$q_{ij}^{1*} = 8.5022, \text{ for } i = 1,2; j = 1,2; \quad q_{ij}^{2*} = 3.7511, \quad \text{for } i = 1,2; j = 1,2;$$

The equilibrium shipments between the retailers and the demand markets are:

$$q_{jk}^{1*} = 8.1767, \quad \text{for } j = 1,2; \ k = 1,2; \quad q_{jk}^{2*} = 4.0767, \quad \text{for } j = 1,2; \ k = 1,2.$$

Finally, the equilibrium prices are: $\rho_{31}^* = \rho_{32}^* = 187.7466$ and the expected equilibrium demands are: $\hat{d}_1 = \hat{d}_2 = 24.5068$.

Supply Chain Example 2

For the same network structure and cost and demand functions, we now assume that the relevant parameters are changed as follows: $\alpha_i \sim [0,4]$ for $i = 1,2$; $\beta_{ij}^u \sim [0,2]$ for $i = 1,2$; $j = 1,2$; $u = 1,2$; $\eta_j \sim [0,6]$ for $j = 1,2$.

The equilibrium shipments between manufacturers and retailers are now:

$$q_{ij}^{1*} = 8.6008, \quad \text{for } i = 1,2;\ j = 1,2; \quad q_{ij}^{2*} = 3.3004, \quad \text{for } i = 1,2;\ j = 1,2;$$

whereas the equilibrium shipments between the retailers and the demand markets are:

$$q_{jk}^{1*} = 7.9385, \quad \text{for } j = 1,2;\ k = 1,2; \quad q_{jk}^{2*} = 3.9652, \quad \text{for } j = 1,2;\ k = 1,2.$$

The equilibrium prices are: $\rho_{31}^* = \rho_{32}^* = 188.0963$ and the expected equilibrium demands are: $\hat{d}_1 = \hat{d}_2 = 23.8074$.

Table: Supply Chain Performance Measure

	$\mathcal{E}^0_{SCN}$	$\mathcal{E}_w$ $(w = 0.95)$	$\hat{\mathcal{E}}^{SCN}$ $(\epsilon = 0.5)$
Example 1	0.1305	0.1270	0.0635
Example 2	0.1266	0.1194	0.0597

Observe that first example leads to a better measure of performance since the uncertain parameters do not have as great of an impact as in the second one for the cost functions under the given threshold level.

- Algorithm
- A Weighted Supply Chain Performance Measure
- Numerical Examples

6. Identification of Critical Nodes and Links in Financial Networks
   - The Financial Network Model
   - The Financial Network Performance Measure
   - The Importance of a Financial Network Component

7. Conclusions and Future Research Plan

Fin. Net. Model

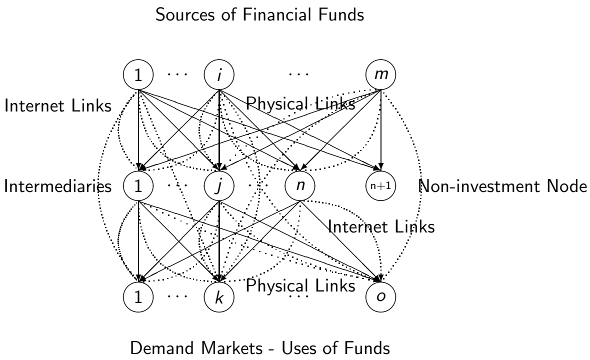


Figure: The Structure of the Financial Network with Intermediation and with Electronic Transactions

The financial network consists of m sources of financial funds, n financial intermediaries, and o demand markets.

The Financial Network Performance Measure

The financial network performance measure, $\mathcal{E}^{FN}$, for a given network topology G, and demand price functions $\rho_{3k}(d)$ $(k = 1, 2, \ldots, o)$, and available funds held by source agents S, is defined as follows:

$$\mathcal{E}^{FN} = \frac{\sum_{k=1}^{o} \frac{d_k^*}{\rho_{3k}(d^*)}}{o}, \quad (41)$$

where o is the number of demand markets in the financial network, and d_k^* and $\rho_{3k}(d^*)$ denote the equilibrium demand and the equilibrium price for demand market k, respectively.

Importance of a Financial Network Component

The importance of a financial network component $g \in G$, $I(g)$, is measured by the relative financial network performance drop after g is removed from the network:

$$I(g)^{FN} = \frac{\triangle \mathcal{E}^{FN}}{\mathcal{E}^{FN}} = \frac{\mathcal{E}^{FN}(G) - \mathcal{E}^{FN}(G-g)}{\mathcal{E}^{FN}(G)} \tag{42}$$

where $G - g$ is the resulting financial network after component g is removed from network G.

- Algorithm
- A Weighted Supply Chain Performance Measure
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6. Identification of Critical Nodes and Links in Financial Networks
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7. Conclusions and Future Research Plan

Conclusions I

- Proposed a network efficiency/performance measure that captures the important network information, such as flows, costs, and behaviors; contains the L-M measure as a special case.
- The proposed measure is well-defined even in the case of disconnected O/D pairs.
- Extended the network measure to a dynamic setting with time-varying demands.
- Defined a network robustness measure based on the proposed network measure. Theoretical results were obtained for networks with special structures.

Conclusions II

- Constructed relative cost indices to assess network robustness with alternative user behaviors.
- Studied the environmental impact of transportation networks with degradable links.
- Developed a novel supply chain network model to study the demand-side as well as the supply-side risks, with the supply-side risks modeled as uncertain parameters in the underlying cost functions.
- Proposed a weighted supply chain performance and robustness measure based on the our network performance/efficiency measure described.

Conclusions III

- Theoretical results and computational procedure is discussed.
- Applied the network measure to study financial networks with intermediation and electronic transactions.

Future Research Plan

- To explore other metrics to analyze supply chain performance more comprehensively.
- To explore vulnerability and traceability issues in food supply chain networks.
- To study the supply chain network performance and robustness under merge and acquisition.
- To conduct additional empirical research to guide the future development of my theoretical frameworks.

Thank You!