Modeling of Supply Chain Risk under Disruptions with Performance Measurement and Robustness Analysis

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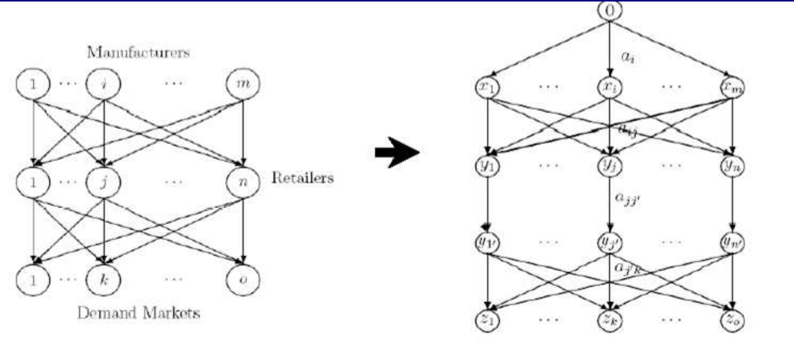
Motivation for Research on Supply Chain Under Disruptions

The Equivalence of Decentralized Supply Chains and Transportation Networks



Nagurney, *Transportation Research E* (2006).

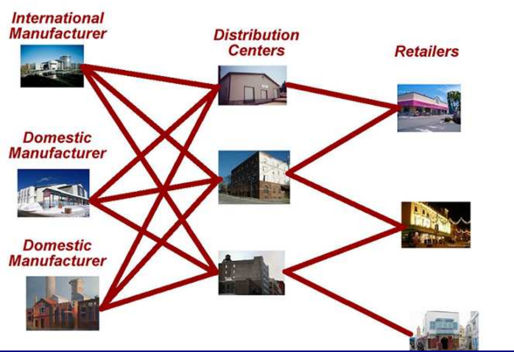
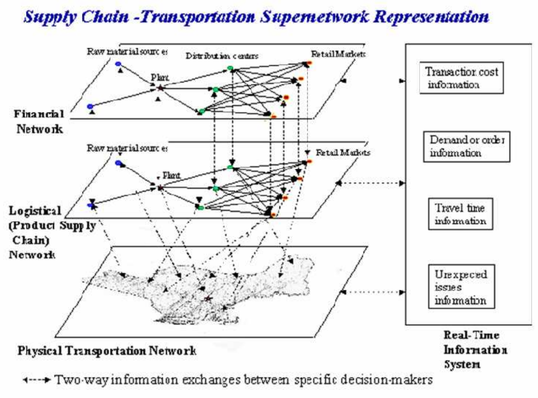


Figure: Depiction of a Global Supply Chain Network

Supply chain networks depend on infrastructure networks for their effective and efficient operations from: manufacturing and logistical networks, to transportation networks, to electric power networks, financial networks, and telecommunication networks, most, notably, the Internet.



The economic and financial troubles of the automobile companies in the United States among the "Big Three" are creating a domino effect throughout the supply chain and the vast network of auto supplier firms. For example, GM alone has approximately 2,000 suppliers, whereas Ford has about 1,600 suppliers, and Chrysler about 900 suppliers. Although Ford is in better shape in terms of the cash the company has, it shares most of the same big parts suppliers, so a disruption in the supply chain that a bankruptcy would invariably cause would hurt Ford too, and even halt production temporarily.

- The West Coast port lockout in 2002, which resulted in a 10 day shutdown of ports in early October, typically, the busiest month. 42% of the US trade products and 52% of the imported apparel go through these ports, including Los Angeles. Estimated losses were one billion dollars per day.

As summarized by Sheffi (2005), one of the main characteristics of disruptions in supply networks is "the seemingly unrelated consequences and vulnerabilities stemming from global connectivity."

Indeed, supply chain disruptions may have impacts that propagate not only locally but globally and, hence, a holistic, system-wide approach to supply chain network modeling and analysis is essential in order to be able to capture the complex interactions among decision-makers.

Contributions of This Research

- Developed a multi-tiered, multi transportation modal supply chain network with interactions among various decision-makers.
- The model captures the supply-side risks together with uncertain demand.
- The mean-variance approach is used to model individual's attitude towards risks.
- Developed a weighted measure to study the supply chain network performance.

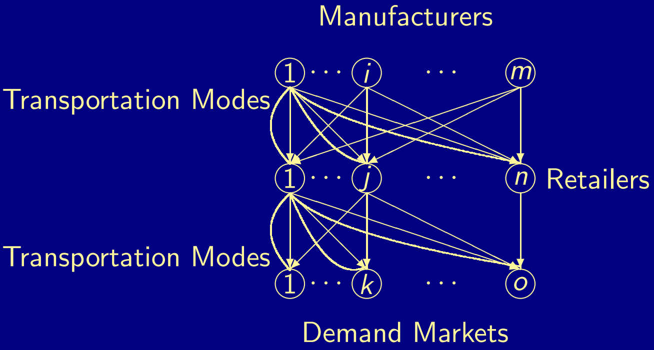


Figure: The Multitiered Network Structure of the Supply Chain

Assumptions

- Manufacturers and retailers are multicriteria decision-makers
- Manufacturers and retailers try to
  - Maximize profit
  - Minimize risk
  - Individual weight assigned to the risk level according to decision maker's attitude towards risk
- Nash Equilibrium

For each manufacturer i, there is a random parameter α_i that reflects the impact of disruption to his production cost function. The expected production cost function is given by:

$$\hat{F}_i(Q^1) \equiv \int f_i(Q^1, \alpha_i) d\mathcal{F}_i(\alpha_i), \quad i = 1, \ldots, m.$$

The variance of the above production cost function is denoted by $VF_i(Q^1)$ where $i = 1, \ldots, m$.
We assume that each manufacturer has g types of transportation modes available to ship the product to the retailers, the cost of which is also subject to disruption impacts. The expected transportation cost function is given by:

$$\hat{C}_{ij}^u(q_{ij}^u) \equiv \int_{\beta_{ij}^u} c_{ij}^u(q_{ij}^u, \beta_{ij}^u) d\mathcal{F}_{ij}^u(\beta_{ij}^u), \; i = 1, \ldots, m; \; j = 1, \ldots, n; \; u = 1, \ldots, g.$$

We further denote the variance of the above transportation cost function as $VC_{ij}^u(Q^1)$ where $i = 1, \ldots, m$; $j = 1, \ldots, n$; $u = 1, \ldots, g$.

Manufacturer's Maximization Problem

$$\text{Maximize} \quad \sum_{j=1}^{n}\sum_{u=1}^{g} \rho_{1ij}^{u*} q_{ij}^{u} - \hat{F}_i(Q^1) - \sum_{j=1}^{n}\sum_{u=1}^{g} \hat{C}_{ij}^{u}(q_{ij}^{u})$$

$$-\theta_i \left[\sum_{i=1}^{m} VF_i(Q^1) + \sum_{j=1}^{n}\sum_{u=1}^{g} VC_{ij}^{u}(q_{ij}^{u}) \right]$$

Nonnegative weight θ_i is assigned to the variance of the cost functions for each manufacturer to reflect his attitude towards disruption risks.

We assume that for each manufacturer, the production cost function and the transaction cost function without disruptions are continuously differentiable and convex. Hence, the optimality conditions for all manufacturers simultaneously (cf. Bazaraa, Sherali, and Shetty (1993) and Nagurney (1999)) can be expressed as the following VI:

The Optimal Conditions for All Manufacturers

Determine $Q^{1*} \in R_+^{mng}$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^{u*}}$$

$$+\frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^{u*}}) - \rho_{1ij}^{u*}] \times [q_{ij}^u - q_{ij}^{u*}] \geq 0, \quad \forall Q^1 \in R_+^{mng}.$$

A random risk/disruption related random parameter η_j is associated with the handling cost of retailer j. The expected handling cost is:

$$\hat{C}_j^1(Q^1, Q^2) \equiv \int c_j(Q^1, Q^2, \eta_j) d\mathcal{F}_j(\eta_j), \ j = 1, \ldots, n$$

The variance of the handling cost function is denoted by $VC_j^1(Q^1, Q^2)$ where $j = 1, \ldots, n$.

Retailer's Maximization Problem

The objective function for distributor j; $j = 1, \ldots, n$ can be expressed as follows:

$$\text{Maximize} \ \sum_{k=1}^{o} \sum_{v=1}^{h} \rho_{2jk}^{v*} q_{jk}^{v} - \hat{C}_j^1(Q^1, Q^2) - \sum_{i=1}^{m} \sum_{u=1}^{g} \rho_{1ij}^{u*} q_{ij}^{u} - \varpi_j VC_j^1(Q^1, Q^2)$$

subject to:

$$\sum_{k=1}^{o} \sum_{v=1}^{h} q_{jk}^{v} \leq \sum_{i=1}^{m} \sum_{u=1}^{g} q_{ij}^{u}$$

and the nonnegativity constraints: $q_{ij}^u \geq 0$ for all i, j, and u; $q_{jk}^v \geq 0$ for all j, k, and v.

We assume that, for each retailer, the handling cost without disruptions is continuously differentiable and convex.

The Optimal Conditions for All Retailers

Determine $(Q^{1*}, Q^{2*}, \gamma^*) \in R_+^{mng+noh+n}$ satisfying:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}\left[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \rho_{1ij}^{u*} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*\right]$$

$$\times \left[q_{ij}^u - q_{ij}^{u*}\right] + \sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}[-\rho_{2jk}^{v*} + \gamma_j^* + \frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right]\times\left[\gamma_j - \gamma_j^*\right] \geq 0, \ \forall (Q^1, Q^2, \gamma) \in R_+^{mng+noh+n}$$

The Market Stochastic Economic Equilibrium Conditions

For any retailer with associated demand market k; $k = 1, \ldots, o$:

$$\hat{d}_k(\rho_3^*) \begin{cases} \leq \sum_{j=1}^{o} \sum_{v=1}^{h} q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^* = 0, \\ = \sum_{j=1}^{o} \sum_{v=1}^{h} q_{jk}^{v*}, & \text{if} \quad \rho_{3k}^* > 0, \end{cases}$$

$$\rho_{2jk}^{v*} + c_{jk}^{v}(Q^{2*}) \begin{cases} \geq \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*} = 0, \\ = \rho_{3k}^*, & \text{if} \quad q_{jk}^{v*} > 0. \end{cases}$$

The above market equilibrium conditions are equivalent to the following VI problem, after taking the expected value and summing over all retailers/demand markets k:

Equivalent VI Problem

Determine $(Q^{2*}, \rho_3^*) \in R_+^{noh+o}$ satisfying:

$$\sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*]$$

$$+\sum_{k=1}^{o}\sum_{j=1}^{n}\sum_{v=1}^{h}(\rho_{2jk}^{v*}+c_{jk}^{v}(Q^{2*})-\rho_{3k}^*)\times[q_{jk}^{v}-q_{jk}^{v*}] \geq 0, \ \forall \rho_3 \in R_+^{o}, \forall Q^2 \in R_+^{noh},$$

where ρ_3 is the o-dimensional vector with components: $\rho_{31}, \ldots, \rho_{3o}$ and Q^2 is the noh-dimensional vector.

Remark:

We are interested in the cases where the expected demands are positive, that is, $\hat{d}_k(\rho_3) > 0$, $\forall \rho_3 \in R^o_+$ for $k = 1, \ldots, o$. Furthermore, we assume that the unit transaction costs: $c^v_{jk}(Q^2) > 0$, $\forall j, k$, $\forall Q^2 \neq 0$.
Under the above assumptions, we can show that $\rho^*_{3k} > 0$ and $\hat{d}_k(\rho^*_3) = \sum_{j=1}^n \sum_{k=1}^o \sum_{v=1}^h q^{v*}_{jk}$, $\forall k$.

Definition: Supply Chain Network Equilibrium with Uncertainty and Expected Demands

The equilibrium state of the supply chain network with disruption risks and expected demands is one where the flows of the product between the tiers of the decision-makers coincide and the flows and prices satisfy the sum of conditions of manufacturers, distributors, and demand markets.

Theorem: VI Formulation of the Supply Chain Network Equilibrium with Uncertainty and Expected Demands

Determine $(Q^{1*}, Q^{2*}, \gamma^*, \rho_3^*) \in R_+^{mng+noh+n+o}$ *satisfying:*

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{u=1}^{g}[\frac{\partial \hat{F}_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial \hat{C}_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u} + \theta_i(\frac{\partial VF_i(Q^{1*})}{\partial q_{ij}^u} + \frac{\partial VC_{ij}^u(q_{ij}^{u*})}{\partial q_{ij}^u})$$

$$+\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{ij}^u} - \gamma_j^*] \times [q_{ij}^u - q_{ij}^{u*}]$$

$$+\sum_{j=1}^{n}\sum_{k=1}^{o}\sum_{v=1}^{h}[\frac{\partial \hat{C}_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v} + \varpi_j \frac{\partial VC_j^1(Q^{1*}, Q^{2*})}{\partial q_{jk}^v}$$

$$+\gamma_j^* + c_{jk}^v(Q^{2*}) - \rho_{3k}^*] \times \left[q_{jk}^v - q_{jk}^{v*}\right]$$

$$+\sum_{j=1}^{n}\left[\sum_{i=1}^{m}\sum_{u=1}^{g} q_{ij}^{u*} - \sum_{k=1}^{o}\sum_{v=1}^{h} q_{jk}^{v*}\right] \times \left[\gamma_j - \gamma_j^*\right] + \sum_{k=1}^{o}(\sum_{j=1}^{n}\sum_{v=1}^{h} q_{jk}^{v*} - \hat{d}_k(\rho_3^*)) \times [\rho_{3k} - \rho_{3k}^*] \geq 0,$$

$$\forall (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}.$$

where $\mathcal{K} \equiv \{(Q^1, Q^2, \gamma, \rho_3) \| (Q^1, Q^2, \gamma, \rho_3) \in R_+^{mng+noh+n+o}\}$.

Algorithm–The Modified Projection Method (Korpelevich(1977))

Step 0: Initialization

Start with an $x^0 \in \mathcal{K}$. Set $k := 1$ and select ρ, such that $0 < \rho < \frac{1}{L}$, where L is the Lipschitz constant for function F in the VI governing the supply chain network.

Step 1: Construction and Computation

Compute $\hat{x}^{k-1}$ by solving the VI subproblem:

$$\langle (\hat{x}^{k-1} + (\rho F(x^{k-1}) - x^{k-1}))^T, x - \hat{x}^{k-1} \rangle \geq 0, \ \ \forall x \in \mathcal{K}.$$

Step 2: Adaptation

Compute x^k by solving the VI problem:

$$\langle (x^{k-1} + (\rho F(\hat{x}^{k-1}) - \hat{x}^{k-1}))^T, x - x^k \rangle \geq 0, \ \ \forall x \in \mathcal{K}.$$

Step 3: Convergence Verification

If $|x^k - x^{k-1}| \leq \epsilon$, for $\epsilon > 0$, a prespecified tolerence, then stop; otherwise, set $k := k + 1$ and go to Step 1.

A Supply Chain Network Performance Measure

The supply chain network performance measure, $\mathcal{E}$, for a given supply chain, and expected demands: $\hat{d}_k$; $k = 1, 2, \ldots, o$, is defined as follows:

$$\mathcal{E} \equiv \frac{\sum_{k=1}^{o} \frac{\hat{d}_k}{\rho_{3k}}}{o},$$

where o is the number of demand markets in the supply chain network, and $\hat{d}_k$ and ρ_{3k} denote, respectively, the expected equilibrium demand and the equilibrium price at demand market k.

Assume that all the random parameters take on a given threshold probability value; say, for example, 95%. Moreover, assume that all the cumulative distribution functions for random parameters have inverse functions. Hence, we have that: $\alpha_i = \mathcal{F}_i^{-1}(.95)$, for $i = 1, \ldots, m$; $\beta_{ij}^u = \mathcal{F}_{ij}^{u^{-1}}(.95)$, for $i = 1, \ldots, m$; $j = 1, \ldots, n$, and so on.

Supply Chain Robustness Measurement

Let $\mathcal{E}_w$ denote the supply chain performance measure with random parameters fixed at a certain level as described above. Then, the supply chain network robustness measure, $\mathcal{R}$, is given by the following:

$$\mathcal{R} = \mathcal{E}^0 - \mathcal{E}_w,$$

where $\mathcal{E}^0$ gauges the supply chain performance based on the supply chain model, but with weights related to risks being zero.

$\mathcal{E}^0$ examines the "base" supply chain performance while $\mathcal{E}_w$ assesses the supply chain performance measure at some prespecified uncertainty level. If their difference is small, a supply chain maintains its functionality well and we consider the supply chain to be robust.

Note that different supply chains may have different requirements regarding the performance and robustness concepts introduced in the previous sections. For example, in the case of a supply chain of a toy product one may focus on how to satisfy demand in the most cost efficient way and not care too much about supply chain robustness. A medical/healthcare supply chain, on the other hand, may have a requirement that the supply chain be highly robust when faced with uncertain conditions. Hence, in order to be able to examine and to evaluate the different application-based supply chains from both perspectives, we now define a weighted supply chain performance measure as follows:

A Weighted Supply Chain Performance Measure

$$\hat{\mathcal{E}} = (1-\epsilon)\mathcal{E}^0 + \epsilon(-\mathcal{R}),$$

where $\epsilon \in [0,1]$ is the weight that is placed on the supply chain robustness.

Numerical Example 1

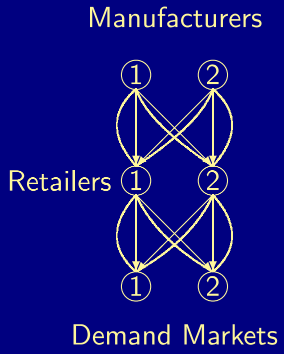


Figure: The Supply Chain Network for the Numerical Examples

For illustration purposes, we assumed that all the random parameters followed uniform distributions. The relevant parameters are as follows:
$\alpha_i \sim [0, 2]$ for $i = 1, 2$; $\beta^u_{ij} \sim [0, 1]$ for $i = 1, 2$; $j = 1, 2$; $u = 1, 2$;
$\eta_j \sim [0, 3]$ for $j = 1, 2$.
Demand functions are assumed followed a uniform distribution given by $[200 - 2\rho_{3k}, 600 - 2\rho_{3k}]$, for $k = 1, 2$. Hence, the expected demand functions are:

$$\hat{d}_k(\rho_3) = 400 - 2\rho_{3k}, \quad \text{for } k = 1, 2.$$

The production cost functions for the manufacturers are given by:

$$f_1(Q^1, \alpha_1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{1j})^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{1j})(\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{2j}) + 2\alpha_1(\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{1j}),$$

$$f_2(Q^1, \alpha_2) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{2j})^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{1j})(\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{2j}) + 2\alpha_2(\sum_{j=1}^{2}\sum_{u=1}^{2} q^u_{2j}).$$

The expected production cost functions for the manufacturers are given by:

$$\hat{F}_1(Q^1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u),$$

$$\hat{F}_2(Q^1) = 2.5(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u)^2 + (\sum_{j=1}^{2}\sum_{u=1}^{2} q_{1j}^u)(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u) + 2(\sum_{j=1}^{2}\sum_{u=1}^{2} q_{2j}^u).$$

The variances of the production cost functions for the manufacturers are given by:
$VF_1(Q^1) = \frac{4}{3}(\sum_{j=1}^2 \sum_{u=1}^2 q_{1j}^u)^2$; $VF_2(Q^1) = \frac{4}{3}(\sum_{j=1}^2 \sum_{u=1}^2 q_{2j}^u)^2$.
The transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$c_{ij}^1(q_{ij}^1, \beta_{ij}^1) = .5(q_{ij}^1)^2 + 3.5\beta_{ij}^1 q_{ij}^1, \text{ for } i = 1, 2; j = 1, 2,$$

$$c_{ij}^2(q_{ij}^2, \beta_{ij}^2) = (q_{ij}^2)^2 + 5.5\beta_{ij}^2 q_{ij}^2, \text{ for } i = 1, 2; j = 1, 2.$$

The expected transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$\hat{C}^1_{ij}(q^1_{ij}) = .5(q^1_{ij})^2 + 1.75q^1_{ij}, \text{ for } i = 1, 2; j = 1, 2,$$

$$\hat{C}^2_{ij}(q^2_{ij}) = .5(q^2_{ij})^2 + 2.75q^2_{ij}, \text{ for } i = 1, 2; j = 1, 2.$$

The variances of the transaction cost functions faced by the manufacturers and associated with transacting with the retailers are given by:

$$VC^1_{ij}(q^1_{ij}) = 1.0208(q^1_{ij})^2, \text{ for } i = 1, 2; j = 1, 2,$$

$$VC^2_{ij}(q^2_{ij}) = 2.5208(q^2_{ij})^2, \text{ for } i = 1, 2; j = 1, 2.$$

The handling costs of the retailers, in turn, are given by:

$$c_j(Q^1, Q^2, \eta_j) = .5(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij})^2 + \eta_j(\sum_{i=1}^{2}\sum_{u=1}^{2} q^u_{ij}), \text{ for } j = 1, 2.$$

The expected handling costs of the retailers are given by:

$$\hat{C}_j^1(Q^1, Q^2) = .5(\sum_{i=1}^{2}\sum_{u=1}^{2} q_{ij}^u)^2 + 1.5(\sum_{i=1}^{2}\sum_{u=1}^{2} q_{ij}^u), \text{ for } j = 1, 2.$$

The variance of the handling costs of the retailers are given by:

$$VC_j(Q^1, Q^2) = \frac{3}{4}(\sum_{i=1}^{2}\sum_{u=1}^{2} q_{ij}^u)^2, \text{ for } j = 1, 2.$$

The unit transaction costs from the retailers to the demand markets are given by:

$$c_{jk}^1(Q^2) = .3q_{jk}^1, \quad \text{for } j = 1, 2; \; k = 1, 2,$$

$$c_{jk}^2(Q^2) = .6q_{jk}^2, \quad \text{for } j = 1, 2; \; k = 1, 2.$$

We assumed that the manufacturers and the retailers placed zero weights on the disruption risks to compute $\mathcal{E}^0$.

The equilibrium shipments between manufacturers and retailers are:

$q_{ij}^{1*} = 8.5022$, for $i = 1, 2$; $j = 1, 2$; $q_{ij}^{2*} = 3.7511$, for $i = 1, 2$; $j = 1, 2$;

The equilibrium shipments between the retailers and the demand markets are:

$q_{jk}^{1*} = 8.1767$, for $j = 1, 2$; $k = 1, 2$; $q_{jk}^{2*} = 4.0767$, for $j = 1, 2$; $k = 1, 2$.

Finally, the equilibrium prices are: $\rho_{31}^* = \rho_{32}^* = 187.7466$ and the expected equilibrium demands are: $\hat{d}_1 = \hat{d}_2 = 24.5068$.

Numerical Example 2

For the same network structure and cost and demand functions, we now assume that the relevant parameters are changed as follows: $\alpha_i \sim [0, 4]$ for $i = 1, 2$; $\beta_{ij}^u \sim [0, 2]$ for $i = 1, 2$; $j = 1, 2$; $u = 1, 2$; $\eta_j \sim [0, 6]$ for $j = 1, 2$.

The equilibrium shipments between manufacturers and retailers are now:

$$q_{ij}^{1*} = 8.6008, \quad \text{for } i = 1, 2;\ j = 1, 2; \quad q_{ij}^{2*} = 3.3004, \quad \text{for } i = 1, 2;\ j = 1, 2;$$

whereas the equilibrium shipments between the retailers and the demand markets are:

$$q_{jk}^{1*} = 7.9385, \quad \text{for } j = 1, 2;\ k = 1, 2; \quad q_{jk}^{2*} = 3.9652, \quad \text{for } j = 1, 2;\ k = 1, 2.$$

The equilibrium prices are: $\rho_{31}^* = \rho_{32}^* = 188.0963$ and the expected equilibrium demands are: $\hat{d}_1 = \hat{d}_2 = 23.8074$.

Table: Supply Chain Performance Measure

	$\mathcal{E}^0$	$\mathcal{E}_w$ $(w = 0.95)$	$\hat{\mathcal{E}}$ $(\epsilon = 0.5)$
Example 1	0.1305	0.1270	0.0635
Example 2	0.1266	0.1194	0.0597

Observe that first example leads to a better measure of performance since the uncertain parameters do not have as great of an impact as in the second one for the cost functions under the given threshold level.

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