

# A Network Economic Game Theory Model of a Service-Oriented Internet with Price and Quality Competition in Both Content and Network Provision

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- NGI is expected to be service-oriented.
- Services are available at different quality levels and different costs.
- NGI includes multi-tier service providers
  - Content Service Providers  $\Rightarrow$  Content Provider (CP),
  - Network Service Providers  $\Rightarrow$  Network Provider (NP).



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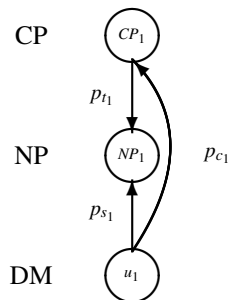
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- We perform sensitivity analysis in order to investigate the impact of the transfer prices.



# The Basic Model Description

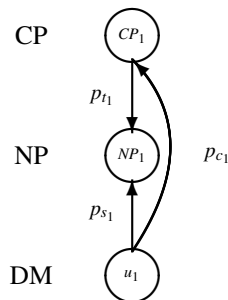
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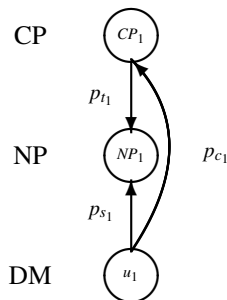


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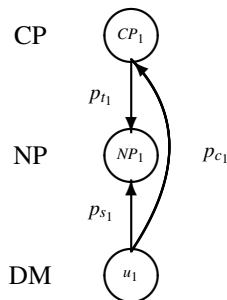


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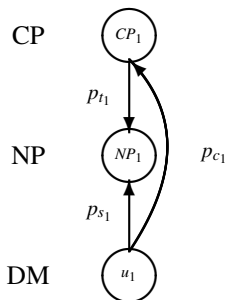
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$$U_{NP_1} = U_{NP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{s_1} + p_{t_1})d_{111} - CS_1 = (p_{s_1} + p_{t_1} - R)d_{111} - Rq_{s_1}^2.$$



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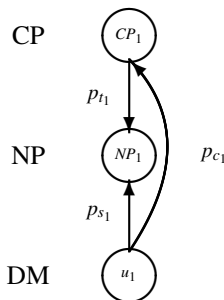
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# Analysis of Two-sided Pricing in the Basic Model

$$\text{Maximize } U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) = (p_{c_1} - p_{t_1})d_{111} - Kq_{c_1}^2.$$

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## Theorem

The network provider will benefit from charging the content provider if

$$4\alpha R - \gamma^2 > 0, \text{ and } \alpha > \beta.$$

## Definition: Nash Equilibrium in Price and Quality

$$U_{CP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) = \max_{(p_{c_1}, q_{c_1}) \in \mathcal{S}_{CP}} U_{CP_1}(p_{c_1}, q_{c_1}, p_{s_1}^*, q_{s_1}^*),$$

$$U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) = \max_{(p_{s_1}, q_{s_1}) \in \mathcal{S}_{NP}} U_{NP_1}(p_{c_1}^*, q_{c_1}^*, p_{s_1}, q_{s_1}).$$

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## Theorem: VI Formulation of Nash Equilibrium for CP and NP

$$\begin{aligned} & (-d_{111} + \beta(p_{c_1}^* - p_{t_1})) \times (p_{c_1} - p_{c_1}^*) + (2Kq_{c_1}^* + \delta(p_{t_1} - p_{c_1}^*)) \times (q_{c_1} - q_{c_1}^*) \\ & + (-d_{111} + \alpha(p_{s_1}^* + p_{t_1} - R)) \times (p_{s_1} - p_{s_1}^*) + (2Rq_{s_1}^* + \gamma(R - p_{s_1}^* - p_{t_1})) \times (q_{s_1} - q_{s_1}^*) \geq 0, \\ & \forall (p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1}) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}. \end{aligned}$$

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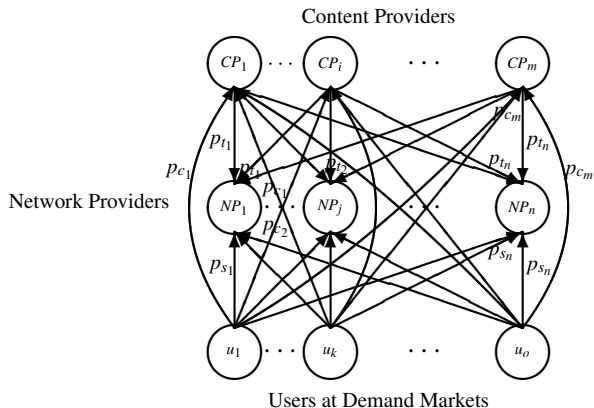
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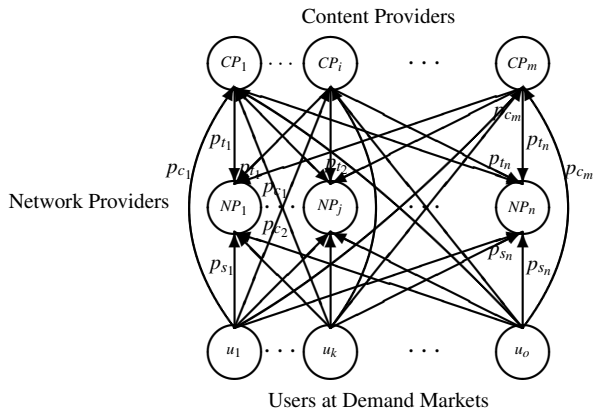
## Theorem: Uniqueness of the Nash Equilibrium

The Nash equilibrium  $(p_{c_1}^*, q_{c_1}^*, p_{s_1}^*, q_{s_1}^*) \in \mathcal{S}_{CP} \times \mathcal{S}_{NP}$  satisfying variational inequality is unique, if the function  $F = -\nabla U(p_{c_1}, q_{c_1}, p_{s_1}, q_{s_1})$  is strictly monotone over the feasible set  $\mathcal{S}_{NP} \times \mathcal{S}_{CP}$ .

# The Network of Oligopoly Model



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Demand function  $d_{ijk}$  for the content produced by content provider  $i$  and transmitted by network provider  $j$  to demand market  $k$ :

$$d_{ijk} = d_{ijk}(p_s, p_s, p_c, q_c), \quad \forall i, j, k.$$

# The Behavior of the Providers

## Content Providers

Each  $CP_i$  has a production cost  $CC_i$ :

$$CC_i = f_{c_i}(SCP_i, q_{c_i}), \quad i = 1, \dots, M.$$

The utility of  $CP_i$ :

$$U_{CP_i} = \sum_{j=1}^N (p_{c_i} - p_{t_j}) \sum_{k=1}^O d_{ijk} - f_{c_i}(SCP_i, q_{c_i}).$$

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Each  $NP_j$  incurs a transmission cost  $CS_j$ :

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# Nash Equilibrium

## Definition: Nash Equilibrium in Price and Quality

$$U_{CP_i}(p_{c_i}^*, \hat{p}_{c_i}^*, q_{c_i}^*, \hat{q}_{c_i}^*, p_s^*, q_s^*) \geq U_{CP_i}(p_{c_i}, \hat{p}_{c_i}^*, q_{c_i}, \hat{q}_{c_i}^*, p_s^*, q_s^*), \quad \forall (p_{c_i}, q_{c_i}) \in \mathcal{K}_i^1,$$

$$\hat{p}_{c_i}^* \equiv (p_{c_1}^*, \dots, p_{c_{i-1}}^*, p_{c_{i+1}}^*, \dots, p_{c_m}^*) \text{ and } \hat{q}_{c_i}^* \equiv (q_{c_1}^*, \dots, q_{c_{i-1}}^*, q_{c_{i+1}}^*, \dots, q_{c_m}^*).$$

$$U_{NP_j}(p_c^*, q_c^*, p_{s_j}^*, \hat{p}_{s_j}^*, q_{s_j}^*, \hat{q}_{s_j}^*) \geq U_{NP_j}(p_{s_j}, p_c^*, \hat{q}_c^*, p_{s_j}^*, q_{s_j}, \hat{q}_{s_j}^*), \quad \forall (p_{s_j}, q_{s_j}) \in \mathcal{K}_j^2,$$

$$\hat{p}_{s_j}^* \equiv (p_{s_1}^*, \dots, p_{s_{j-1}}^*, p_{s_{j+1}}^*, \dots, p_{s_n}^*) \text{ and } \hat{q}_{s_j}^* \equiv (q_{s_1}^*, \dots, q_{s_{j-1}}^*, q_{s_{j+1}}^*, \dots, q_{s_n}^*).$$

# The Equilibrium Conditions of the Service-Oriented Internet

## Theorem: Variational Inequality Formulation of Nash Equilibrium

$$\begin{aligned}
 & \sum_{i=1}^M \left[ - \sum_{j=1}^N \sum_{k=1}^O d_{ijk} - \sum_{j=1}^N \sum_{k=1}^O \frac{\partial d_{ijk}}{\partial p_{c_i}} \times (p_{c_i}^* - p_{t_j}) + \frac{\partial f_{c_i}(SCP_i, q_{c_i}^*)}{\partial SCP_i} \cdot \frac{\partial SCP_i}{\partial p_{c_i}} \right] \times (p_{c_i} - p_{c_i}^*) \\
 & \quad + \sum_{i=1}^M \left[ - \sum_{j=1}^N \sum_{k=1}^O \frac{\partial d_{ijk}}{\partial q_{c_i}} \times (p_{c_i}^* - p_{t_j}) + \frac{\partial f_{c_i}(SCP_i, q_{c_i}^*)}{\partial q_{c_i}} \right] \times (q_{c_i} - q_{c_i}^*) \\
 & + \sum_{j=1}^N \left[ - \sum_{i=1}^M \sum_{k=1}^O d_{ijk} - \sum_{i=1}^M \sum_{k=1}^O \frac{\partial d_{ijk}}{\partial p_{s_j}} \times (p_{s_j}^* + p_{t_j}) + \frac{\partial f_{s_j}(TNP_j, q_{s_j}^*)}{\partial TNP_j} \cdot \frac{\partial TNP_j}{\partial p_{s_j}} \right] \times (p_{s_j} - p_{s_j}^*) \\
 & \quad + \sum_{j=1}^N \left[ - \sum_{i=1}^M \sum_{k=1}^O \frac{\partial d_{ijk}}{\partial q_{s_j}} \times (p_{s_j}^* + p_{t_j}) + \frac{\partial f_{s_j}(TNP_j, q_{s_j}^*)}{\partial q_{s_j}} \right] \times (q_{s_j} - q_{s_j}^*) \geq 0, \\
 & \quad \forall (p_c, q_c, p_s, q_s) \in \mathcal{K}^3.
 \end{aligned}$$

# Euler Method

We recall the Euler method for the solution of the Variational Inequality Problem. Specifically, iteration  $\tau$  of the Euler method is given by:

$$X^{\tau+1} = p_{\mathcal{K}}(X^{\tau} - a_{\tau}F(X^{\tau})).$$

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## Theorem: Convergence

In the service-oriented Internet model, let  $F(X) = -\nabla U(p_c, q_c, p_s, q_s)$  be strictly monotone at any equilibrium pattern. Also, assume that  $F$  is uniformly Lipschitz continuous. Then, there exists a unique equilibrium price and quality pattern  $(p_c^*, q_c^*, p_s^*, q_s^*) \in \mathcal{K}$  and any sequence generated by the Euler method, where  $\{a_\tau\}$  satisfies  $\sum_{\tau=0}^{\infty} a_\tau = \infty$ ,  $a_\tau > 0$ ,  $a_\tau \rightarrow 0$ , as  $\tau \rightarrow \infty$  converges to  $(p_c^*, q_c^*, p_s^*, q_s^*)$ .

# Example 1

The demand functions:

$$d_{111} = 100 - 2.8p_{s_1} - 2.1p_{c_1} + 1.3p_{c_2} + 1.62q_{s_1} + 1.63q_{c_1} - .42q_{c_2},$$

$$d_{211} = 112 - 2.8p_{s_1} + 1.3p_{c_1} - 2.7p_{c_2} + 1.62q_{s_1} - .42q_{c_1} + 1.58q_{c_2}.$$

The cost functions:

$$CC_1 = 1.7q_{c_1}^2, \quad CC_2 = 2.4q_{c_2}^2, \quad CS_1 = 2.2(d_{111} + d_{211} + q_{s_1}^2).$$

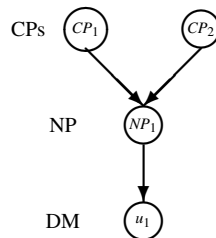
The utility functions, with  $p_{t_1} = 33$ :

$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} - CC_1, \quad U_{CP_2} = (p_{c_2} - p_{t_1})d_{211} - CC_2.$$

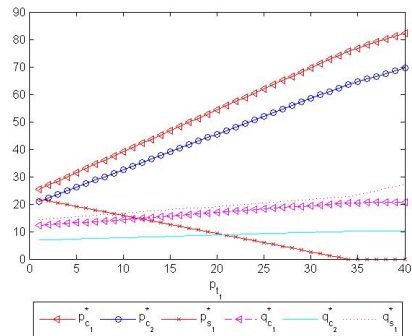
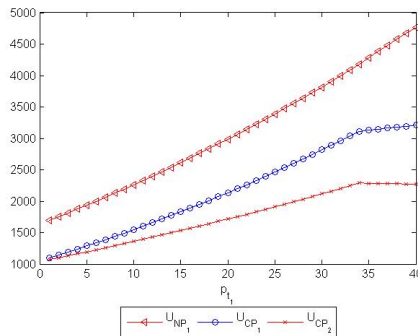
$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{211}) - CS_1.$$

The computed equilibrium solution:

$$p_{c_1}^* = 75.68, \quad p_{c_2}^* = 63.62, \quad p_{s_1}^* = 0, \quad q_{c_1}^* = 20.46, \quad q_{c_2}^* = 10.08, \quad q_{s_1}^* = 22.68,$$



# Example 1: Sensitivity Analysis



## Example 2

The demand functions:

$$d_{111} = 100 - 1.8p_{s_1} + .5p_{s_2} - 1.83p_{c_1} + 1.59q_{s_1} - .6q_{s_2} + 1.24q_{c_1},$$

$$d_{121} = 100 + .5p_{s_1} - 1.5p_{s_2} - 1.83p_{c_1} - .6q_{s_1} + 1.84q_{s_2} + 1.24q_{c_1}.$$

The cost functions:

$$CS_1 = 1.7(d_{111} + q_{s_1}^2), \quad CS_2 = 1.8(d_{121} + q_{s_2}^2).$$

$$CC_1 = 1.84[d_{111} + d_{121} + q_{c_1}^2].$$

The utility functions, with  $p_{t_1} = p_{t_2} = 0$ :

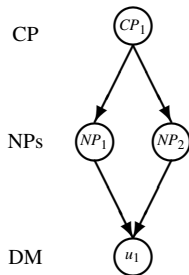
$$U_{CP_1} = (p_{c_1} - p_{t_1})d_{111} + (p_{c_1} - p_{t_2})d_{121} - CC_1.$$

$$U_{NP_1} = (p_{s_1} + p_{t_1})d_{111} - CS_1, \quad U_{NP_2} = (p_{s_2} + p_{t_2})d_{121} - CS_2.$$

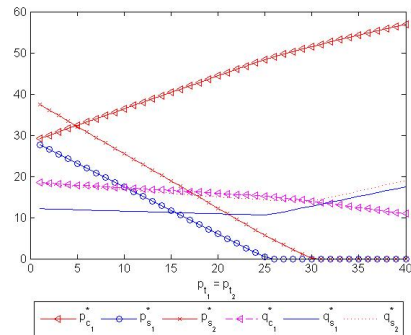
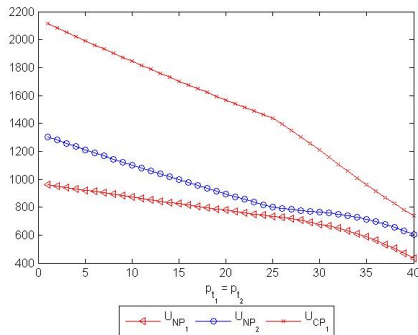
The equilibrium solution:

$$p_{c_1}^* = 29.19, \quad p_{s_1}^* = 27.66, \quad p_{s_2}^* = 37.38,$$

$$q_{c_1}^* = 18.43, \quad q_{s_1}^* = 12.14, \quad q_{s_2}^* = 18.18.$$



## Example 2: Sensitivity Analysis





## Example 3

The utility functions of the content providers:

$$U_{CP_1} = (p_{c_1} - p_{t_1})(d_{111} + d_{112} + d_{113}) + (p_{c_1} - p_{t_2})(d_{121} + d_{122} + d_{123}) - CC_1,$$

$$U_{CP_2} = (p_{c_2} - p_{t_1})(d_{211} + d_{212} + d_{213}) + (p_{c_2} - p_{t_2})(d_{221} + d_{222} + d_{223}) - CC_2.$$

The utility functions, with  $p_{t_1} = 23$  and  $p_{t_2} = 21$ :

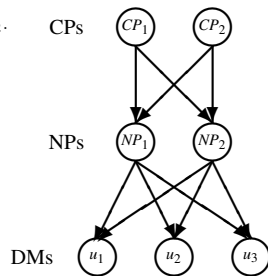
$$U_{NP_1} = (p_{s_1} + p_{t_1})(d_{111} + d_{112} + d_{113} + d_{211} + d_{212} + d_{213}) - CS_1,$$

$$U_{NP_2} = (p_{s_2} + p_{t_2})(d_{121} + d_{122} + d_{123} + d_{221} + d_{222} + d_{223}) - CS_2.$$

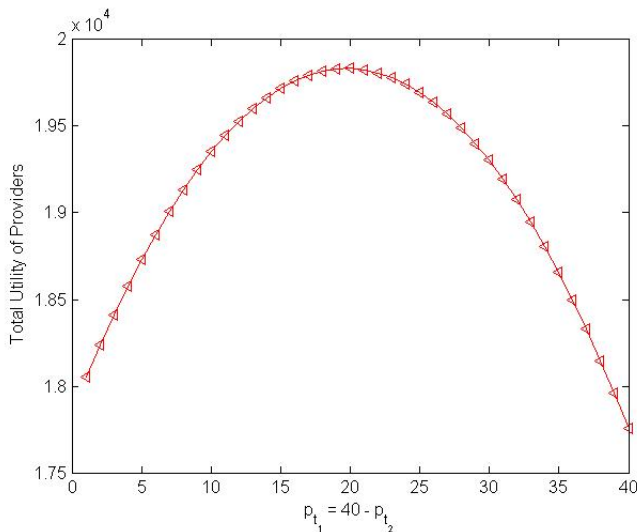
The equilibrium solution:

$$p_{c_1}^* = 40.57, \quad p_{c_2}^* = 41.49, \quad p_{s_1}^* = 8.76, \quad p_{s_2}^* = 5.35,$$

$$q_{c_1}^* = 13.96, \quad q_{c_2}^* = 12.76, \quad q_{s_1}^* = 36.67, \quad q_{s_2}^* = 12.15,$$



## Example 3: Sensitivity Analysis



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- We used the Euler method to solve numerical examples.
- Sensitivity analysis shows that the overall effect of implementing network neutrality regulations (e.g., having  $p_{tj} = 0$ ) may still be both positive and negative depending on the parameter values and the model structure.



# THANK YOU!



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