Networks Against Time: From Food to Pharma

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Acknowledgments

I would like to thank Professor Dr. Patrick Qiang for inviting me to speak to you today.

It is an honor and pleasure to be speaking to you today on your campus.

Special acknowledgments and thanks to my students and collaborators who have made research and teaching always stimulating and rewarding.

Outline

- Background and Motivation
- Time as a Challenge and Competitive Advantage
- Fascinating Facts About Food Perishability
- Why User Behavior Must be Captured in Supply Chain Network Analysis and Design
- Supply Chain Network Theory
- Methodology – The Variational Inequality Problem
- Variational Inequalities and Optimization Theory
- Variational Inequalities and Game Theory
- The Pharmaceutical Industry, Issues, and a Full Model
- Some Other Issues in Supply Chain Networks that We Have Explored
- Summary, Conclusions, and Suggestions for Future Research

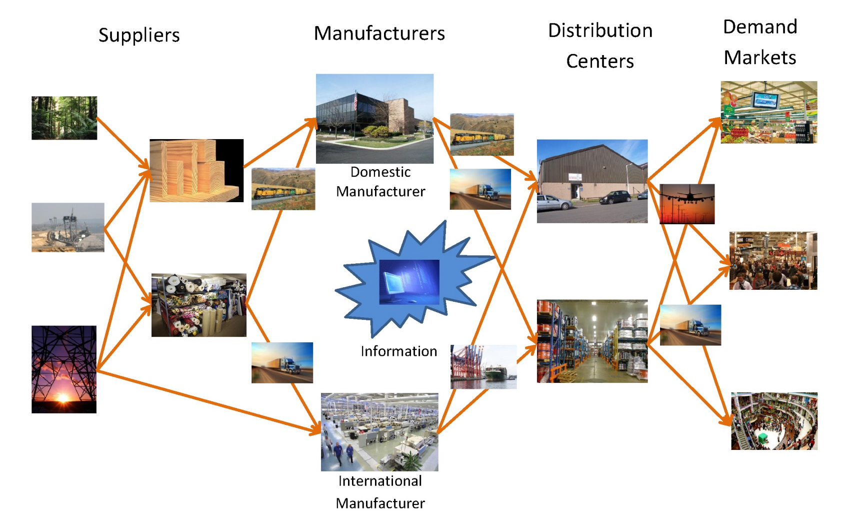
Background and Motivation

Supply chains are the *critical infrastructure and backbones* for the production, distribution, and consumption of goods as well as services in our globalized *Network Economy*.

Supply chains, in their most fundamental realization, *consist of manufacturers and suppliers, distributors, retailers, and consumers at the demand markets*.

Today, supply chains may span thousands of miles across the globe, involve numerous suppliers, retailers, and consumers, and be underpinned by multimodal transportation and telecommunication networks.

A General Supply Chain

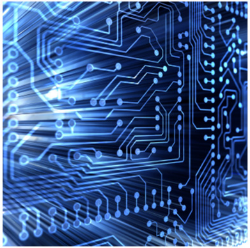


Examples of Supply Chains

- food and food products
- high tech products
- automotive
- energy (oil, electric power, etc.)
- clothing and toys
- healthcare supply chains
- humanitarian relief
- supply chains in nature.

Food Supply Chains

High Tech Products



Automotive Supply Chains

Energy Supply Chains

Clothing and Toys

Healthcare Supply Chains

Humanitarian Relief

Supply Chains in Nature

Characteristics of Supply Chains and Networks Today

- *large-scale nature* and complexity of network topology;
- *congestion*, which leads to nonlinearities;
- *alternative behavior of users of the networks*, which may lead to paradoxical phenomena;
- *possibly conflicting criteria associated with optimization*;
- *interactions among the underlying networks themselves*, such as the Internet with electric power networks, financial networks, and transportation and logistical networks;
- recognition of *their fragility and vulnerability*;
- policies surrounding networks today may have major impacts not only economically, but also *socially, politically, and security-wise*.

Time as a Challenge and Competitive Advantage

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- Delivery times are becoming a strategy, as important as productivity, quality, and even innovation.
- Practitioners realize that speed and consistency of delivery time are two essential components of customer satisfaction, along with price.

Time as a Challenge and Competitive Advantage

Stalk, Jr., in his *Harvard Business Review* 1988 article, "Time - The next source of competitive advantage," utilized the term *time-based competition*, to single out time as the major factor for sustained competitive advantage.

Today, time-based competition has emerged as a paradigm for strategizing about and operationalizing supply chain networks in which efficiency and timeliness matter.

Time and Perishable Products

Added challenges arise in the case of *Perishable Products*, which, by definition, are time-sensitive.

Benjamin Franklin wrote in 1748 in his "Advice to a Young Tradesman," *Remember that Time is Money*.

It may also be said that *time is life*, since time-sensitive products, such as vaccines and medicines, as well as, at the most fundamental level, food and water, are of a life-sustaining, if not, life-saving, nature,

Time and Perishable Products

Classical examples of perishable goods include fresh produce in the form of fruits and vegetables, meat and dairy products, medicines and vaccines, radioisotopes, cut flowers, and even human blood.

We take the broader perspective of products being perishable not only in terms of their characteristics (such as their chemistry and the underlying physics) and *supply* (that is, the manner of procurement/production/processing, storage, transportation, etc.) aspects, but also in terms of the *demand* for the products.

Time and Perishable Products

We include under the *perishable product* umbrella products that are *discarded* (or replaced) relatively quickly after purchase, because of changing consumer tastes, such as *fast* fashion apparel, or those that become obsolete (as in certain high technology products).

Such an approach follows from Whitin (1957), who considered the deterioration of fashion goods at the end of a prescribed shortage period.

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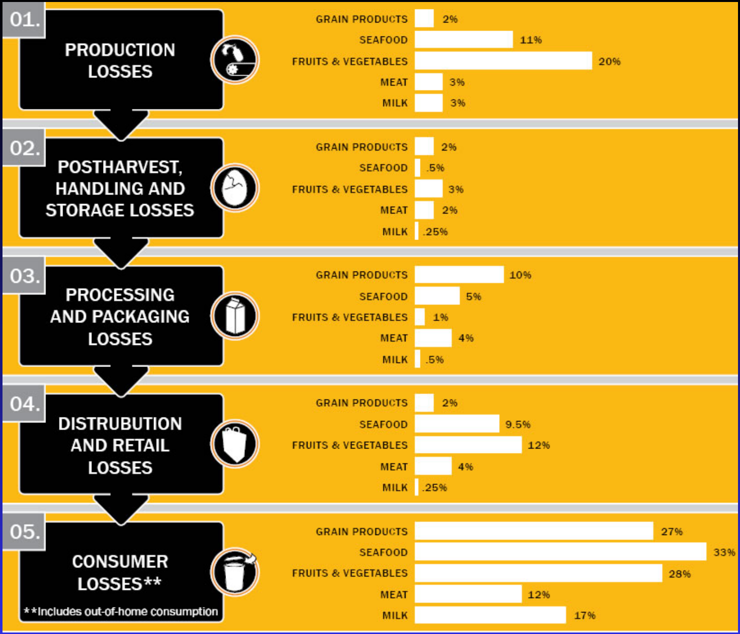
- *Incurred waste discarding cost*: Discarding of the waste may impose additional costs.
- *Safety and environmental impact*: Perished products and the associated waste may be hazardous and may pollute.
- *Demand management*: Demand may be uncertain or known (as in scheduled treatments) and fixed. It may be price-sensitive (fashion apparel, consumer goods and pharma).

Fascinating Facts About Food Perishability

THE

SHELF LIFE OF FOOD

Foods unopened, uncut or uncooked unless stated otherwise	COUNTER/PANTRY (1 DAY → 1 MONTH)	REFRIGERATOR (1 DAY → 3 MONTHS)	FREEZER (1 MONTH → 1 YEAR)
APPLES	2-4 weeks	1-2 months	8-12 months
BANANAS	2-7 days	5-9 days	2-3 months
CANTALOUPE	Until ripe	1 week	8-12 months
CARROTS	Up to 4 days	4-5 weeks	8-12 months
CUCUMBERS	1-3 days	1 week	8-12 months
EGGS	Few hours	3-4 weeks	Do ~~not~~ freeze
MILK	Few hours	5-7 days	1 month
YOGURT	Few hours	2-3 weeks	1-2 months
BACON	2 hours	2 weeks	4 months



Source: Food and Agriculture Organization 2011

ABOUT 10 PERCENT OF THE U.S. ENERGY BUDGET GOES TO BRINGING FOOD TO OUR TABLES.

Source: Webber, Michael, "How to Make the Food System More Energy Efficient," *Scientific American*, December 29, 2011.

ONE INDUSTRY CONSULTANT ESTIMATES THAT UP TO ONE IN SEVEN TRUCKLOADS OF PERISHABLES DELIVERED TO SUPERMARKETS IS THROWN AWAY.

Source: Beswick, P. et al, "A Retailer's Recipe for Fresher Food and Far Less Shrink," Oliver Wyman, Boston. ergoeditorial.biz/worksamples/OW%20grocery%20shrinkage.pdf.

FOR THE AVERAGE U.S. HOUSEHOLD OF FOUR, FOOD WASTE TRANSLATES INTO AN ESTIMATED $1,350 TO $2,275 IN ANNUAL LOSSES.

Source: Bloom, *American Wasteland*, 187. Another report using updated USDA consumer loss numbers and 2011 prices estimates $1,600 in annual losses per household of four: Clean Metrics, "The Climate Change and Economic Impacts of Food Waste in the United States," http://www.cleanmetrics.com/pages/ClimateChangeImpactofUSFoodWaste.pdf.

Source: Food and Agriculture Organization 2011

Supply Chains Are Network Systems

Supply chains are, in fact, *Complex Network Systems*.

Hence, *any formalism that seeks to model supply chains and to provide quantifiable insights and measures must be a system-wide one and network-based.*

Such crucial issues as the stability and resiliency of supply chains, as well as their adaptability and responsiveness to events in *a global environment of increasing risk and uncertainty* can only be rigorously examined from the view of supply chains as network systems.

Supply chains may be characterized by *decentralized decision-making* associated with the different economic agents or by *centralized* decision-making.

Why User Behavior Must be Captured in Supply Chain Network Analysis and Design

Supply Chain Network Design Must Capture the Behavior of Users

Behavior on Congested Networks

Decision-makers select their cost-minimizing routes.

Decentralized		Selfish		User-Optimized U–O
vs.	⟺	vs.	⟺	vs.
Centralized		Unselfish		S–O System-Optimized

Flows are routed so as to minimize the total cost to society.

Two fundamental principles of travel behavior, due to Wardrop (1952), with terms coined by Dafermos and Sparrow (1969).

User-optimized (U-O) (network equilibrium) Problem – each user determines his/her cost minimizing route of travel between an origin/destination, until an equilibrium is reached, in which no user can decrease his/her cost of travel by unilateral action (in the sense of Nash).

System-optimized (S-O) Problem – users are allocated among the routes so as to minimize the total cost in the system, where the total cost is equal to the sum over all the links of the link's user cost times its flow.

The U-O problems, under certain simplifying assumptions, possesses optimization reformulations. But now we can handle cost asymmetries, multiple modes of transport, and different classes of travelers, without such assumptions.

We Can State These Conditions Mathematically!

The U-O and S-O Conditions

Definition: U-O or Network Equilibrium – Fixed Demands

A path flow pattern x^**, with nonnegative path flows and O/D pair demand satisfaction, is said to be U-O or in equilibrium, if the following condition holds for each O/D pair* $w \in W$ *and each path* $p \in P_w$*:*

$$C_p(x^*) \begin{cases} = \lambda_w, & \text{if} \quad x_p^* > 0, \\ \geq \lambda_w, & \text{if} \quad x_p^* = 0. \end{cases}$$

Definition: S-O Conditions

A path flow pattern x with nonnegative path flows and O/D pair demand satisfaction, is said to be S-O, if for each O/D pair $w \in W$ and each path $p \in P_w$:

$$\hat{C}'_p(x) \begin{cases} = \mu_w, & \text{if} \quad x_p > 0, \\ \geq \mu_w, & \text{if} \quad x_p = 0, \end{cases}$$

where $\hat{C}'_p(x) = \sum_{a \in \mathcal{L}} \frac{\partial \hat{c}_a(f_a)}{\partial f_a} \delta_{ap}$, and μ_w is a Lagrange multiplier.

The importance of behavior will now be illustrated through a famous example known as the Braess paradox which demonstrates what can happen under *U-O* as opposed to *S-O* behavior.

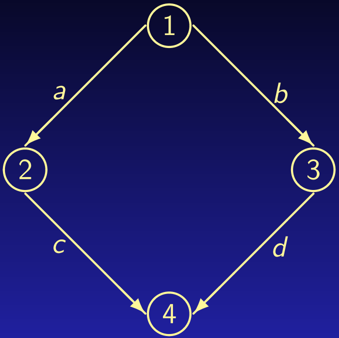
Although the paradox was presented in the context of transportation networks, it is relevant to other network systems in which decision-makers act in a noncooperative (competitive) manner.

The Braess (1968) Paradox

Assume a network with a single O/D pair (1,4). There are 2 paths available to travelers: $p_1 = (a, c)$ and $p_2 = (b, d)$.

For a travel demand of **6**, the equilibrium path flows are $x^*_{p_1} = x^*_{p_2} = 3$ and

The equilibrium path travel cost is $C_{p_1} = C_{p_2} = 83.$



$$c_a(f_a) = 10f_a, \quad c_b(f_b) = f_b + 50,$$

$$c_c(f_c) = f_c + 50, \quad c_d(f_d) = 10f_d.$$

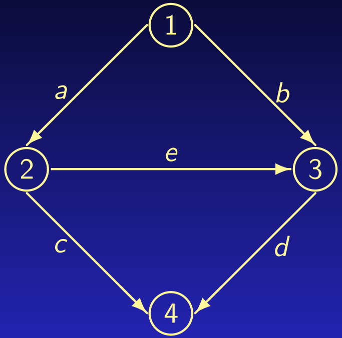
Adding a Link Increases Travel Cost for All!

Adding a new link creates a new path $p_3 = (a, e, d)$.

The original flow distribution pattern is no longer an equilibrium pattern, since at this level of flow the cost on path p_3, $C_{p_3} = 70$.

The new equilibrium flow pattern network is
$x^*_{p_1} = x^*_{p_2} = x^*_{p_3} = 2$.

The equilibrium path travel cost:
$C_{p_1} = C_{p_2} = C_{p_3} = 92$.



$c_e(f_e) = f_e + 10$

The 1968 Braess article has been translated from German to English and appears as:

"On a Paradox of Traffic Planning,"

D. Braess, A. Nagurney, and T. Wakolbinger (2005) Transportation Science **39**, 446-450.

Über ein Paradoxon aus der Verkehrsplanung

Von D. Braess, Münster [1])

Eingegangen am 28. März 1968

Zusammenfassung: Für die Straßenverkehrsplanung möchte man den Verkehrsfluß auf den einzelnen Straßen des Netzes abschätzen, wenn die Zahl der Fahrzeuge bekannt ist, die zwischen den einzelnen Punkten des Straßennetzes verkehren. Welche Wege am günstigsten sind, hängt nun nicht nur von der Beschaffenheit der Straße ab, sondern auch von der Verkehrsdichte. Es ergeben sich nicht immer optimale Fahrzeiten, wenn jeder Fahrer nur für sich den günstigsten Weg heraussucht. In einigen Fällen kann sich durch Erweiterung des Netzes der Verkehrsfluß sogar so umlagern, daß größere Fahrzeiten erforderlich werden.

Summary: For each point of a road network let be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of the traffic flow. Whether a street is preferable to another one depends not only upon the quality of the road but also upon the density of the flow. If every driver takes that path which looks most favorable to him, the resultant running times need not be minimal. Furthermore it is indicated by an example that an extension of the road network may cause a redistribution of the traffic which results in longer individual running times.

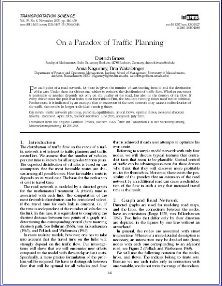
1. Einleitung

Für die Verkehrsplanung und Verkehrssteuerung interessiert, wie sich der Fahrzeugstrom auf die einzelnen Straßen des Verkehrsnetzes verteilt. Bekannt sei dabei die Anzahl der Fahrzeuge für alle Ausgangs- und Zielpunkte. Bei der Berechnung wird davon ausgegangen, daß von den möglichen Wegen jeweils der günstigste gewählt wird. Wie günstig ein Weg ist, richtet sich nach dem Aufwand, der zum Durchfahren nötig ist. Die Grundlage für die Bewertung des Aufwandes bildet die Fahrzeit.

Für die mathematische Behandlung wird das Straßennetz durch einen gerichteten Graphen beschrieben. Zur Charakterisierung der Bögen gehört die Angabe des Zeitaufwandes. Die Bestimmung der günstigen Stromverteilungen kann als gelöst betrachtet werden, wenn die Bewertung konstant ist, d. h., wenn die Fahrzeiten unabhängig von der Größe des Verkehrsflusses sind. Sie ist dann äquivalent mit der bekannten Aufgabe, den kürzesten Abstand zweier Punkte eines Graphen und den zugehörigen kritischen Pfad zu bestimmen [1], [5], [7].

Will man das Modell aber realistischer gestalten, ist zu berücksichtigen, daß die benötigte Zeit stark von der Stärke des Verkehrs abhängt. Wie die folgenden Untersuchungen zeigen, ergeben sich dann gegenüber dem Modell mit konstanter (belastungsunabhängiger) Bewertung z. T. völlig neue Aspekte. Dabei erweist sich schon eine Präzisierung der Problemstellung als notwendig; denn es ist zwischen dem Strom zu unterscheiden, der für alle am günstigsten ist,, und dem, der sich einstellt, wenn jeder Fahrer nur seinen eigenen Weg optimalisiert.

[1]) Priv.-Doz. Dr. Dietrich Braess, Institut für numerische und instrumentelle Mathematik, 44 Münster, Hüfferstr. 3 a.



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On a Paradox of Traffic Planning

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For each point of a road network, let there be given the number of cars starting from it, and the destination of the cars. Under these conditions one wishes to estimate the distribution of traffic flow. Whether one street is preferable to another depends not only on the quality of the road, but also on the density of the flow. If every driver takes the path that looks most favorable to him, the resultant running times need not be minimal. Furthermore, it is indicated by an example that an extension of the road network may cause a redistribution of the traffic that results in longer individual running times.

Key words: traffic network planning; paradox; equilibrium; critical flows; optimal flows; existence theorem
History: Received: April 2005; revision received: June 2005; accepted: July 2005.

Translated from the original German: Braess, Dietrich. 1968. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung* **12** 258–268.

1. Introduction

The distribution of traffic flow on the roads of a traffic network is of interest to traffic planners and traffic controllers. We assume that the number of vehicles per unit time is known for all origin-destination pairs. The expected distribution of vehicles is based on the assumption that the most favorable routes are chosen among all possible ones. How favorable a route is depends on its travel cost. The basis for the evaluation of cost is travel time.

The road network is modeled by a directed graph for the mathematical treatment. A (travel) time is associated with each link. The computation of the most favorable distribution can be considered solved if the travel time for each link is constant, i.e., if the time is independent of the number of vehicles on the link. In this case, it is equivalent to computing the shortest distance between two points of a graph and determining the corresponding critical (here meaning shortest) path. See Bellman (1958), von Falkenhausen (1963), and Pollack and Wiebenson (1960).

In more realistic models, however, one has to take into account that the travel time on the links will strongly depend on the traffic flow. Our investigations will show that we will encounter new effects compared to the model with flow-independent costs. Specifically, a more precise formulation of the problem will be required. We have to distinguish between flow that will be optimal for all vehicles and flow that is achieved if each user attempts to optimize his own route.

Referring to a simple model network with only four nodes, we will discuss typical features that contradict facts that seem to be plausible. Central control of traffic can be advantageous even for those drivers who think that they will discover more profitable routes for themselves. Moreover, there exists the possibility of the paradox that an extension of the road network by an additional road can cause a redistribution of the flow in such a way that increased travel time is the result.

2. Graph and Road Network

Directed graphs are used for modeling road maps, and the links, the connections between the nodes, have an orientation (Berge 1958, von Falkenhausen 1966). Two links that differ only by their direction are depicted in the figures by one line without an arrowhead.

In general, the nodes are associated with street intersections. Whenever a more detailed description is necessary, an intersection may be divided into (four) nodes with each one corresponding to an adjacent road; see Figure 2 (Pollack and Wiebenson 1960).

We will use the following notation for the nodes, links, and flows. The indices belong to finite sets. Because we use each index only in connection with one variable, we do not write the range of the indices.

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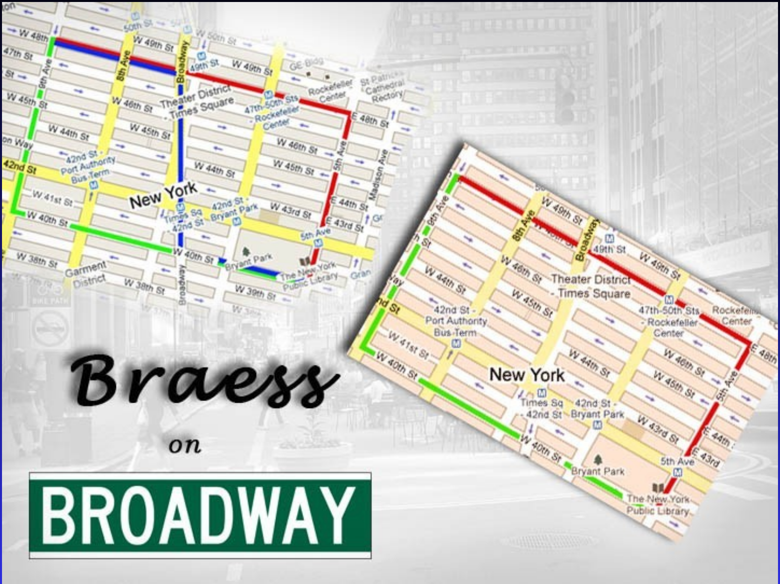
The Braess Paradox Around the World

1969 - Stuttgart, Germany - The traffic worsened until a newly built road was closed.

1990 - Earth Day - New York City - 42^{nd} Street was closed and traffic flow improved.

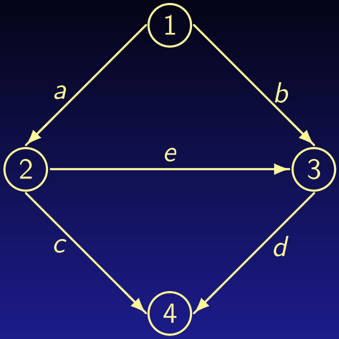
2002 - Seoul, Korea - A 6 lane road built over the Cheonggyecheon River that carried 160,000 cars per day and was perpetually jammed was torn down to improve traffic flow.


Braess
on
BROADWAY

Interview on Broadway for *America Revealed* on March 15, 2011

Under S-O behavior, the total cost in the network is minimized, and the new route p_3, under the same demand, would not be used.

The Braess paradox never occurs in S-O networks.



Recall the Braess network with the added link e.

What happens as the demand increases?

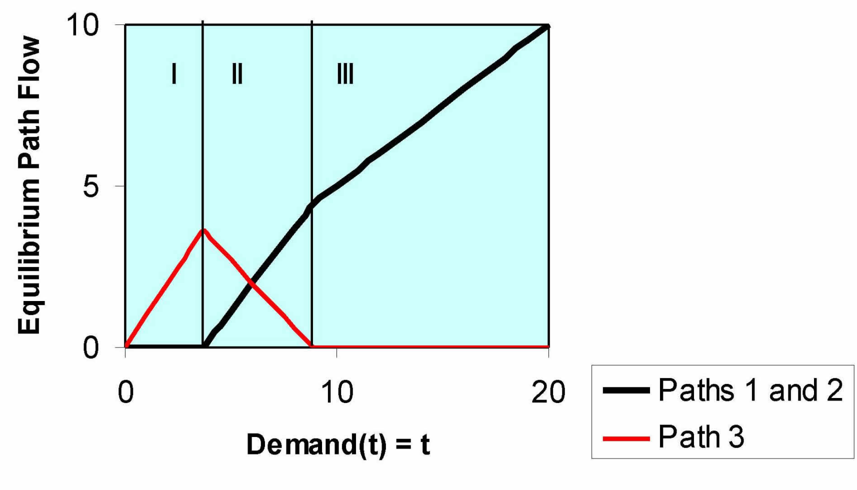
For Networks with Time-Dependent Demands

We Use Evolutionary Variational Inequalities

Radcliffe Institute for Advanced Study – Harvard University 2005-2006

Research with Professor David Parkes of Harvard University and Professor Patrizia Daniele of the University of Catania, Italy

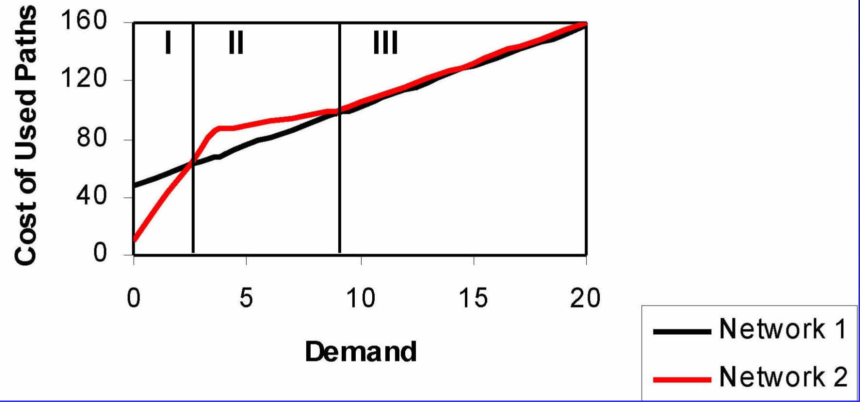
The U-O Solution of the Braess Network with Added Link (Path) and Time-Varying Demands Solved as an *Evolutionary Variational Inequality* (Nagurney, Daniele, and Parkes, *Computational Management Science* (2007)).



In Demand Regime I, Only the New Path is Used.

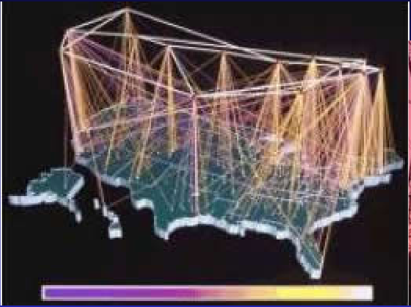
In Demand Regime II, the travel demand lies in the range [2.58, 8.89], and *the Addition of a New Link (Path) Makes Everyone Worse Off*!

In Demand Regime III, when the travel demand exceeds 8.89, *Only the Original Paths are Used!*



The new path is never used, under U-O behavior, when the demand exceeds 8.89, even out to infinity!

Other Networks that Behave like Traffic Networks



The Internet and electric power networks and even supply chains!

Supply Chain Network Theory

Supply Chain Network Theory

Supply chain network theory is an integrated theory that includes:

- *network theory* – in order to identify the structure of the supply chain and relationships
- *optimization theory* – in order to capture the decision-maker's criteria in the form of objective functions, the decision variables that they control, and the underlying recourse constraints, and
- *game theory* – so that cooperation, if appropriate, as well as the reality of competition can be modeled.

For full background, see the book: Anna Nagurney, *Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits*, Edward Elgar Publishing, Cheltenham, England (2006).
The fundamental methodology that we utilize for the integration is that of *Variational Inequality Theory*.

Methodology - The Variational Inequality Problem

Methodology - The Variational Inequality Problem

We utilize the theory of variational inequalities for the formulation, analysis, and solution of both centralized and decentralized supply chin network problems.

Definition: The Variational Inequality Problem

The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, *is to determine a vector* $X^* \in \mathcal{K}$, *such that:*

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where F *is a given continuous function from* $\mathcal{K}$ *to* R^N, $\mathcal{K}$ *is a given closed convex set, and* $\langle \cdot, \cdot \rangle$ *denotes the inner product in* R^N.

Methodology - The Variational Inequality Problem

The vector X consists of the decision variables – typically, the flows (products, prices, etc.).

$\mathcal{K}$ is the feasible set representing how the decision variables are constrained – for example, the flows may have to be nonnegative; budget constraints may have to be satisfied; similarly, quality and/or time constraints.

The function F that enters the variational inequality represents functions that capture the bahvior in the form of the functions such as costs, profits, risk, etc.

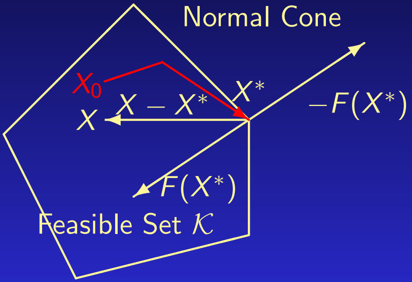
The variational inequality problem contains, as special cases, such mathematical programming problems as:

- systems of equations,
- optimization problems,
- complementarity problems,
- game theory problems, operating under Nash equilibrium,
- and is related to the fixed point problem.

Hence, it is a natural methodology for a spectrum of supply chain network problems from centralized to decentralized ones as well as to design problems.

Geometric Interpretation of VI($F, \mathcal{K}$) and a Projected Dynamical System (Dupuis and Nagurney, Nagurney and Zhang)

In particular, $F(X^*)$ is "orthogonal" to the feasible set $\mathcal{K}$ at the point X^*.



Associated with a VI is a Projected Dynamical System, which provides a natural underlying dynamics associated with travel (and other) behavior to the equilibrium.

To model the *dynamic behavior of complex networks*, including supply chains, we utilize *projected dynamical systems* (PDSs) advanced by Dupuis and Nagurney (1993) in *Annals of Operations Research* and by Nagurney and Zhang (1996) in our book *Projected Dynamical Systems and Variational Inequalities with Applications*.

Such nonclassical dynamical systems are now being used in

evolutionary games (Sandholm (2005, 2011)),

ecological predator-prey networks (Nagurney and Nagurney (2011a, b)), and

even *neuroscience* (Girard et al. (2008)).

Variational Inequalities and Optimization Theory

Variational Inequalities and Optimization Theory

Optimization problems, including constrained and unconstrained, can be formulated as variational inequality problems (see Nagurney (1999)). The relationship between variational inequalities and optimization problems is as follows.

Proposition

Let X^* be a solution to the optimization problem:

$$\text{Minimize} \quad f(X)$$

subject to:

$$X \in \mathcal{K},$$

where f is continuously differentiable and $\mathcal{K}$ is closed and convex. Then X^* is a solution of the variational inequality problem: determine $X^* \in \mathcal{K}$, such that

$$\langle \nabla f(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $\nabla f(X)$ is the gradient vector of f with respect to X.

Proposition

If $f(X)$ is a convex function and X^* is a solution to $\mathrm{VI}(\nabla f, \mathcal{K})$, then X^* is a solution to the optimization problem:

$$\textit{Minimize} \quad f(X)$$

subject to:

$$X \in \mathcal{K}.$$

In the case that the feasible set $\mathcal{K} = R^n$, then the unconstrained optimization problem is also a variational inequality problem.

The variational inequality problem can be reformulated as an optimization problem under certain symmetry conditions. The definitions of positive semidefiniteness, positive definiteness, and strongly positive definiteness are presented next, followed by stating the above relationship in a theorem.

Optimization and Supply Chain Networks

The types of optimization problems that are of relevance to supply chain networks, include:

- the minimization of costs,
- the maximization of profits,
- the minimization of risk, the minimization of pollution emissions,
- the minimization of delay,
- or a combination thereof, subject to the constraints being met.

Variational Inequalities and Game Theory

Variational Inequalities and Game Theory

The Nobel laureate John Nash (1950, 1951) developed noncooperative game theory, involving multiple players, each of whom acts in his/her own interest.

In particular, consider a game with m players, each player i having a strategy vector $X_i = \{X_{i1}, ..., X_{in}\}$ selected from a closed, convex set $\mathcal{K}^i \subset R^n$. Each player i seeks to maximize his/her own utility function, U_i: $\mathcal{K} \rightarrow R$, where $\mathcal{K} = \mathcal{K}^1 \times \mathcal{K}^2 \times \ldots \times \mathcal{K}^m \subset R^{mn}$.

The utility of player i, U_i, depends not only on his/her own strategy vector, X_i, but also on the strategy vectors of all the other players, $(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_m)$.

An equilibrium is achieved if no one can increase his/her utility by unilaterally altering the value of its strategy vector. The formal definition of Nash equilibrium is:

Definition: Nash Equilibrium

A Nash equilibrium is a strategy vector

$$X^* = (X_1^*, \ldots, X_m^*) \in \mathcal{K},$$

such that

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in \mathcal{K}^i, \forall i,$$

where $\hat{X}_i^* = (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_m^*)$.

In other words, under Nash equilibrium, no unilateral deviation in strategy by any single player is profitable for that player.

Variational Inequalities and Game Theory

Given continuously differentiable and concave utility functions, U_i, $\forall i$, the Nash equilibrium problem can be formulated as a variational inequality problem defined on $\mathcal{K}$ (cf. Hartman and Stampacchia (1966), Gabay and Moulin (1980), and Nagurney (1999)).

Theorem: Variational Inequality Formulation of Nash Equilibrium

Under the assumption that each utility function U_i is continuously differentiable and concave, X^* is a Nash equilibrium if and only if $X^* \in \mathcal{K}$ is a solution of the variational inequality

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad X \in \mathcal{K},$$

where $F(X) \equiv (-\nabla_{X_1} U_1(X), \ldots, -\nabla_{X_m} U_m(X))$ and $\nabla_{X_i} U_i(X) = (\frac{\partial U_i(X)}{\partial X_{i1}}, \ldots, \frac{\partial U_i(X)}{\partial X_{in}})$.

Game Theory and Supply Chain Networks

In game theory models of supply chain networks, each decision-maker (such as a firm) has his/her own objective function and constraints.

In the case of oligopolies, for example, the objective functions could be profits and the strategic variables the product flows.

Examples of oligopolies are:

- airlines
- freight carriers
- automobile manufacturers
- oil companies
- beer / beverage companies
- wireless communications
- fast fashion brands
- certain financial institutions.

Supply Chain Networks – Optimization Models

Blood Supply Chains for the Red Cross

A. Nagurney, A. H. Masoumi, and M. Yu, "Supply Chain Network Operations Management of a Blood Banking System with Cost and Risk Minimization," *Computational Management Science* **9(2)** (2012), pp 205-231.

Blood Supply Chains for the Red Cross

- The shelf life of platelets is 5 days and of red blood cells is 42.
- Over 39,000 donations are needed everyday in the US, and the blood supply is frequently reported to be just 2 days away from running out (American Red Cross (2010)).
- Some hospitals have delayed surgeris due to blood shortages on 120 days in a year (Whitaker et al. (2007)).
- The national estimate for the number of units blood products outdated by blood centers and hospitals was 1,276,000 out of 15,688,000 units (Whitaker et al. (2007)).

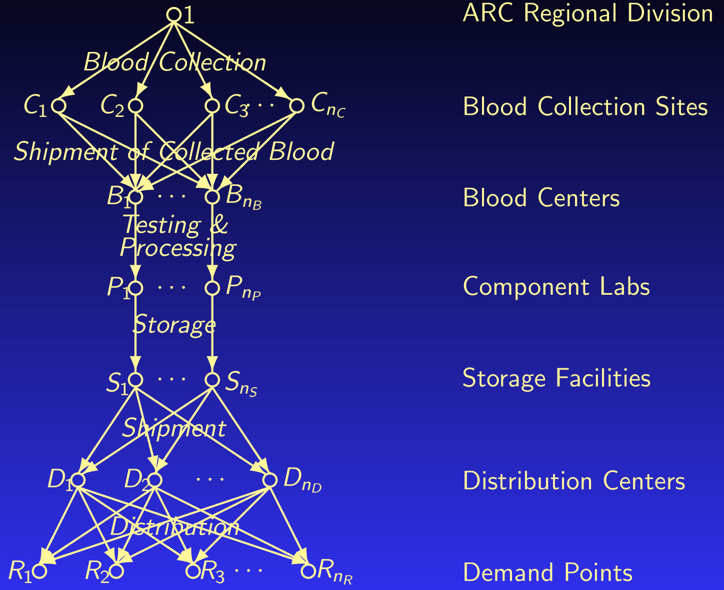
The American Red Cross is the major supplier of blood products to hospitals and medical centers satisfying over 45% of the demand for blood components nationally (Walker (2010)).

Blood Supply Chains for the Red Cross

The hospital cost of a unit of red blood cells in the US had a 6.4% increase from 2005 to 2007.

In the US, this criticality has become more of an issue in the Northeastern and Southwestern states since this cost is meaningfully higher compared to that of the Southeastern and Central states.

Supply Chain Network Topology for a Regionalized Blood Bank



Blood Supply Chains for the Red Cross

We developed a supply chain network optimization model for the management of the procurement, testing and processing, and distribution of a perishable product – that of human blood.

Novel features of the model include:

- It captures *perishability of this life-saving product* through the use of arc multipliers;
- It contains *discarding costs* associated with waste/disposal;
- It handles *uncertainty* associated with demand points;
- It assesses *costs associated with shortages/surpluses at the demand points*, and
- It quantifies the *supply-side risk* associated with procurement.

Medical Nuclear Supply Chains

We developed a medical nuclear supply chain network design model which captures the decay of the radioisotope molybdenum.

"Medical Nuclear Supply Chain Design: A Tractable Network Model and Computational Approach," A. Nagurney and L. S. Nagurney, *International Journal of Production Economics* **140(2)** (2012), pp 865-874.

Medical Nuclear Supply Chains

In our medical nuclear supply chain models we capture the radioactive decay through the use of arc multipliers.

Hence, the framework for both our blood supply chain work and medical nuclear work is that of *generalized* networks.

We will highlight how this is done in a full developed game theory model for pharmaceuticals later in this presentation. We use a similar approach for capturing food perishability.

Medical Nuclear Supply Chains

Medical nuclear supply chains are essential supply chains in healthcare and provide the conduits for products used in nuclear medical imaging, which is routinely utilized by physicians for diagnostic analysis for both cancer and cardiac problems.

Such supply chains have unique features and characteristics due to the products' time-sensitivity, along with their hazardous nature.

Salient Features:

- complexity
- economic aspects
- underlying physics of radioactive decay
- importance of considering both waste management and risk management.

Medical Nuclear Supply Chains

Over 100,000 hospitals in the world use radioisotopes (World Nuclear Association (2011)).

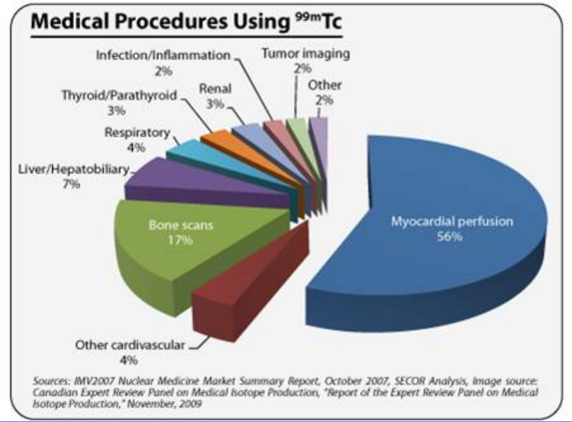
Technetium, ^{99m}Tc, which is a decay product of Molybdenum-99, ^{99}Mo, is the most commonly used medical radioisotope, used in more than 80% of the radioisotope injections, with more than 30 million procedures worldwide each year.

The half-life of Molybdenum-99 is 66 hours.

Each day, 41,000 nuclear medical procedures are performed in the United States using Technetium-99m.

Medical Nuclear Supply Chains

A radioactive isotope is bound to a pharmaceutical that is injected into the patient and travels to the site or organ of interest in order to construct an image for medical diagnostic purposes.



Medical Nuclear Supply Chains

For over two decades, all of the Molybdenum necessary for US-based nuclear medical diagnostic procedures comes from foreign sources.

Medical Nuclear Supply Chains

^{99}Mo Supply Chain Challenges:

- The majority of the reactors are between 40 and 50 years old. Several of the reactors currently used are due to be retired by the end of this decade(Seeverens (2010) and OECD Nuclear Energy Agency (2010a)).

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Medical Nuclear Supply Chains

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^{99}Mo Supply Chain Challenges:

- The majority of the reactors are between 40 and 50 years old. Several of the reactors currently used are due to be retired by the end of this decade(Seeverens (2010) and OECD Nuclear Energy Agency (2010a)).
- Limitations in processing capabilities make the world critically vulnerable to Molybdenum supply chain disruptions.
- The number of generator manufacturers is under a dozen (OECD Nuclear Energy Agency (2010b)).
- Long-distance transportation of the product during raises safety and security risks, and also results in greater decay of the product.

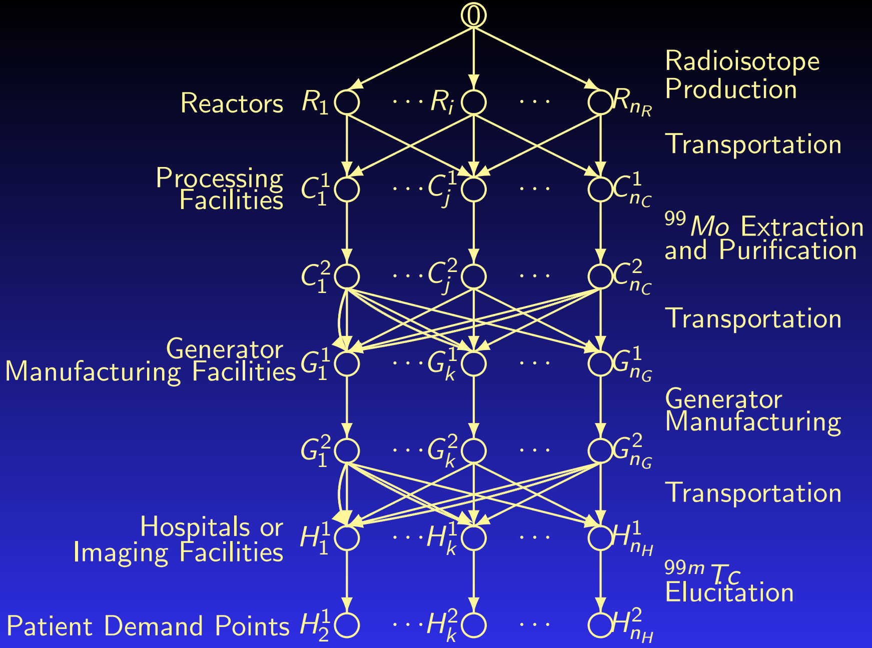


Figure 1: The Medical Nuclear Supply Chain Network Topology

Supply Chain Networks – Game Theory Models

Electric Power Supply Chains

We developed *an empirical, large-scale electric supply chain network equilibrium model*, formulated it as a VI problem, and were able to solve it by *exploiting the connection between electric power supply chain networks and transportation networks* using our proof of a hypothesis posed in the classic book, *Studies in the Economics of Transportation*, by Beckmann, McGuire, and Winsten (1956).

The paper, "An Integrated Electric Power Supply Chain and Fuel Market Network Framework: Theoretical Modeling with Empirical Analysis for New England," by Zugang Liu and Anna Nagurney was published in *Naval Research Logistics* **56** (2009), pp 600-624.

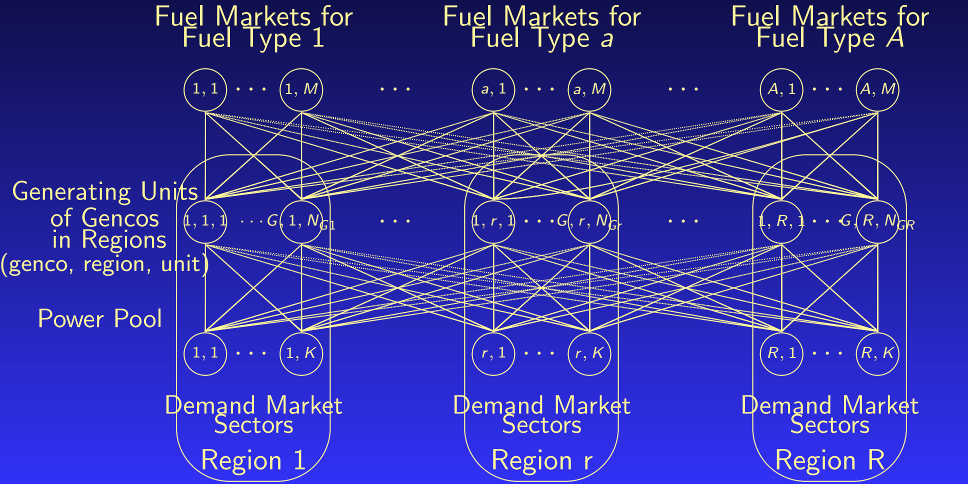
An Empirical Example of an Electric Power Supply Chain for New England

There are 82 generating companies who own and operate 573 generating units. We considered 5 types of fuels: natural gas, residual fuel oil, distillate fuel oil, jet fuel, and coal. The whole area was divided into 10 regions:

1. Maine,
2. New Hampshire,
3. Vermont,
4. Connecticut (excluding Southwest Connecticut),
5. Southwestern Connecticut (excluding the Norwalk-Stamford area),
6. Norwalk-Stamford area,
7. Rhode Island,
8. Southeastern Massachusetts,
9. Western and Central Massachusetts,
10. Boston/Northeast Massachusetts.

Graphic of New England

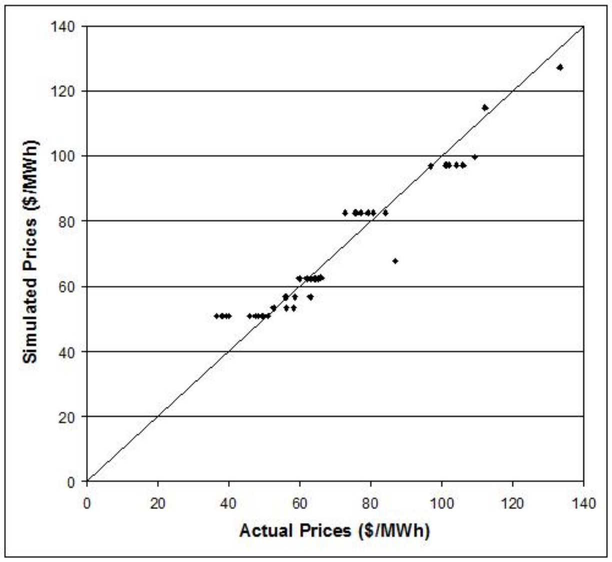
The Electric Power Supply Chain Network with Fuel Supply Markets



We tested the model on the data of July 2006 which included $24 \times 31 = 744$ hourly demand/price scenarios. We sorted the scenarios based on the total hourly demand, and constructed the load duration curve. We divided the duration curve into 6 blocks ($L_1 = 94$ hours, and $L_w = 130$ hours; $w = 2, ..., 6$) and calculated the average regional demands and the average weighted regional prices for each block.

The empirical model had on the order of $20,000$ *variables.*

Actual Prices Vs. Simulated Prices ($/Mwh)



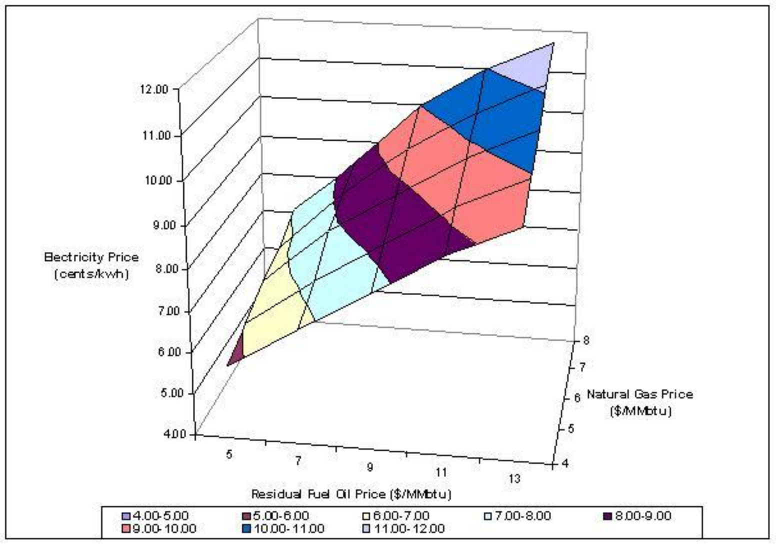
Sensitivity Analysis

We used the same demand data, and then varied the prices of natural gas and residual fuel oil. We assumed that the percentage change of distillate fuel oil and jet fuel prices were the same as that of the residual fuel oil price.

The next figure presents the average electricity price for the two peak blocks under oil/gas price variations.

The surface in the figure represents the average peak electricity prices under different natural gas and oil price combinations.

Sensitivity Analysis



Food Supply Chains

Food is something anyone can elate to.

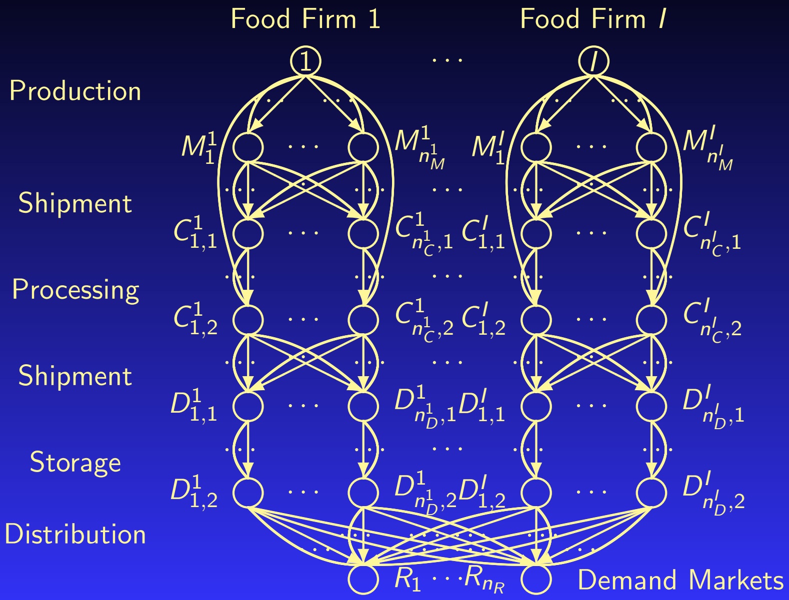
Fresh Produce Food Supply Chains

We developed a fresh produce supply chain network oligopoly model that

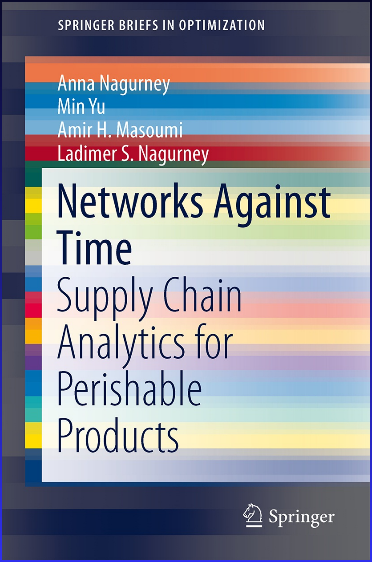
1. captures the deterioration of fresh food along the entire supply chain from a network perspective;
2. handles the exponential time decay through the introduction of arc multipliers;
3. formulates oligopolistic competition with product differentiation;
4. includes the disposal of the spoiled food products, along with the associated costs;
5. allows for the assessment of alternative technologies involved in each supply chain activity.

Reference: "Competitive Food Supply Chain Networks with Application to Fresh Produce," Min Yu and Anna Nagurney, *European Journal of Operational Research* **224(2)**, (2013), pp 273-282.

Fresh Produce Food Supply Chains



A variety of perishable product supply chain models, computational procedures, and applications can be found in our new book:



The Pharmaceutical Industry, Issues, and a Full Model

The Pharmaceutical Industry

Pharmaceutical, that is, medicinal drug, manufacturing is *an immense global industry*.

In 2003, worldwide pharmaceutical industry sales were at $491.8 billion, an increase in sales volume of 9% over the preceding year with *US being the largest national market, accounting for 44% of global industry sales*.

In 2011, the global pharmaceutical industry is expected to experience *growth of 5-7% on sales of approximately $880 billion* (Zacks Equity Research (2011)).

The Pharmaceutical Industry

Although pharmaceutical supply chains have begun to be coupled with sophisticated technologies in order to improve both the quantity and the quality of their associated products, despite all the advances in manufacturing, storage, and distribution methods, *pharmaceutical drug companies are far from effectively satisfying market demands on a consistent basis*.

In fact, it has been argued that pharmaceutical drug supply chains are *in urgent need of efficient optimization techniques in order to reduce costs and to increase productivity and responsiveness* (Shah (2004) and Papageorgiou (2009)).

Pharmaceutical Product Perishability

Product perishability is another critical issue in pharmaceutical / drug supply chains.

• In a 2003 survey, the estimated incurred *due to the expiration of branded products in supermarkets and drug stores was over 500 million dollars*.

• In 2007, in a warehouse belonging to the Health Department of Chicago,*over one million dollars in drugs, vaccines, and other medical supplies were found spoiled, stolen, or unaccounted for*.

• In 2009, CVS pharmacies in California, as a result of a settlement of a lawsuit filed against the company, had to offer promotional coupons to customers who had identified expired drugs, including expired baby formula and children's medicines, *in more than 42 percent of the stores* surveyed the year before.

Pharmaceutical Product Perishability

Other instances of medications sold more than a year past their expiration dates have occurred in other pharmacies across the US.

According to the Harvard Medical School (2003), since a law was passed in the US in 1979, drug manufacturers are required to stamp an expiration date on their products. This is the date at which the manufacturer can still guarantee the full, that is, 100%, potency and safety of the drug, assuming, of course, that proper storage procedures have been followed.

For example, certain medications, including insulin, must be stored under appropriate environmental conditions, and exposure to water, heat, humidity or other factors can adversely affect how certain drugs perform in the human body.

Product Shortages

Ironically, whereas some drugs may be unsold and unused and / or past their expiration dates, *the number of drugs that were reported in short supply in the US in the first half of 2011 has risen to 211 – close to an all-time record –* with only 58 in short supply in 2004.

According to the Food and Drug Administration (FDA), hospitals have reported shortages of drugs used in a wide range of applications, ranging from cancer treatment to surgery, anesthesia, and intravenous feedings.

Some Consequences of Product Shortages

The consequences of such shortages *include the postponement of surgeries and treatments, and may also result in the use of less effective or costlier substitutes*.

According to the American Hospital Association, all US hospitals have experienced drug shortages, and 82% have reported delayed care for their patients as a consequence (Szabo (2011)).

H1N1 (Swine) Flu

As of May 2, 2010, worldwide, more than 214 countries and overseas territories or communities reported laboratory confirmed cases of pandemic influenza H1N1 2009, including over 18,001 deaths (www.who.int).

Parts of the globe experienced serious flu vaccine shortages, both seasonal and H1N1 (swine) ones, in late 2009.

An Example of a Critical Medicine Shortage – Cytarabine

In the past year, the US experienced shortages of *critical drug, cytarabine, due to manufacturer production problems*.

Due to the severity of this medical crisis for leukemia patients, Food and Drug Administration is exploring the possibility of importing this medical product (Larkin (2011)).

Hospira re-entered the market in March 2011 and has made the manufacture of cytarabine a priority ahead of other products.

Some Possible Causes of Shortages

While the causes of many shortages are complex, most cases appear to be related to manufacturers' decisions to cease production in the presence of financial challenges.

It is interesting to note that, among curative cancer drugs, only the older generic, yet, less expensive, ones, have experienced shortages.

As noted by Shah (2004), pharmaceutical companies secure notable returns solely in the early lifetime of a successful drug, before competition takes place. This competition-free time-span, however, has been observed to be shortening, from 5 years to only 1-2 years.

Some Possible Causes of Shortages

Hence, the low profit margins associated with such drugs may be forcing pharmaceutical companies to make a difficult decision: whether to lose money by continuing to produce a lifesaving product or to switch to a more profitable drug.

Unfortunately, the FDA cannot force companies to continue to produce low-profit medicines even if millions of lives rely on them.

On the other hand, where competition has been lacking, shortages of some other lifesaving drugs have resulted in spikes in prices, ranging from a 100% to a 4,500% increase with an average of 650% (Schneider (2011)).

Economic and Financial Pressures

Pharmaceutical companies are expected to suffer a significant decrease in their revenues as a result of losing patent protection for ten of the best-selling drugs by the end of 2012 (De la Garza (2011)).

These include Lipitor and Plavix, that, presently, generate more than $142 billion in sales, are expected, over the next five years, to be faced with generic competition.

In 2011, pharmaceutical products valued at more than $30 billion are losing patent protection, with such products generating more than $15 billion in sales in 2010.

Safety Issues

• More than 80% of the ingredients of drugs sold in the US are made overseas, mostly in remote facilities located in China and India that are rarely – if not ever – visited by government inspectors.

• Supply chains of generic drugs, which account for 75 percent of the prescription medicines sold in the US, are, typically, more susceptible to falsification with the supply chains of some of the over-the-counter products, such as vitamins or aspirins, also vulnerable to adulteration.

• The amount of counterfeit drugs in the European pharmaceutical supply chains has considerably increased.

In the past, product recalls were mainly related to local errors in design, manufacturing, or labeling, *a single product safety issue may result in huge global consequences*.

Waste and Environmental Impacts

Another pressure faced by pharmaceutical firms is the environmental impact of their medical waste, which includes the perished excess medicine, and inappropriate disposal on the retailer / consumer end.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

The supply chain generalized network oligopoly model has the following novel features:

1. it handles the perishability of the pharmaceutical product through the introduction of arc multipliers;

2. it allows each firm to minimize the discarding cost of waste / perished medicine;

3. it captures product differentiation under oligopolistic competition through the branding of drugs, which can also include generics as distinct brands.

References can be found in our paper, "A Supply Chain Generalized Network Oligopoly Model for Pharmaceuticals Under Brand Differentiation and Perishability," A. H. Masoumi, M. Yu, and A. Nagurney, *Transportation Research E* **48** (2012), 762-780.

Our proposed supply chain network model can be applied to similar cases of oligopolistic competition in which a finite number of firms provide perishable products.

However, proper minor modifications may have to be made in order to address differences in the supply chain network topologies in related industries.

Some Examples of Oligopolies

- airlines
- freight carriers
- automobile manufacturers
- oil companies
- beer / beverage companies
- wireless communications
- fast fashion brands
- certain food companies.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

We consider I pharmaceutical firms, with a typical firm denoted by i.

The firms compete non-cooperatively, in an oligopolistic manner, and the consumers can differentiate among the products of the pharmaceutical firms through their individual product brands.

The supply chain network activities include manufacturing, shipment, storage, and, ultimately, the distribution of the brand name drugs to the demand markets.

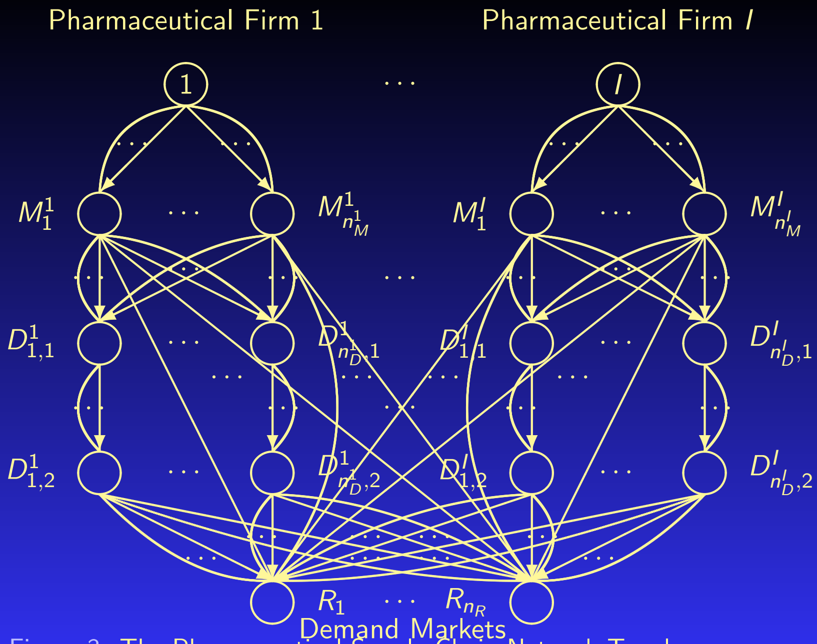


Figure 3: The Pharmaceutical Supply Chain Network Topology

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

Each pharmaceutical firm i; $i = 1, \ldots, I$, utilizes n^i_M manufacturing plants and n^i_D distribution / storage facilities, and the goal is to serve n_R demand markets consisting of pharmacies, retail stores, hospitals, and other medical centers.

L^i denotes the set of directed links corresponding to the sequence of activities associated with firm i. Also, $G = [N, L]$ denotes the graph composed of the set of nodes N, and the set of links L, where L contains all sets of L_is: $L \equiv \cup_{i=1,\ldots,I} L^i$.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

In the Figure, the first set of links connecting the top two tiers of nodes corresponds to the process of production of the drugs at each of the manufacturing units of firm i; $i = 1, \ldots, I$. Such facilities are denoted by $M_1^i, \ldots, M_{n_M^i}^i$, respectively, for firm i.

We emphasize that the manufacturing facilities may be located not only in different regions of the same country but also in different countries.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

The next set of nodes represents the distribution centers, and, thus, the links connecting the manufacturing nodes to the distribution centers are shipment-type links. Such distribution nodes associated with firm i; $i = 1, \ldots, I$ are denoted by $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$ and represent the distribution centers that the produced drugs are shipped to, and stored at, before being delivered to the demand markets.

There are alternative shipment links to denote different possible modes of transportation. In the shipment of pharmaceuticals that are perishable one may wish, for example, to ship by air, but at a higher cost.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

The next set of links connecting nodes $D^i_{1,1}, \ldots, D^i_{n^i_D,1}$ to $D^i_{1,2}, \ldots, D^i_{n^i_D,2}$; $i = 1, \ldots, I$ represents the process of storage.

Since drugs may require different storage conditions / technologies before being ultimately shipped to the demand markets, we represent these alternatives through multiple links at this tier.

The last set of links connecting the two bottom tiers of the supply chain network corresponds to distribution links over which the stored products are shipped from the distribution / storage facilities to the demand markets. Here we also allow for multiple modes of shipment / transportation.

A Generalized Network Oligopoly Model for Pharmaceutical Supply Chains

T here are direct links connecting manufacturing units with various demand markets in order to capture the possibility of direct mail shipments from manufacturers and the costs should be adjusted (see below) accordingly.

While representing a small percentage of the total filled prescriptions (about 6.1 percent in 2004), mail-order pharmacy sales remained the fastest-growing sector of the US prescription drug retail market in 2004, increasing by 18 percent over the preceding year (The Health Strategies Consultancy LLC (2005)).

How We Handle Perishability

Although pharmaceutical products may have different life-times, we can assign a multiplier to each activity / link of the supply chain to represent the fraction of the product that may perish / be wasted / be lost over the course of that activity.

The fraction of lost product depends on the type of the activity since various processes of manufacturing, shipment, storage, and distribution may result in dissimilar amounts of losses.

How We Handle Perishability

In addition, this fraction need not be the same among various links of the same tier in the supply chain network since different firms and even different units of the same firm may experience non-identical amounts of waste, depending on the brand of drug, the efficiency of the utilized technology, and the experience of the staff, etc.

Also, such multipliers can capture pilferage / theft, a significant issue in drug supply chains.

How We Handle Perishability

We associate with every link a in the supply chain network, a multiplier α_a, which lies in the range of (0,1]. The parameter α_a may be interpreted as a throughput factor corresponding to link a meaning that $\alpha_a \times 100\%$ of the initial flow of product on link a reaches the successor node of that link.

Let f_a denote the (initial) flow of product on link a with f'_a denoting the final flow on link a; i.e., the flow that reaches the successor node of the link after wastage has taken place. Therefore, we have:

$$f'_a = \alpha_a f_a, \qquad \forall a \in L. \tag{1}$$

Consequently, the waste / loss on link a, denoted by w_a, which is the difference between the initial and the final flow, can be derived as:

$$w_a = f_a - f'_a = (1 - \alpha_a) f_a, \qquad \forall a \in L. \tag{2}$$

How We Handle Perishability

The parameter α_a is assumed to be constant and known a priori. We can construct a total discarding cost function, $\hat{z}_a$, associated with discarding the medical waste, which is a function of the flow, f_a, and is assumed to be convex and continuously differentiable:

$$\hat{z}_a = \hat{z}_a(f_a), \qquad \forall a \in L. \tag{3}$$

How We Handle Perishability

Let x_p represent the (initial) flow of product on path p joining an origin node, i, with a destination node, R_k. The path flows must be nonnegative, that is,

$$x_p \geq 0, \qquad \forall p \in P_k^i; \ i = 1, \ldots, I; k = 1, \ldots, n_R, \qquad (4)$$

where P_k^i is the set of all paths joining the origin node i; $i = 1, \ldots, I$ with destination node R_k.

Also, μ_p denotes the multiplier corresponding to the throughput on path p, defined as the product of all link multipliers on links comprising that path, that is,

$$\mu_p \equiv \prod_{a \in p} \alpha_a, \qquad \forall p \in P_k^i; \ i = 1, \ldots, I; k = 1, \ldots, n_R. \qquad (5)$$

How We Handle Perishability

We define the multiplier, α_{ap}, which is the product of the multipliers of the links on path p that precede link a in that path, as follows:

$$\alpha_{ap} \equiv \begin{cases} \delta_{ap} \prod_{a' < a} \alpha_{a'}, & \text{if } \{a' < a\} \neq \emptyset, \\ \\ \delta_{ap}, & \text{if } \{a' < a\} = \emptyset, \end{cases} \quad (6)$$

where $\{a' < a\}$ denotes the set of the links preceding link a in path p, and $\emptyset$ denotes the null set. In addition, δ_{ap} is defined as equal to 1 if link a is contained in path p, and 0, otherwise. As a result, α_{ap} is equal to the product of all link multipliers preceding link a in path p. If link a is not contained in path p, then α_{ap} is set to zero. If a belongs to the first set of links; i.e., the manufacturing links, this multiplier is equal to 1.

How We Handle Perishability

Hence, the relationship between the link flow, f_a, and the path flows can be expressed as:

$$f_a = \sum_{i=1}^{I} \sum_{k=1}^{n_R} \sum_{p \in P_k^i} x_p \ \alpha_{ap}, \qquad \forall a \in L. \tag{7}$$

How We Handle Perishability

Note that the arc multipliers may be obtained from historical and statistical data.

They may also, in the case of certain perishable products, be related to an exponential time decay function where the time, in our framework, is associated with each specific link activity (see, for instance, Blackburn and Scudder (2009) and Bai and Kendall (2009)).

For example, Nagurney and Nagurney (2011) constructed explicit arc multipliers for molybdenum, which is used in nuclear medicine, which were based on the physics of time decay for this pharmaceutical product used in cancer and cardiac diagnostics, among other procedures.

How We Handle Perishability

Let d_{ik} denote the demand for pharmaceutical firm i's brand drug; $i = 1, \ldots, I$, at demand market R_k; $k = 1, \ldots, n_R$. The consumers differentiate the products by their brands.

The following equation reveals the relationship between the path flows and the demands in the supply chain network:

$$\sum_{p \in P_k^i} x_p \mu_p = d_{ik}, \quad i = 1, \ldots, I; k = 1, \ldots, n_R, \qquad (8)$$

that is, the demand for a brand drug at the demand market R_k is equal to the sum of all the final flows – subject to perishability – on paths joining (i, R_k). We group the demands d_{ik}; $i = 1, \ldots, I; k = 1, \ldots, n_R$ into the $n_R \times I$-dimensional vector d.

The Demand Price Functions

A demand price function is associated with each firm's pharmaceutical at each demand market. We denote the demand price of firm i's product at demand market R_k by ρ_{ik} and assume that

$$\rho_{ik} = \rho_{ik}(d), \quad i = 1, \ldots, I; k = 1, \ldots, n_R. \tag{9}$$

The Total Cost Functions

The total operational cost on link a may, in general, depend upon the product flows on all the links, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{10}$$

where f is the vector of all the link flows. The total cost on each link is assumed to be convex and continuously differentiable.

X_i denotes the vector of path flows associated with firm i; $i = 1, \ldots, I$, where $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$, and $P^i \equiv \cup_{k=1,\ldots,n_R} P_k^i$. In turn, n_{P^i}, denotes the number of paths from firm i to the demand markets. Thus, X is the vector of all the firm' strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$.

The Profit Function

The profit function of firm i, denoted by U_i, is expressed as:

$$U_i = \sum_{k=1}^{n_R} \rho_{ik}(d) \sum_{p \in P_k^i} \mu_p x_p - \sum_{a \in L^i} \hat{c}_a(f) - \sum_{a \in L^i} \hat{z}_a(f_a). \quad (11)$$

In lieu of the conservation of flow expressions (7) and (8), and the functional expressions (3), (9), and (10), we may define $\hat{U}_i(X) = U_i$ for all firms i; $i = 1, \ldots, I$, with the I-dimensional vector $\hat{U}$ being the vector of the profits of all the firms:

$$\hat{U} = \hat{U}(X). \quad (12)$$

Supply Chain Generalized Network Cournot-Nash Equilibrium

In the Cournot-Nash oligopolistic market framework, each firm selects its product path flows in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below.

Definition 1: Supply Chain Generalized Network Cournot-Nash Equilibrium
A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a supply chain generalized network Cournot-Nash equilibrium if for each firm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{13}$$

where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{P^i}}\}$.

An equilibrium is established if no firm can unilaterally improve its profit by changing its production path flows, given the production path flow decisions of the other firms.

Next, we present the variational inequality formulations of the Cournot-Nash equilibrium for the pharmaceutical supply chain network under oligopolistic competition satisfying Definition 1, in terms of both path flows and link flows (see Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), and Nagurney (2006)).

The Variational Inequality Formulation

Theorem 1

Assume that, for each pharmaceutical firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K$ is a supply chain generalized network Cournot-Nash equilibrium according to Definition 1 if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*)^T, X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{14}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i.

The Variational Inequality Formulation

Variational inequality (14), in turn, for our model, is equivalent to the variational inequality: determine $x^* \in K^1$ such that:

$$\sum_{i=1}^{I}\sum_{k=1}^{n_R}\sum_{p\in P_k^i}\left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p}+\frac{\partial \hat{Z}_p(x^*)}{\partial x_p}-\rho_{ik}(x^*)\mu_p-\right.$$

$$\left.\sum_{l=1}^{n_R}\frac{\partial \rho_{il}(x^*)}{\partial d_{ik}}\mu_p\sum_{p\in P_l^i}\mu_p x_p^*\right]\times[x_p-x_p^*]\geq 0,\quad \forall x\in K^1, \qquad (15)$$

where $K^1 \equiv \{x|x \in R_+^{n_P}\}$, and, for notational convenience, we denote:

$$\frac{\partial \hat{C}_p(x)}{\partial x_p}\equiv\sum_{b\in L^i}\sum_{a\in L^i}\frac{\partial \hat{c}_b(f)}{\partial f_a}\alpha_{ap} \text{ and } \frac{\partial \hat{Z}_p(x)}{\partial x_p}\equiv\sum_{a\in L^i}\frac{\partial \hat{z}_a(f_a)}{\partial f_a}\alpha_{ap}. \qquad (16)$$

The Variational Inequality Formulation

Variational inequality (15) can also be re-expressed in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$*, such that:*

$$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{n_R} \left[-\rho_{ik}(d^*) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d^*)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ik} - d_{ik}^*] \geq 0, \quad \forall (f, d) \in K^2, \tag{17}$$

where $K^2 \equiv \{(f, d) | x \geq 0,$ *and* (7) *and* (8) *hold*$\}$.

The Variational Inequality Formulation

Variational inequalities (15) and (17) can be put into standard form (see Nagurney (1999)): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{18}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. Let: $X \equiv x$ and

$$F(X) \equiv \Big[\frac{\partial \hat{C}_p(x)}{\partial x_p} + \frac{\partial \hat{Z}_p(x)}{\partial x_p} - \rho_{ik}(x)\mu_p - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x)}{\partial d_{ik}} \mu_p \sum_{p \in P_l^i} \mu_p x_p;\ p \in P_k^i;$$

and $\mathcal{K} \equiv K^1$.

The Variational Inequality Formulation

Similarly, for the variational inequality in terms of link flows, if we define the column vectors: $X \equiv (f, d)$ and $F(X) \equiv (F_1(X), F_2(X))$:

$$F_1(X) = \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} + \frac{\partial \hat{z}_a(f_a)}{\partial f_a};\ a \in L^i;\ i = 1, \ldots, I \right],$$

$$F_2(X) = \left[-\rho_{ik}(d) - \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(d)}{\partial d_{ik}} d_{il};\ i = 1, \ldots, I;\ k = 1, \ldots, n_R \right],$$

and let $\mathcal{K} \equiv K^2$.

Relationship of the Model to Others in the Literature

Relationship of the Model to Others in the Literature

The above model is now related to several models in the literature.

If the arc multipliers are all equal to 1, in which case the product is not perishable, then the model is related to the sustainable fashion supply chain network model of Nagurney and Yu in the *International Journal of Production Economics* **135** (2012), 532-540. In that model, however, the other criterion, in addition to the profit maximization one, was emission minimization, rather than waste cost minimization, as in the model in this paper.

Relationship of the Model to Others in the Literature

If the demands are fixed, and there is a single organization, but there are additional processing tiers, as well as capacity investments as variables, the model is the medical nuclear supply chain design model of Nagurney and Nagurney, *International Journal of Production Economics* (2012).

Relationship of the Model to Others in the Literature

If the demands are fixed, and there is a single organization, but there are additional processing tiers, as well as capacity investments as variables, the model is the medical nuclear supply chain design model of Nagurney and Nagurney, *International Journal of Production Economics* (2012).

If there is only a single organization / firm, and the demands are subject to uncertainty, with the inclusion of expected costs due to shortages or excess supplies, the total operational cost functions are separable, and a criterion of risk is added, then the model above is related to the blood supply chain network operations management model of Nagurney, Masoumi, and Yu, *Computational Management Science* (2012).

Relationship of the Model to Others in the Literature

If the product is homogeneous, and all the arc multipliers are, again, assumed to be equal to 1, and the total costs are assumed to be separable, then the above model collapses to the supply chain network oligopoly model of Nagurney (2010) in which synergies associated with mergers and acquisitions were assessed.

The Original Supply Chain Network Oligopoly Model

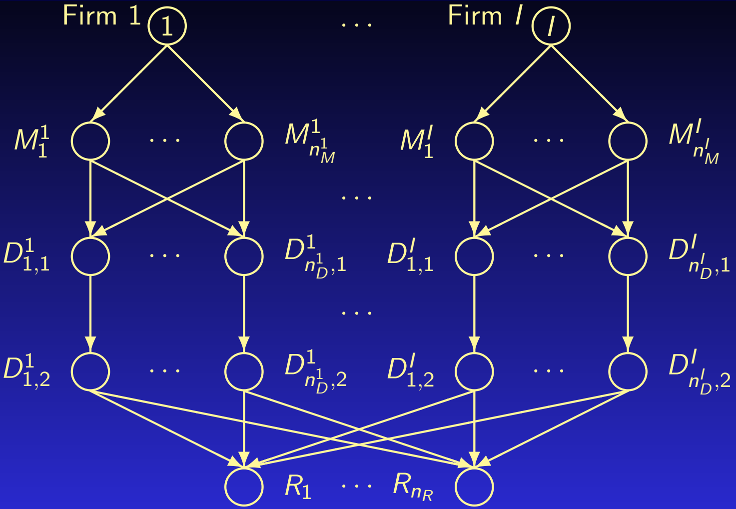


Figure 4: Supply Chain Network Structure of the Oligopoly Without Perishability; Nagurney, *Computational Management Science* **7**(2010), 377-401.

Mergers Through Coalition Formation

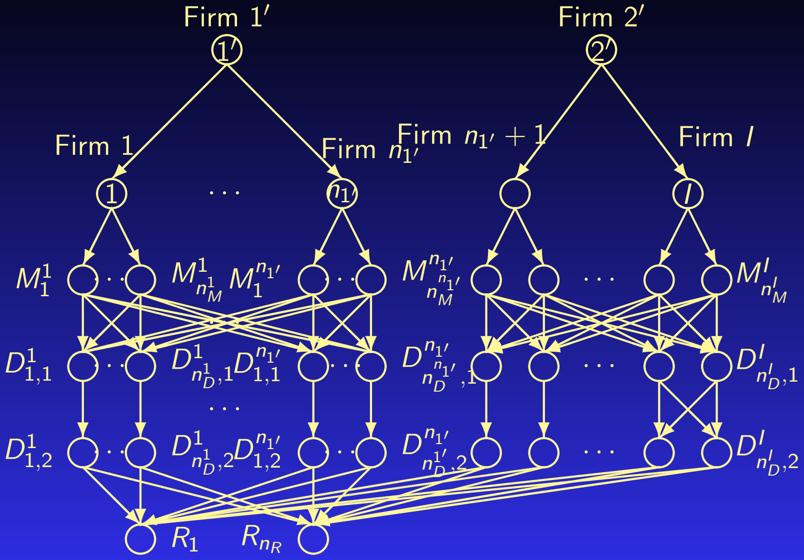


Figure 5: Mergers of the First $n_{1'}$ Firms and the Next $n_{2'}$ Firms

A Simple Perishable Product Numerical Example

A Simple Perishable Product Numerical Example

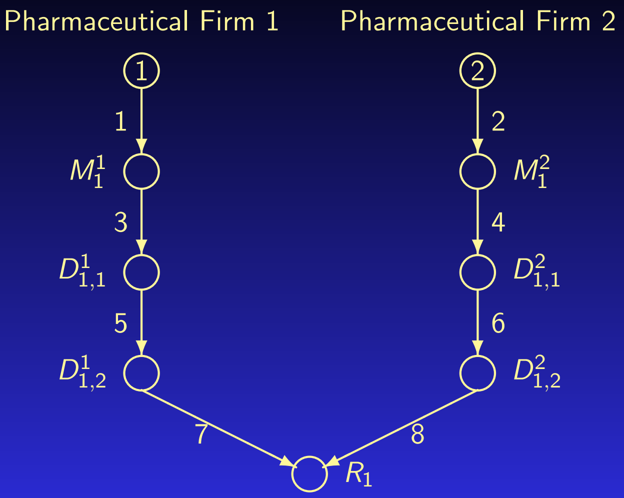


Figure 6: Supply Chain Network Topology for the Pharmaceutical Duopoly in the Illustrative Example

A Simple Perishable Product Numerical Example

In this example, two pharmaceutical firms compete in a duopoly with a single demand market (See Figure). The two firms produce differentiated, but substitutable, brand drugs 1 and 2, corresponding to Firm 1 and Firm 2, respectively.

The total cost functions on the various links of manufacturing, shipment, storage, and distribution are:

$$\hat{c}_1(f_1) = 5f_1^2 + 8f_1,\ \hat{c}_2(f_2) = 7f_2^2 + 3f_2,\ \hat{c}_3(f_3) = 2f_3^2 + f_3,$$

$$\hat{c}_4(f_4) = 2f_4^2 + 2f_4,$$

$$\hat{c}_5(f_5) = 3f_5^2 + 4f_5,\ \hat{c}_6(f_6) = 3.5f_6^2 + f_6,\ \hat{c}_7(f_7) = 2f_7^2 + 5f_7,$$

$$\hat{c}_8(f_8) = 1.5f_8^2 + 4f_8.$$

The arc multipliers are given by:

$$\alpha_1 = .95,\ \alpha_2 = .98,\ \alpha_3 = .99,\ \alpha_4 = 1.00,\ \alpha_5 = .99,\ \alpha_6 = .97,$$

$$\alpha_7 = 1.00,\ \alpha_8 = 1.00.$$

A Simple Perishable Product Numerical Example

The total discarding cost functions on the links are assumed identical, that is,

$$\hat{z}_a(f_a) = .5f_a^2, \quad \forall a.$$

The firms compete in the demand market R_1, and the consumers reveal their preferences for the two products through the following nonseparable demand price functions:

$$\rho_{11}(d) = -3d_{11} - d_{21} + 200, \quad \rho_{21}(d) = -4d_{21} - 1.5d_{11} + 300.$$

In this supply chain network, there exists one path corresponding to each firm, denoted by p_1 and p_2.

A Simple Perishable Product Numerical Example

Thus, variational inequality (15), here takes the form:

$$\left[\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} + \frac{\partial \hat{Z}_{p_1}(x^*)}{\partial x_{p_1}} - \rho_{11}(x^*)\mu_{p_1} - \frac{\partial \rho_{11}(x^*)}{\partial d_{11}}\mu_{p_1} \times \mu_{p_1} x^*_{p_1}\right]$$

$$\times [x_{p_1} - x^*_{p_1}]$$

$$+\left[\frac{\partial \hat{C}_{p_2}(x^*)}{\partial x_{p_2}} + \frac{\partial \hat{Z}_{p_2}(x^*)}{\partial x_{p_2}} - \rho_{21}(x^*)\mu_{p_2} - \frac{\partial \rho_{21}(x^*)}{\partial d_{21}}\mu_{p_2} \times \mu_{p_2} x^*_{p_2}\right]$$

$$\times [x_{p_2} - x^*_{p_2}] \geq 0, \ \forall x \in K^1.$$

A Simple Perishable Product Numerical Example

Under the assumption that $x^*_{p_1} > 0$ and $x^*_{p_2} > 0$, the two expressions on the left-hand side of inequality (27) must be equal to zero, that is:

$$\left[\frac{\partial \hat{C}_{p_1}(x^*)}{\partial x_{p_1}} + \frac{\partial \hat{Z}_{p_1}(x^*)}{\partial x_{p_1}} - \rho_{11}(x^*)\mu_{p_1} - \frac{\partial \rho_{11}(x^*)}{\partial d_{11}}\mu_{p_1} \times \mu_{p_1} x^*_{p_1}\right]$$

$$\times [x_{p_1} - x^*_{p_1}] = 0,$$

and

$$\left[\frac{\partial \hat{C}_{p_2}(x^*)}{\partial x_{p_2}} + \frac{\partial \hat{Z}_{p_2}(x^*)}{\partial x_{p_2}} - \rho_{21}(x^*)\mu_{p_2} - \frac{\partial \rho_{21}(x^*)}{\partial d_{21}}\mu_{p_2} \times \mu_{p_2} x^*_{p_2}\right]$$

$$\times [x_{p_2} - x^*_{p_2}] = 0.$$

Since each of the paths flows must be nonnegative, we know that the term preceding the multiplication sign in both of the above must be equal to zero.

A Simple Perishable Product Numerical Example

Calculating the values of the multipliers from (6), and then, substituting those values, as well as, the given functions into (16), we can determine the partial derivatives of the total operational cost and the total discarding cost functions. Furthermore, the partial derivatives of the given demand price functions can be calculated and substituted into the above. Applying (5), the path multipliers are equal to:

$$\mu_{p_1} = \alpha_1 \times \alpha_3 \times \alpha_5 \times \alpha_7 = .95 \times .99 \times .99 \times 1 = .93,$$

$$\mu_{p_2} = \alpha_2 \times \alpha_4 \times \alpha_6 \times \alpha_8 = .98 \times 1 \times .97 \times 1 = .95.$$

Simple arithmetic calculations, with the above substitutions, yield the below system of equations:

$$\begin{cases} 31.24x^*_{p_1} + 0.89x^*_{p_2} = 168.85, \\ \\ 1.33x^*_{p_1} + 38.33x^*_{p_2} = 274.46. \end{cases}$$

A Simple Perishable Product Numerical Example

Thus, the equilibrium solution corresponding to the path flow of brand drugs produced by firms 1 and 2 is:

$$x^*_{p_1} = 5.21, \quad x^*_{p_2} = 6.98.$$

Using (7), the equilibrium link flows can be calculated as:

$$f^*_1 = 5.21, \quad f^*_3 = 4.95, \quad f^*_5 = 4.90, \quad f^*_7 = 4.85,$$

$$f^*_2 = 6.98, \quad f^*_4 = 6.84, \quad f^*_6 = 6.84, \quad f^*_8 = 6.64.$$

From (8), the equilibrium values of demand for products of the two pharmaceutical firms are equal to:

$$d^*_{11} = 4.85, \quad d^*_{21} = 6.64.$$

Finally, the equilibrium prices of the two branded drugs are:

$$\rho_{11} = 178.82, \quad \rho_{21} = 266.19.$$

Note that, even though the price of Firm 2's product is observed to be higher, the market has a slightly stronger tendency toward this product as opposed to the product of Firm 1.

This is due to the willingness of the consumers to spend more on one product which can be a consequence of the reputation, or the perceived quality, of Firm 2's brand drug.

The Algorithm with Explicit Formulae

The Algorithm

We recall the Euler method, which is induced by the general iterative scheme of Dupuis and Nagurney (1993). Its realization for the solution of the supply chain generalized network oligopoly model with brand differentiation governed by variational inequality (15) induces subproblems that can be solved explicitly and in closed form.

Specifically, iteration τ of the Euler method (see also Nagurney and Zhang (1996)) is given by:

$$X^{\tau+1} = P_{\mathcal{K}}(X^{\tau} - a_{\tau} F(X^{\tau})),$$

where $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$ and F is the function that enters the variational inequality problem (18).

The Algorithm

As shown in Dupuis and Nagurney (1993) and Nagurney and Zhang (1996), for convergence of the general iterative scheme, which induces the Euler method, the sequence $\{a_\tau\}$ must satisfy: $\sum_{\tau=0}^{\infty} a_\tau = \infty$, $a_\tau > 0$, $a_\tau \to 0$, as $\tau \to \infty$.

Conditions for convergence of this scheme as well as various applications to the solutions of network oligopolies can be found in Nagurney and Zhang (1996), Nagurney, Dupuis, and Zhang (1994), Nagurney (2010a), and Nagurney and Yu (2011).

Explicit Formulae for the Euler Method Applied to the Supply Chain Generalized Network Oligopoly Variational Inequality (15)

The elegance of this procedure for the computation of solutions to our supply chain generalized network oligopoly model with product differentiation can be seen in the following explicit formulae. We have the following closed form expressions for all the path flows $p \in P^i_k, \forall i, k$:

$$x_p^{\tau+1} = \max$$

$$\{0, x_p^{\tau} + a_{\tau}(\rho_{ik}(x^{\tau})\mu_p + \sum_{l=1}^{n_R} \frac{\partial \rho_{il}(x^{\tau})}{\partial d_{ik}} \mu_p \sum_{p \in P^i_l} \mu_p x_p^{\tau} - \frac{\partial \hat{C}_p(x^{\tau})}{\partial x_p} - \frac{\partial \hat{Z}_p(x^{\tau})}{\partial x_p})\}.$$

Numerical Cases

Case I

This case is assumed occur in the third quarter of 2011 prior to the expiration of the patent for Lipitor.

Firm 1 represents a multinational pharmaceutical giant, hypothetically, Pfizer, Inc., which still possesses the patent for Lipitor, the most popular brand of cholesterol-lowering drug.

Firm 2, on the other hand, which might represent, for example, Merck & Co., Inc., been producing Zocor, another cholesterol regulating brand, whose patent expired in 2006.

The Pharmaceutical Supply Chain Network Topology for Case I

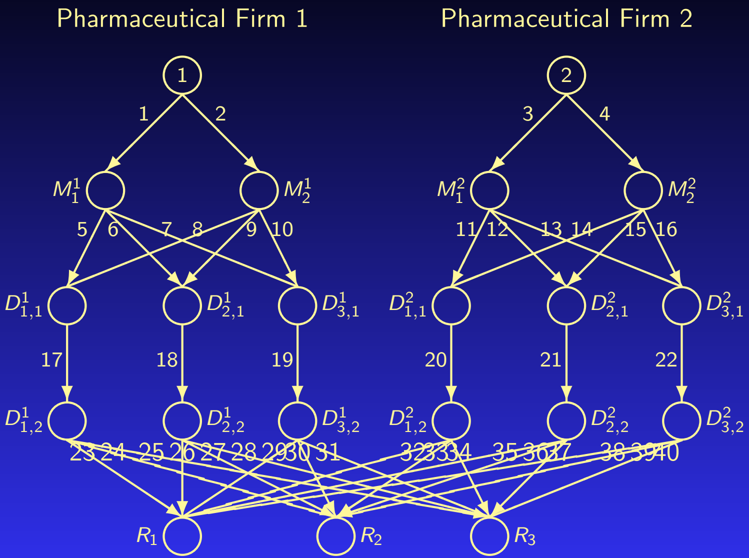


Figure 7: Case I Supply Chain Network

Case I (cont'd)

The demand price functions were as follows:

$$\rho_{11}(d) = -1.1d_{11} - 0.9d_{21} + 275;\ \rho_{21}(d) = -1.2d_{21} - 0.7d_{11} + 210;$$

$$\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} + 255;\ \rho_{22}(d) = -1.0d_{22} - 0.5d_{12} + 200;$$

$$\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} + 265;\ \rho_{23}(d) = -1.5d_{23} - 0.4d_{13} + 186.$$

The Euler method for the solution of variational inequality was implemented in Matlab. The results can be seen in the following tables.

Link Multipliers, Total Cost Functions and Link Flow Solution for Case I

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	13.73
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	10.77
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	8.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	10.55
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	5.21
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	3.36
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	4.47
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	3.02
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	3.92
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	3.50
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	3.10
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	2.36
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.63
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	3.79
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	3.12
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	3.43
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	8.20
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	7.25
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	7.97
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	6.85

Link Multipliers, Total Cost Functions and Solution for Case I (cont'd)

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	5.42
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	6.00
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	3.56
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.66
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	2.82
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	3.34
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.24
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	2.59
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	3.45
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.28
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	3.09
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.54
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	3.43
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	0.75
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.72
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	2.64
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	0.95
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.47
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	2.47
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	0.00

Case I: Result Analysis

The computed equilibrium demands for each of the two brands were:

$$d_{11}^* = 10.32,\ d_{21}^* = 7.66,$$

$$d_{12}^* = 4.17,\ d_{22}^* = 8.46,$$

$$d_{13}^* = 8.41,\ d_{23}^* = 1.69.$$

The incurred equilibrium prices associated with the branded drugs at each demand market were as follows:

$$\rho_{11}(d^*) = 256.75,\ \ \rho_{21}(d^*) = 193.58,$$

$$\rho_{12}(d^*) = 244.48,\ \ \rho_{22}(d^*) = 189.46,$$

$$\rho_{13}(d^*) = 251.52,\ \ \rho_{23}(d^*) = 180.09.$$

Case I: Result Analysis

Firm 1, which produces the top-selling product, captures the majority of the market share at demand markets 1 and 3, despite the higher price. In fact, it has almost entirely seized demand market 3 forcing several links connecting Firm 2 to demand market 3 to have insignificant flows including link 40 with a flow equal to zero.

Firm 2 dominates demand market 2, due to the consumers' willingness to lean towards this product there, perhaps as a consequence of the lower price, or the perception of quality, etc.

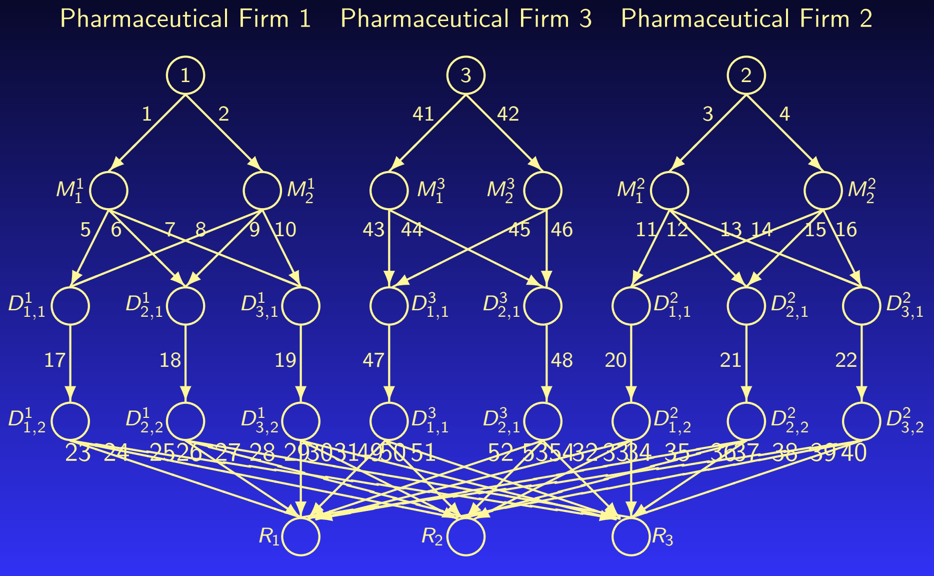
The profits of the two firms are:

$$U_1(X^*) = 2,936.52 \text{ and } \quad U_2(X^*) = 1,675.89.$$

A Case Study –Case II

In this case, we consider the scenario in which Firm 1 has just lost the exclusive patent right of its highly popular cholesterol regulator. A manufacturer of generic drugs, say, Sanofi, here denoted by Firm 3, has recently introduced a generic substitute for Lipitor by reproducing its active ingredient Atorvastatin (Smith (2011)). Firm 3 is assumed to have two manufacturing plants, two distribution centers as well as two storage facilities in order to supply the same three demand markets as in Case I (See Figure).

The Pharmaceutical Supply Chain Network Topology for Cases II and III



Case II

Firm 1 has just lost the exclusive patent right of its highly popular cholesterol regulator. A manufacturer of generic drugs, say, Ranbaxy Laboratories, here denoted by Firm 3, has recently introduced a generic substitute for Lipitor by reproducing its active ingredients.

The demand price functions for the products of Firm 1 and 2 will stay the same as in Case I. The demand price functions corresponding to the product of Firm 3 are as follows:

$$\rho_{31}(d) = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 150;$$

$$\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 130;$$

$$\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 133.$$

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	13.73
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	10.77
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	8.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	10.55
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	5.21
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	3.36
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	4.47
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	3.02
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	3.92
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	3.50
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	3.10
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	2.36
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.63
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	3.79
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	3.12
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	3.43
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	8.20
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	7.25
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	7.97
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	6.85
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	5.42
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	6.00
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	3.56
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.66
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	2.82
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	3.34
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.24

Link Multipliers, Total Cost Functions and Link Flow Solution for Case II

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	2.59
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	3.45
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.28
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	3.09
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.54
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	3.43
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	0.75
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.72
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	2.64
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	0.95
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.47
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	2.47
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	0.00
41	.97	$3f_{41}^2 + 12f_{41}$	$.3f_{41}^2$	6.17
42	.96	$2.7f_{42}^2 + 10f_{42}$	$.4f_{42}^2$	6.23
43	.98	$1.1f_{43}^2 + 6f_{43}$	$.45f_{43}^2$	3.23
44	.98	$.9f_{44}^2 + 5f_{44}$	$.45f_{44}^2$	2.75
45	.97	$1.3f_{45}^2 + 6f_{45}$	$.5f_{45}^2$	3.60
46	.99	$1.5f_{46}^2 + 7f_{46}$	$.55f_{46}^2$	2.38
47	.98	$1.5f_{47}^2 + 4f_{47}$	$.4f_{47}^2$	6.66
48	.98	$2.1f_{48}^2 + 6f_{48}$	$.45f_{48}^2$	5.05
49	.99	$.6f_{49}^2 + 3f_{49}$	$.55f_{49}^2$	3.79
50	1.00	$.7f_{50}^2 + 2f_{50}$	$.7f_{50}^2$	1.94
51	.98	$.6f_{51}^2 + 7f_{51}$	$.45f_{51}^2$	0.79
52	.99	$.9f_{52}^2 + 9f_{52}$	$.5f_{52}^2$	1.43
53	1.00	$.55f_{53}^2 + 6f_{53}$	$.55f_{53}^2$	1.23
54	.98	$.8f_{54}^2 + 4f_{54}$	$.5f_{54}^2$	2.28

Link Multipliers, Total Cost Functions and Solution for Case II (cont'd)

Case II: Result Analysis

The equilibrium product flows of Firms 1 and 2 on links 1 through 40 are identical to the corresponding values in Case I.

When the new product produced by Firm 3 is just introduced, the manufacturers of the two existing products will not experience an immediate impact on their respective demands of branded drugs.

The equilibrium computed demands for the products of Firms 1 and 2 at the demand markets will remain as in Case I, and the equilibrium amounts of demand for the new product of Firm 3 at each demand market is equal to:

$$d_{31}^* = 5.17, \quad d_{32}^* = 3.18, \quad \text{and } d_{33}^* = 3.01.$$

Case II: Result Analysis

The equilibrium prices associated with the branded drugs 1 and 2 at the demand markets will not change, whereas the incurred equilibrium prices of generic drug 3 are as follows:

$$\rho_{31}(d^*) = 133.02, \quad \rho_{32}(d^*) = 120.30, \quad \text{and } \rho_{33}(d^*) = 123.55,$$

which is significantly lower than the respective prices of its competitors in all the demand markets.

Thus, the profit that Firm 3 derived from manufacturing and delivering the new generic substitute to these 3 markets is:

$$U_3(X^*) = 637.38,$$

while the profits of Firms 1 and 2 remain unchanged.

Case III

The generic product of Firm 3 has now been well-established, and has affected the behavior of the consumers through the demand price functions of the relatively more recognized products of Firms 1 and 2. The demand price functions associated are now given by:

Firm 1: $\rho_{11}(d) = -1.1d_{11} - 0.9d_{21} - 1.0d_{31} + 192;$

$$\rho_{21}(d) = -1.2d_{21} - 0.7d_{11} - 0.8d_{31} + 176;$$

$$\rho_{31} = -0.9d_{31} - 0.6d_{11} - 0.8d_{21} + 170;$$

Firm 2: $\rho_{12}(d) = -0.9d_{12} - 0.8d_{22} - 0.7d_{32} + 166;$

$$\rho_{22}(d) = -1.0d_{22} - 0.5d_{12} - 0.8d_{32} + 146;$$

$$\rho_{32}(d) = -0.8d_{32} - 0.5d_{12} - 0.6d_{22} + 153;$$

Firm 3: $\rho_{13}(d) = -1.4d_{13} - 1.0d_{23} - 0.5d_{33} + 173;$

$$\rho_{23}(d) = -1.5d_{23} - 0.4d_{13} - 0.7d_{33} + 164;$$

$$\rho_{33}(d) = -0.9d_{33} - 0.7d_{13} - 0.5d_{23} + 157.$$

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
1	.95	$5f_1^2 + 8f_1$	$.5f_1^2$	8.42
2	.97	$7f_2^2 + 3f_2$	$.4f_2^2$	6.72
3	.96	$6.5f_3^2 + 4f_3$	$.3f_3^2$	6.42
4	.98	$5f_4^2 + 7f_4$	$.35f_4^2$	8.01
5	1.00	$.7f_5^2 + f_5$	$.5f_5^2$	3.20
6	.99	$.9f_6^2 + 2f_6$	$.5f_6^2$	2.07
7	1.00	$.5f_7^2 + f_7$	$.5f_7^2$	2.73
8	.99	$f_8^2 + 2f_8$	$.6f_8^2$	1.85
9	1.00	$.7f_9^2 + 3f_9$	$.6f_9^2$	2.44
10	1.00	$.6f_{10}^2 + 1.5f_{10}$	$.6f_{10}^2$	2.23
11	.99	$.8f_{11}^2 + 2f_{11}$	$.4f_{11}^2$	2.42
12	.99	$.8f_{12}^2 + 5f_{12}$	$.4f_{12}^2$	1.75
13	.98	$.9f_{13}^2 + 4f_{13}$	$.4f_{13}^2$	2.00
14	1.00	$.8f_{14}^2 + 2f_{14}$	$.5f_{14}^2$	2.84
15	.99	$.9f_{15}^2 + 3f_{15}$	$.5f_{15}^2$	2.40
16	1.00	$1.1f_{16}^2 + 3f_{16}$	$.6f_{16}^2$	2.60
17	.98	$2f_{17}^2 + 3f_{17}$	$.45f_{17}^2$	5.02
18	.99	$2.5f_{18}^2 + f_{18}$	$.55f_{18}^2$	4.49
19	.98	$2.4f_{19}^2 + 1.5f_{19}$	$.5f_{19}^2$	4.96
20	.98	$1.8f_{20}^2 + 3f_{20}$	$.3f_{20}^2$	5.23
21	.98	$2.1f_{21}^2 + 3f_{21}$	$.35f_{21}^2$	4.11
22	.99	$1.9f_{22}^2 + 2.5f_{22}$	$.5f_{22}^2$	4.56
23	1.00	$.5f_{23}^2 + 2f_{23}$	$.6f_{23}^2$	2.44
24	1.00	$.7f_{24}^2 + f_{24}$	$.6f_{24}^2$	1.47
25	.99	$.5f_{25}^2 + .8f_{25}$	$.6f_{25}^2$	1.02
26	.99	$.6f_{26}^2 + f_{26}$	$.45f_{26}^2$	2.48
27	.99	$.7f_{27}^2 + .8f_{27}$	$.4f_{27}^2$	1.31

Link Multipliers, Total Cost Functions and Link Flow Solution for Case III

Link a	α_a	$\hat{c}_a(f_a)$	$\hat{z}_a(f_a)$	f_a^*
28	.98	$.4f_{28}^2 + .8f_{28}$	$.45f_{28}^2$	0.66
29	1.00	$.3f_{29}^2 + 3f_{29}$	$.55f_{29}^2$	2.29
30	1.00	$.75f_{30}^2 + f_{30}$	$.55f_{30}^2$	1.29
31	1.00	$.65f_{31}^2 + f_{31}$	$.55f_{31}^2$	1.28
32	.99	$.5f_{32}^2 + 2f_{32}$	$.3f_{32}^2$	2.74
33	.99	$.4f_{33}^2 + 3f_{33}$	$.3f_{33}^2$	0.00
34	1.00	$.5f_{34}^2 + 3.5f_{34}$	$.4f_{34}^2$	2.39
35	.98	$.4f_{35}^2 + 2f_{35}$	$.55f_{35}^2$	1.82
36	.98	$.3f_{36}^2 + 2.5f_{36}$	$.55f_{36}^2$	0.00
37	.99	$.55f_{37}^2 + 2f_{37}$	$.55f_{37}^2$	2.21
38	1.00	$.35f_{38}^2 + 2f_{38}$	$.4f_{38}^2$	3.46
39	1.00	$.4f_{39}^2 + 5f_{39}$	$.4f_{39}^2$	0.00
40	.98	$.55f_{40}^2 + 2f_{40}$	$.6f_{40}^2$	1.05
41	.97	$3f_{41}^2 + 12f_{41}$	$.3f_{41}^2$	8.08
42	.96	$2.7f_{42}^2 + 10f_{42}$	$.4f_{42}^2$	8.13
43	.98	$1.1f_{43}^2 + 6f_{43}$	$.45f_{43}^2$	4.21
44	.98	$.9f_{44}^2 + 5f_{44}$	$.45f_{44}^2$	3.63
45	.97	$1.3f_{45}^2 + 6f_{45}$	$.5f_{45}^2$	4.62
46	.99	$1.5f_{46}^2 + 7f_{46}$	$.55f_{46}^2$	3.19
47	.98	$1.5f_{47}^2 + 4f_{47}$	$.4f_{47}^2$	8.60
48	.98	$2.1f_{48}^2 + 6f_{48}$	$.45f_{48}^2$	6.72
49	.99	$.6f_{49}^2 + 3f_{49}$	$.55f_{49}^2$	3.63
50	1.00	$.7f_{50}^2 + 2f_{50}$	$.7f_{50}^2$	3.39
51	.98	$.6f_{51}^2 + 7f_{51}$	$.45f_{51}^2$	1.41
52	.99	$.9f_{52}^2 + 9f_{52}$	$.5f_{52}^2$	1.12
53	1.00	$.55f_{53}^2 + 6f_{53}$	$.55f_{53}^2$	2.86
54	.98	$.8f_{54}^2 + 4f_{54}$	$.5f_{54}^2$	2.60

Link Multipliers, Total Cost Functions and Solution for Case III (cont'd)

Case III: Results

The computed equilibrium demands and sales prices for the products of Firms 1, 2, and 3 are as follows:

$$d_{11}^* = 7.18, \quad d_{21}^* = 7.96, \quad d_{31}^* = 4.70,$$
$$d_{12}^* = 4.06, \quad d_{22}^* = 0.00, \quad d_{32}^* = 6.25,$$
$$d_{13}^* = 2.93, \quad d_{23}^* = 5.60, \quad \text{and } d_{33}^* = 3.93.$$

$$\rho_{11}(d^*) = 172.24, \quad \rho_{21}(d^*) = 157.66, \quad \rho_{31}(d^*) = 155.09,$$
$$\rho_{12}(d^*) = 157.97, \quad \rho_{22}(d^*) = 138.97, \quad \rho_{32}(d^*) = 145.97,$$
$$\rho_{13}(d^*) = 161.33, \quad \rho_{23}(d^*) = 151.67, \quad \text{and } \rho_{33}(d^*) = 148.61.$$

The computed amounts of firms' profits:

$U_1(X^*) = 1,199.87, \quad U_2(X^*) = 1,062.73, \quad$ and $U_3(X^*) = 980.83.$

Case III: Result Analysis

As a result of the consumers' growing inclination towards the generic substitute of the previously popular Lipitor, Firm 2 has lost its entire share of market 2 to its competitors, resulting in zero flows on several links. Similarly, Firm 1 now has declining sales of its brand in demand markets 1 and 3.

As expected, the introduction of the generic substitute has also caused remarkable drops in the prices of the existing brands. Interestingly, the decrease in the price of Lipitor in demand markets 2 and 3 exceeds 35%.

Note that simultaneous declines in the amounts of demand and sales price has caused a severe reduction in the profits of Firms 1 and 2. This decline for Firm 1 is observed to be as high as 60%.

Validation of Results: Observations

As noted by Johnson (2011), the market share of a branded drug may decrease by as much as 40%-80% after the introduction of its generic rival. Thus, the model captures the observed decrease in the US market share.

Validation of Results: Observations

As noted by Johnson (2011), the market share of a branded drug may decrease by as much as 40%-80% after the introduction of its generic rival. Thus, the model captures the observed decrease in the US market share.

The reduction in demand and price due to the patent expiration has been observed in the market sales. The US sales of Lipitor have dropped over 75% (Forbes (2012) and Firecepharma (2012)).

Paths Definition and Optimal Path Flow Pattern - Firm 1

	Path Definition	Path Flow
O/D Pair $(1, R_1)$	$p_1 = (1, 5, 17, 23)$	$x^*_{p_1} = 1.87$
	$p_2 = (1, 6, 18, 26)$	$x^*_{p_2} = 1.46$
	$p_3 = (1, 7, 19, 29)$	$x^*_{p_3} = 1.57$
	$p_4 = (2, 8, 17, 23)$	$x^*_{p_4} = 0.73$
	$p_5 = (2, 9, 18, 26)$	$x^*_{p_5} = 1.17$
	$p_6 = (2, 10, 19, 29)$	$x^*_{p_6} = 0.87$
O/D Pair $(1, R_2)$	$p_7 = (1, 5, 17, 24)$	$x^*_{p_7} = 0.89$
	$p_8 = (1, 6, 18, 27)$	$x^*_{p_8} = 0.57$
	$p_9 = (1, 7, 19, 30)$	$x^*_{p_9} = 0.66$
	$p_{10} = (2, 8, 17, 24)$	$x^*_{p_{10}} = 0.68$
	$p_{11} = (2, 9, 18, 27)$	$x^*_{p_{11}} = 0.82$
	$p_{12} = (2, 10, 19, 30)$	$x^*_{p_{12}} = 0.71$
O/D Pair $(1, R_3)$	$p_{13} = (1, 5, 17, 25)$	$x^*_{p_{13}} = 0.60$
	$p_{14} = (1, 6, 18, 28)$	$x^*_{p_{14}} = 0.16$
	$p_{15} = (1, 7, 19, 31)$	$x^*_{p_{15}} = 0.64$
	$p_{16} = (2, 8, 17, 25)$	$x^*_{p_{16}} = 0.49$
	$p_{17} = (2, 9, 18, 28)$	$x^*_{p_{17}} = 0.53$
	$p_{18} = (2, 10, 19, 31)$	$x^*_{p_{18}} = 0.72$

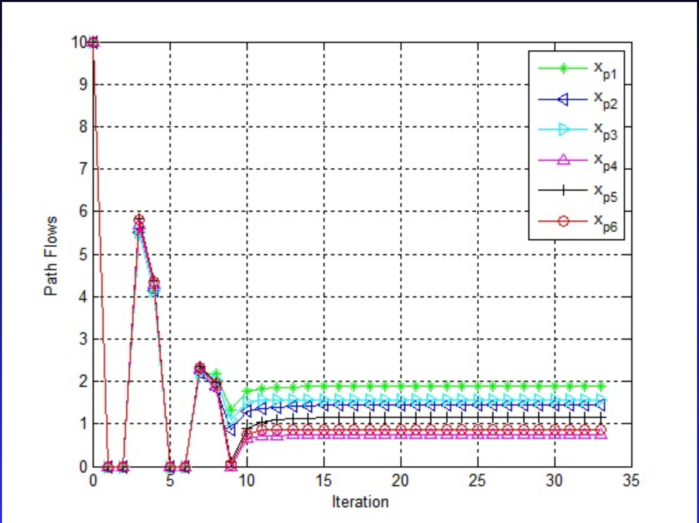
Paths Definition and Optimal Path Flow Pattern - Firm 2

	Path Definition	Path Flow
O/D Pair $(2, R_1)$	$p_{19} = (3, 11, 20, 32)$	$x^*_{p_{19}} = 1.26$
	$p_{20} = (3, 12, 21, 35)$	$x^*_{p_{20}} = 0.77$
	$p_{21} = (3, 13, 22, 38)$	$x^*_{p_{21}} = 1.51$
	$p_{22} = (4, 14, 20, 32)$	$x^*_{p_{22}} = 1.63$
	$p_{23} = (4, 15, 21, 35)$	$x^*_{p_{23}} = 1.16$
	$p_{24} = (4, 16, 22, 38)$	$x^*_{p_{24}} = 2.12$
O/D Pair $(2, R_2)$	$p_{25} = (3, 11, 20, 33)$	$x^*_{p_{25}} = 0.00$
	$p_{26} = (3, 12, 21, 36)$	$x^*_{p_{26}} = 0.00$
	$p_{27} = (3, 13, 22, 39)$	$x^*_{p_{27}} = 0.00$
	$p_{28} = (4, 14, 20, 33)$	$x^*_{p_{28}} = 0.00$
	$p_{29} = (4, 15, 21, 36)$	$x^*_{p_{29}} = 0.00$
	$p_{30} = (4, 16, 22, 39)$	$x^*_{p_{30}} = 0.00$
O/D Pair $(2, R_3)$	$p_{31} = (3, 11, 20, 34)$	$x^*_{p_{31}} = 1.26$
	$p_{32} = (3, 12, 21, 37)$	$x^*_{p_{32}} = 1.05$
	$p_{33} = (3, 13, 22, 40)$	$x^*_{p_{33}} = 0.57$
	$p_{34} = (4, 14, 20, 34)$	$x^*_{p_{34}} = 1.26$
	$p_{35} = (4, 15, 21, 37)$	$x^*_{p_{35}} = 1.29$
	$p_{36} = (4, 16, 22, 40)$	$x^*_{p_{36}} = 0.54$

Paths Definition and Optimal Path Flow Pattern - Firm 3

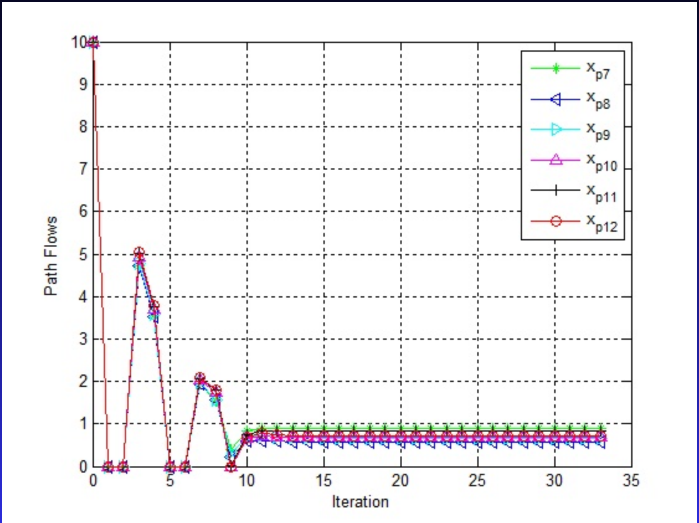
	Path Definition	Path Flow
O/D Pair $(3, R_1)$	$p_{37} = (41, 43, 47, 49)$	$x^*_{p_{37}} = 1.87$
	$p_{38} = (41, 44, 48, 52)$	$x^*_{p_{38}} = 1.78$
	$p_{39} = (42, 45, 47, 49)$	$x^*_{p_{39}} = 0.70$
	$p_{40} = (42, 46, 48, 52)$	$x^*_{p_{40}} = 0.68$
O/D Pair $(3, R_2)$	$p_{41} = (41, 43, 47, 50)$	$x^*_{p_{41}} = 1.61$
	$p_{42} = (41, 44, 48, 53)$	$x^*_{p_{42}} = 1.46$
	$p_{43} = (42, 45, 47, 50)$	$x^*_{p_{43}} = 2.07$
	$p_{44} = (42, 46, 48, 53)$	$x^*_{p_{44}} = 1.90$
O/D Pair $(3, R_3)$	$p_{45} = (41, 43, 47, 51)$	$x^*_{p_{45}} = 0.84$
	$p_{46} = (41, 44, 48, 54)$	$x^*_{p_{46}} = 0.53$
	$p_{47} = (42, 45, 47, 51)$	$x^*_{p_{47}} = 1.46$
	$p_{48} = (42, 46, 48, 54)$	$x^*_{p_{48}} = 1.33$

Path Flow Trajectories



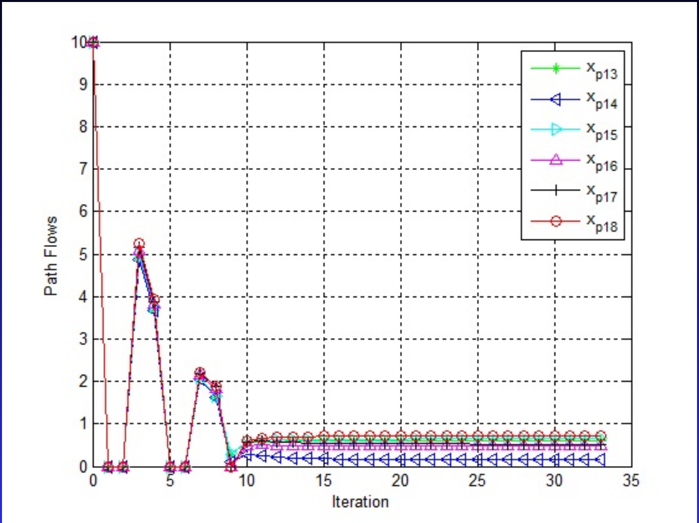
The Trajectories of Product Flows on Paths $p_1 - p_6$

Path Flow Trajectories



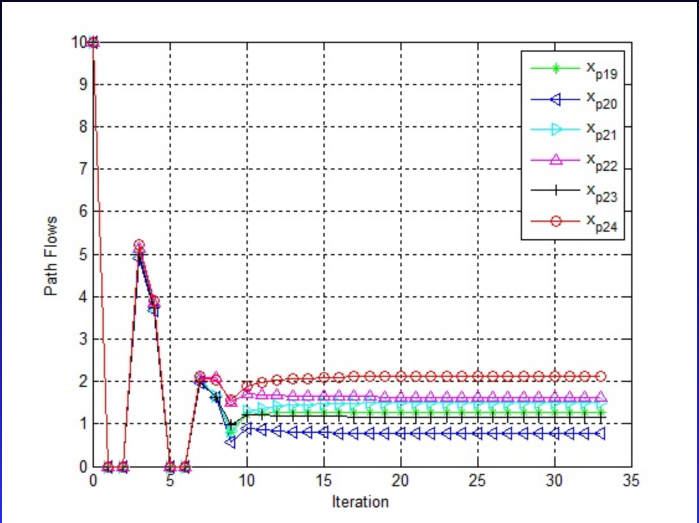
The Trajectories of Product Flows on Paths $p_7 - p_{12}$

Path Flow Trajectories



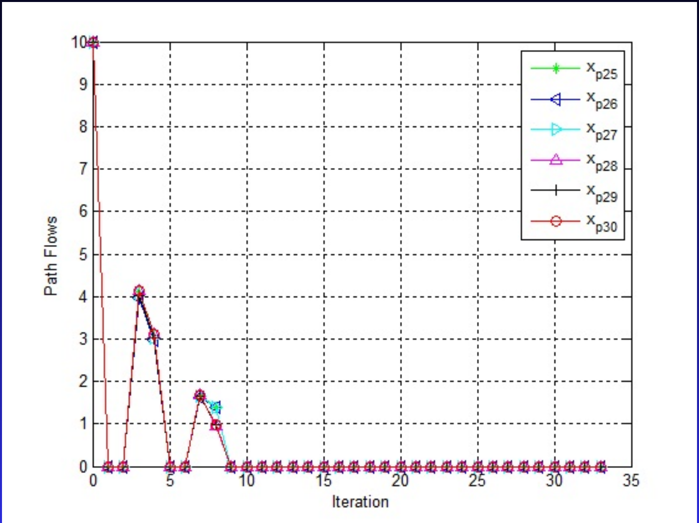
The Trajectories of Product Flows on Paths $p_{13} - p_{18}$

Path Flow Trajectories



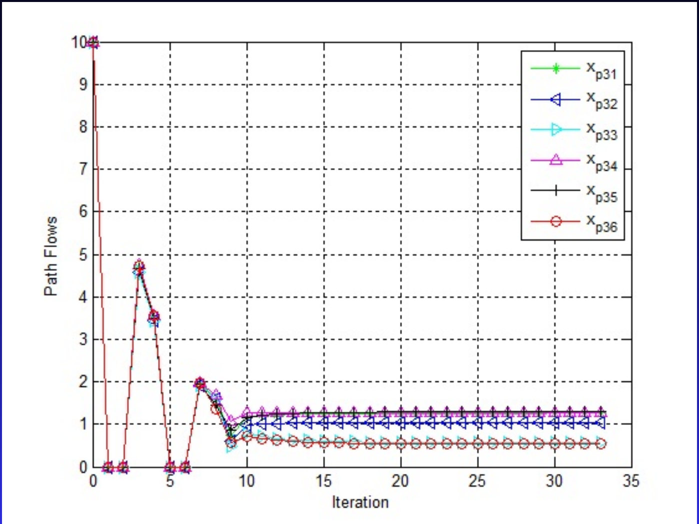
The Trajectories of Product Flows on Paths $p_{19} - p_{24}$

Path Flow Trajectories



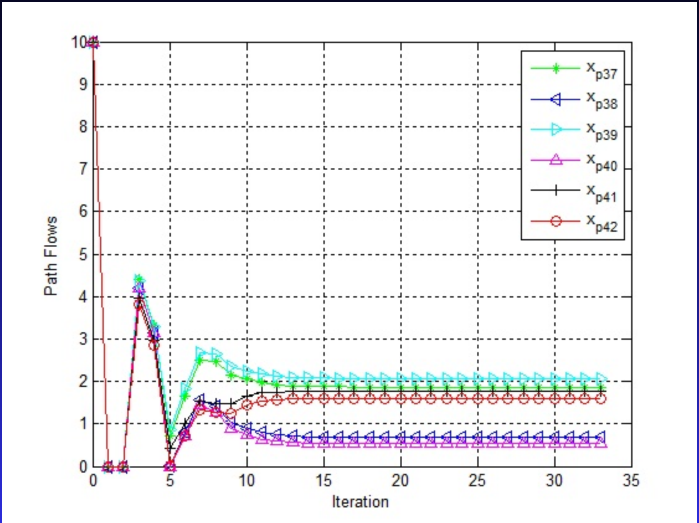
The Trajectories of Product Flows on Paths $p_{25} - p_{30}$

Path Flow Trajectories



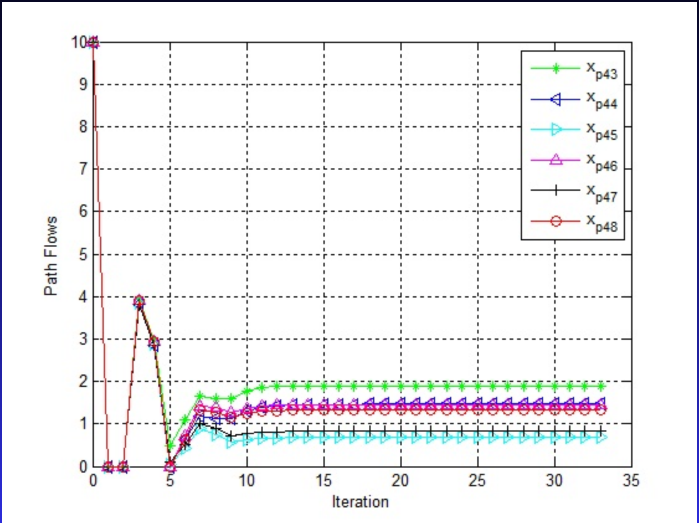
The Trajectories of Product Flows on Paths $p_{31} - p_{36}$

Path Flow Trajectories



The Trajectories of Product Flows on Paths $p_{37} - p_{42}$

Path Flow Trajectories

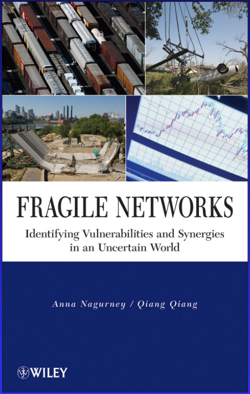


The Trajectories of Product Flows on Paths $p_{43} - p_{48}$

Some Other Issues in Supply Chain Networks that We Have Explored

Teaming in Humanitarian Operations and Network Synergies

A successful team depends on the ability to measure the anticipated synergy of the proposed team, which can be viewed as a merger (cf. Chang (1988)).



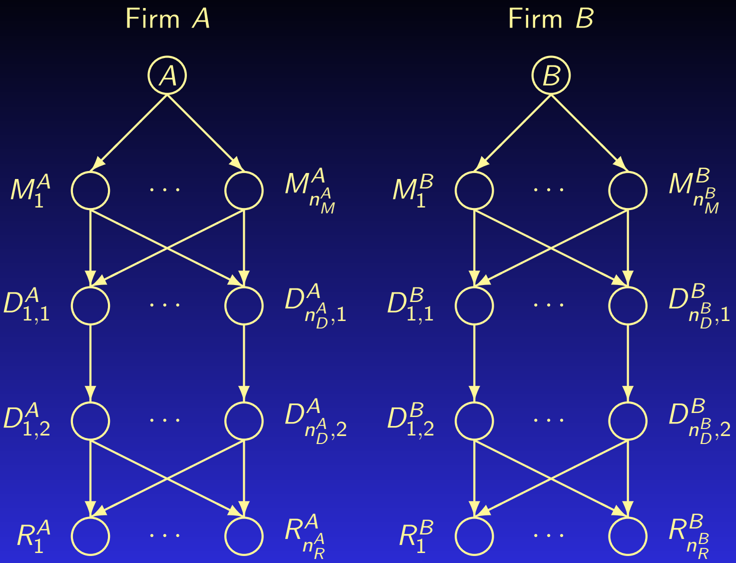


Figure 8: Case 0: Organizations A and B Prior to a Horizontal Merger

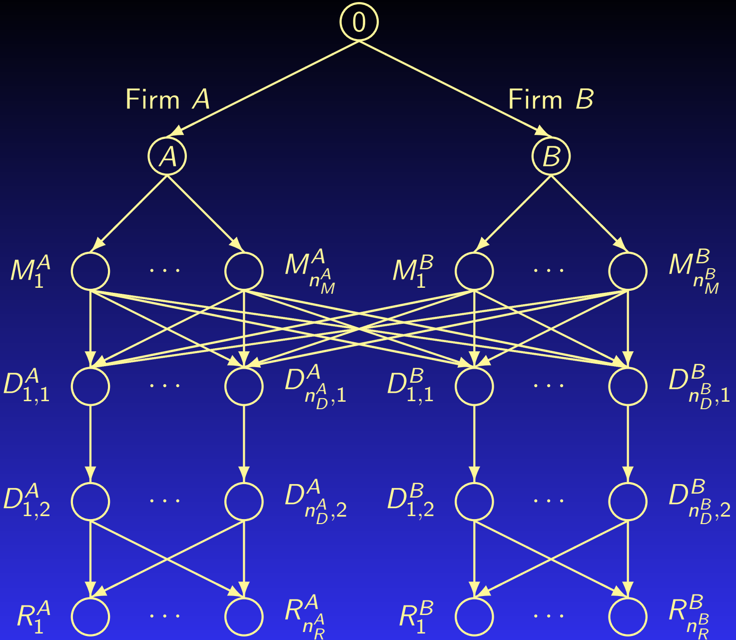


Figure 9: Case 1: Organizations A and B Team Up or “Merge”

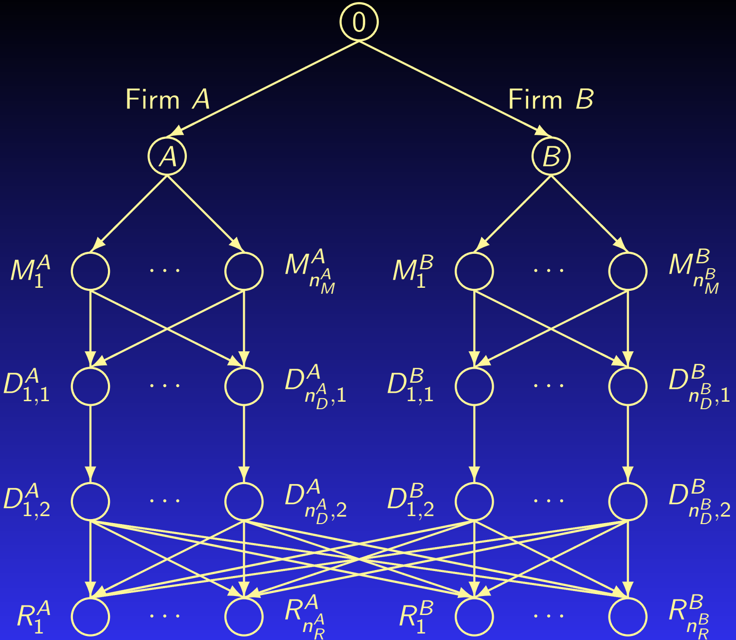


Figure 10: Case 2: Organizations A and B Merge

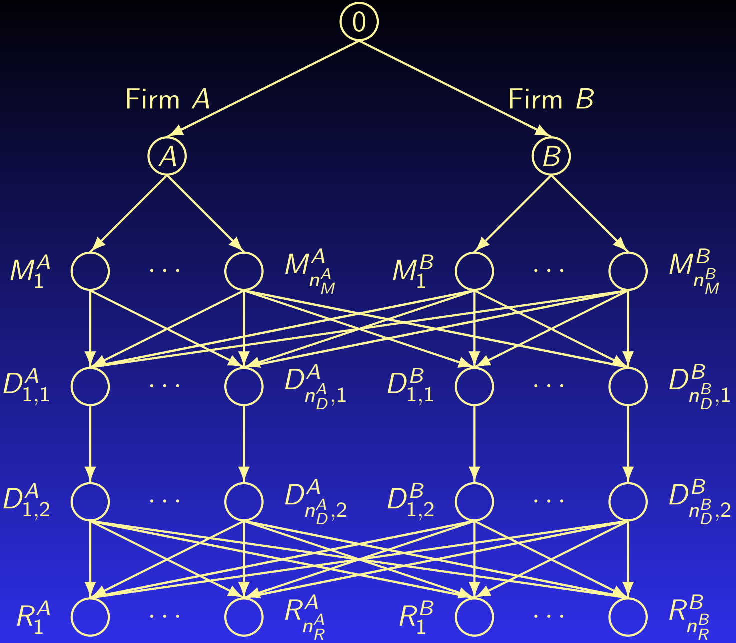


Figure 11: Case 3: Firms A and B Merge

Synergy Measure

The measure that we utilized to capture the gains, if any, associated with a horizontal merger Case i; $i = 1, 2, 3$ is as follows:

$$\mathcal{S}^i = \left[\frac{TC^0 - TC^i}{TC^0} \right] \times 100\%,$$

where TC^i is the total cost associated with the value of the objective function $\sum_{a \in L^i} \hat{c}_a(f_a)$ for $i = 0, 1, 2, 3$ evaluated at the optimal solution for Case i. Note that $\mathcal{S}^i$; $i = 1, 2, 3$ may also be interpreted as *synergy*.

Bellagio Conference on Humanitarian Logistics

See: http://hlogistics.isenberg.umass.edu/

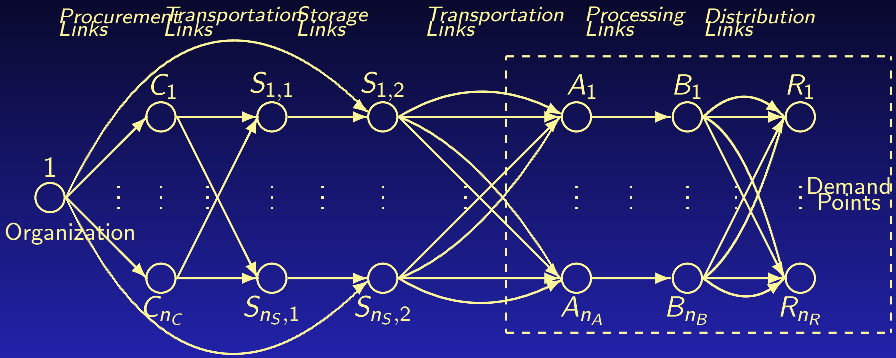


Figure 12: Network Topology of the Integrated Disaster Relief Supply Chain

A. Nagurney, A. H. Masoumi, and M. Yu, "An Integrated Disaster Relief Supply Chain Network Model with Time Targets and Demand Uncertainty," Isenberg School of Management, UMass

Summary, Conclusions, and Suggestions for Future Research

- We emphasized the *importance of capturing behavior* in supply chain modeling, analysis, and design.

- We developed an *integrated framework for the modeling of competition in pharmaceutical supply chains with brand differentiation and perishability* with outsourcing.

- The model is formulated and solved as a variational inequality problem.

- We also related the model to several others in the literatures with applications ranging from medical nuclear supply chains to blood supply chains.

- The framework *can be applied in numerous situations*, with some minot modifications, to capture oligopolistic competition for perishable and time-sensitive products.

- In addition, we have been heaving involved in *constructing mathematical models that capture synergies in mergers and acquisitions with the inclusion of risk* as well as exchange rate risk associated with outsourcing.
- Our research in supply chains has also led us to other time-sensitive products, such as *fast fashion*.
- We have also worked on models where guaranteed time of delivery is a strategic variable.
- Finally, we have done some modeling of the disequilibrium dynamics and equilibrium states in ecological predator-prey networks, that is, supply chains in nature.

We expect that future research will include supply chain network design for robustness and resiliency.

And for this important research we will certainly collaborate with one of your faculty members, Dr. Patrick Qiang!

THANK YOU!

For more information, see: http://supernet.isenberg.umass.edu